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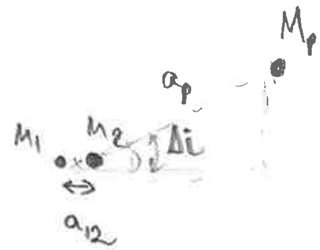
For a mutually inclined CBP orbiting a contact binary, what is the precession timescale, & is it short enough that assuming transits all thru Kepler data is obviously dumb?

From scratch: $\vec{\tau} = \frac{d\vec{L}}{dt}$, so $\frac{|\vec{L}|}{|\vec{\tau}|} \overset{?}{\approx} t_{prec} \doteq \frac{L}{(dL/dt)}$



$\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{F} = \vec{F}_g = \vec{F}_1 + \vec{F}_2 = GM_p \left(\frac{M_1}{|\vec{r}_1 - \vec{r}_p|^2} \hat{r}_{1p} + \frac{M_2}{|\vec{r}_2 - \vec{r}_p|^2} \hat{r}_{2p} \right)$

ok. But see Sec 3.4 of Martin & Triaud (2014).

They cite analytics (Farago & Laskar 2010, Leung & Hoi Lee 2013) & numerics (Doolin & Blundel 2011). M&T (2014) state:

"The variation that most significantly affects the transit probability is a precession of the longitude of the ascending node, Ω_p, and the mutual inclination, I_p".

Quick timescales — of order years — perturb the ~~binary~~ orbital elements.

M & T 2014 also comment:

"The systems alternate between being in an out of transitability, which implies that there are several systems (whose parameters may resemble very closely the current Kepler detections) that remained out of transitability for the duration of the survey"

(e.i.?? angle of periastron?)

Farago & Laskar (2010) gives the precession rate of the nodes of a massless particle's orbital around a circular binary:

$$\dot{\Omega} = -\frac{3}{4} n_1 \left(\frac{a_1}{a_2}\right)^{7/2} \frac{\beta_1}{M_{01}} \frac{\cos i_2}{(1-e_2^2)^2} \qquad (2.24)$$

binary total mass $M_{01} \equiv m_0 + m_1$

$\beta_1 = \frac{m_0 m_1}{m_0 + m_1}$

n_1 mean motion of the binary,

$n_1 \equiv \left(\frac{GM_{01}}{a_1^3}\right)^{1/2}$

& have an energy integral

$z^2 - e_1^2(4x^2 - y^2) = h = \text{const.}$

$-4e_1^2 \leq h \leq 1$.

"This precession is equivalent to the precession generated by the quadrupolar potential of a circular & homogeneous ring of mass β_1 and of radius a_1, following an idea going back to Gauss [cf Touma, Tremaine, & Kazandjian 2009]".

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If the binary is elliptical, no longer quadrupolar torque of a circular ring. This leads to a messier period of motion:

$$T = \frac{16}{3n_1} \frac{M_{01}}{\beta_1} \left(\frac{a_2}{a_1}\right)^{7/2} \frac{K(k^2)\,(1-e_2^2)^2}{[(1-e_1^2)(h+4e_1^2)]^{1/2}} \quad (2.32)$$

for $K(k^2)$ an elliptic integral of kind 1 (Eq 2.33)

Again $h \leq 1$ (Eq 2.25), & $1-(1+4e_1^2)x^2 - (1-e_1^2)y^2 = h$

so again $-4e_1^2 \leq h \leq 1$ allowed.

note (2.33) for $K(k^2)$ is just

$$K(k^2) = \begin{cases} \int_0^{\pi/2} \frac{d\phi}{(1-k^2\sin^2\phi)^{1/2}} & \text{if } k^2<1 \\ \int_0^{\phi_0} \frac{d\phi}{(1-k^2\sin^2\phi)^{1/2}} & \text{if } k^2>1 \quad \left(\phi_0 = \pm\arcsin\left(\frac{1}{k}\right)\right) \end{cases}$$

w/ vanishing & other issues

F&L (2010) conclude noting:

- sdB + M HW Virginis (Lee + 2009) is a nontransiting CBP system w/ inner binary period 2.8 hr (vs. P_{CBP} 9.1 & 15.8 yr)
- motion of *s around binary black holes is relevant (Gillessen+ 2009, Mikkola & Merritt 2008).

Are the Leung & Hoi Lee 2013 analytics any nicer?

Not much, also tricky on the interpretation... (Eq 37 in their paper seems like it has slightly diff period scaling)...

Take F&L (2010)'s (2.24):

$$\dot{\Omega} = -\frac{3}{4} n_1 \left(\frac{a_1}{a_2}\right)^{7/2} \frac{\beta_1}{M_{01}} \frac{\cos i_2}{(1-e_2^2)^2}.$$

$$= -\frac{3}{4} \frac{(GM_{01})^{1/2}}{a_1^{3/2}} \frac{a_1^{7/2}}{a_2^{7/2}} \frac{m_0 m_1}{(m_0+m_1)^2} \frac{\cos i_2}{(1-e_2^2)^2} \ldots$$

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$$\dot{\Omega} = -\frac{3}{4} G^{1/2} \frac{a_1^2}{a_2^{7/2}} \frac{m_0 m_1}{(m_0+m_1)^{3/2}} \frac{\cos i_2}{(1-e_2^2)^2}$$

$$\tau_{prec} \equiv |\dot{\Omega}|^{-1}$$

$$\boxed{\tau_{prec} = \frac{4}{3G^{1/2}} \frac{a_2^{7/2}}{a_1^2} \frac{(m_0+m_1)^{3/2}}{m_0 m_1} \frac{(1-e_2^2)^2}{\cos i_2}} \quad (A)$$

N.b. a_1 given by Kepler's 3rd, $\frac{P_{EB}^2}{a_1^3} = \text{const} = \left(\frac{G(m_0+m_1)}{4\pi^2}\right)^{-1} = \frac{4\pi^2}{G(m_0+m_1)}$

(since ~~the~~ particle is massless).

$$\hookrightarrow a_1 = \left(\frac{4\pi^2}{G(m_0+m_1)}\right)^{-1} \left(\frac{1}{P_{EB}^2}\right)^{-1}$$

$$a_1 = \left[\frac{G(m_0+m_1)}{4\pi^2} P_{EB}^2\right]^{1/3}$$

What we're really interested in is the case where the assumption that "all transits will be visible in the Kepler data" is a bad one.

↳ ToDo: rerun w/ finer inclination grid (x2 or x3)

uh... no... (well already done)

ToDo:
derive upper bound on $\cos i_2$ s.t. $\tau_{prec} < 100\,\text{yr}$, given numerically sane values of everything else as a prefactor term!

...

1.e

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$$\cos i_2 = \frac{4}{3G^{1/2}} \frac{a_2^{7/2}}{a_1^2} \frac{(m_0+m_1)^{3/2}}{m_0 m_1} \frac{(1-e_2^2)^2}{\tau_{prec}}.$$

want $\tau_{prec} = |\dot{\Omega}|^{-1}$ s.t. "no more than $\sim 2°$ over 4 yr"

$$\rightarrow \tau_{prec} : \left|\frac{\Delta\Omega}{\Delta t}\right|^{-1} = \left|\frac{2°}{4\,yr}\right|^{-1} = \left|\frac{0.0349\,rad}{4\,yr}\right|^{-1} = 114.6\,\frac{yr}{rad} \approx 100\,yr\,~~rad^{-1}~~$$

unitless.

imposing $\tau_{prec} > 100\,yr$,

$$\cos i_2 < \frac{4}{3G^{1/2}\tau_{prec}} \frac{a_2^{7/2}}{a_1^2} \frac{(m_0+m_1)^{3/2}}{m_0 m_1} (1-e_2^2)^2 \quad (\star)$$

(n.b. this upper bound is probably stupid & unhelpful b/c it won't apply for an actual physical system but ok)

note $a_1 = \left(\frac{G(m_0+m_1)}{4\pi^2} P^2\right)^{1/3}$

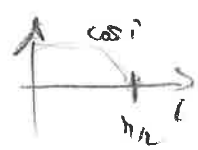
[this (pg 3) & pg 4 are deriving a useless upper bound]

$$\cos i_2 < \frac{4}{3G^{1/2}\tau_{prec}} a_2^{7/2} \left(\frac{G \ldots}{4 \ldots}\right) \ldots (1-e_2^2)^2$$

$$< \frac{4}{3\tau_{prec}} G^{-5/6} (4\pi^2)^{2/3} \frac{(m_0+\ldots}{m_0 \ldots} \ldots {}^{7/2}$$

$$< \frac{2^{10/3}\pi^{4/3}}{3G^{5/6}} \frac{\overbrace{(m_0+m_1)^{5/6}}^{m_{frac}}}{m_0 m_1} \frac{\ldots}{P_{EB}^{4/3}} (1-e_2^2)^2 a_2^{7/2} \frac{1}{\tau_{prec}}$$

[want ~~smallest~~ biggest $\cos i$ for $\tau_{prec} = 100\,yr$]



We're interested in $e_2 \le 0.1$,
($e_2 = 0$ makes max $\cos i$)

$\tau_{prec} \propto \frac{1}{\cos i}$

$0.1 \le \frac{P_{EB}}{day} \le 10$,

low end limits $\cos i$

at $\cos i = 0$, $\tau_{prec} \to \infty$ (does not precess)

$m_0, m_1 \in [0.1, 2]\,M_\odot$

$\rightarrow m_{frac}$ min at $m_0 = m_1 = 0.1\,M_\odot$

at $\cos i = 1$, maximal precession (smallest precession period)

$\lfloor a_2 = 100\,a_1$. (generally $a_2 = x a_1$ is easier...)

3.

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From (*) pg 3,

$$\cos i_2 < \frac{4}{3G^{1/2}\tau_{prec}} \frac{a_2^{7/2}}{a_1^{2}} \frac{(m_0+m_1)^{3/2}}{m_0 m_0} (1-e_2^2)^2$$

Write $a_2 = x a_1$ ($x \gtrsim 4$, $x \in \mathbb{R}$), & $\lesssim 200$ for search

$$< \frac{4}{3G^{1/2}\tau_{prec}} x^{7/2} a_1^{3/2} \frac{(m_0+m_1)^{3/2}}{m_0 m_1} (1-e_2^2)^2$$

Now note $a_1 = \left(\frac{G(m_0+m_1)}{4\pi^2} P_{EB}^2\right)^{1/3}$, so $a_1^{3/2} = \left(\frac{G(m_0+m_1)}{4\pi^2}\right)^{1/2} P_{EB}$

$$\cancel{< \frac{4}{3G^{1/6}\tau_{prec}} x^{7/2} \frac{1}{(4\pi^2)^{1/3}} P_{EB} \frac{(m_0+m_1)^{11/6}}{m_0 m_1} (1-e_2^2)^2}$$

$m_{frac} \approx 20\, M_\odot^{-1/6}$

$$\cancel{\cos i_2 < \frac{2^{4/3} x^{7/2}}{3G^{1/6}\pi^{2/3}\tau_{prec}} P_{EB}^{2/3} \frac{(m_0+m_1)^{11/6}}{m_0 m_1} (1-e_2^2)^2}$$

RHS maximized (smallest "τ_{prec}" if it were physical)

- large x (e.g., 100, 200 ...)
- $P_{EB} = 10$ day
- $e_2 = 0$
- $\tau_{prec} = 100$ yr

$$< \frac{4x^{7/2}}{3\cancel{G^{1/2}}\tau_{prec}} \frac{\cancel{G^{1/2}}(m_0+m_1)^{1/2}}{2\pi} P_{EB} \frac{(m_0+m_1)^{3/2}}{m_0 m_1} (1-e_2^2)^2$$

$$< \frac{2x^{7/2}}{3\pi\tau_{prec}} \underbrace{\frac{(m_0+m_1)^2}{m_0 m_1}}_{m_{frac,max} = 22.05} P_{EB} (1-e_2^2)^2$$

Very helpful! (...)

very good. $\cos i_2 < 12810.14$.

(A) start w/ eqn on pg 2 :

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$$\tau_{prec} = \frac{4}{3G^{1/2}} \frac{a_2^{7/2}}{a_1^{2}} \frac{(m_0+m_1)^{3/2}}{m_0 m_0} \frac{(1-e_2^2)^2}{\cos i_2}$$

write $a_2 = x\, a_1$ $\quad (x \in [4, \sim 200],\ x \in \mathbb{R})$

$$\tau_{prec} = \frac{4}{3G^{1/2}} a_1^{3/2} x^{7/2} \frac{(m_0+m_1)^{3/2}}{m_1 m_0} \frac{(1-e_2^2)^2}{\cos i_2}$$

recalling $a_1^{3/2} = \left(\frac{G(m_0+m_1)}{4\pi^2}\right)^{1/2} P_{EB}$, get

$$\tau_{prec} = \frac{4x^{7/2}}{3\cancel{G}^{1/2}} \frac{\cancel{G}^{1/2}}{2\pi} P_{EB} \frac{(m_0+m_1)^2}{m_1 m_0} \frac{(1-e_2^2)^2}{\cos i_2}$$

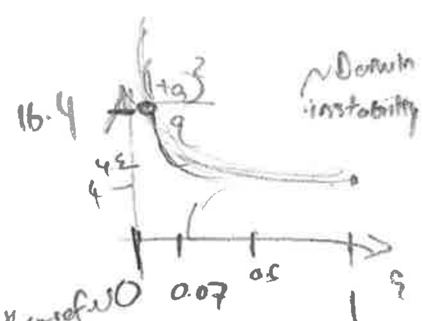
$$= \frac{2x^{7/2}}{3\pi} \underbrace{P_{EB} \frac{(m_0+m_1)^2}{m_1 m_0}}_{} \frac{(1-e_2^2)^2}{\cos i_2}$$

indept [could express as $q = \frac{m_0}{m_1}$, $M_{tot} = m_0 + m_1$]. Could define s.t. $m_0 > m_1$, $q = \frac{m_1}{m_0} < 1$.

$$= \frac{2x^{7/2}}{3\pi} (1\,\text{day}) \left(\frac{P_{EB}}{1\,\text{day}}\right) \underbrace{\frac{(m_0+m_1)^2}{m_1 m_0}} \frac{(1-e_2^2)^2}{\cos i_2}$$

$$\underbrace{\frac{m_0^2\left(1+\frac{m_1}{m_0}\right)^2}{m_1 m_0}}_{\frac{(1+q)^2}{q}}$$

$$\tau_{prec} = \frac{2x^{7/2}}{3\pi} P_{EB} \frac{(1+q)^2}{q} \frac{(1-e_2^2)^2}{\cos i_2}$$



$$= \frac{2}{3\pi} 10^{7/2} \left(\frac{x}{10}\right)^{7/2} 3\,\text{day} \left(\frac{P_{EB}}{3\,\text{day}}\right) (4.5) \frac{1}{\cos i_2}$$

$$= 22.0\ \text{yr} \left(\frac{x}{10}\right)^{7/2} \left(\frac{P_{EB}}{3\,\text{day}}\right) \frac{(1+q)^2}{4q} \frac{(1-e_2^2)^2}{\cos i_2}$$

$q = 1$
$e_2 = 0$
$\cos i_2 = 1$

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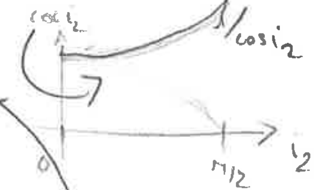
(*) $T_{prec} = 22.05\,\mathrm{yr} \left(\frac{x}{10}\right)^{7/2} \left(\frac{P_{EB}}{3d}\right) \underbrace{\frac{(1+q)^2}{4q}}_{\geq 1} \frac{(1-e_2^2)^2}{\cos i_2}$



won't be sensitive to very eccentric systems

If, at

$x \approx 5$ (inner stability limit? check Holman & Wiegert?),

worst q ratio (4)

circular

$P_{EB} = 3$ day

↳ what $\cos i_2$ is needed?

Fabrycky 2014, quoting

n.b. Eq 4 of Martin, Mazeh & Holman & Wiegert (1999):

$$a_{crit,CB} = a_{in} \left(1.6 + 5.1\,e_{in} + 4.12\,\mu_{in} - 4.27\,e_{in}\mu_{in} - 2.22\,e_{in}^2 - 5.09\,\mu_{in}^2 + 4.61\,e_{in}^2\mu_{in}^2\right)$$

$$\mu_{in} \equiv \text{"}\frac{M_2}{M_1+M_2}\text{"} = \frac{m_0}{m_1+m_0} = \frac{m_0}{m_0\left(\frac{m_1}{m_0}+1\right)} = \frac{1}{1+q}$$

$e_{in} = 0$

$$\hookrightarrow a_{crit,CB} = a_{in} \left(1.6 + 4.12\left(\frac{1}{1+q}\right) - 5.09\left(\frac{1}{1+q}\right)^2\right)$$

for $q=1$, $e_{in}=0$,

AT the $e_2=0$, $q=1$ limit, $P_{EB} = 3$ day (for $x = 2.3875$, the H&W 1999 limit)

$$T_{prec} = 0.147\,\mathrm{yr} \left(\frac{P_{EB}}{3\,\mathrm{day}}\right)^1 \frac{1}{\cos i_2}$$

uh... need (for $T_{prec} \approx 100\,\mathrm{yr}$) $\frac{1}{\cos i_2} \gtrsim \frac{100}{0.147}$, or $\boxed{i_2 \gtrsim \arccos\left(\frac{1}{682}\right) = 89.92°}$

$\cos i_2 \lesssim \frac{0.147\,\mathrm{yr}}{100\,\mathrm{yr}}$

(0.02° on plot)

6.

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Rewrite (*) from pg 6: requiring

$$\cos i_2 \lesssim \frac{22.05\ \text{yr}}{100\ \text{yr}} \left(\frac{x}{10}\right)^{7/2} \frac{P_{EB}}{3\,d} \frac{(1+q)^2}{4q} \left(1-e_2^2\right)^2$$

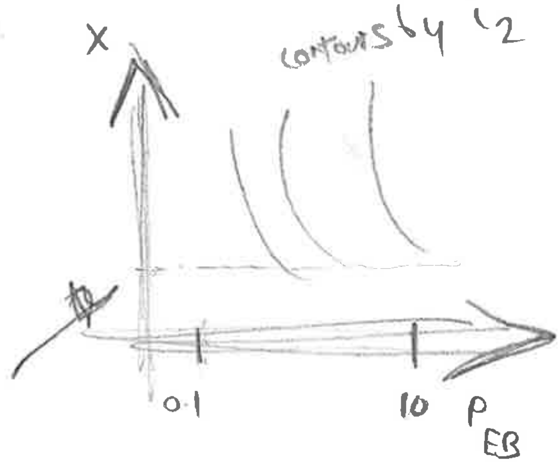
$$\left[\, i_2 \gtrsim \arccos\left[\ \cdots\ \right] \right]$$

$q = 1,\ e_2 = 0$

$$\cos i_2 \lesssim 0.2205 \left(\frac{x}{10}\right)^{7/2} \frac{P_{EB}}{3\,\text{day}}$$

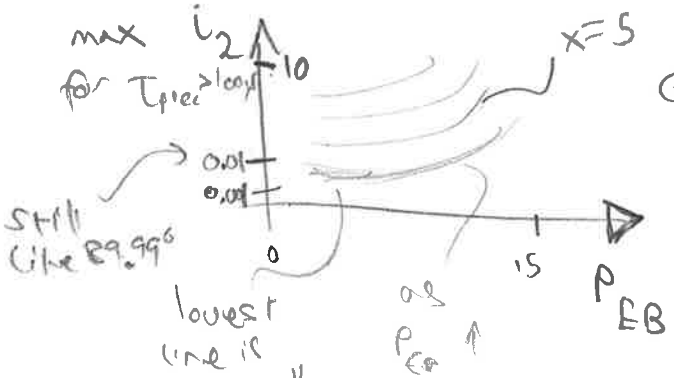
$$i_2 \gtrsim \arccos\Big(\underbrace{0.2205 \left(\frac{x}{10}\right)^{7/2} \frac{P_{EB}}{3\,\text{day}}}_{\text{"arg"}}\Big)$$

↳ see plot in precession_timescale_Farago_Laskar_2010.ipynb.



TODO

Might want to present as:



Contour or IMSHOW w/ colorbar "x"

unclear if max allowed i_2 ↑ or ↓

x[100]
Y[80]

$\cos\theta = \#$
$\theta = \cos^{-1}(\#)$