$$\begin{array}{ccc}
\hat{U} & \iff & \sum_{i} \operatorname{tr}_{1} \left[(\hat{U}_{i} \otimes \hat{\rho}_{i})(\ldots)(\hat{U}_{i}^{\dagger} \otimes \hat{I}) \right] \\
& = \sum_{i} \hat{U}_{i} \operatorname{tr}_{1} \left[(\hat{I} \otimes \hat{\rho}_{i})(\ldots) \right] \hat{U}_{i}^{\dagger}
\end{array}$$