Sansing's Method

17/03/13

Represent data $I(t)$ as a Fourier series: (n.b. have N data pts)

$$I(t) = \sum_{n=1}^{N/2} a_n \cos\left(\frac{2\pi n (t-\tau_i)}{\tau_f - \tau_i}\right) + \sum_{n=1}^{N/2} b_n \sin\left(\frac{2\pi n (t-\tau_i)}{\tau_f-\tau_i}\right). \qquad (3)$$

For us,

$$F_{obs} = \bar{F}_\star + \Delta F + \bar{F}_\star (I^S \ldots)$$

$\tau_{i,f}$: start & end time of dataset in analysis (one quarter)

just write

$$I(t) = \underbrace{I(t)^S}_{\text{signal: EB transits.}} + \cancel{I(t)^{CBP}} + \underbrace{I(t)^N}_{\text{noise + CBP signal (latter} \ll \text{EB signal)}}$$

~~CBP signal much smaller than EB signal~~

$$T_w \equiv |\tau_f - \tau_i|$$

Fourier transform is linear, so the Fourier coefficients can be split up as:

$$a_n = a_n^S + \cancel{a_n^{CBP}} + a_n^N; \qquad b_n = b_n^S + \cancel{b_n^{CBP}} + b_n^N$$

absorbed to N

Take

τ_i = time of first biggest minimum ("primary transit")

τ_f = time of last bigger minimum (" " ")

$\Rightarrow T_w = P_{EB} \times n$, for some $n \in \mathbb{N}$.

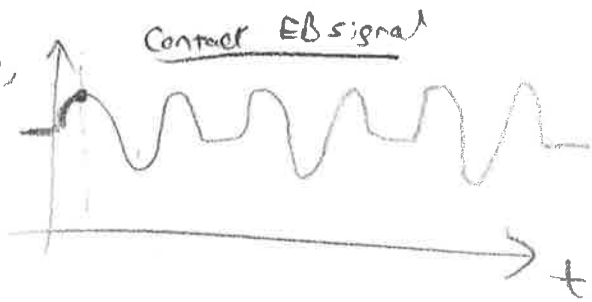
Benefits:

1) Frequency of signal is confined to single frequency bin w/ no broadening, since T_w is a multiple of P_{EB}.

2) Since time-window starts in center of an eclipse, the signal coeffs a_n^S & b_n^S (which describe odd & even contribns to data) also describe the symmetry of the signal.

If signal is actually **even** (e.g. a transiting, e=0 planet) $b_n^S = 0 \; \forall n$.

$\Rightarrow$ noise & signal are perfectly separated in b_n coefficient space.



sec 3.3: assume symmetric (extend later).

Let N_0 be the # of symmetric signals (e.g. planet orbits) within the data window.

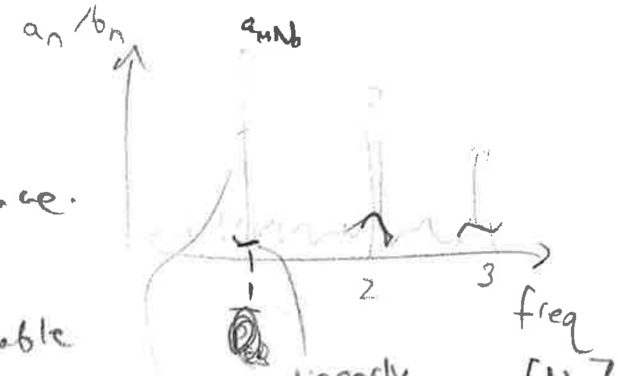
Then ⑤ becomes

(actually assuming noise is even too, s.t. $b_n = b_n^S + b_n^{CBP} + b_n^N = 0$ ...)

$$I(t) = \sum_{n=1}^{N/2} \underbrace{a_n^{\mathcal{N}}}_{\text{let } a_n^N = a_n^{noise} + a_n^{CBP}} \cos\left(\frac{2\pi n t}{T_w}\right) + \sum_{m=1}^{[N/(2N_0)]} a_{mN_0}^S \cos\left(\frac{2\pi (m N_0) t}{T_w}\right)$$

N_0: # of fully symmetric signals (e.g. planet orbits) in data window

This identifies the signal & noise in Fourier space.



isolate these (mostly signal + some noise).

linearly interpolate here for noise contribn

$$a_{mN_0} = a_{mN_0}^S + a_{mN_0}^{\mathcal{N}}$$

<u>Reconstruct Signal</u>

Signal now appears in only $\sim 1/(2N_0)$ of available Fourier space.

To reconstruct signal, (subtracting noise components in Fourier space), write

$$a_{mN_0}^S = a_{mN_0} - \underbrace{a_{mN_0}^{\mathcal{N}}}_{\text{estimate this by linear interpol}^n}$$

$$a_{mN_0}^S \approx a_{mN_0} - \left(a_{mN_0-1} + a_{mN_0+1}\right)/2 . \quad \text{(Eq 8)}$$

Note that the a_x coefficients here come from data via:

$$a_x = \sum_{j=1}^{N} I(t_j) \cos\left(\frac{2\pi x\, t_j}{T_w}\right) \; ; \quad 0 \leq t_j \leq T_w . \quad \text{(Eq 9)}$$

For actual Kepler data, which is nonuniformly sampled, the coefficients will be ~different (similar for nonsymmetric transits).

Note first though that real space signal should be reconstructable as

$$I(t)^S \approx \sum_{m=1}^{[N/(2N_0)]} a_{mN_0}^S \cos\left(\frac{2\pi m t}{T_0}\right)$$

for T_0 "the orbital time of the planet" (i.e. $\frac{t}{T_0} = \frac{t - \tau_i}{(\tau_f - \tau_i)}$, I think...)

- For an arbitrary shape, just do all the same things w/ cosines & sines, other than only cosines.

For nonuniform samples, Eq (9) doesn't hold.

Instead, need to do a joint least squares fit of sine & cosine func^ns to the data.

Johan notes:

Well, the Fourier expansion is linear. You can actually formulate the fit in terms of matrices. Write data as a matrix multiplic^n,

$$\vec{I}_i \approx \overleftrightarrow{C}_{ni}\,\vec{a}_n + \overleftrightarrow{S}_{ni}\,\vec{b}_n = \overleftrightarrow{T}_{ki}\,\vec{R}_k$$

for $C_{ni} \equiv \cos\left(\frac{2\pi n t_i}{T_W}\right)$; $S_{ni} \equiv \sin\left(\frac{2\pi n t_i}{T_W}\right)$,

T_{ki} "their joined matrix"; $\vec{R}_k = [\vec{a}_n, \vec{b}_n]$ is the vector of coefficients to fit for.

If you minize the difference squared btwn this expansion & the data, the soln to the vector $\vec{R}_k$ can be written

$$\vec{R}_k \approx \underbrace{\left(\overleftrightarrow{T}^{T}\overleftrightarrow{T}\right)^{-1}}_{\text{" } \hat{\mathbb{I}} \text{ when data uniformly sampled.}} \overleftrightarrow{T}^{T}\,\vec{I}\,.$$

Note by "joined matrix", something means

$$\overleftrightarrow{T}_{ki} \equiv \left[\overleftrightarrow{C}_{ni}, \overleftrightarrow{S}_{ni}\right] \quad \text{s.t.} \quad \begin{array}{l} \dim(C) = N \times N \\ \dim(a_n) = \cancel{N\times N}\ N \times 1 \end{array}$$

(N ↑ rows, 1 ↑ cols)

$$\overleftrightarrow{T}_{ki}\,\vec{R}_k = \begin{bmatrix} \overleftrightarrow{C}_{ni} \\ \overleftrightarrow{S}_{ni} \end{bmatrix}_{N\times 2N} \left[\vec{a}_n, \vec{b}_n\right]_{2N\times 1}.$$