



\iff

$$\begin{aligned}
 C^0 \bar{C}^1 \hat{U}^2 &= |0\rangle\langle 0|^0 \otimes \hat{I}^{1,2} + |1\rangle\langle 1|^0 \otimes \bar{C}^1 \hat{U}^2 \\
 &= \hat{I} \otimes \hat{I} \otimes \hat{I} + |1\rangle\langle 1| \otimes |0\rangle\langle 0| \otimes (\hat{U} - \hat{I})
 \end{aligned}$$