

Dear Dr. Endl,

We thank you for organizing the review of our paper, and thank the referee for constructive criticism. We have considered the referee's remarks carefully and revised our manuscript accordingly.

Below, we have reproduced the referee's report, along with our point-by-point responses. A list of changes is also appended to the manuscript.

Sincerely,

Luke Bouma

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REFeree COMMENT:

The authors present a number of new and high-quality transit observations of the hot Jupiter WASP-4b observed by TESS. After comparing the predicted mid-transit times (obtained only considering the pre-existing transits in the literature) with the measured values from the TESS light curves, the authors conclude that the TESS transits occur  $\sim 82$  seconds earlier than predicted. They examine different possibilities and find that the 2 most favored scenarios to explain this deviation are orbital decay and apsidal precession. However, the authors caution that more observations are needed to confirm these results. Also, they redetermine various parameters of the system, finding them in general agreement with literature values.

In general, the manuscript is scientifically accurate and the results presented in it certainly will be of interest to other investigators in the field. Then, I consider this paper worthy of being published after the concerns described below be addressed.

Below are my comments, beginning with the major points. I recommend the authors take them into consideration when revising their manuscript.

- > RESPONSE: We thank the referee for this summary. In our revision,
- > we have added weight to a third possible explanation that we
- > previously mentioned but did not sufficiently emphasize: a massive
- > outer companion, perhaps on an eccentric orbit, might be able to
- > explain both the transit and radial velocity observations. We have
- > described this possibility throughout the text where appropriate.
- > We also added our own re-analysis of the extant RV data (now
- > section 4.3, and figure 6) to check on the Knutson et al (2014)
- > result that the star's acceleration towards our line of sight is
- > only weakly detected, and is an order of magnitude too small to
- > explain the observed period derivative from the transit times.

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REFeree COMMENT:

Major points:

1) In section 3.1, the authors claim "...We included data from peer-reviewed literature for which the analysis was based on observations of a single, complete transit,...". However, the transits listed in Table 2 corresponding to the epochs  $E=-827$  ( $t_{\text{tra}}=2454697.79817$ ; Winn et al. 2009),  $E=-804$  ( $t_{\text{tra}}=2454728.57767$ ; Hoyer et al. 2013),  $E=-537$  ( $t_{\text{tra}}=2455085.88418$ ; Dragomir et al. 2011),  $E=-208$  ( $t_{\text{tra}}=2455526.16356$ ; Ranjan et al. 2014) are incomplete light curves, and those from the epochs  $E=-795$  ( $2454740.62125$ ; Hoyer et al. 2013),

E=-251 (2455468.61943; Hoyer et al. 2013) and E=21 (2455832.61815; Hoyer et al. 2013) present scarcity of after ingress/before egress data-points. On one hand, given that several partial light curves have been included in their study, the use of the word "complete" to indicate the characteristics of the transits in their sample, must be avoided.

On the other hand, at least two other transits included in their sample show visible anomalies, probably produced by the pass of the planet in front of one or several cold spots during transit (E=-561,  $t_{tra}=2455053.76774$  and E=-526,  $t_{tra}=2455100.60595$ , both from Sanchis-Ojeda et al. 2011). Different studies have proven that mid transit-times measured from partial light curves (Sada et al. 2012, Barros et al. 2013, Nikolov et al. 2015, Mancini et al. 2018) or light curves with anomalies due to the presence of spots (Oshagh et al. 2013, Mazeh et al. 2015, Ioannidis et al. 2016), usually present larger uncertainties and untrustable values. Taking in consideration the conclusions of these previous works, it is possible that the exclusion of incomplete/with anomalies transits might affect the outcome of their analysis and conclusions, particularly with respect to orbital decay and apsidal precession. Then, in order to warrant confident results, I would like to see the same analysis but without including partial transits or with visible spot-crossing events on them.

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> RESPONSE: We thank the referee for this suggestion. We have removed
> the word "complete" when describing the transit timing dataset. We
> have performed the check requested: We repeated the analysis, but
> omitting epochs -827, -804, -537, and -208 because of gaps in their
> coverage. We also omitted epochs -526 and -561 because of the
> visible spot anomalies during the transits.
>
> The resulting best-fit transit timing model parameters were all
> within  $1\sigma$  of the values quoted in Table 4. The confidence
> intervals, goodness-of-fit statistics  $\chi^2$ , and model comparison
> statistics (AIC, BIC) did not appreciably change.
>
> The most important of these statistics are summarized below for the
> referee's inspection, and this exercise is described in the new
> subsection "Systematic Concerns" at the end of Section 3.
>
> ORIGINAL NUMBERS:
> #####
>
> quadratic model:
> dP/dt =  $-4.001e-10 + (3.748e-11) - (3.812e-11)$ 
> dP/dt =  $-12.62 + (1.18) - (1.20)$  millisec/yr
>
> -----chi2 values for best fit parameters-----
>
> linear model:      chi2: 174.446 dof: 61 AIC: 1.78e+02 BIC: 1.83e+02
> quadratic model:  chi2: 62.5811 dof: 60 AIC: 6.86e+01 BIC: 7.50e+01
> precession model: chi2: 64.329  dof: 58 AIC: 7.43e+01 BIC: 8.50e+01
>
> -----quad vs precession-----
> delta_AIC = AIC_prec-AIC_quad = 5.75
> delta_BIC = BIC_prec-BIC_quad = 10.03
> approx Bayes Factor =  $\exp(\Delta BIC/2) = 1.14e+04$ 
>
>
> NUMBERS WHEN OMITTING PARTIAL TRANSITS AND OBVIOUS SPOT CROSSINGS:
> #####
>
> quadratic model:
> dP/dt =  $-4.098e-10 + (3.909e-11) - (3.862e-11)$ 
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> dP/dt = -12.93 +(1.23) -(1.22) millisec/yr
>
> -----chi2 values for best fit parameters-----
>
> linear model:      chi2: 163.285 dof: 55 AIC: 1.67e+02 BIC: 1.71e+02
> quadratic model:   chi2: 53.5164 dof: 54 AIC: 5.95e+01 BIC: 6.56e+01
> precession model:  chi2: 56.4648 dof: 52 AIC: 6.65e+01 BIC: 7.67e+01
>
> -----quad vs precession-----
> delta_AIC = AIC_prec-AIC_quad = 6.95
> delta_BIC = BIC_prec-BIC_quad = 11.03
> approx Bayes Factor = exp(deltaBIC/2) = 3.10e+04

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#### REFeree COMMENT

2) As mentioned in 1), another possible interpretation for explaining the deviation from a linear ephemeris of the TESS mid-transit times could be the presence of anomalies in the light curves due to stellar activity. It has been well established that WASP-4 is an active star with a rotation period (P) in between 20-40 days (P ~ 22 days and P ~ 34 days were determined by Sanchis-Ojeda et al. 2011 and Hoyer et al. 2013, respectively), which is expected for a main-sequence G7 star. Given that the TESS mid-transits span around 30 days, wouldn't it be possible that these measurements be affected by the existence of unseen stellar spots? Regarding this point, I would like to see an extra paragraph in the Section "Interpretation" with a discussion about this scenario.

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> RESPONSE: We have considered this possibility, and have now added
> the following text in Section 3.3:

> To explore the effect of possible spot-crossing events on the TESS
> transit time measurements, we performed a separate test. We
> injected triangular spot-anomalies with amplitude $0.03\%$ and
> duration 30 minutes at random phases into each transit. The
> amplitudes were larger than the spot-crossing anomalies observed
> by \cite{southworth_high-precision_2009} and
> \cite{sanchis-ojeda_starspots_2011}, and the durations were
> comparable. Spots of these amplitudes resemble the anomalies
> present in transits ``1360.54'' and ``1372.58'' of Figure
> \ref{fig:lightcurves}: they are visible, but at low confidence.
>
> With spots injected, we repeated our measurement of the transit
> times. On average, the measured transit times did not change
> after injecting spots, because the flux deviations are equally
> likely to occur in the first and second halves of the transit.
> For individual transits, there were no cases for which the timing
> deviation was larger than one minute. The largest shifts occur
> when the spot anomaly occurs during transit ingress or egress, in
> which case the measured mid-time is shifted either late or early
> by between 30 and 50 seconds \cite[qualitatively similar to]{ioannidis_how_2016}.
>
> Therefore the TESS observations could hypothetically all be skewed
> early if there were spot-crossing events during every egress. Two
> arguments rule out this possibility. (1) The lightcurve residuals
> do not show evidence for these events. (2) The stellar rotation
> period is between 20 and 40 days, and the sky-projected stellar
> obliquity is less than 10 degrees
> \cite{triaud_spin-orbit_2010,sanchis-ojeda_starspots_2011,hoyer_tramos_2013}.
> Since the planet orbits every 1.3 days, requiring that spot
> anomalies always occur during egress would be equivalent to
> requiring a stellar spot distribution that is exquisitely (and

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> thus implausibly) distributed to match the planet egress times.

REFeree COMMENT

-- Minor points:

0. TITLE:

i) As it is, the title implies a very strong asseveration. It would be more appropriate "Possible Early Arrival of WASP-4b for the TESS Mission" or something similar.

> RESPONSE: The main result of the paper is that TESS saw WASP-4b  
> transit 81.6 +/- 11.7 seconds early ( $\sim 7$  sigma). Given the  
> statistical significance, we would prefer to keep the title as-is.

REFeree COMMENT

1. INTRODUCTION:

i) Please, include here a reference for TESS.

> RESPONSE: Done.

REFeree COMMENT

2. NEW TRANSITS AND SYSTEM PARAMETERS:

Section 2.1:

i) It could be informative to provide a reference for the "Deep Space Network".

> RESPONSE: We are not aware of any paper that suits the purpose  
> here, so we added a citation to the URL of the Deep Space Network.

REFeree COMMENT

ii) barycentric Julian date --> Barycentric Julian Date

> RESPONSE: Done.

REFeree COMMENT

iii) For the casual reader, it would be important to briefly specify what "Threshold Crossing Events" means, and what the flags "Reaction Wheel Desaturation Event" and "Manual Exclude" represent. Footnotes including this information would be appropriate. Also, could the authors describe what a "Presearch Data Conditioning" lightcurve is?

> RESPONSE: We have clarified that a "Presearch Data Conditioning"  
> lightcurve is at a stage of process in which systematic trends  
> have been removed through the decorrelation process described by  
> Smith et al (2017).  
>  
> We now omit explicit mention of "Treshold Crossing Events" in  
> favor of a more qualitative description.  
>  
> As described in the text, the flags "Reaction Wheel Desaturation  
> Event" and "Manual Exclude" together mean that the data are  
> potentially of poor quality due to "momentum dumps". We referred  
> the reader to the data release notes for an explanation of what a  
> "momentum dump" is, and have added text to this footnote to

> improve its clarity.

REFeree COMMENT

iv) It might be worthwhile to mention what "ramp-like systematic effects" are and also include a reference.

> RESPONSE: We have changed the wording from "ramp-like  
> systematic effects" to "correlated red noise". No reference is  
> needed.

REFeree COMMENT

v) It would be important to include in this section an explanation of how photometric data-point errors are estimated.

> RESPONSE: Section 2.2, paragraph 3, final sentence explains  
> this: "The uncertainty in each data point was set equal to the  
> root-mean-square (rms) level of the out-of-transit data." We  
> have added the word "photometric" to the above sentence, to  
> clarify that we mean the photometric data points.

REFeree COMMENT

Section 2.2:

i) Could the authors explicitly indicate, in the first paragraph, the number of transits observed by TESS finally used to create the phase-folded light curve? Furthermore, the manuscript would be more readable if the main text refers first to Figure 1 instead of Figure 2.

> RESPONSE: We have clarified that the phase-folded lightcurve  
> comes from 18 transits. We have left the figure ordering  
> as-is.

REFeree COMMENT

ii) The authors say that the phase-folded lightcurve was fitted by using the implementation of Kreidberg (2015) of the analytic model of Mandel & Agol (2002). Is this referring to the BATMAN code? If it is so, It would be good to mention the code's name in the text. Also, I would appreciate if, in the main text, the authors clarify from which source are the values used as initial photometric parameters (planet to star radius ratio, orbital distance to stellar radius ratio, inclination, etc). On the other hand, for highlighting the results obtained with TESS data, it would be important to visualize all the transit parameters derived from section 2.2 in an independent table. Furthermore, to see the agreement between the transit parameters reported in previous studies and those resulting from this work, it would be useful to present together the values obtained in both cases.

> RESPONSE: We now mention the name "BATMAN". We do the same with  
> "emcee".  
>  
> We did not explicitly describe our choice for the initial  
> parameters, because the converged best-fit values from the chain  
> do not depend on them. For reproducibility purposes, similar  
> best-fit values can be obtained independent of initial parameter  
> choices. Barring an overruling request from the referee, we will  
> not mention our explicit choice in the text (it was to use values  
> from Hoyer et al., 2013).  
>  
> The transit parameters from the phase-folded fit:  $R_p/R_{\text{star}}$ ,

> inclination,  $a/R_{\text{star}}$ , and limb darkening parameters are given in  
 > the first five lines of Table 1.  
 >  
 > The transit mid-times from the independent fits to each transit  
 > are given in Table 2.  
 >  
 > The parameters which are not reported are the depths of the  
 > individual transits, and the linear fit parameters. Since the  
 > focus of this study is the transit times, we omitted these  
 > additional parameters for the sake of concision.  
 >  
 > Regarding the suggestion to add a table comparing our  
 > stellar/planet parameters with those found by other investigators:  
 > again, because the focus of the paper is the transit times, we  
 > have left the interested reader to explore these differences by  
 > pointing them to the relevant studies in the last paragraph of  
 > Section 2.3. From e.g., Table 8 of Southworth+2009, we see that  
 > our values do agree with those of previous studies, and so do not  
 > merit further attention, given the main focus of the present work.

#### REFeree COMMENT

iii) In the caption of Figure 2, it would be important to specify which is the bin size used to determine the yellow points. Furthermore, in Figure 2, the photometric error of the unbinned data-points should be included.

> RESPONSE: We have added a representative error bar to Figure 2  
 > which shows the photometric error of the unbinned data points.  
 >  
 > We have opted to not explicitly specify the bin size. It can be  
 > inferred from inspection of the plot, and is only relevant for  
 > visualization purposes.

#### REFeree COMMENT

iv) To estimate transit times for each individual lightcurve, did the authors use the same code as for the phase-folded light curve? In this case, how were the mid-transit times' uncertainties determined?

> RESPONSE: This is discussed in paragraphs 3 and 4 of Section 2.2:  
 > ""To measure the transit times, we returned to the 'cleaned' PDC  
 > time series and fitted the data within four transit durations of  
 > each transit separately. We used four free parameters: the time of  
 > mid-transit  $t_{\text{tra}}$ , the planet-to-star radius ratio, and the  
 > slope and intercept of a linear trend to account for any slow  
 > variations unrelated to the transit. We fixed the remaining  
 > parameters at the values that had been determined from the  
 > phase-folded TESS lightcurve. The uncertainty in each  
 > added photometric data point was set equal to the  
 > root-mean-square (rms) level of the out-of-transit data.  
 >  
 > To verify that the measured uncertainties are estimated accurately,  
 > we computed the reduced  $\chi^2$  for a linear ephemeris fit to the  
 > measured TESS mid-transit times. We found that  $\chi^2 = 9.2$ ,  
 > with  $n=16$  degrees of freedom. The variance of the  $\chi^2$   
 > distribution is  $2n$ , so we would expect  $\chi^2 = 16 \pm 5.7$ .  
 > Visually inspecting the residuals showed that the error variance  
 > had been overestimated, so we multiplied the measured TESS errors  
 > by a factor  $f=0.76$ , forcing a reduced  $\chi^2$  of unity. This  
 > lowered the mean uncertainty of the transit midtimes from  $29.8$  to  
 >  $22.6$  seconds. We verified that omitting this step did not

> appreciably alter any of our conclusions.""

> In short, we used the same transit model as before (the BATMAN  
> code), but with the addition of the slope and intercept of a  
> linear trend as free parameters. Our proceduring for  
> determining the mid-transit time uncertainties is also  
> described.

#### REFeree COMMENT

v) Is  $\chi^2=9.2$  the reduced chi-square? If it is so, It should be clarified by using a different symbol or by adding a sub-index, for example. Please, fix this in the rest of the manuscript when necessary.

> RESPONSE:  $\chi^2 = 9.2$  is the  $\chi^2$  value. We have  
> corrected the typo in the text, in which we wrote "reduced  
>  $\chi^2$ ", which likely led to the confusion!

#### REFeree COMMENT

vi)  $\chi^2 = 16$  {plus minus} 5.7 -->  $\chi^2 = 9.2$  {plus minus} 5.7

> RESPONSE: For 18 transits, and 2 degrees of freedom, we  
> would expect  $\chi^2 = 16 \pm 5.7$ .

#### REFeree COMMENT

vii) The note at the end of Table 2 explains what the parameter  $\sigma_{t_0}$  represents, but it's not clear which is its relation with the mid-transit times' measurements presented in the same table.

> RESPONSE: This was a typo --  $\sigma_{t_0}$  should have read  
>  $\sigma_{t_{\rm tra}}$ . We have added a sentence to summarize  
> the complete procedure in the note at the end of Table 2.

#### REFeree COMMENT

Section 2.3:

i) The authors should include the references for each of the mentioned catalogs and also for the Yonsei Yale isochrones.

> RESPONSE: Done, thank you.

#### REFeree COMMENT

ii) How did the authors estimate the errors of the stellar and planet parameters?

> RESPONSE: We have added the following sentence in the last  
> paragraph of Section 2.3: "The uncertainties in our derived  
> stellar and planetary parameters are propagated according to  
> standard analytic formulae, under the assumption of uncorrelated  
> and normally distributed initial measurements."

#### REFeree COMMENT

### 3. TIMING ANALYSIS

Section 3.1:

i) The sentence "..Since those data points carry significant weight in the

analysis, we checked that the timestamps in their data represent mid-exposure times, that the barycentric correction was performed correctly, and that the time system of the final results was BJD(TDB).." is confusing, because seems to imply that these checks were only performed on the Huitson et al. mid-transit times and not in all the literature mid-transit times. It would be recommendable to clarify this point.

> RESPONSE: we have modified the wording slightly to clarify. The  
 > point is that for these Gemini South observations, we confirmed  
 > with the authors through back and forth correspondence that the  
 > details of their time-keeping were sound. We made this extra  
 > effort because of the weight of the Huitson+(2017) times in the  
 > analysis.  
 >  
 > For the remaining data points, we used the references stated in  
 > Section 3.1 and Table 2 to confirm that we were using with  
 > mid-exposure times, and corrected from HJD\_UTC to BJD\_TDB as  
 > appropriate. However the point is indeed that because of the  
 > weight of the Huitson et al (2017) times, we made an particular  
 > extra effort to confirm their accuracy with the original authors.

#### REFeree COMMENT

ii) Could the authors specify what is the meaning of each of the variables in Equation (1)?

> RESPONSE: Done.

#### REFeree COMMENT

Section 3.2:

i) How did you fit the mid times of the preTESS measurements? Did you use a weighted least-square fit?

> RESPONSE: Yes, this was through weighted least-squares. We have changed the  
 > wording accordingly.

#### REFeree COMMENT

ii) Figure 3:

I would recommend changing the "title" indicated in bold font in the caption, by a more explicit sentence about what is shown in the plot. To give an example, a possibility could be "Observed minus predicted mid-transit times".

> RESPONSE: Though we appreciate the suggestion, we have opted to  
 > keep the "title" as is. The nuance of what "predicted" means for  
 > this plot (a linear ephemeris, based only on the pre-TESS times)  
 > is sufficient that we find it to be better left in the body of the  
 > caption.

#### REFeree COMMENT

Also: ii-1) In the legends of both panels, could the authors specify which is the bin size of the TESS light curves? (Please, fix this in all the manuscript).

> RESPONSE: We have added the following sentence in the caption:  
 > "The binned TESS point is the weighted average of 18 TESS  
 > transits". The caption of Figure 4 already stated the same. We  
 > have changed the words "Dots" to "Symbols" in the caption of



> Figure 6 in order to clarify this for the reader.

REFeree COMMENT

ii-2) "The red band shows the average deviation of the TESS transits,..." -->  
"The red band shows the average deviation of the TESS transits {plus  
minus} $1\sigma$ ,..."

> RESPONSE: Done.

REFeree COMMENT

iii) It would be important to introduce references for equations (4) and (5).  
Also, for the casual reader, It might be informative to give a brief  
explanation of what sidereal and anomalistic periods are.

> RESPONSE: Equations (4) and (5) are generic equations for the  
> Taylor series expansion of the transit and occultation times as a  
> function of epoch, and so do not require references. We have  
> added the following sentence to describe the sidereal and  
> anomalistic periods: "The sidereal period is the duration required  
> to return to the same orientation with respect to the stars; the  
> slightly longer anomalistic period is the duration required to  
> reach a fixed longitude with respect to the rotating line of  
> apsides."

REFeree COMMENT

iv) Figure 4: In the caption, could the authors explicitly indicate what the  
plot shows in each of its panels (top and bottom)?

> RESPONSE: Done.

REFeree COMMENT

v) Similar to i) How did you fit the mid-transit times and occultations shown  
in Figure 4?

> RESPONSE: We have added the following sentences in Section 3.2:  
> "We fit each model by assuming a Gaussian likelihood and sampling  
> over the posterior probability distributions. The prior for the  
> quadratic model allowed the period derivative to have any sign."  
> We have clarified the wording throughout from "decreasing-period  
> model" to "quadratic model" to reflect the latter point.

REFeree COMMENT

vi) In the caption of Table 4, please be more explicit, "Best-fit model  
parameters" of what?

> RESPONSE: The caption now reads "Best-fit transit timing model  
> parameters".

REFeree COMMENT

vii) Given that the mid-transit times values used in this work are taken from  
different authors with different methodologies to perform the fitting and  
detrending of the light curves, could the authors mention how this could affect  
their results?

> RESPONSE: We have added the following paragraphs at the end of  
> Section 3.3:

> There is a final concern that is difficult to address. We collected  
 > the mid-transit time values derived by different authors, who used  
 > heterogeneous methods to fit and detrend their lightcurves. We have  
 > also assumed that these authors have correctly documented the time  
 > systems in which the data are reported. Further, though many choices  
 > in transit-fitting (e.g., parametrization of limb-darkening and  
 > eccentricity) do not affect transit mid-time measurements, different  
 > detrending approaches can asymmetrically warp transits and shift  
 > mid-transit times. The magnitude of this systematic effect is hard to  
 > quantify, but the situation is fairly clear from  
 > Figure~\ref{fig:times}. Many independent authors provided transit  
 > measurements shortly after WASP-4b's discovery, and the data are  
 > consistent with each other. \citet{huitson\_gemini\_2017} provided the  
 > most important data from epochs 0-1000. If their data were  
 > systematically affected by detrending choices or time-system confusion  
 > at the level of several times their reported uncertainties, then it  
 > is possible that the orbital period is constant despite the evidence in  
 > the TESS data. For this reason, we paid careful attention to the  
 > \citet{huitson\_gemini\_2017} data set, and corresponded with the  
 > authors to confirm that their results are not affected by systematic  
 > effects of the required amplitude.  
 >  
 > None of the concerns mentioned in this subsection seem likely to  
 > explain the observed timing variations. We proceed by considering  
 > possible astrophysical explanations.

#### REFeree COMMENT

#### 4. INTERPRETATION

Referee comment:

Section 4.1:

i) First line: The word "caused" is repeated twice.

> RESPONSE: Corrected, thank you.

REFeree COMMENT:

ii) In the caption of Figure 5 should be indicated what open circles represent.

> RESPONSE: Done.

REFeree COMMENT:

iii) The characteristic timescale value calculated in equation (13), is it computed by considering a theoretical  $dP/dt$  value or from the one estimated through the observations?

> RESPONSE: Changed wording to remove any ambiguity.

REFeree COMMENT:

iv) The authors mention that there are about 20 Hot Jupiters for which the theoretical timescale is shorter, where are the timescale values obtained from? Are they estimated from equation (13)? If this is the case, how do the theoretical  $dP/dt$ s are computed?

> RESPONSE: As noted in the caption of Figure 5, the decay timescale values  
 > are obtained from Equation 15, assuming  $Q_{\text{star}} = 10^7$ . We have slightly

> modified the text for clarity on this point.

REFeree COMMENT:

v) In the top panels of Figures 4 and 6, the orbital decay model evolution shows that at the beginning the O-C values increase until, at some point, they reach a maximum and after that, they start decreasing. What is the physical explanation for that behavior? If the orbital period is diminishing, wouldn't it be expected to see only a decreasing in the O-C values? Why the O-C values would increase at these first epochs?

> RESPONSE: To make Figure 4 and 6, we start with the observed  
> transit and occultation mid-times, as a function of epoch number.  
> We then fit a line to these data. If the data were generated by a  
> constant period orbit (with gaussian uncertainties) the residuals  
> to such a fit would show no correlation with epoch number.  
> However if the data were generated by a planet with a constant  
> period derivative (with gaussian uncertainties), the residuals to  
> such a fit would always be a parabola. This is because the linear  
> fit to the data will have contributions from the beginning,  
> middle, and end of the transit timeseries. Thus the best-fit  
> linear ephemeris (to data that actually have a decreasing period)  
> has a best-fit period somewhere between the minimum and maximum  
> period. At the earliest epochs, the best-fit period is an  
> underestimate, so you see the increase in O-C. At the latest  
> epochs, the best-fit period is an overestimate, so you see the  
> decrease.

REFeree COMMENT:

Section 4.2:

i) It might be informative to explicitly include the equation (14) of Ragozzine & Wolf (2009) to see of which parameters is depending on.

> RESPONSE: we have opted against this suggestion, chiefly to  
> encourage the interested reader to refer to the original study!

REFeree COMMENT:

ii) Similar to Figure 3, I would recommend changing the title indicated in bold font in Figure 6, by a more explicit sentence about what is shown in the plot. For example "Different apsidal precession and orbital decay models for the O-C values" or something like that.

> RESPONSE: As in Figure 3, we appreciate the suggestion, but have  
> opted to keep the "title" as is. We have appended the sentence  
> "The two models may begin to diverge in the mid-2020s" as further  
> clarification for the reader.

REFeree COMMENT:

iii)  $M_b$  is indicated as the mass of the hypothetical planet WASP-4c, however, the  $M_b$  symbol is not present in equation (19).

> RESPONSE: We have corrected the wording accordingly.

REFeree COMMENT:

iv) Why do the authors assume  $k_2 = 0.6$  for the Love number? Is this value randomly chosen?

> RESPONSE: We have added the phrase "similar to Jupiter". Section 4.2  
> already notes that our measured value for  $k_2p$  under the precession  
> hypothesis (Equation 18) is imprecise.

REFeree COMMENT:

v) Would it be possible to introduce a definition of the reduced mass presented in equation 21? Also, could the authors specify which values they used for  $L$ ,  $M_c$ , and  $R_c$  and provide references?

> RESPONSE: We have clarified the origin of the values used when  
> evaluating Equation 21. The tricky ones -- the mass and radius of  
> the convective envelope -- came from the MESA/MIST projects, for  
> which we have added appropriate citations. Our thanks to the  
> referee for pointing out this omission. We have opted to not  
> explicitly define the reduced mass, as it is standard knowledge in  
> the gravitational two-body problem.

REFeree COMMENT:

i) "WASP-4 timing anomaly" --> "WASP-4b timing anomaly"

> RESPONSE: Corrected.

REFeree COMMENT:

ii) In the caption of Figure 7: "with standard deviation ( $\sigma_{\text{predicted}}$ )" -->  
"with standard deviation ( $\sigma_{\text{pre-TESS}}$ )"

> RESPONSE: Corrected.

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OTHER CHANGES:

\* Added the following coauthors: Fei Dai, Natalia Guerrero, Bill Wohler,  
Douglas Caldwell.

\* Minor edits in text throughout, as indicated with the `\added{}`,  
`\deleted{}`, and `\replaced{...}` commands.

\* Minor tweaks to figures.