Formal Languages Homework 5

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1 Problem 4.1.1

Prove that the following languages are not regular languages

1.1 e). $\{0^n10^n \mid n \geq 1\}$

Using the pumping lemma, suppose that $x = \epsilon, y = 0^n$, and $z = 10^n$. Note that $|xy| \leq n$, and $y \neq \epsilon$. However, note that the pumping lemma requires that for all $k \geq 0$, the string is still in our language L. Lets choose $k = 0$. Then the string is essentially 10^n, where the number of 0's on either side of the 1 do not match. Therefore, the language is not regular.

1.2 f). $\{0^n1^{2n} \mid n \geq 1\}$

Let $x = \epsilon, y = 0^n, z = 1^{2n}$. By choosing a $k = 0$, we have $0^{n*0} = \epsilon$. Since the number of 1's is not a square of the number of 0's, the language is not regular.

2 Problem 4.1.2

Prove that the following are not regular languages

2.1 e). The set of strings of 0's and 1's that are of the form ww, that is, some string repeated

Let $x = \epsilon$, and $y = w$, and $z = w$. Then yz should be in the language. However, by choosing any $k \neq 1$ gives us a string not equal to ww, and thus not in the language. Therefore the language is not regular.

3 Problem 4.2.4

Which of the following identities are true?

3.1 a). $(L/a)a = L$ (the left side represents the concatenation of the languages L/a and $\{a\}$)

Note that (L/a) is the set of words in L such that wa is in L. So if there is a word in L such that it ends in some other symbol, such as b, then it will not be in (L/a). Thus concatentating a again on the end will not restore the set to L. Thus, false.

3.2 b). $a(a \backslash L) = L$ (again, concatenation with $\{a\}$, this time on the left is intended)

The argument for this identity follows the same as above, except on the left. Thus, false.

3.3 c). $(La)/a = L$

(La) is the set of all $w \in L$ with an a concatenated on the right side. And $(La)/a$ is all $w \in L$ where wa is also in L. Since we are appending an a to every $w \in L$, then we take the quotient, where we only keep w's such that they have an a on the right side, then yes, it returns the original L. Thus true.

3.4 d). $a \setminus (aL) = L$

The argument follows the same from above, except considering for the left side of the w's. Thus true.

4 Problem 4.4.2

Given the following transition table of a DFA

	0	1
$\rightarrow A$	B	E
B	C	F
$*C$	D	H
D	E	H
E	F	I
$*F$	G	B
G	H	B
H	I	C
$*I$	A	E

4.1 a). Draw the table of distinguishabilities for this automaton

B	X							
C	X	X						
D		X	X					
E	X		X	X				
F	X	X		X	X			
G		X	X		X	X		
H	X		X	X		X	X	
I	X	X		X	X		X	X
	A	B	C	D	E	F	G	H

4.2 b). Construct the minimum-state equivalent DFA