Numerical Analysis Homework 1

Liam Dillingham

September 15, 2019

1 Problem 1

Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$. Approximate $\sqrt{0.5}, \sqrt{0.75}, \sqrt{1.25}$ and $\sqrt{1.5}$ using $P_3(x)$ and the actual errors.

$P_3(x) = 1 + \frac{1}{2}(x-1) = \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$ for $x_0 = 0$
$\sqrt{0.5} \approx 0.7109$. actual value $= 1.2247$. Error: *actual* - *approx* $= 0.5138$
$\sqrt{0.75} \approx 0.8662$. actual value $= 1.3229$. Error $= 0.4567$
$\sqrt{1.25} \approx 1.1182$. actual value $= 1.5$. Error $= 0.3818$
$\sqrt{1.5} \approx 1.3311$. actual value $= 1.5811$. Error $= 0.25$

2 Problem 2

Compute the absolute error and relative error in approximations of p by p^* for $p = 8!$, $p^* = 39900$

---

actual value of $8! = 40320$
absolute error: $|p - p^*| = 420$
relative error: $\frac{|p-p^*|}{|p|} = 0.0104$

3 Problem 3

Find the largest interval in which p^* must lie to approximate p with relative error at most 10^{-4} for each value of p, where $p = \pi$

After some algebra, $p^* \in \pi \pm \frac{\pi}{10^4} = [3.1412785, 3.1419068]$

4 Problem 4

Perform the following computations on $4/5 + 1/3$

4.1 exactly

$4/5 + 1/3 = 17/15$

4.2 using three-digit rounding arithmetic

$4/5 + 1/3 = 0.800 + 0.333 = 1.13$

4.3 Compute relative errors in 3-digit rounding arithmetic

$\frac{|1.133333-1.13|}{|1.133333|} = 0.00294$

5 Problem 4

Find two distinct 3-decimal digit floats in $[1, 100]$, say a and b, such that $a < b$, so that $fl(fl(a+b)/2)$ is not in the interval $[a, b]$. Now with these values of a and b, find $fl(a + fl(fl(b-a)/2))$

Let $a = 0.101 * 10^1$ and $b = 0.102 * 10^1$. Then $fl(a+b) = 0.103 * 10^1$, and $fl(fl(a+b)/2) = 0.515 * 10^0$. Note that $0.515 * 10^0$ is not in $[a, b]$.
Now to compute $fl(a + fl(fl(b-a)/2))$.
$fl(a + fl(fl((0.102 * 10^1) - (0.101 * 10^1))/2))$
$= fl(a + fl(fl(0.001 * 10^1)/2))$
$= fl(a + fl(0.1 * 10^{-1}/2))$
$= fl(a + fl(0.05 * 10^{-1}))$
$= fl(a + 0.5 * 10^{-2})$
$= fl((0.101 * 10^1) + (0.5 * 10^{-2}))$
$= fl((0.101 * 10^1) + (0.5 * 10^{-2}))$
$= fl(1.015)$
$= 0.101 * 10^1$

6 Problem 6

Use the Bisection method to find solution accurate to within 10^{-2} for $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on the interval $[0, 2]$

---

for the interval $[a, b] = [0, 2]$, $a > 0$ and $b < 0$ so we have a root bracketing interval. $p_0 = \frac{0+2}{2} = 1$
$f(p) * f(a) = 3 * 4 > 0$ so $a = 1$
$p = \frac{1+2}{2} = 1.5$, and $f(1.5) * f(1) = -0.6875 * 3 < 0$, so $b = 1.5$
$p = \frac{1+1.5}{2} = 1.25$, and $f(1.25) * f(1) = 1.29 > 0$, so $a = 1.25$
$p = \frac{1.25+1.5}{2} = 1.375$ and $f(1.375) * f(1.25) = 0.31 * 1.29 > 0$ so $a = 1.375$
$p = \frac{1.375+1.5}{2} = 1.4375$ and $f(1.4375) * f(1.375) = -0.187 * 0.31 < 0$ so $b = 1.4375$
$p = \frac{1.375+1.4375}{2} = 1.406$.
$\frac{|1.414-1.406|}{|1.414|} = 0.0057 < 10^{-2}$, so $p^* = 1.406$.