

Formal Languages Homework 8

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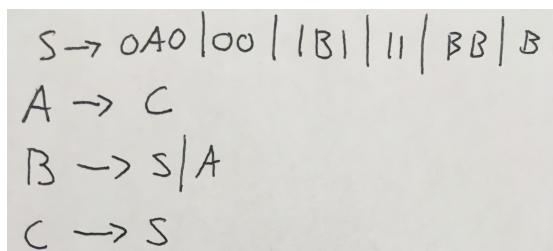
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1 Problem 7.1.3

Repeat Exercise 7.1.2 for the following grammar:

$$\begin{aligned} S &\rightarrow 0A0 \mid 1B1 \mid BB \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \mid \epsilon \end{aligned}$$

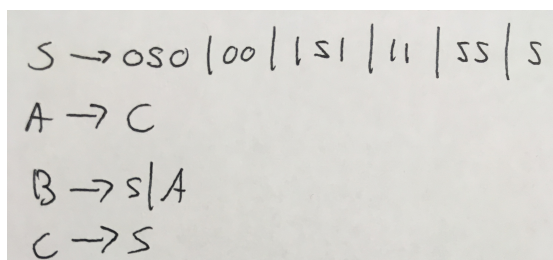
1.1 a). Eliminate ϵ -productions



Handwritten derivation of the grammar after eliminating ϵ -productions. The original grammar is shown, and the resulting grammar is derived by replacing C with S in all productions where it appears as a non-terminal.

$$\begin{aligned} S &\rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \end{aligned}$$

1.2 b). Eliminate any unit productions in the resulting grammar



Handwritten derivation of the grammar after eliminating unit productions. The original grammar is shown, and the resulting grammar is derived by replacing A with S in all productions where it appears as a non-terminal.

$$\begin{aligned} S &\rightarrow 0S0 \mid 00 \mid 1S1 \mid 11 \mid SS \mid S \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \end{aligned}$$

1.3 c). Eliminate any useless symbols in the resulting grammar

$$S \rightarrow OSO \mid OO \mid ISI \mid II \mid SS \mid S$$

1.4 d). Put the resulting grammar into Chomsky Normal Form

$$\begin{aligned} S &\rightarrow OS_1 \mid O_2 \mid I_1 S_2 \mid I_2 \mid SS \mid S \\ S_1 &\rightarrow SO_1 \\ O_1 &\rightarrow O \\ O_2 &\rightarrow O_1 O_1 \\ I_1 &\rightarrow I \\ S_2 &\rightarrow SI_1 \\ I_2 &\rightarrow I_1 I_1 \end{aligned}$$

2 Problem 7.2.1

Use the CFL pumping lemma to show each of these languages not to be *context-free*

2.1 a). $\{a^i b^j c^k \mid i < j < k\}$

2.2 b). $\{a^n b^n c^i \mid i \leq n\}$

3 Problem 7.3.2

Consider the following two languages:

$$\begin{aligned} L_1 &= \{a^n b^{2n} c^m \mid n, m \geq 0\} \\ L_2 &= \{a^n b^m c^{2m} \mid n, m \geq 0\} \end{aligned}$$

3.1 a). Show that each of these languages is context-free by giving grammars for each

3.2 b). Is $L_1 \cap L_2$ a CFL? Justify your answer

$$\begin{aligned}
 L_1: \\
 S &\rightarrow A \mid B \mid \epsilon \\
 A &\rightarrow a A b b C \mid a A b b \mid \epsilon \\
 B &\rightarrow A C \mid C \\
 C &\rightarrow c
 \end{aligned}$$

$$\begin{aligned}
 L_2: \\
 S &\rightarrow A \mid B C C \mid \epsilon \\
 A &\rightarrow a A \mid a A B C C \mid \epsilon \\
 B &\rightarrow b B C C \mid \epsilon \\
 C &\rightarrow c
 \end{aligned}$$