Formal Languages Homework 1

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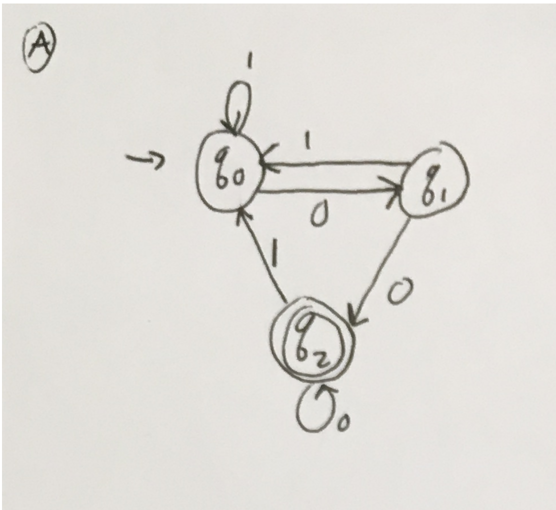
January 31, 2019

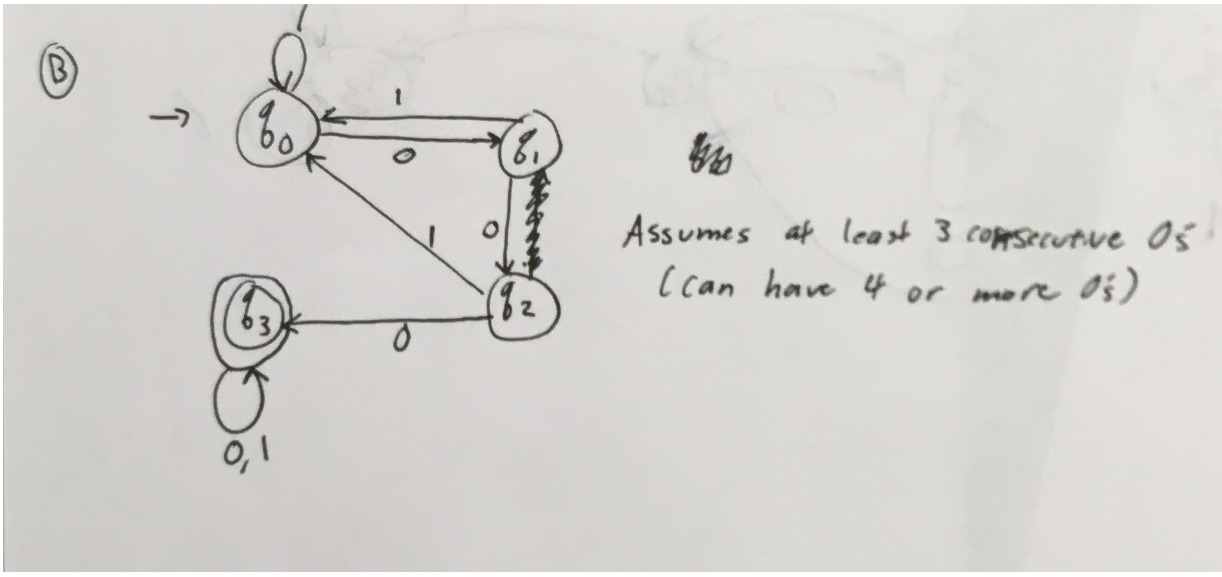
1 Problem 2.2.4

Give DFA's accepting the following languages over the alphabet $\{0, 1\}$:

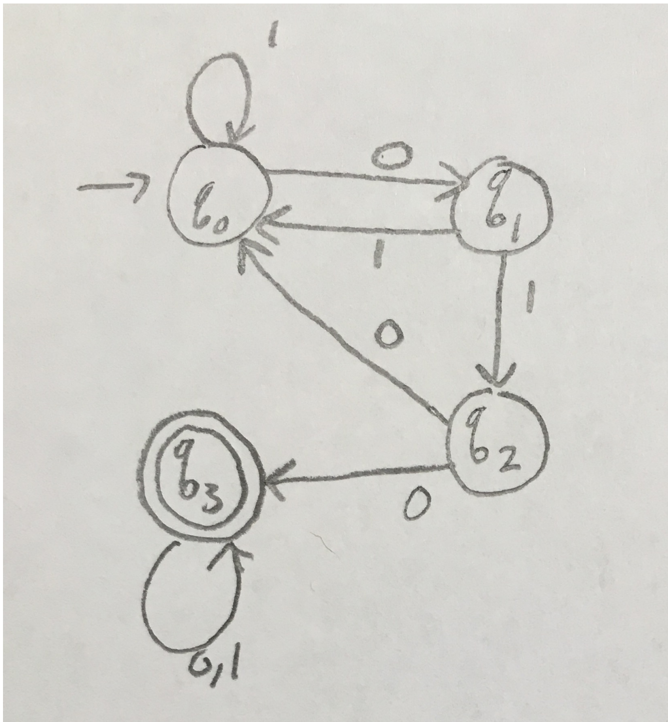
1.1 a). The set of all strings ending in 00.



1.2 b). the set of all strings with three consecutive 0's (not necessarily at the end)



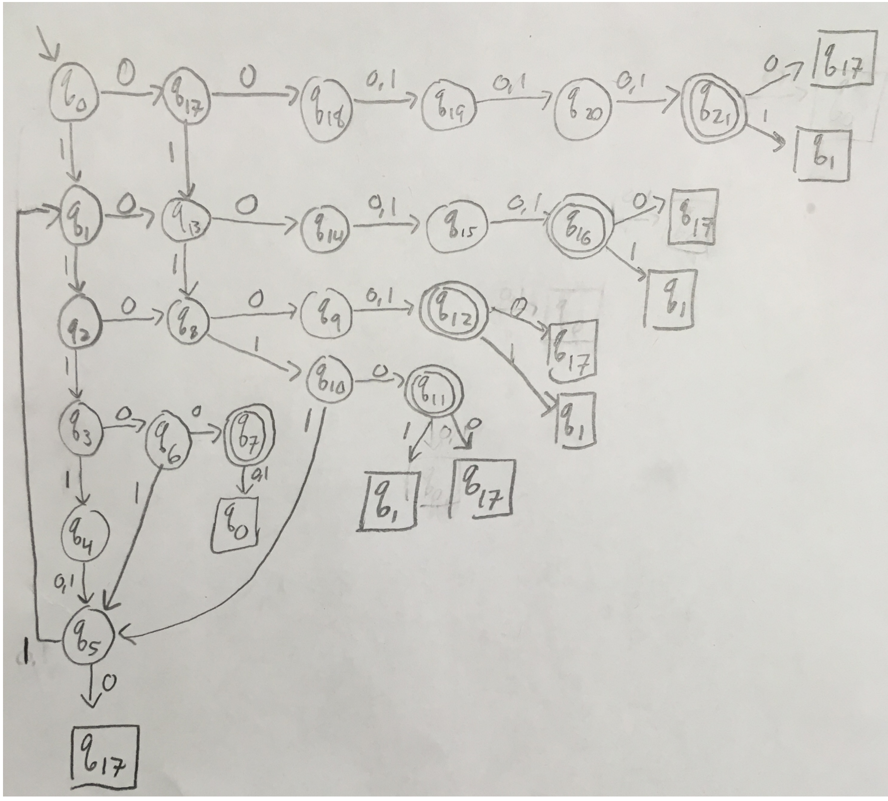
1.3 c). The set of strings with 011 as a substring



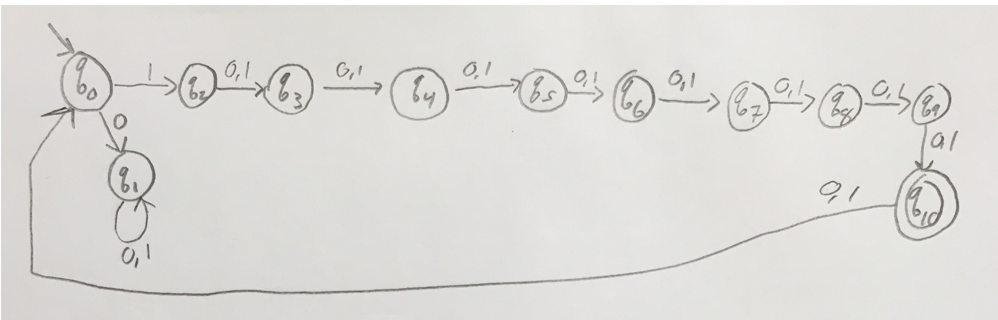
2 Problem 2.2.5

Give DFA's accepting the following lagnuages over the alphabet $\{0, 1\}$.

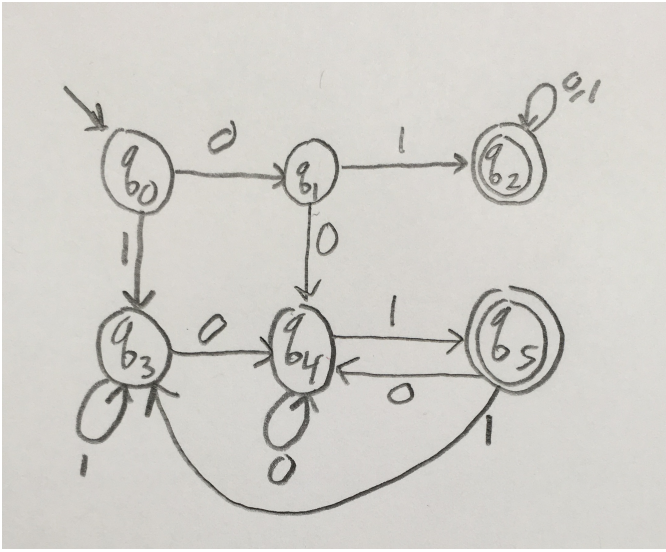
2.1 a). The set of all strings such that each block of five consecutive symbols contains at least two 0's.



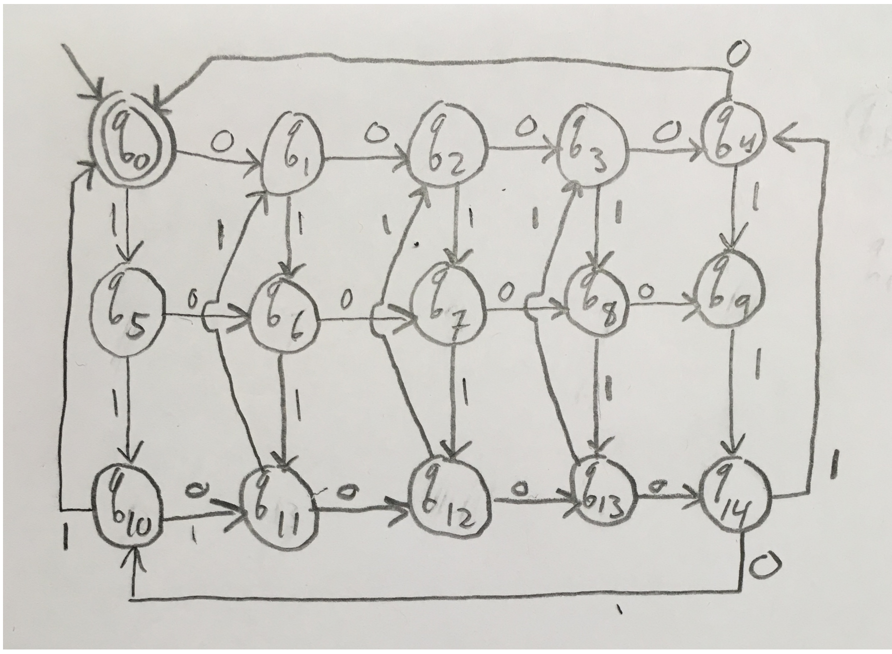
2.2 b). The set of all strings whose tenth symbol from the right end is a 1.



2.3 c). The set of strings that either begin or end (or both) with 01.



2.4 d). The set of strings such that the number of 0's is divisible by five, and the number of 1's is divisible by 3.



3 Problem 2.2.7

Let A be a DFA and q a particular state of A, such that $\delta(q,a) = q$ for all input symbols a. Show by induction on the length of the input that for all input strings w, $\hat{\delta}(q,w) = q$.

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basis: Suppose $a = \epsilon$. Then, $\delta(q,\epsilon) = q$ as a result of no inputs being taken.
hypothesis: Suppose there is a string w of length k such that $\hat{\delta}(q,w) = q$.

Proof:
Suppose there is a word v of length $k+1$, such that $v = wa$. Since $\hat{\delta}(q,w) = q$, and $\delta(q,a) = q$, then $\delta(q,v) = \delta(\hat{\delta}(q,w),a) = q$. Thus $\hat{\delta}(q,w) = q$ for w of any input length.

4 Problem 2.2.8

Let A be a DFA and a a particular input symbol of A, such that for all states q of A we have $\delta(q,a) = q$.

4.1 a). Show by induction on n that for all $n \geq 0$, $\delta(q,a^n) = q$, where a^n is the string consisting of n a's.

basis: Suppose $n = 0$. Then, $a = \epsilon$. Thus $\delta(q,a^0) = q$.
hypothesis: Then Assume $\delta(q,a^n) = q$ for some $n > 0$ as well. Show that $\delta(q,a^{n+1}) = q$.

Proof:
Note that $a^{n+1} = a^n a$. By extending the transition function, we have $\hat{\delta}(q,a^{n+1}) = \delta(\hat{\delta}(q,a^n),a)$. Since $\hat{\delta}(q,a^n) = q$, then we have: $\hat{\delta}(q,a^{n+1}) = \delta(q,a) = q$. Thus $\delta(q,a^n) = q$ for all $n \geq 0$.

4.2 b). Show that either $\{a\}^* \subseteq L(A)$ or $\{a\}^* \cap L(A) = \emptyset$

Note that $\{a\}^* = \{\, a \mid a^k ; k \geq 0 \,\}$, that is, $a \in \{a\}^*$ consists of the input symbol a to some non-negative power.

Suppose that $\{a\}^* \cap L(A) = \emptyset$ and $q \in F$. However, since $\delta(q,a) = q$, then $a \in L(A)$. Thus, we have a contradiction, and $\{a\}^* \cap L(A) \neq \emptyset$. It should be worth noting that this only holds true when q is an acceptance state. if it is not, then a is not in $L(A)$. However, the problem doesn't clarify if q is an accepting state.