

Formal Languages Homework 8

Liam Dillingham

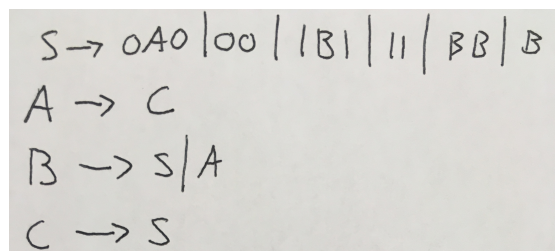
April 10, 2019

1 Problem 7.1.3

Repeat Exercise 7.1.2 for the following grammar:

$$\begin{aligned} S &\rightarrow 0A0 \mid 1B1 \mid BB \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \mid \epsilon \end{aligned}$$

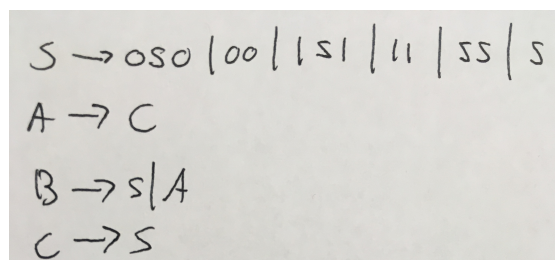
1.1 a). Eliminate ϵ -productions



Handwritten derivation of the grammar after eliminating ϵ -productions. The original grammar is shown, and the resulting grammar is derived by substituting S for C in all productions, since $C \rightarrow \epsilon$ is the only ϵ -production.

$$\begin{aligned} S &\rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \end{aligned}$$

1.2 b). Eliminate any unit productions in the resulting grammar



Handwritten derivation of the grammar after eliminating unit productions. The original grammar is shown, and the resulting grammar is derived by substituting S for C in all productions, since $C \rightarrow S$ is the only unit production.

$$\begin{aligned} S &\rightarrow 0S0 \mid 00 \mid 1S1 \mid 11 \mid SS \mid S \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \end{aligned}$$

1.3 c). Eliminate any useless symbols in the resulting grammar

$$S \rightarrow OSO \mid OO \mid ISI \mid II \mid SS \mid S$$

1.4 d). Put the resulting grammar into Chomsky Normal Form

$$\begin{aligned} S &\rightarrow OS_1 \mid O_2 \mid I_1 S_2 \mid I_2 \mid SS \mid S \\ S_1 &\rightarrow SO_1 \\ O_1 &\rightarrow O \\ O_2 &\rightarrow O_1 O_1 \\ I_1 &\rightarrow I \\ S_2 &\rightarrow SI_1 \\ I_2 &\rightarrow II_1 \end{aligned}$$

2 Problem 7.2.1

Use the CFL pumping lemma to show each of these languages not to be *context-free*

2.1 a). $\{a^i b^j c^k \mid i < j < k\}$

2.2 b). $\{a^n b^n c^i \mid i \leq n\}$

3 Problem 7.3.2

Consider the following two languages:

$$\begin{aligned} L_1 &= \{a^n b^{2n} c^m \mid n, m \geq 0\} \\ L_2 &= \{a^n b^m c^{2m} \mid n, m \geq 0\} \end{aligned}$$

3.1 a). Show that each of these languages is context-free by giving grammars for each

3.2 b). Is $L_1 \cap L_2$ a CFL? Justify your answer