

Formal Languages Homework 6

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March 27, 2019

1 Problem 4.1.1 and 4.1.2

Provide the CFG for the following languages:

- 1.1 b). The set of strings of balanced parentheses. These are the strings of characters "(" and ")" that can appear in a well-formed arithmetic expression.

$$S \rightarrow (S) \mid SS \mid \epsilon.$$

$$V = \{S\}.$$

$$T = \{ "(", ")" \}. \text{ (The quotations are only to make reading easier).}$$

- 1.2 d). $\{0^n 1^m 2^n \mid n \text{ and } m \text{ are arbitrary integers}\}.$

$$V = \{S, A\}.$$

$$T = \{0, 1, 2\}.$$

$$\begin{array}{lcl} S & \rightarrow & 0S2 \mid 0A2 \mid A \mid \epsilon \\ A & \rightarrow & 1 \mid \epsilon \end{array}$$

- 1.3 f). $\{0^n 1^{2n} \mid n \geq 1\}$

$$V = \{S, A\}$$

$$T = \{0, 1\}$$

$$\begin{array}{lcl} S & \rightarrow & 0S11 \mid 0A11 \mid \\ A & \rightarrow & \epsilon \end{array}$$

1.4 h). The set of strings of the form $w1^n$, where w is a string of 0's and 1's of length n .

$$V = \{S, a\}$$

$$T = \{0, 1\}$$

$$\begin{array}{lcl} S & \rightarrow & AS1 \mid \epsilon \\ A & \rightarrow & 0 \mid 1 \end{array}$$

2 Problem 5.1.2

The following grammar generates the language of regular expression $0^*1(0+1)^*$.

$$\begin{array}{lcl} S & \rightarrow & A1B \\ A & \rightarrow & 0A \mid \epsilon \\ B & \rightarrow & 0B \mid 1B \mid \epsilon \end{array}$$

Give leftmost and rightmost derivations of the following strings:

2.1 a). 00101.

Leftmost:

$$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 00\epsilon 1B \Rightarrow 001B \Rightarrow 0010B \Rightarrow 00101B \Rightarrow 00101\epsilon \Rightarrow 00101.$$

Rightmost:

$$S \Rightarrow A1B \Rightarrow A10B \Rightarrow A101B \Rightarrow A101\epsilon \Rightarrow A101 \Rightarrow 0A101 \Rightarrow 00A101 \Rightarrow 00\epsilon 101 \Rightarrow 00101.$$

2.2 b). 1001.

Leftmost:

$$S \Rightarrow A1B \Rightarrow \epsilon 1B \Rightarrow 10B \Rightarrow 100B \Rightarrow 1001B \Rightarrow 1001\epsilon \Rightarrow 1001.$$

Rightmost:

$$S \Rightarrow A1B \Rightarrow A10B \Rightarrow A100B \Rightarrow A1001B \Rightarrow A1001\epsilon \Rightarrow \epsilon 1001 \Rightarrow 1001.$$

2.3 c). 00011.

Leftmost:

$$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 000A1B \Rightarrow 000\epsilon 1B \Rightarrow 00011B \Rightarrow 00011\epsilon \Rightarrow 00011.$$

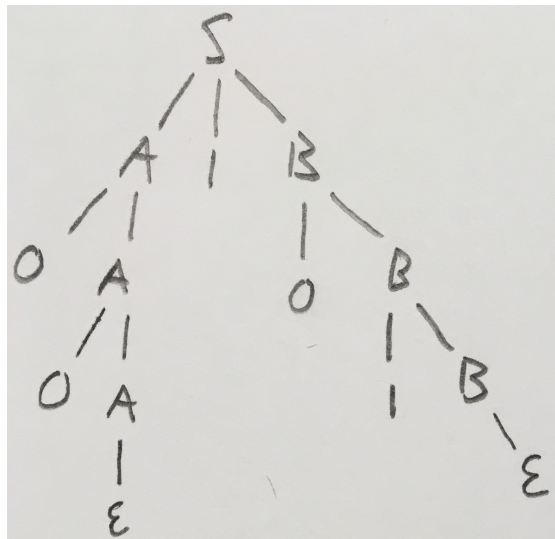
Rightmost:

$$S \Rightarrow A1B \Rightarrow A11B \Rightarrow A11\epsilon \Rightarrow 0A11 \Rightarrow 00A11 \Rightarrow 000A11 \Rightarrow 000\epsilon 11 \Rightarrow 00011.$$

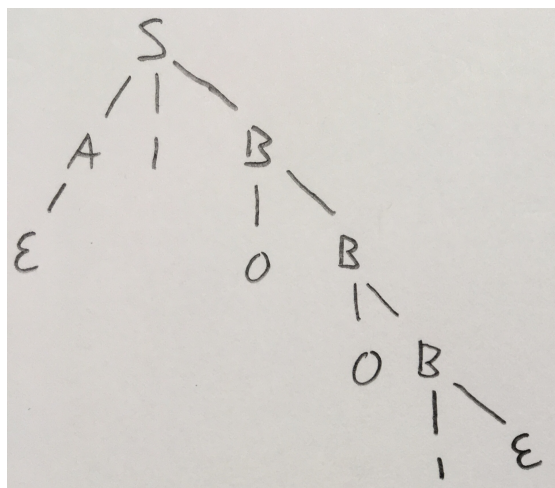
3 Problem 5.2.1

For the grammar and each of the strings in Exercise 5.1.2, give parse trees.

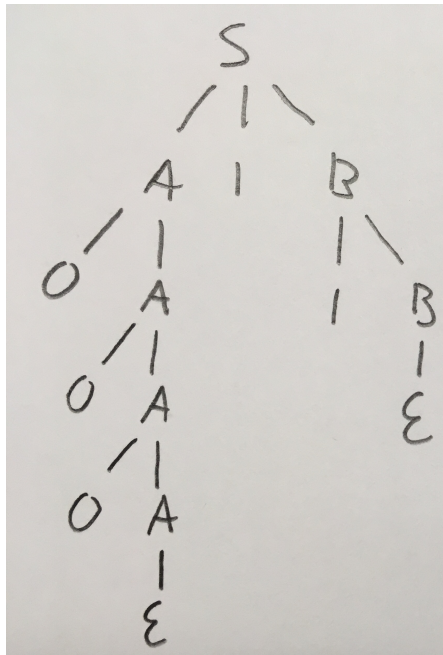
3.1 a). 00101.



3.2 b). 1001.



3.3 c). 00011.



4 Problem 5.4.5

This question concerns the grammar from Exercise 5.1.2

- 4.1 a). Show that this grammar is unambiguous
- 4.2 b). Find a grammar for the same language that *is* ambiguous, and demonstrate its ambiguity

Additional Exercises

5 Problem 1

Design a regular grammar without using a variable goes to ϵ rule ($A \rightarrow \epsilon$) for the set of binary strings beginning with 1, ending with 0, and having even number of 0's and even number of 1's.

6 Problem 2

Design context-free grammars for the following languages

6.1 1). $L = \{a^n b^m c^{2n+m} \mid n, m > 0\}$ where $\Sigma = \{a, b, c\}$

$$\begin{array}{lcl} S & \rightarrow & A \\ A & \rightarrow & aABcc \mid \epsilon \\ B & \rightarrow & bBc \mid \epsilon \end{array}$$

6.2 2). $L = \{a^n b^m c^i \mid m > n + i \text{ and } m, n, i \geq 0\}$, where $\Sigma = a, b, c$

6.3 3). $L = \{a^m b^n \mid 0 \leq n \leq m \leq 3n\}$, where $\Sigma = \{a, b\}$

7 Problem 3

Let G be the grammar

$$\begin{array}{lcl} S & \rightarrow & ASB \mid ab \mid SS \\ A & \rightarrow & aA \mid \epsilon \\ B & \rightarrow & bB \mid \epsilon \end{array}$$

7.1 1). Give a leftmost derivation of $aaabb$

$$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aaASB \Rightarrow aa\epsilon SB \Rightarrow aaabB \Rightarrow aaabbB \Rightarrow aaabb\epsilon \Rightarrow aaabb$$

7.2 2). Give a rightmost derivation of $aaabb$

$$S \Rightarrow ASB \Rightarrow ASbB \Rightarrow ASb\epsilon \Rightarrow Aabb \Rightarrow aAabb \Rightarrow aaAabb \Rightarrow aa\epsilon abb \Rightarrow aaabbb$$

7.3 3). Show that G is ambiguous

We can show that G is ambiguous by showing that there are two distinct leftmost derivations for a given string (ex. $aaabb$).

$$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aaASB \Rightarrow aa\epsilon SB \Rightarrow aaabB \Rightarrow aaabbB \Rightarrow aaabb\epsilon \Rightarrow aaabb.$$

$$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aaASB \Rightarrow aa\epsilon SB \Rightarrow aaASBB \Rightarrow aa\epsilon SBB \Rightarrow aaabBB \Rightarrow aaabbbBB \Rightarrow aaabb\epsilon\epsilon \Rightarrow aaabb.$$

Thus G is ambiguous

7.4 4). Construct an unambiguous grammar equivalent to G

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