

# Formal Languages Homework 6

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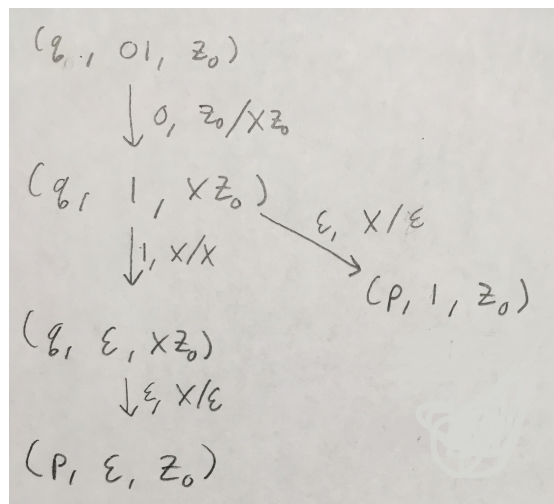
## 1 Problem 6.1.1

Suppose the PDA  $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$  has the following transition function

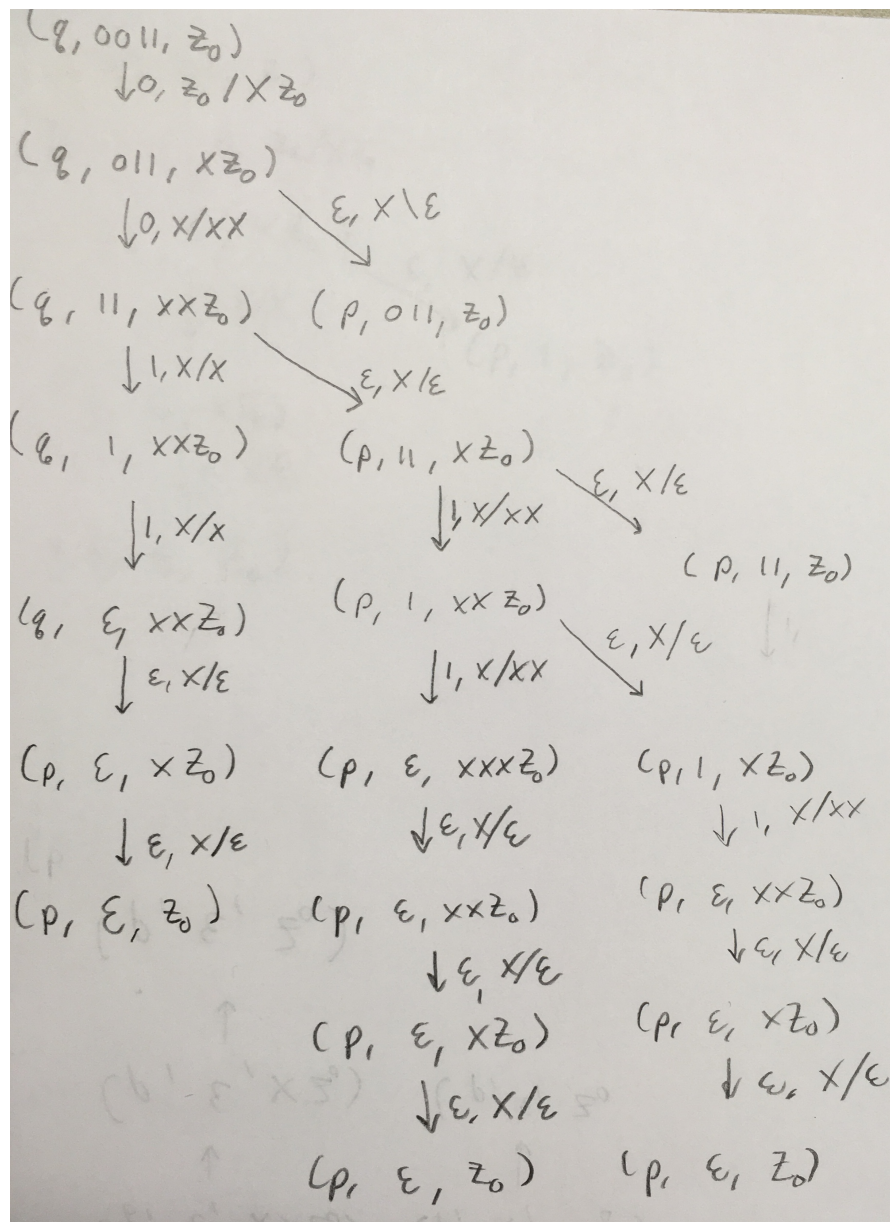
1.  $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$ .
2.  $\delta(q, 0, X) = \{(q, XX)\}$ .
3.  $\delta(q, 1, X) = \{(q, X)\}$ .
4.  $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$ .
5.  $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$ .
6.  $\delta(p, 1, X) = \{(p, XX)\}$ .
7.  $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$ .

Starting from the initial ID  $(q, w, Z_0)$ , show all the reachable ID's when the input  $w$  is:

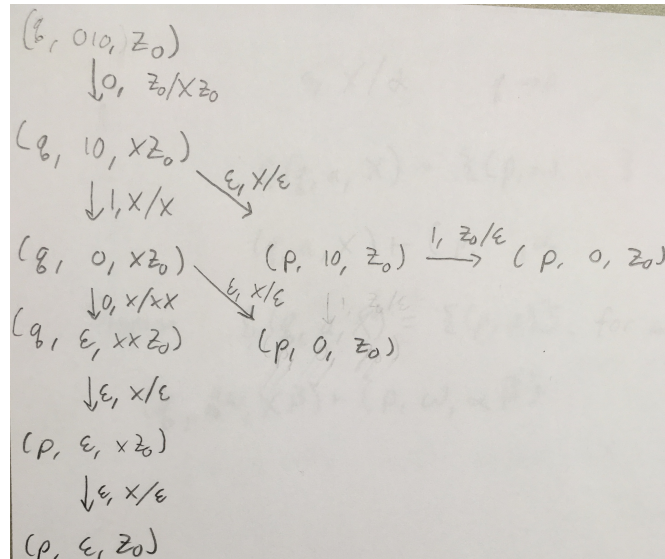
1.1 a). 01.



1.2 b). 0011.



1.3 c). 010.



## 2 Problem 6.2.1

Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

2.1 a).  $\{0^n 1^n \mid n \geq 1\}$

Let the PDA  $P = (\{q_0, q_1, q_2\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, \{q_2\})$ . Then the rules for  $\delta$  are:

1.  $\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$
2.  $\delta(q_0, 0, X) = \{(q_0, XX)\}$
3.  $\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$
4.  $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$
5.  $\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$

2.2 b). The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

Let the PDA  $P = (\{q_0, q_1\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, \{q_1\})$ . Then the rules for  $\delta$  are:

1.  $\delta(q_0, 1, Z_0) = \{(q_0, XZ_0)\}$
2.  $\delta(q_0, 1, X) = \{(q_0, XX)\}$
3.  $\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$
4.  $\delta(q_0, 0, Z_0) = \{(q_0, Z_0)\}$

2.3 c). The set of all strings of 0's and 1's with an equal number of 0's and 1's.

Let the PDA  $P = (\{q_0, q_1, q_2\}, \{0, 1\}, \{Z_0, X\}, \delta, \{q_0, 1_1\}, \{q_2\})$ . Then the rules for  $\delta$  are:

1.  $\delta(q_0, 1, Z_0) = \{(q_0, XZ_0)\}$
2.  $\delta(q_0, 1, X) = \{(q_0, XX)\}$
3.  $\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$
4.  $\delta(q_0, \epsilon, Z_0) = \{(q_2, Z_0)\}$
5.  $\delta(q_1, 0, Z_0) = \{(q_1, XZ_0)\}$
6.  $\delta(q_1, 0, X) = \{(q_1, XX)\}$
7.  $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$
8.  $\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$

### 3 Problem 6.3.2

Convert the grammar to a PDA that accepts the same language by empty stack

$$\begin{array}{lcl} S & \rightarrow & 0S1 \mid A \\ A & \rightarrow & 1A0 \mid S \mid \epsilon \end{array}$$

### 4 Problem 6.4.1

For each of the following PDA's, tell whether or not it is deterministic. Either show that it meets the definition of a DPDA or find a rule or rules that violate it.

#### 4.1 The PDA of Example 6.2

The PDA  $P = (\{q_0, q_1, q_2\}, \{0, 1\}, \{0, 1, Z_0\}, \delta, q_0, Z_0, \{q_2\})$ , where  $\delta$  is defined by the following rules:

These rules are hard to read \_\_\_\_\_

#### 4.2 The PDA of Exercise 6.1.1

#### 4.3 The PDA of Exercise 6.3.3

The PDA  $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0)$ , where  $\delta$  is defined to be:

1.  $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$ .
2.  $\delta(q, 1, X) = \{(q, XX)\}$ .
3.  $\delta(q, 0, X) = \{(p, X)\}$ .

- 4.  $\delta(q, \epsilon, X) = \{(q, \epsilon)\}$ .
  - 5.  $\delta(p, 1, X) = \{(p, \epsilon)\}$ .
  - 6.  $\delta(p, 0, Z_0) = \{(q, Z_0)\}$ .
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## 5 Problem 6.4.2

Give deterministic pushdown automata to accept the following languages:

- 5.1 a).  $\{0^n 1^m \mid n \leq m\}$
- 5.2 b).  $\{0^n 1^m \mid n \geq m\}$
- 5.3 c).  $\{0^n 1^m 0^n \mid n \text{ and } m \text{ are arbitrary}\}$ .