Formal Languages Homework 6

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1 Problem 4.1.1 and 4.1.2

Provide the CFG for the following languages:

1.1 b). The set of strings of balanced parentheses. These are the strings of characters "(" and ")" that can appear in a well-formed arithmetic expression.

$S \to (S) \mid SS \mid \epsilon$.
$V = \{S\}$.
$T = \{$ "(", ")" $\}$. (The quotations are only to make reading easier).

1.2 d). $\{0^n 1^m 2^n \mid n$ and m are abitrary integers $\}$.

$V = \{S, A\}$.
$T = \{0, 1, 2\}$.

$$\begin{array}{rcl} S & \to & 0S2 \mid 0A2 \mid A \mid \epsilon \\ A & \to & 1 \mid \epsilon \end{array}$$

1.3 f). $\{0^n 1^{2n} \mid n \geq 1\}$

$V = \{S, A\}$
$T = \{0, 1\}$

$$\begin{array}{rcl} S & \to & 0S11 \mid 0A11 \\ A & \to & \epsilon \end{array}$$

1.4 h). The set of strings of the form $w1^n$, where w is a string of 0's and 1's of length n.

$V = \{S, a\}$
$T = \{0, 1\}$

$$\begin{array}{rcl} S & \to & AS1 \mid \epsilon \\ A & \to & 0 \mid 1 \end{array}$$

2 Problem 5.1.2

The following grammar generates the language of regular expression $0^*1(0+1)^*$.

$$\begin{array}{rcl} S & \to & A1B \\ A & \to & 0A \mid \epsilon \\ B & \to & 0B \mid 1B \mid \epsilon \end{array}$$

Give leftmost and rightmost derivations of the following strings:

2.1 a). 00101.

Leftmost:
$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 00\epsilon 1B \Rightarrow 001B \Rightarrow 0010B \Rightarrow 00101B \Rightarrow 00101\epsilon \Rightarrow 00101.$
Rightmost:
$S \Rightarrow A1B \Rightarrow A10B \Rightarrow A101B \Rightarrow A101\epsilon \Rightarrow A101 \Rightarrow 0A101 \Rightarrow 00A101 \Rightarrow 00\epsilon 101 \Rightarrow 00101.$

2.2 b). 1001.

Leftmost:
$S \Rightarrow A1B \Rightarrow \epsilon 1B \Rightarrow 10B \Rightarrow 100B \Rightarrow 1001B \Rightarrow 1001\epsilon \Rightarrow 1001.$
Rightmost:
$S \Rightarrow A1B \Rightarrow A10B \Rightarrow A100B \Rightarrow A1001B \Rightarrow A1001\epsilon \Rightarrow \epsilon 1001 \Rightarrow 1001.$

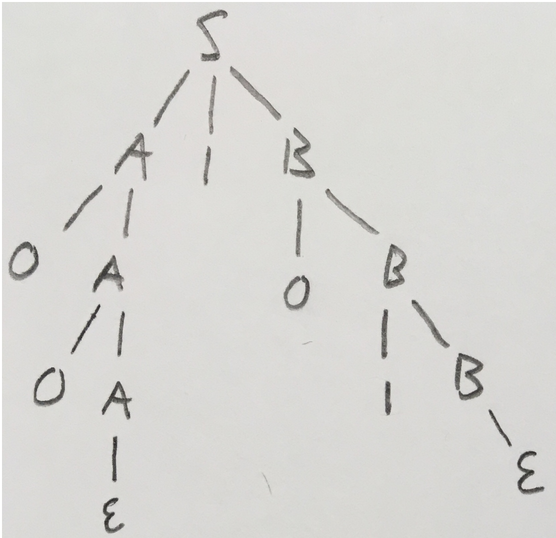
2.3 c). 00011.

Leftmost:
$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 000A1B \Rightarrow 000\epsilon 1B \Rightarrow 00011B \Rightarrow 00011\epsilon \Rightarrow 00011.$
Rightmost:
$S \Rightarrow A1B \Rightarrow A11B \Rightarrow A11\epsilon \Rightarrow 0A11 \Rightarrow 00A11 \Rightarrow 000A11 \Rightarrow 000\epsilon 11 \Rightarrow 00011.$

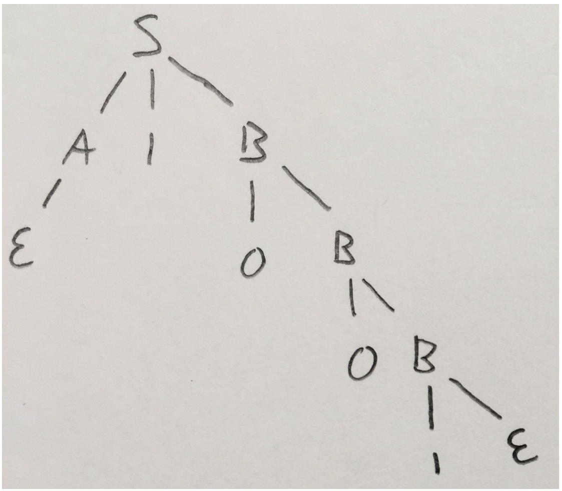
3 Problem 5.2.1

For the grammar and each of the strings in Exercise 5.1.2, give parse trees.

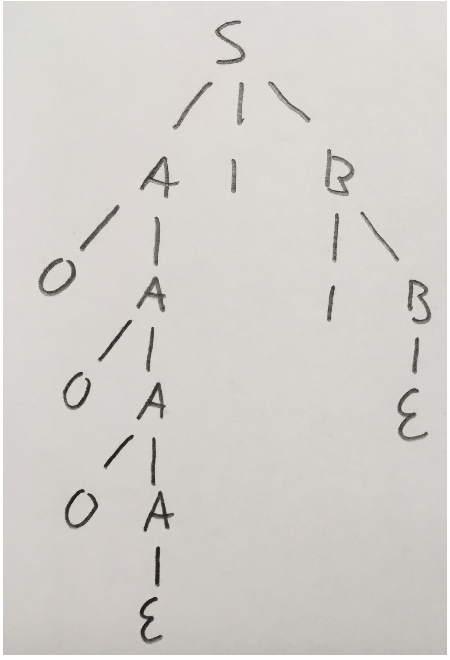
3.1 a). 00101.



3.2 b). 1001.



3.3 c). 00011.



4 Problem 5.4.5

This question concerns the grammar from Exercise 5.1.2

4.1 a). Show that this grammar is unambiguous

Base case: The shortest string we can make. since $S \to A1B$, and it is possible such that $A \to \epsilon$ and $B \to \epsilon$, then the smallest w is $|w| = 1$. And there is only one derivation for this.
Inductive Hypothesis: Assume that for all $w \in \{0,1\}^*$, with $|w| = n$, where $n \geq 1$, w has at most 1 left-most derivation from S, A, B.
Inductive Step: Show that the hypothesis holds for w, $|w| = n + 1$, $n \geq 1$.
Case 1: Show that for each $A \Rightarrow^* w$, there is aunique LM derivation $A \Rightarrow^*_{LM} w$.
Proof: Since $A \Rightarrow^*_{LM} w$, replace A with string y. Then $|y| \leq n$ because $|w| = n + 1$. Note that because $|y| = n$, that y has a unique leftmost derivation from A. Thus $A \Rightarrow^*_{LM} 0y = w$. The same argument follows for B. Since the hypothesis holds true for both A and B, then certainly it holds true for S, since $S \to A1B$. Thus the grammar is unambiguous.

4.2 b). Find a grammar for the same language that *is* ambiguous, and demonstrate its ambiguity

Let G:

$$\begin{array}{rcl} S & \to & A1B \\ A & \to & 0A \mid 00A \mid \epsilon \\ B & \to & bBc \mid \epsilon \end{array}$$

Suppose we want to find the derivation for 0001. We have two derivations:
$S \Rightarrow A1B \Rightarrow 0A1B \Rightarrow 00A1B \Rightarrow 000A1B \Rightarrow 000\epsilon 1B \Rightarrow 0001B \Rightarrow 0001\epsilon \Rightarrow 0001.$
$S \Rightarrow A1B \Rightarrow 00A1B \Rightarrow 000A1B \Rightarrow 000\epsilon 1B \Rightarrow 0001\epsilon \Rightarrow 0001.$

Thus the grammar is ambiguous

Additional Exercises

5 Problem 1

Design a regular grammar without using a variable goes to ϵ rule ($A \to \epsilon$) for the set of binary strings beginning with 1, ending with 0, and having even number of 0's and even number of 1's.

$$\begin{array}{rcl} S & \to & 1A0 \\ A & \to & 1 \end{array}$$

6 Problem 2

Design context-free grammars for the following languages

6.1 1). $L = \{a^n b^m c^{2n+m} \mid n, m > 0\}$ where $\Sigma = \{a, b, c\}$

$$\begin{array}{rcl} S & \to & aAbccc \\ A & \to & aABcc \mid \epsilon \\ B & \to & bBc \mid \epsilon \end{array}$$

$$
\begin{aligned}
S &\rightarrow aAbbBbCc \\
A &\rightarrow aAb \mid \epsilon \\
B &\rightarrow bBb \mid \epsilon \\
C &\rightarrow bCc \mid \epsilon
\end{aligned}
$$

$$
\begin{aligned}
S &\rightarrow aaAb \\
A &\rightarrow aaAb \mid \epsilon
\end{aligned}
$$

6.2 2). $L = \{a^n b^m c^i \mid m > n + i \text{ and } m, n, i \geq 0\}$, where $\Sigma = \{a, b, c\}$

6.3 3). $L = \{a^m b^n \mid 0 \leq n \leq m \leq 3n\}$, where $\Sigma = \{a, b\}$

7 Problem 3

Let G be the grammar

$$
\begin{aligned}
S &\rightarrow ASB \mid ab \mid SS \\
A &\rightarrow aA \mid \epsilon \\
B &\rightarrow bB \mid \epsilon
\end{aligned}
$$

7.1 1). Give a leftmost derivation of aaabb

$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aaASB \Rightarrow aa\epsilon SB \Rightarrow aaabB \Rightarrow aaabbB \Rightarrow aaabb\epsilon \Rightarrow aaabb$

7.2 2). Give a rightmost derivation of aaabb

$S \Rightarrow ASB \Rightarrow ASbB \Rightarrow ASb\epsilon \Rightarrow Aabb \Rightarrow aAabb \Rightarrow aaAabb \Rightarrow aa\epsilon abb \Rightarrow aaabbb$

7.3 3). Show that G is ambiguous

We can show that G is ambiguous by showing that there are two distinct leftmost derivations for a given string (ex. $aaabb$).

$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aaASB \Rightarrow aa\epsilon SB \Rightarrow aaabB \Rightarrow aaabbB \Rightarrow aaabb\epsilon \Rightarrow aaabb.$

$S \Rightarrow ASB \Rightarrow aASB \Rightarrow aaASB \Rightarrow aa\epsilon SB \Rightarrow aaASBB \Rightarrow aa\epsilon SBB \Rightarrow aaabBB \Rightarrow aaabbBB \Rightarrow aaabb\epsilon\epsilon \Rightarrow aaabb.$

Thus G is ambiguous

7.4 4). Construct an unambiguous grammar equivalent to G

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$$
\begin{array}{rcl}
S & \rightarrow & AB \\
A & \rightarrow & aA \mid \epsilon \\
B & \rightarrow & bB \mid \epsilon
\end{array}
$$