eq. (29) of Dobbs-Dixon + 2004 ... 18/10/10.0

$$\frac{da}{dt} = -18\pi \frac{a}{P} \frac{1}{Q_*} \frac{M_p}{M_*} \left(\frac{R_*}{a}\right)^5$$

note $P = \left(a^3 \frac{4\pi^2}{G M_{tot}}\right)^{1/2} = a^{3/2} \left(\frac{4\pi^2}{G M_{tot}}\right)^{1/2}$, so

$$\frac{da}{dt} = -18\pi \, a^{-1/2} \left(\frac{4\pi^2}{G M_{tot}}\right)^{-1/2} \frac{1}{Q_*} \frac{M_p}{M_*} \left(\frac{R_*}{a}\right)^5$$

analytically,

$$\int_{a_0}^{R_*} a^{11/2} \, da = -18\pi \left(\frac{M_p}{M_*}\right) \left(\frac{4\pi^2}{G M_{tot}}\right)^{-1/2} \frac{1}{Q_*} R_*^5 \int_0^{t_{final}} \cancel{a^{-11/2}} \, dt$$

$$\left[\left(\frac{1}{6.5}\right) a^{(11/2+1)}\right]_{a_{initial}}^{R_* = a_{final}} =$$

$$\left(-a_{final}^{13/2} + a_{initial}^{13/2}\right) = \frac{13}{2} \cdot 18\pi \cdot \left(\frac{M_p}{M_*}\right) \left(\frac{4\pi^2}{G M_{tot}}\right)^{-1/2} \frac{R_*^5}{Q_*} \left(t_{final} - t_{initial}\right)$$

$$\therefore \; a_{final} = \left[a_{initial}^{13/2} - \frac{13}{2} \cdot 18\pi \cdot \frac{M_p}{M_*} \left(\frac{G M_{tot}}{4\pi^2}\right)^{1/2} \frac{R_*^5}{Q_*} \left(t_{final} - t_{initial}\right)\right]^{2/13}$$

or, setting $a_{final} = R_*$,

$$\left(t_{final} - t_{initial}\right) = \left(a_{initial}^{13/2} - R_*^{13/2}\right) \frac{2}{13 \cdot 18\pi} \frac{M_*}{M_p} \left(\frac{G M_{tot}}{4\pi^2}\right)^{1/2} \frac{Q_*}{R_*^5}$$