Research Topic Intro

Liam Dillingham

February 19, 2019

The mandelbrot set is a set of numbers whose orbits remain constrained for all iterations give their initial parameters. I want to start by clarifying what an orbit is.

The orbit of a point x_0 under some function f is the sequence of points following x_0 under iteration, where x_0 is called the *seed.* so

$$x_0, f(x_0) = x_1, f(x_1) = x_2, ..., f(x_{n-1}) = x_n$$

.

So we are taking the seed and iterating it, looping the output back into the next iteration. We can label the functions iteration index as follows:

$$f^n(...f^2(f^1(x_0))...)$$

for n iterations. Since we are interested in the mandelbrot set, we can introduce its function and try to analyze the behavior of some of its orbits. The equation is:

$$f_c^n(z) = z^2 + c$$

Where c is a given parameter. It is worth noting that fixed orbits of a function are points such that $f^n(x_0) = x_0$ for all n. Let's graph our function. It is a quadratic function with y-intercept at c. To find the fixed points of our function, we super-impose the line $f(x) = x$, and the intersections are the fixed orbits. What happens when we choose a random seed given our parameter c?

$$f_c(x_0) = x_0^2 + c$$

First, we start at our seed, and move vertically to the point on the graph, $(x_0, f(x_0))$. Then we move horizontaly to our diagonal line to the point $(f(x_0), f(x_0)$. This places us above the point on the quadratic, $(f(x_0), f^2(x_0))$, and we can move down to locate it. By continuing this process, we can find the orbit of our seed x_0.

However, choosing certain seeds, and following the process will result in our orbit exploding to infinity. Especially if there are no intersections between the line and the curve.

So the Mandelbrot set is a set within the complex plane, which, instead of choosing random seeds, we choose the seed of 0, and change our parameter c to test every point in the complex plane. If the orbit of 0 explodes to infinity given a particular c, then that point is not in the Mandelbrot set.