Computational Statistics Homework 1

Liam Dillingham

February 17, 2019

1 Question 1

The function $p(x) = sin(x)$ is a density for $x \in (0, \pi/2)$.

1.1 Describe an inverse CDF transform method to sample random variables with this density. Plot the histogram and the true density for visual verification.

1.2 Set up a rejection sampling method to sample from $p(x)$ using a proposal density $g(x)$. Plot the histogram and the true density for visual verification

Hint: You can take $g(x)$ as the uniform density on the interval $(0, \pi/2)$.

2 Question 2

Determine a method to draw samples from the distribution with PDF: $f(x) \propto exp(-x^4/12)$, for $x \in R$. Turn in derivation and code. Plot histogram and true density for visual verification.

---

3 Question 3

Suppose that $X \in R^{nxn}$ is a random matrix with independent $\mathcal{N}(0, 1)$ entries. Let ℓ_1 be the smallest Eigenvalue of $S_n = X^T X/n$ and let $Y = n\ell_1$. Edelman showed that as $n \to \infty$, the PDF of Y approaches

$$f(y) = \frac{1+\sqrt{y}}{2\sqrt{y}} e^{-(y/2+\sqrt{y})},\ 0 < y < \infty$$

3.1 Develop a method to sample Y from the density $f(y)$ given in the equation above. Show derivation and code. Plot histogram and true density for visual verification.

3.2 Test method by estimating $E(log(Y))$ by simple Monte Carlo, and giving a 99% confidence interval. Edelman found that the answer was roughly -1.68788.