Formal Languages Homework 8

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1 Problem 7.1.3

Repeat Exercise 7.1.2 for the following grammar:

$$
\begin{array}{lcl}
S & \rightarrow & 0A0 \mid 1B1 \mid BB \\
A & \rightarrow & C \\
B & \rightarrow & S \mid A \\
C & \rightarrow & S \mid \epsilon
\end{array}
$$

1.1 a). Eliminate ϵ-productions

$$
\begin{array}{lcl}
S & \rightarrow & 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B \\
A & \rightarrow & C \\
B & \rightarrow & S \mid A \\
C & \rightarrow & S
\end{array}
$$

1.2 b). Eliminate any unit productions in the resulting grammar

$$
\begin{array}{lcl}
S & \rightarrow & 0S0 \mid 00 \mid 1S1 \mid 11 \mid SS \mid S \\
A & \rightarrow & C \\
B & \rightarrow & S \mid A \\
C & \rightarrow & S
\end{array}
$$

1.3 c). Eliminate any useless symbols in the resulting grammar

$$S \to 0S0 \mid 00 \mid 1S1 \mid 11 \mid SS \mid S$$

1.4 d). Put the resulting grammar into Chomsky Normal Form

$$S \to O_1 S_1 \mid O_2 \mid I_1 S_2 \mid I_2 \mid SS \mid S$$

$$S_1 \to S O_1$$

$$O_1 \to 0$$

$$O_2 \to O_1 O_1$$

$$I_1 \to 1$$

$$S_2 \to S I_1$$

$$I_2 \to I_1 I_1$$

2 Problem 7.2.1

Use the CFL pumping lemma to show each of these languages not to be *context-free*

2.1 a). $\{a^i b^j c^k \mid i < j < k\}$

Let's break down the cases here and look at candidates for vwx:

1. alli n a's
2. all in b's
3. all in c's
4. between a's and b's where v contains both a's and b's
5. between a's and b's where x contains both a's and b's
6. between a's and b's where v may contain a's and x may contain b's
7. between b's and c's where v contains both b's and c's
8. between b's and c's where x contains both b's and c's
9. between b's and c's where v may contain b's and x may contain c's

For case 1, select a q (because i is already used) to pump v, x by. Let $q > j$. Then by pumping v and x, we end up with a string such that the length is longer than j, and the string is not in L. The same applies for case 2, if we select $q > k$. For case 3, select $q = 0$, then we transform part of the substring to epsilon. if $|x|$ is such that $k - |x| < j$, then by "eliminating" x by x^0, then the string is no longer in L. For case 6, if we pick $x = \epsilon$ and pump v, we can get an $i \geq j$, which is not in L. The same goes for case 9. For cases 4 and 7, we can simply pump both v and x. Since the size of the left hand substring (i.e. i, or j) will be growing faster, then at some point it will violate the rules for L. For the last cases, 5 and 8, if we select and $x = \epsilon$ and pump like usual, we will find a violation after a certain i.

2.2 b). $\{a^n b^n c^i \mid i \leq n\}$

This question also contains the cases from the previous question. For cases 1 and 2, we can simply pump the string until it breaks the equality between $|a^n|$ and $|b^n|$. For case 3, pump i until it is larger than n. For cases 4 and 7, simply allow $v = \epsilon$ and by pumping x we either violate the length equality between a and b, or we pump c such that its length is greater than n. For case 5 since both v and x contain a's, by pumping we break equality. The same reasoning applies to case 8. for cases 6 and 9, let $v = \epsilon$, and pump x, and we will break the conditions on L for both.

3 Problem 7.3.2

Consider the following two languages:

$$L_1 = \{a^n b^{2n} c^m \mid n, m \geq 0\}$$
$$L_2 = \{a^n b^m c^{2m} \mid n, m \geq 0\}$$

3.1 a). Show that each of these languages is context-free by giving grammars for each

L_1:

$S \rightarrow A \mid B \mid \varepsilon$

$A \rightarrow aAbbC \mid aAbb \mid \varepsilon$

$B \rightarrow AC \mid C$

$C \rightarrow c$

L_2:

$S \rightarrow A \mid BCC \mid \varepsilon$

$A \rightarrow aA \mid aABCC \mid \varepsilon$

$B \rightarrow bBCC \mid \varepsilon$

$C \rightarrow c$

3.2 b). Is $L_1 \cap L_2$ a CFL? Justify your answer

It seems like $L_1 \cap L_2 = \{a^n b^{2nm} c^{2m} \mid n, m \geq 0\}$. And yes, because unlike the previous questions, there are no constrains on the substrings in relation (read: context) to one another, such as the length of one substring being greater than another or etc.