

# Formal Languages Homework 8

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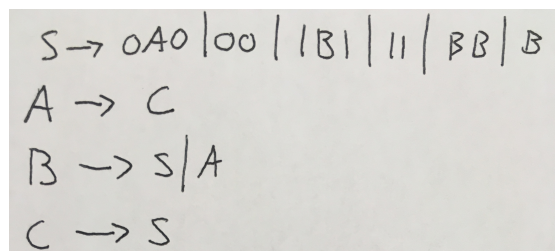
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## 1 Problem 7.1.3

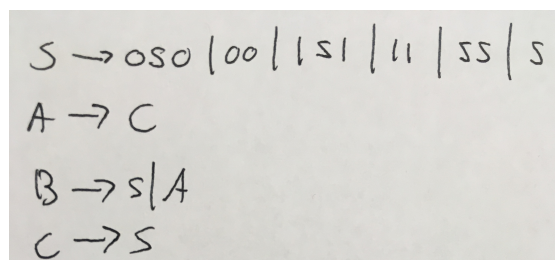
Repeat Exercise 7.1.2 for the following grammar:

$$\begin{aligned} S &\rightarrow 0A0 \mid 1B1 \mid BB \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \mid \epsilon \end{aligned}$$

1.1 a). Eliminate  $\epsilon$ -productions


$$\begin{aligned} S &\rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB \mid B \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \end{aligned}$$

1.2 b). Eliminate any unit productions in the resulting grammar


$$\begin{aligned} S &\rightarrow 0S0 \mid 00 \mid 1S1 \mid 11 \mid SS \mid S \\ A &\rightarrow C \\ B &\rightarrow S \mid A \\ C &\rightarrow S \end{aligned}$$

1.3 c). Eliminate any useless symbols in the resulting grammar

$$S \rightarrow OSO \mid OO \mid ISI \mid II \mid SS \mid S$$

1.4 d). Put the resulting grammar into Chomsky Normal Form

$$\begin{aligned} S &\rightarrow OS_1 \mid O_2 \mid I_1 S_2 \mid I_2 \mid SS \mid S \\ S_1 &\rightarrow SO_1 \\ O_1 &\rightarrow O \\ O_2 &\rightarrow O_1 O_1 \\ I_1 &\rightarrow I \\ S_2 &\rightarrow SI_1 \\ I_2 &\rightarrow I_1 I_1 \end{aligned}$$

## 2 Problem 7.2.1

Use the CFL pumping lemma to show each of these languages not to be *context-free*

### 2.1 a). $\{a^i b^j c^k \mid i < j < k\}$

Let's break down the cases here and look at candidates for  $vwx$ :

1. all in  $a$ 's
2. all in  $b$ 's
3. all in  $c$ 's
4. between  $a$ 's and  $b$ 's where  $v$  contains both  $a$ 's and  $b$ 's
5. between  $a$ 's and  $b$ 's where  $x$  contains both  $a$ 's and  $b$ 's
6. between  $a$ 's and  $b$ 's where  $v$  may contain  $a$ 's and  $x$  may contain  $b$ 's
7. between  $b$ 's and  $c$ 's where  $v$  contains both  $b$ 's and  $c$ 's
8. between  $b$ 's and  $c$ 's where  $x$  contains both  $b$ 's and  $c$ 's
9. between  $b$ 's and  $c$ 's where  $v$  may contain  $b$ 's and  $x$  may contain  $c$ 's

For case 1, select a  $q$  (because  $i$  is already used) to pump  $v, x$  by. Let  $q > j$ . Then by pumping  $v$  and  $x$ , we end up with a string such that the length is longer than  $j$ , and the string is not in  $L$ . The same applies for case 2, if we select  $q > k$ . For case 3, select  $q = 0$ , then we transform part of the substring to epsilon. if  $|x|$  is such that  $k - |x| < j$ , then by "eliminating"  $x$  by  $x^0$ , then the string is no longer in  $L$ . For case 6, if we pick  $x = \epsilon$  and pump  $v$ , we can get an  $i \geq j$ , which is not in  $L$ . The same goes for case 9. For cases 4 and 7, we can simply pump both  $v$  and  $x$ . Since the size of the left hand substring (i.e.  $i$ , or  $j$ ) will be growing faster, then at some point it will violate the rules for  $L$ . For the last cases, 5 and 8, if we select and  $x = \epsilon$  and pump like usual, we will find a violation after a certain  $i$ .

### 2.2 b). $\{a^n b^n c^i \mid i \leq n\}$

This question also contains the cases from the previous question. For cases 1 and 2, we can simply pump the string until it breaks the equality between  $|a^n|$  and  $|b^n|$ . For case 3, pump  $i$  until it is larger than  $n$ . For cases 4 and 7, simply allow  $v = \epsilon$  and by pumping  $x$  we either violate the length equality between  $a$  and  $b$ , or we pump  $c$  such that its length is greater than  $n$ . For case 5 since both  $v$  and  $x$  contain  $a$ 's, by pumping we break equality. The same reasoning applies to case 8. for cases 6 and 9, let  $v = \epsilon$ , and pump  $x$ , and we will break the conditions on  $L$  for both.

### 3 Problem 7.3.2

Consider the following two languages:

$$L_1 = \{a^n b^{2n} c^m \mid n, m \geq 0\}$$

$$L_2 = \{a^n b^m c^{2m} \mid n, m \geq 0\}$$

- 3.1 a). Show that each of these languages is context-free by giving grammars for each

$L_1$ :

$$\begin{aligned} S &\rightarrow A \mid B \mid \epsilon \\ A &\rightarrow a A b b C \mid a A b b \mid \epsilon \\ B &\rightarrow A C \mid C \\ C &\rightarrow c \end{aligned}$$

$L_2$ :

$$\begin{aligned} S &\rightarrow A \mid B C C \mid \epsilon \\ A &\rightarrow a A \mid a A B C C \mid \epsilon \\ B &\rightarrow b B C C \mid \epsilon \\ C &\rightarrow c \end{aligned}$$

- 3.2 b). Is  $L_1 \cap L_2$  a CFL? Justify your answer

It seems like  $L_1 \cap L_2 = \{a^n b^{2nm} c^{2m} \mid n, m \geq 0\}$ . And yes, because unlike the previous questions, there are no constraints on the substrings in relation (read: context) to one another, such as the length of one substring being greater than another or etc.