$$\Longleftrightarrow$$

$$C^{0,1}\hat{X}^2 = |0\rangle\langle 0|^0 \otimes \hat{I}^{1,2} + |1\rangle\langle 1|^0 \otimes C^1\hat{X}^2$$
$$= \hat{I} \otimes \hat{I} \otimes \hat{I} + |1\rangle\langle 1| \otimes |1\rangle\langle 1| \otimes (\hat{\sigma}_x - \hat{I})$$