6. One-Sample Estimation

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PROBLEM 6.1

$n = 36,\ \bar{x} = 2.6,\ \sigma = 0.3$

a) 95% confidence interval : Find $z_{0.025} = 1.96$

$$\rightarrow\ 2.6 - \frac{1.96}{\sqrt{36}}(0.3) < \mu < 2.6 + \frac{61.9}{\sqrt{36}}(0.3)$$

$$2.50 < \mu < 2.70$$

99% confidence interval : Find $z_{0.005} = 2.58$

$$\rightarrow\ 2.6 - \frac{2.58}{\sqrt{36}}(0.3) < \mu < 2.6 + \frac{2.58}{\sqrt{36}}(0.3)$$

$$2.471 < \mu < 2.729$$

b) 95% confident that the error does not exceed :

$$e_1 = \frac{z_{0.025} \cdot \sigma}{\sqrt{n}} = \frac{(1.96)(0.3)}{\sqrt{36}} = 0.098$$

99% confident that the error does not exceed

$$e_2 = \frac{z_{0.005} \cdot \sigma}{\sqrt{n}} = \frac{(2.58)(0.3)}{\sqrt{36}} = 0.129$$

c) Sample size to ensure 95% confident that the error does not exceed 0.05 is:

$$n = \left(\frac{z_{0.025}\ \sigma}{e}\right)^2 = \left(\frac{(1.96)(0.3)}{(0.05)}\right)^2$$

$= 138.5$

$\rightarrow n = 139$ will satisfy

PROBLEM 6.2

$\sigma = 40, n = 30, \bar{x} = 780$

96% confidence $\rightarrow$ Find $z_{0.02} = 2.05$

Interval:

$$780 - 2.05 \frac{40}{\sqrt{30}} < \mu < 780 + 2.05 \frac{40}{\sqrt{30}}$$

$$765.03 < \mu < 794.97$$

PROBLEM 6.3

$n = 25, \sigma^2 = 4, \bar{x} = 6.2$

Upper 95% bound for the reaction time:

$$\bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} = 6.2 + z_{0.05} \sqrt{\frac{4}{25}}$$

$$= 6.2 + (1.645)\sqrt{\frac{4}{25}} = 6.8616$$

PROBLEM 6.4

95% confidence interval : $\bar{x} - t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \frac{s}{\sqrt{n}}$.

where $\bar{x} = 10$ and $s = 0.283$, $t_{\frac{\alpha}{2}} = t_{0.025} = 2.447$

Thus : $10 - (2.447)\frac{0.283}{} < \mu < 10 + (2.447)\frac{0.283}{}$

$$\ldots \frac{}{\sqrt{7}} < \mu < \ldots \frac{}{\sqrt{7}}$$

$$9.74 < \mu < 10.26$$

PROBLEM 6.5

$\bar{x} = 1.0056$, $S = 0.0245$

99% confidence interval: $\bar{x} - t_{0.005} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{0.005} \frac{s}{\sqrt{n}}$

$t_{0.005} = 3.355$, with $v = 8$

Thus $1.0056 - (3.355) \frac{0.0245}{\sqrt{9}} < \mu < 1.0056 + (3.355) \frac{0.0245}{\sqrt{9}}$

$$0.0978 < \mu < 1.033$$

PROBLEM 6.6

$\bar{x} = 3.787$, $S = 0.971$, $t_{0.025} = 2.145$

95% prediction interval:

$$\bar{x} - t_{0.025} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{0.025} \frac{s}{\sqrt{n}}$$

$$3.787 - 2.145 \frac{0.971}{\sqrt{15}} < \mu < 3.787 + 2.145 \frac{0.971}{\sqrt{15}}$$

$$3.249 < \mu < 4.325$$

PROBLEM 6.7

$\bar{x} = 501$, $S = 112$

Large sample size $\rightarrow$ use $z_{0.005} = 2.575$

Then the 99% confidence interval is:

$$501 - 2.575\frac{112}{\sqrt{500}} < \mu < 501 + 2.575\frac{112}{\sqrt{500}}$$

$$488.1 < \mu < 513.9$$

PROBLEM 6.8

$n = 50,\ \bar{x} = 174.5,\ s = 6.9$

a) Large sample size : use z_α

98% confidence interval:

$$\bar{x} - z_{0.001}\frac{s}{\sqrt{n}} < \mu < \bar{x} + z_{0.001}\frac{s}{\sqrt{n}}$$

$$174.5 - (3.090)\cdot\frac{6.9}{\sqrt{50}} < \mu < 174.5 + (3.090)\frac{6.9}{\sqrt{50}}$$

$$171.48 < \mu < 177.51$$

b) Error : $e = \dfrac{z_{0.01}\cdot(6.9)}{\sqrt{50}} = 3.015$

PROBLEM 6.9

Point estimate = $\hat{p} = \dfrac{340}{500} = 0.68$

a) 95% confidence interval:

$$\hat{p} - z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{0.025}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.68 - (1.96)\sqrt{\frac{(0.68)(1-0.68)}{500}} < p < 0.68 + 1.96\sqrt{\frac{(0.68)(1-.68)}{500}}$$

$$0.6391 < p < 0.7209$$

b) $e = z_{0.025}\sqrt{\dfrac{\hat{p}\hat{q}}{500}} = (1.96)\sqrt{\dfrac{(0.68)(1-0.68)}{500}}$

$= 0.041$

c) $n = \dfrac{z_{0.025}^2\,\hat{p}\hat{q}}{e^2} = \dfrac{(1.96)^2(0.68)(1-.68)}{(0.041)^2}$

$\simeq 497.3$

PROBLEM 6.10

point estimate $\hat{p} = \dfrac{228}{1000} = 0.228 \rightarrow \hat{q} = 0.772$

99% confidence interval:

$$\hat{p} - z_{0.005}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{0.005}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.772 - 2.58\sqrt{\frac{(0.228)(0.772)}{1000}} < p < 0.772 + 2.58\sqrt{\frac{(.228)(.772)}{1000}}$$

$$0.738 < p < 0.806$$

PROBLEM 6.11

a) Point estimate $\hat{p} = \dfrac{114}{200} = 0.57 \Rightarrow \hat{q} = 0.43$

$\Rightarrow$ 96 % confidence interval:

$$\hat{p} - z_{0.002}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{0.02}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.57 - (2.88)\sqrt{\frac{(.57)(.43)}{200}} < p < 0.57 + (2.88)\sqrt{\frac{(.57)(.43)}{200}}$$

$$0.469 < p < 0.671$$

b) $$e = z_{0.002}\sqrt{\frac{\hat{p}\hat{q}}{n}} = (2.88)\sqrt{\frac{(.57)\cdot(.43)}{200}}$$

$$= 0.101$$

PROBLEM 6.12

Point estimate $\hat{p} = \frac{24}{100} = .24 \Rightarrow \hat{q} = .76$

a) 99 % confident interval:

$$\hat{p} - z_{0.005}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{0.005}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.76 - 2.58\sqrt{\frac{(.24)(.76)}{100}} < p < 0.76 + 2.58\sqrt{\frac{(.24)(.76)}{100}}$$

$$0.650 < p < 0.870$$

b) $$e = z_{0.005}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 2.58\sqrt{\frac{(.24)(.76)}{100}}$$

$$= 0.110$$