



$$\begin{aligned}
 C^{0,1} \hat{X}^2 &= |0\rangle\langle 0|^0 \otimes \hat{I}^{1,2} + |1\rangle\langle 1|^0 \otimes C^1 \hat{X}^2 \\
 &= \hat{I} \otimes \hat{I} \otimes \hat{I} + |1\rangle\langle 1| \otimes |1\rangle\langle 1| \otimes (\hat{\sigma}_x - \hat{I})
 \end{aligned}$$