Formal Languages Homework 6

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April 4, 2019

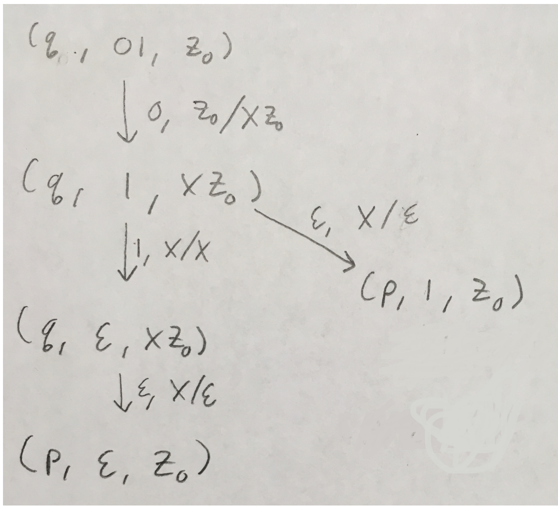
1 Problem 6.1.1

Suppose the PDA $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0, \{p\})$ has the following transition function

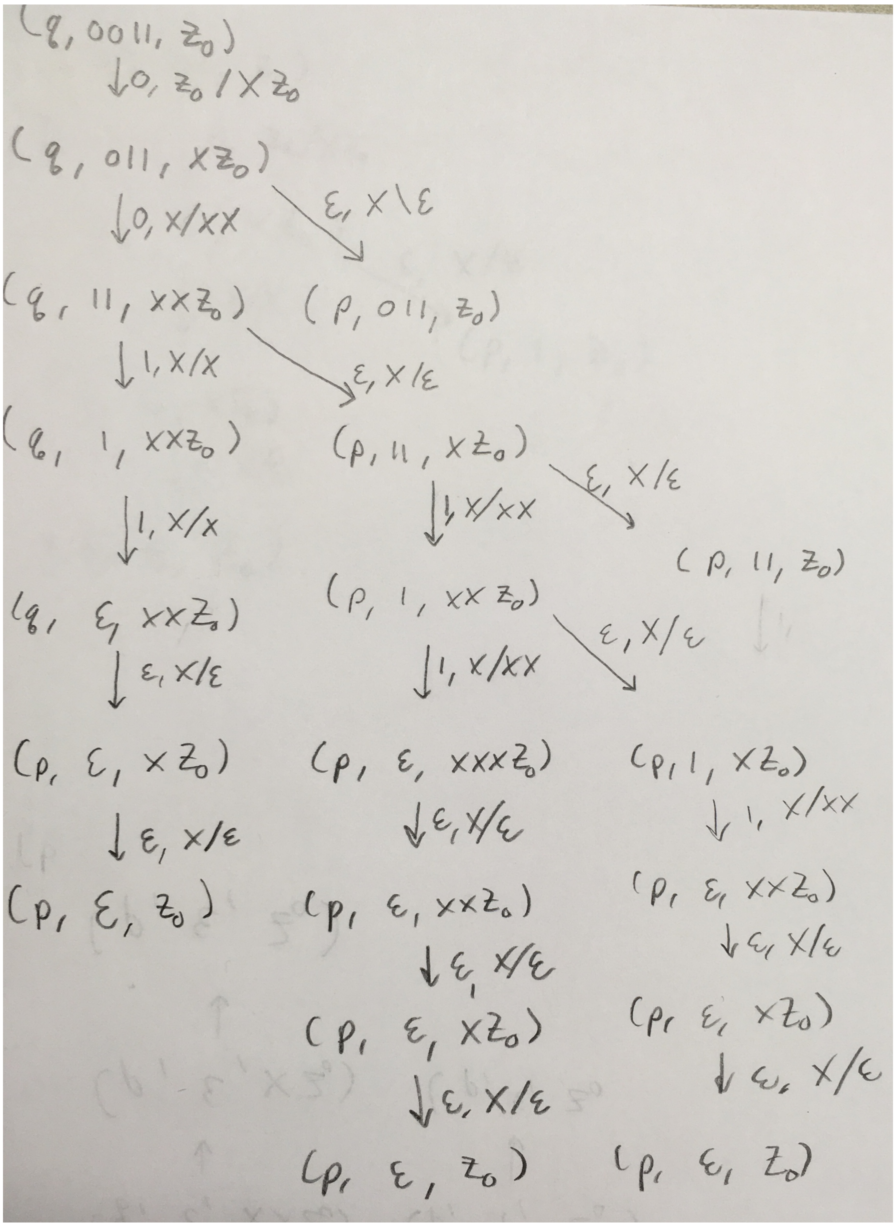
1. $\delta(q, 0, Z_0) = \{(q, XZ_0)\}$.
2. $\delta(q, 0, X) = \{(q, XX)\}$.
3. $\delta(q, 1, X) = \{(q, X)\}$.
4. $\delta(q, \epsilon, X) = \{(p, \epsilon)\}$.
5. $\delta(p, \epsilon, X) = \{(p, \epsilon)\}$.
6. $\delta(p, 1, X) = \{(p, XX)\}$.
7. $\delta(p, 1, Z_0) = \{(p, \epsilon)\}$.

Starting from the initial ID (q, w, Z_0), show all the reachable ID's when the input w is:

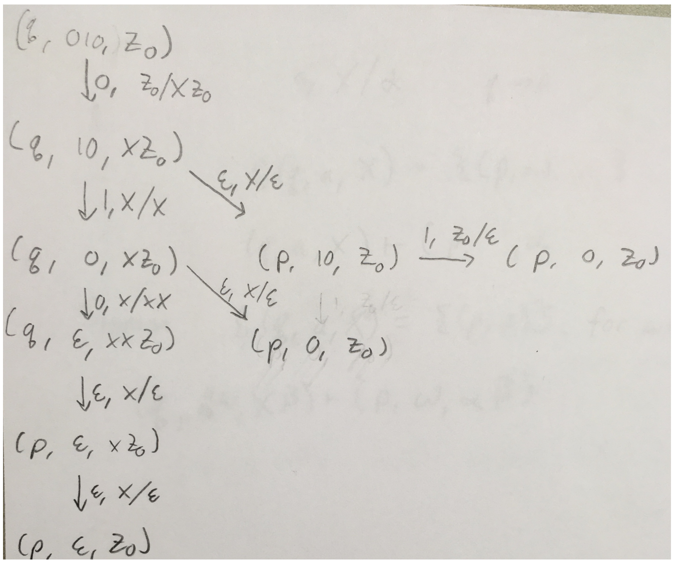
1.1 a). 01.



1.2 b). 0011.


(q, 0011, Z0)
0, Z0/XZ0
(q, 011, XZ0)
0, X/XX
ε, X\ε
(q, 11, XXZ0)
(p, 011, Z0)
1, X/X
ε, X/ε
(q, 1, XXZ0)
(p, 11, XZ0)
ε, X/ε
1, X/X
1, X/XX
(p, 11, Z0)
(q, ε, XXZ0)
(p, 1, XXZ0)
ε, X/ε
ε, X/ε
1, X/XX
(p, ε, XZ0)
(p, ε, XXXZ0)
(p, 1, XZ0)
ε, X/ε
ε, X/ε
1, X/XX
(p, ε, Z0)
(p, ε, XXZ0)
(p, ε, XXZ0)
ε, X/ε
ε, X/ε
(p, ε, XZ0)
(p, ε, XZ0)
ε, X/ε
ε, X/ε
(p, ε, Z0)
(p, ε, Z0)

1.3 c). 010.



2 Problem 6.2.1

Design a PDA to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

2.1 a). $\{0^n 1^n \mid n \geq 1\}$

Let the PDA $P = (\{q_0, q_1, q_2\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, \{q_2\})$. Then the rules for δ are:

1. $\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_0, 0, X) = \{(q_0, XX)\}$
3. $\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$
4. $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$
5. $\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$

2.2 b). The set of all strings of 0's and 1's such that no prefix has more 1's than 0's.

Let the PDA $P = (\{q_0, q_1\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, \{q_1\})$. Then the rules for δ are:

1. $\delta(q_0, 1, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_0, 1, X) = \{(q_0, XX)\}$
3. $\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$
4. $\delta(q_0, 0, Z_0) = \{(q_0, Z_0)\}$

2.3 c). The set of all strings of 0's and 1's with an equal number of 0's and 1's.

Let the PDA $P = (\{q_0, q_1, q_2\}, \{0, 1\}, \{Z_0, X\}, \delta, \{q_0, 1_1\}, \{q_2\})$. Then the rules for δ are:

1. $\delta(q_0, 1, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_0, 1, X) = \{(q_0, XX)\}$
3. $\delta(q_0, 0, X) = \{(q_0, \epsilon)\}$
4. $\delta(q_0, \epsilon, Z_0) = \{(q_2, Z_0)\}$
5. $\delta(q_1, 0, Z_0) = \{(q_1, XZ_0)\}$
6. $\delta(q_1, 0, X) = \{(q_1, XX)\}$
7. $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$
8. $\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$

3 Problem 6.3.2

Convert the grammar to a PDA that accepts the same language by empty stack
The rules for the transition function δ are:

$$\begin{array}{rcl} S & \rightarrow & 0S1 \mid A \\ A & \rightarrow & 1A0 \mid S \mid \epsilon \end{array}$$

1. $\delta(q, \epsilon, S) = \{(q, A), (q, 0S1)\}$
2. $\delta(q, \epsilon, A) = \{(q, \epsilon), (q, 1A0), (q, S)\}$
3. $\delta(q, 0, 0) = \{(q, \epsilon)\}$
4. $\delta(q, 1, 1) = \{(q, \epsilon)\}$

4 Problem 6.4.1

For each of the following PDA's, tell whether or not it is deterministic. Either show that it meets the definition of a DPDA or find a rule or rules that violate it.

4.1 The PDA of Example 6.2

Because the PDA must choose when to move to the next state and process the other half of the string to verify a match, the PDA cannot be deterministic. If it were a DPDA, there would be no choice.

4.2 The PDA of Exercise 6.1.1

Did not have enough time to attempt

4.3 The PDA of Exercise 6.3.3

The PDA $P = (\{q, p\}, \{0, 1\}, \{Z_0, X\}, \delta, q, Z_0)$, where δ is defined to be:

1. $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$.
2. $\delta(q, 1, X) = \{(q, XX)\}$.
3. $\delta(q, 0, X) = \{(p, X)\}$.
4. $\delta(q, \epsilon, X) = \{(q, \epsilon)\}$.
5. $\delta(p, 1, X) = \{(p, \epsilon)\}$.
6. $\delta(p, 0, Z_0) = \{(q, Z_0)\}$.

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5 Problem 6.4.2

Give deterministic pushdown automata to accept the following languages:

5.1 a). $\{0^n 1^m \mid n \leq m\}$

$P = (\{q_0, q_1, q_3, q_4\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, \{q_4\})$. In this problem, we consider q_3 as the dead state, that is, if the stack is empty before all of the 1's are processed, then we goto the deadstate.

1. $\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_0, 0, X) = \{(q_0, XX)\}$
3. $\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$
4. $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$
5. $\delta(q_1, \epsilon, Z_0) = \{(q_3, Z_0)\}$
6. $\delta(q_1, 1, Z_0) = \{(q_4, Z_0)\}$
7. $\delta(q_4, 1, Z_0) = \{(q_4, Z_0)\}$

5.2 b). $\{0^n 1^m \mid n \geq m\}$

$P = (\{q_0, q_1, q_3\}, \{0,1\}, \{Z_0, X\}, \delta, q_0, \{q_3\})$.

1. $\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_0, 0, X) = \{(q_0, XX)\}$
3. $\delta(q_0, 1, X) = \{(q_1, \epsilon)\}$
4. $\delta(q_1, 1, X) = \{(q_1, \epsilon)\}$
5. $\delta(q_1, \epsilon, X) = \{(q_3, \epsilon)\}$
6. $\delta(q_1, \epsilon, Z_0) = \{(q_3, Z_0)\}$

5.3 c). $\{0^n 1^m 0^n \mid n$ and m are arbitrary $\}$.

$P = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{Z_0, X\}, \delta, q_0, \{q_3\})$.

1. $\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_0, 0, X) = \{(q_0, XX)\}$
3. $\delta(q_0, \epsilon, Z_0) = \{(q_1, Z_0)\}$
4. $\delta(q_0, \epsilon, X) = \{(q_1, X)\}$
5. $\delta(q_1, 1, X) = \{(q_1, X)\}$
6. $\delta(q_1, 0, X) = \{(q_2, \epsilon)\}$
7. $\delta(q_1, \epsilon, Z_0) = \{(q_2, Z_0)\}$
8. $\delta(q_2, 0, X) = \{(q_2, \epsilon)\}$
9. $\delta(q_2, \epsilon, Z_0) = \{(q_3, Z_0)\}$