7. One and Two Sample Test of Hypotheses

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PROBLEM 7.1

- Null hypothesis : $\mu = 670$

  Alternative hypothesis : $\mu > 670$

  Standardised statistic :

$$z = \frac{\bar{x} - \mu_0}{\sigma}\sqrt{n} = \frac{725 - 670}{102}\sqrt{40} = 3.41$$

Rejection region: $z > z_{\alpha=0.01} = 2.33$

$z > z_\alpha \Rightarrow$ We can reject H_0

Conclude: The average weekly earnings for male social workers are higher than that of women, the prob that we made an incorrect decision is $\alpha = 0.01$

PROBLEM 7.2

$n = 64$, $\bar{x} = 5.23$, $\sigma = 0.24 \Rightarrow z = -9$

$z_{0.05} = 1.64$

$z < -z_{0.05} \Rightarrow H_0: \mu = 5.5$ is rejected

(Left-tailed hypothesis testing)

PROBLEM 7.3

Null : $H_0 : \mu = 8$, Alternative $H: \mu \neq 8 \rightarrow$ Two-tailed tes

$n = 18$, $s = 0.5$, $\bar{x} = 7.8$, $\alpha = 0.05$

$z = -1.70$, $t_{\alpha/2} = 2.110$

$z > -t_{\alpha/2} \Rightarrow$ Do not reject H_0

→ Not enough evidence

PROBLEM 7.4

Null : $H_0 : \mu = 10$, Alternative $H: \mu \neq 10 \rightarrow$ Two-taile

$n = 10$, $s = 0.23$, $\bar{x} = 10.06$, $\alpha = 0.01$

$z = 0.82$, $t_{0.005} = 3.250$

Rejection region : $z > 3.250$ or $z < -3.250$

→ Do not reject H_0

→ Not enough evidence

PROBLEM 7.5

Null : $H_0 : \mu = 220$, Alternative: $H: \mu > 220 \rightarrow$ Right-ta

$n = 20$, $s = 24.5$, $\bar{x} = 244$, $\alpha = 0.05$

Rejection region: $z > t_{0.05} = 1.729$

$z = 4.381 \Rightarrow$ Reject H_0

→ We can conclude that the avg. sodium content i great than 220mg.

PROBLEM 7.6

Null : $H_0 : \mu = 880$, Alternative $H : \mu \neq 880$

Two-tailed test : $n = 50$, $s = 21$, $\bar{x} = 871$, $\alpha = 0.0$

Rejection region : $z > z_{0.025} = 1.96$

or $z < -z_{0.025} = -1.96$

$$z = \frac{871 - 880}{21}\sqrt{50} = -3.03 < -z_{0.025}$$

→ We can reject H_0

→ The average has changed in recent months.

PROBLEM 7.7

$n = 500$, $p_0 = .94$

Verify: $np_0 = (500)(.94) = 470 \geq 5$, $n(1-p_0) = (500)(1-.94) = 30 \geq 5$ } normal distribut

Null : $H_0 : p = .94$, Alternative : $H : p > .94$

Right-tailed test : $\hat{p} = \frac{475}{500} = .95$, $\alpha = 0.0$

Rejection region : $z > z_{0.01} = 2.33$

$$z = \frac{(.95 - .94)}{\sqrt{(.94)(.06)}}\sqrt{500} = 0.94 < z_{0.01}$$

→ H_0 is not rejected

→ Not enough evidence to support the claim.

PROBLEM 7.8

$n = 100$, $p_0 = 1/8$

Verify: $\begin{cases} np_0 = 12.5 \geq 5 \\ n(1-p_0) = 87.5 \geq 5 \end{cases}$ → normal distribution

Null: $H_0: p = 1/8$, Alternative: $H: p \neq \frac{1}{8}$

Two-tailed test: $\hat{p} = \frac{5}{100} = 0.05$, $\alpha = 0.05$

$z_{0.025} = 1.96$

Rejection region: $z < -1.96$ or $z > 1.96$

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)}}\sqrt{n} = \frac{(0.05) - (1/8)}{\sqrt{\frac{1}{8} \cdot \frac{7}{8}}}\sqrt{100} = -2.2$$

< -1.9

→ H_0 is rejected

→ the claim is true

PROBLEM 7.9

$n = 200$, $p_0 = 0.4$

Verify $\begin{cases} np_0 = 80 \geq 5 \\ n(1-p_0) = 120 \geq 5 \end{cases}$ → Normal distribution

Null: $H_0: \mu = (.4)$, Alternative: $H: \mu \neq (.4)$

Two-tailed test: $\hat{p} = 9/20 = 0.45$, $\alpha = 0.05$

Rejection region: $z > z_{0.025} = 1.96$

or $z < -z_{0.025} = -1.96$

$$z = \frac{0.45 - 0.4}{\sqrt{(.4)(.6)}}\sqrt{200} = 1.44$$

→ H_0 is not rejected

→ Not enough evidence to support the claim

PROBLEM 7.10

$n = 200$, $p_0 = 0.6$

Verify: $np_0 = 120 \geq 5$, $n(1-p_0) = 80 \geq 5$ → Normal distribution

Null: $H_0: p = 0.6$, Alternative: $p > 0.6$

Right-tailed test: $\hat{p} = 110/200$, $\alpha = 0.05$

Rejection region: $z > z_{0.05} = 1.645$

$$z = \frac{110/200 - 0.6}{\sqrt{(0.6)(0.4)}} \sqrt{200} \approx -1.64$$

→ Do not reject H_0.

PROBLEM 7.11

$n = 49$, $n_2 = 44$, $\bar{x}_1 = 22.1$

$s_1 = 4.8$, $s_2 = 5.4$, $\bar{x}_2 = 19.8$

$H_0: \mu_1 = \mu_2$, $H_1: \mu_1 > \mu_2$

Right-tailed test, $\alpha = 0.10$

Rejection region: $z > z_\alpha = 1.28$

$$z = \frac{22.1 - 19.8}{\sqrt{\frac{(4.8)^2}{49} + \frac{(5.4)^2}{44}}} \approx 2.161 > z_\alpha$$

→ Reject H_0.

PROBLEM 7.12

$n_1 = 100$, $\bar{x}_1 = 2.70$, $s_1^2 = 0.36$

$n_2 = 100$, $\bar{x}_2 = 2.54$, $s_2^2 = 0.40$

$H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$

Two-tailed test, $\alpha = 0.05$

Rejection region: $z > z_{\alpha/2} = 1.96$

or $z < -z_{\alpha/2} = -1.96$

$$z = \frac{2.70 - 2.54}{\sqrt{\frac{(0.36)^2}{100} + \frac{(0.40)^2}{100}}} \approx 2.97$$

→ H_0 is rejected

PROBLEM 7.13

$n_1 = 50$, $\bar{x}_1 = 86.7$, $s_1 = 6.28$

$n_2 = 50$, $\bar{x}_2 = 77.8$, $s_2 = 5.61$

$H_0: \mu_1 - \mu_2 = 12$, $H_1: \mu_1 - \mu_2 > 12$

Right-tailed test: $\alpha = 0.05$

Rejection region: $z > z_\alpha = 1.28$

$$t_1 = \frac{86.7 - 77.8 - 12}{\sqrt{\frac{(6.28)^2}{50} + \frac{(5.61)^2}{50}}} = -2.60$$

→ H_0 is rejected

PROBLEM 7.15

$$\hat{p}_1 = \frac{361}{1245} \approx 0.29$$

$$\hat{p}_2 = \frac{341}{1065} \approx 0.32$$

$H_0: p_1 = p_2$, $H_1: p_1 < p_2$

$$\bar{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2} = 0.304$$

$$z = \frac{0.29 - 0.32}{\sqrt{(0.304)(0.696)\left(\frac{1}{1245} + \frac{1}{1065}\right)}} \approx -1.56$$

Rejection region: $z < -z_\alpha = -2.33$ → False

→ H_0 is not rejected.