$$\iff C^0 \hat{S}^{1,2} = |0\rangle\langle 0|^0 \otimes \hat{I}^{1,2} + |1\rangle\langle 1|^0 \otimes \hat{S}^{1,2}$$