17/05/02 BEB & FP checks

Follow Bryson (2013) example:

Aperture w/ target star, const flux F & background binary Δm magnitudes fainter than the target ☆.

Flux ratio of bkgnd ☆ to target ☆ is $\Delta F = 100^{-\Delta m/5}$.

Say bkgnd binary has fractional eclipse depth d_{back}.

Total OOT flux: $F^{out} = F + F\Delta F$.

In transit flux: $F^{in} = F + (1 - d_{back}) F\Delta F$.

↳ observed fractional depth in aperture is

$$d_{obs} = 1 - \frac{F^{in}}{F^{out}} = 1 - \frac{1 + (1 - d_{back})\Delta F}{1 + \Delta F} = \frac{d_{back}\Delta F}{1 + \Delta F}.$$

$$\hookrightarrow \quad d_{obs}(1 + \Delta F) = d_{back}\,\Delta F$$

$$d_{obs} = (d_{back} - d_{obs})\,\Delta F$$

here, $d_{obs} \simeq 5 * 10^{-4}$. 2MASS pt source $\Delta m \gtrsim 5$.

$$d_{back} = d_{obs}\,\frac{1 + \Delta F}{\Delta F}$$

Note centroid x, c_x is (for flux as a func^n of x on image)

$$c_x = \frac{\int_{\text{bounds of aperture}} x\, f(x)\, dx}{\int_{\text{bounds of aperture}} f(x)\, dx}$$

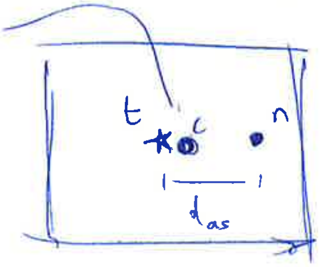
similar for c_y.

Toy model:

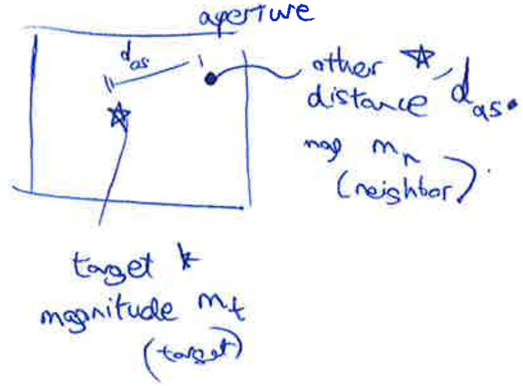
- Better: assume neighbor is just in x dirn:

$\vec{r}_c = (x_c, y_c)$

$\vec{r}_t = (x_t, y_t) \equiv (0,0)$



$\vec{r}_n = (x_n, y_n)$



here $f(x) = f_t\,\delta(0) + f_n\,\delta(x - x_n)$

so

$$x_c \equiv \frac{\int_{\text{aperture}} dx \; x\, f(x)}{\int_{\text{aperture}} f(x)\, dx}.$$

~~Ok~~ Jk, you want $\vec{r}_c = (0,0)$ (out of transit).

Convert magnitude of ★ to flux:

$$m_x - m_{x0} = -\frac{5}{2}\log_{10}\left(\frac{f_x}{f_{x0}}\right)$$

for us

$$f_K = 10^{-\frac{2}{5}(m_K - 12)} f_{12}$$

for $f_{12} = 1.74 \cdot 10^5\, e^-/s$.

m_K kep mag.

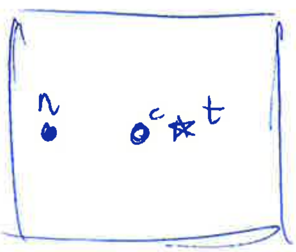
f_K kep flux.

New picture:

$\vec{r}_t = (x_t, y_t)$

$\vec{r}_n = (x_n, y_n)$

$\vec{r}_c = (0,0)$



$f(x) = f_t\,\delta(x - x_t) + f_n\,\delta(x - x_n)$

$$x_c = \frac{\int dx\, f(x)\, x}{\int f(x)\, dx} = \frac{f_t x_t + f_n x_n}{f_t + f_n}.$$

(Generally, N neighbors positions $\vec{r}_{n,i} = (x_n, y_n)_i$

means

$$x_c = \frac{f_t x_t + \sum_i f_{n,i} x_{n,i}}{f_t + \sum_i f_{n,i}}, \quad y_c = \frac{f_t y_t + \sum_i f_{n,i} y_{n,i}}{f_t + \sum_i f_{n,i}}.)$$

~~One will have~~ For a measured centroid shift $\Delta r_c = \sqrt{\Delta x_c^2 + \Delta y_c^2} = \left((x_c^{oot} - x_c^{it})^2 + \dots\right)^{1/2}$

we want to know if it's consistent w/ a nbhr ★ doing the dipping work.

Simplify & ~~xxxx~~ say

$$\Delta r_c = \Delta x_c = x_c^{oot} - x_c^{it}.$$

If you have two targets, there are 2 possibilities: either target t dims, or neighbor n dims.

~~Want~~ Need flux in & out of tra for t & n for both cases. You have observed depth δ_{obs}.

$$f^{oot} = f_t^{oot} + f_n^{oot}$$

$$\delta_{obs} \equiv 1 - \frac{f^{it}}{f^{oot}}$$

$$\Rightarrow f^{it} = f^{oot}(1-\delta_{obs})$$

Case 1:

target is dimming

$$\delta_t \equiv 1 - \frac{f_t^{it}}{f_t^{oot}}$$

being nearer!

know: f_t^{oot}, f_n^{oot}, δ_{obs}

Eq (1): $f^{oot} = f_t^{oot} + f_n^{oot}$

Eq (2): $f^{it} = f_t^{it} + f_n^{oot} = f_t^{oot}(1-\underset{\text{unknown}}{\delta_t}) + f_n^{oot}$

Eq (3): $f^{it} = f^{oot}(1-\delta_{obs})$

combine (2) & (3) to get δ_t, then find f_t^{it}.

$$f^{oot}(1-\delta_{obs}) = f_t^{oot}(1-\delta_t) + f_n^{oot}$$

$$1 - \frac{f^{oot}(1-\delta_{obs}) - f_n^{oot}}{f_t^{oot}} = \delta_t$$

$$\Rightarrow f_t^{it} = f_t^{oot}(1-\delta_t)$$

$$= \cancel{f_t^{oot}}\left(\frac{f^{oot}(1-\delta_{obs}) - f_n^{oot}}{\cancel{f_t^{oot}}}\right)$$

$$\boxed{f_t^{it} = f^{oot}(1-\delta_{obs}) - f_n^{oot}} \quad \text{(Case 1)}$$

which means for the centroid

then for centroid: 17/8/02

$$\Delta x_c = x_c^{oot} - x_c^{it}$$

$$= \left(\frac{f_t^{oot} x_t + f_n^{oot} x_n}{f_t^{oot} + f_n^{oot}} \right) - \left(\frac{f_t^{it} x_t + f_n^{oot} x_n}{f_t^{it} + f_n^{oot}} \right)$$

$$\Delta x_c = (\ldots) - \frac{\left[f^{oot}(1-\delta_{obs}) - f_n^{oot} \right] x_t + f_n^{oot} x_n}{f^{oot}(1-\delta_{obs})}$$ (case 1, target is being transited)

Note: ~~Since the point is~~ that we're setting $x_c^{oot} = 0 = f_t^{oot} x_t + f_n^{oot} x_n$.

Ask:
1) Is case 1 consistent w/ measured centroid shift magnitude?
2) Is case 2 (nbhr ★ is transited) consistent, for reasonable values of δ_n, w/ measured centroid shift?

Case 2:

(1): $f^{oot} = f_t^{oot} + f_n^{oot}$

(2): $f^{it} = f_t^{oot} + f_n^{it} = f_t^{oot} + f_n^{oot}(1-\delta_n)$

(3): $f^{it} = f^{oot}(1-\delta_{obs})$

(2) & (3) $\rightarrow$ $f^{oot}(1-\delta_{obs}) = f_t^{oot} + f_n^{oot}(1-\delta_n)$

$$1 - \frac{f^{oot}(1-\delta_{obs}) - f_t^{oot}}{f_n^{oot}} = \delta_n$$

$$\Rightarrow f_n^{it} = f_n^{oot}(1-\delta_n) = \frac{\cancel{f_n^{oot}}}{\cancel{f_n^{oot}}} \left[f^{oot}(1-\delta_{obs}) - f_t^{oot} \right]$$

and

$$\Delta x_c = x_c^{oot} - x_c^{it} = (\ldots) - \frac{f_t^{oot} x_t + f_n^{it} x_n}{f_t^{oot} + f_n^{it}}$$

$$|\Delta x_c| = \frac{f_t^{oot} x_t + \left[f^{oot}(1-\delta_{obs}) - f_t^{oot} \right] x_n}{f_t^{oot} + f_n^{it}}$$

So case 2 simplifies to

$$|\Delta x_c| = \frac{f_t^{oot} x_t + \left[f^{oot}(1-\delta_{obs}) - f_t^{oot}\right] x_n}{f_t^{oot} + f_n^{it}}$$

$\left[\text{CROWDSAP} \approx 1 \right. \Longrightarrow f^{oot} = f_t^{oot} + f_n^{oot} \approx f_t^{oot}$

Assume $x_t \approx 0$ (i.e. that the centroid is very close to the KIC target. Since most of flux in aperture is from target this is OK).

Then

$$\boxed{|\Delta x_c| = \frac{\left[f^{oot}(1-\delta_{obs}) - f_t^{oot}\right] x_n}{f_t^{oot} + f_n^{it}} \quad \text{(Case 2)}}$$