Formal Languages Homework 10

Liam Dillingham

April 29, 2019

1 Problem 9.1.1

What strings are:

1.1 w_{37}?

37 in binary is 100101. then $1w = 100101$, so $w = 00101$.

1.2 w_{100}?

100 in binary is 1100100. then $1w = 1100100$, so $w = 100100$.

2 Problem 9.2.1

Show that the halting problem, the set of (M, w) pairs such that M halts (with or without accepting) when given input w is r.e. but not recursive

---

For a given input w, the TM M must decide whether or not the string is in the language. Let L_u be the set of pairs of (M, w) such that M halts on w. By encoding M, then we can rename our set X to L_u, that is, $L_u(M, w) = \{(M, w) \mid M \text{ halts on } w\}$. Then $\overline{L_u}$ is the set of pairs of (M, w) such that M does *not* halt on w. If L_u is recursive, then all pairs of (M, w) are in L_u, since a recursive language always halts. However, $\overline{L_u}$ would have to be recursive as well, since the compliment of a recursive language must also be recursive. Yet $\overline{L_u}$ is precisely the language of the pairs of (M, w) which do not halt. Since a recursive language must halt eventually, then determining the set of pairs which halt is recursievely enumerable, since determining which ones do not halt requires infinite computation.

3 Problem 9.2.2

Using *Ackermann's function*:

1. $A(0, y) = 1$ for any $y \geq 0$
2. $A(1, 0) = 2$
3. $A(x, 0) = x + 2$ for $x \geq 2$
4. $A(x+1, y+1) = A(A(x, y+1), y)$ for any $x \geq 0$, and $y \geq 0$

3.1 a). Evaluate $A(2, 1)$

$A(2,1) = A(1+1, 0+1)$ Rule #4 $\Rightarrow A(1+1, 0+1) = A(A(1, 0+1), 0)$
$A(1+1, 0+1) = A(A(1, 0+1), 0) = A(A(1,1), 0)$
$A(1,1) = A(0+1, 0+1) = A(A(0, 0+1), 0) = A(A(0,1), 0)$
$A(0,1) = 1$. Now substitute
$A(1,1) = A(0+1, 0+1) = A(A(0,1), 0) = A(1,0) = 2$. And substituting again,
$A(2,1) = A(1+1, 0+1) = A(A(1, 0+1), 0) = A(A(1,1), 0) = A(2,0) = x + 2 = 4$

3.2 b). What function of x is $A(x, 2)$?

After running a python script I wrote, I got the answers for the following input:
$A(0,1) = 1; A(1,1) = 2; A(2,1) = 4; A(3,1) = 6; A(4,1) = 8$. In addition, the value of y does not affect the output of the function, it merely adds unneccesary complexity. By observation, we can see that the only variable in the entire rules list is x. Everything else is a constant. Thus the function of x is $2x$.

3.3 c). Evaluate $A(4, 3)$

Note to grader: I initially computed this by hand for some time, but after deriving the truth of the function, it is as simple as plugging in the x value
Using the derived function of x from the previous question, $A(4, 3) = 8$

```python
import sys
sys.setrecursionlimit(8000)

def A(x, y):

    # rule 1
    if x == 0 and y >= 0:
        return 1

    # rule 2
    if x == 1 and y == 0:
        return 2

    # rule 3
    if x >= 2 and y == 0:
        return x + 2
```

```
    # rule 4
    if x >= 0 and y >= 0:
        x = x - 1
        y = y - 1

        return A(A(x, y + 1), y)

    raise Exception('Something terrible happened!')

print(A(16,2))
```