
@VH

MECHANICS

PRELIMINARIES

SI UNIT

1. Length m
2. Mass kg
3. Time s
4. Thermodynamic temperature K
5. Eletric current A
6. Luminous intensity Cd
7. Amount of substances mol

DERIVED UNITS

Quantity	Derived unit		Base units
frequency	Hz	$f = \frac{1}{T}$	s^{-1}
velocity	ms^{-1}	$v = s/t$	ms^{-1}
acceleration	ms^{-2}	$a = v/t$	ms^{-2}
force	N	$F = ma$	$kg\,ms^{-2}$
energy	J	$E = \frac{1}{2}mv^2$	$kg\,m^2s^{-2}$
power	W	$P = W/\Delta t$	$kg\,m^2\,s^{-3}$
electric charge	C	$q = It$	As
potential difference	V	$V = E/q$	$kg\,m^2s^{-3}A^{-1}$
electric resistance	Ω	$R = V/I$	$kg\,m^2\,s\,A^{-1}$
specific heat capacity	$Jkg^{-1}K^{-1}$	$c = \frac{Q}{m\Delta t}$	$m^2s^{-2}K^{-1}$

Significant Figures

1. The final result of a multiplication or division should have no more digits than the numerical value with the fewest sig. figures. ex: $(11.3) \times \underbrace{(6.8)}_{\text{2 sig. fig}} \approx 77$

2. When adding or subtracting numbers, the final result should contain no more decimal places than the number with the fewest decimal places.

ex: $3.6 - \underbrace{0.57}_{\text{1 dec.}} \approx \underbrace{3.0}_{\text{1 dec.}}$

$$\underbrace{35.\overset{\text{1 dec.}}{6} \quad 3.\overset{\text{2 dec.}}{56} \quad 0.\overset{\text{5 dec.}}{00356}}_{\text{3 sig. fig}}$$

not.sig: for "cosmetic" purpose

locate dec. point

zeros b.w non zero numbers

0. 0 0 4 0 0 4 5 0 0

≥ 10 → sig.

≠0 integers always sig.

trailing zeros with decimal point

percent Uncertainty

$$\frac{\text{abs uncertainty}}{\text{measured value}} \times 100\%$$

ex: $5.48 \pm 0.25 \rightarrow \frac{0.25}{5.48} \times 100\% \approx 4.6\%$

MECHANICS

KINEMATICS

Basic quantities

$\vec{r} := x\hat{i} + y\hat{j} + z\hat{k}$

Savg := total distance traveled along its path divided by the time it takes to travel this distance

1. Average speed $\qquad s_{avg} = \dfrac{\text{total distance}}{\Delta t}$ $\quad \int_{t_1}^{t_2} |v|\,dt$

:= displacement divided by the elapsed time

2. Average velocity $\qquad v_{avg} = \dfrac{\Delta \vec{r}}{\Delta t}$

3. Instantaenous velocity $\qquad v = \dfrac{d\vec{r}}{dt}$ $\quad$ → rate of change of position

4. Average acceleration $\qquad a_{avg} = \dfrac{\Delta \vec{v}}{\Delta t}$

5. Instantaneous acceleration $\qquad a = \dfrac{d\vec{v}}{dt}$ $\quad$ → rate of change of velocity

acceleration := the change in velocity divided by the time taken to make this change

KINEMATICS EQUATIONS FOR CONST ACCELERATION

1. $v = v_0 + at$
2. $x - x_0 = v_0 t + \frac{1}{2} at^2$
3. $v^2 = v_0^2 + 2a(x - x_0)$
4. $x - x_0 = \frac{1}{2}(v_0 + v)t$
5. $x - x_0 = vt - \frac{1}{2} at^2$

RELATIVE MOTION:

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

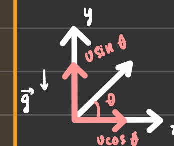
UNIFORM CIRCULAR MOTION

Centripetal acceleration $|\vec{a}| = \frac{v^2}{r}$

Angular speed $\omega = \frac{v}{r}$

PROJETILE MOTION

$$x - x_0 = (v_0 \cos \theta_0) t$$
$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2$$
$$v_y = v_0 \sin \theta_0 - gt$$
$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)$$



$$\begin{cases} x = v_{0x} t \\ y = v_{0y} t - \frac{1}{2} g t^2 \end{cases}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

Trajectory $y = (\tan \theta_0) x - \frac{g x^2}{2(v_0 \cos \theta_0)^2}$

Horizontal Range $R = \frac{v_0^2}{g} \sin 2\theta_0$

MECHANICS

DYNAMICS

Newton's First Law	"Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acting on it."
Newton's Second Law	"The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object." $\vec{F}_{net} = m\vec{a}$
Newton's Third Law	"When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction." $\vec{F}_{BC} = -\vec{F}_{CB}$
Some definitions	Inertia Reference Frames := RFs in which Newtonian mechanic holds are called inertia RFs.
	Mass: The mass of a body is the characteristic of that body that relates to the net force causing the acceleration. ⟨or: mass is a measure of the inertia of an object⟩

Momentum of an object is defined as the product of its mass and its velocity.

Torque := the moment of the force about the axis.

A coordinate transformation relates to the coordinates of the same event on different coordinate systems.

Inertia := the resistance to change in motion of an object.

Some types of force

Centripetal force

$$\vec{F}_c = m\vec{a}_c = -\frac{mv^2}{r}\hat{r}$$

Centrifugal force

$$\vec{F}_{CF} = \frac{mv^2}{r}\hat{r}$$

Friction forces

- static : $F_{fr} = F_{\parallel}$ → parallel component → $F_{fr} \leq \mu_s N$
- kinetic : $\vec{F}_{fr} = -\mu_k N \hat{v}$
- rolling : $\vec{F}_{fr} = -\mu_r N \hat{v}$ → coefficient of friction

Generally: $\mu_s > \mu_k$

$\vec{f}_s$ and $\vec{f}_k$ are always // surface, opposed to the attempted sliding

Gravitational force

$$\vec{F}_g = m\vec{g}$$

Normal force

When the body presses against a surface, the surface deforms and pushes on the body with a normal force $\vec{F}_N$ that is ⊥ surface.

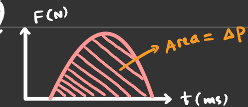
MOMENTUM $\vec{p} = m\vec{v}$

FIRST THEOREM ABOUT MOMENTUM (Second form of Newton's Second Law)

"The rate of change of momentum of an object is equal to the net force applied to it."

$$\sum \vec{F} = \frac{\Delta\vec{P}}{\Delta t}$$

$\Delta\vec{P}$:= impulse = $(F \times \Delta t)$



LAW OF CONSERVATION OF MOMENTUM

"The total momentum of an isolated system of objects remains constant."

$$m\sum \vec{v}_i = M\sum \vec{v}_f \qquad \begin{cases} \sum \vec{F} = \frac{d}{dt}\sum \vec{p} = 0 \\ \Rightarrow \sum \vec{p} = \text{const} \end{cases}$$

TORQUE, ANGULAR MOMENTUM

Torque $\boxed{\vec{\tau} = \vec{r} \times \vec{F}} = rF\sin\theta$

level arm

$\circlearrowright \; \tau < 0$

$\circlearrowleft \; \tau > 0$

Rotational equilvalence of Newton's second law: $\boxed{\sum \tau = I\alpha}$

$$= \underbrace{\sum mr^2}_{\text{moment of inertia}} \alpha$$

angular accele. $\frac{\Delta w}{\Delta t}$

position vector

momentum of particle w/r to the point

Angular momentum $\boxed{\vec{L} = \vec{r} \times \vec{p}}$, $L = I w$

RELATION:

By Newton's 2nd Law of motion: $\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt}$

$$\rightarrow \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt} \quad (1)$$

On the other hand: $\frac{d}{dt}(\vec{r} \times \vec{p}) = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}}$

$$= \vec{v} \times (m\vec{v}) + \vec{r} \times \dot{\vec{p}}$$

$$= 0 + \vec{r} \times \dot{\vec{p}} \quad (2)$$

$$(1)(2) \Rightarrow \underbrace{\vec{r} \times \vec{F}}_{\vec{\tau}} = \frac{d}{dt}\underbrace{(\vec{r} \times \vec{p})}_{\vec{L}}$$

$$\Rightarrow \boxed{\vec{\tau} = \dot{\vec{L}} \equiv \frac{d\vec{L}}{dt}}$$

THEOREM: For a symmetric homogeneous rigid body in pure rotation about its symmetric axis, the net force acting on the body equals to the rate of change of the total angular momentum of the body.

COORDINATE TRANSFORMATION

< References: The Lorentz Transformation - Halliday's Book, P.1129; Galelian and Lorentz Transformation - D.C. Giancolli's Book, Appendix E >

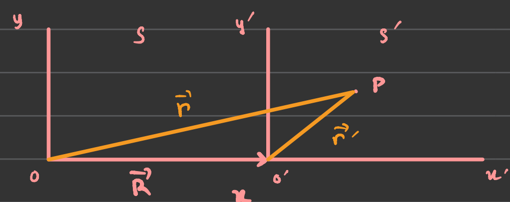
LAW OF CONSERVATION OF ANGULAR MOMENTUM

"The total momentum of a rotating object remains constant if the net torque acting on it is zero."

$$\vec{J} = \frac{d\vec{L}}{dt} = 0 \Rightarrow L = \text{const}$$

WHY? Derive the law of velocity addition (for slow motions only)

HOW? Consider two RFs, S and S'. Assume that S' moves to the right (x-direction) at speed V with respect to S. For simplicity, let t = 0 (initial time) for S and S'.



Galelian trans. eqs:

$$\begin{cases} x' = x - V.t' \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

As time is assumed to be absolute in Galelian-physics

PROB: $\exists$ P in S', what is its coordinates in S?

(x', y', z') at time t'

Let $\vec{R} = \vec{OO'}$, $\vec{V} = \frac{d\vec{R}}{dt}$. Have: $\vec{r} = \vec{r}' + \vec{R}$

$$\Rightarrow \frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \frac{d\vec{R}}{dt} \xrightarrow{t=t'} \frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt'} + \frac{d\vec{R}}{dt}$$

$$\Rightarrow \vec{v} = \vec{v}' + \vec{V}$$

MECHANICS

ENERGY AND CONSERVATIVE FORCE

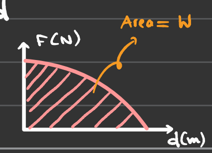
WORK

Work := energy transferred to or from an object by means of a force acting on the object.

Work done by a force $\vec{F}$ in displacement $d\vec{r}$ is: $dW = \vec{F} \cdot d\vec{r}$

or $W = F_{\parallel} d$

If $\alpha = (\vec{F}, d\vec{r})$, $ds = |d\vec{r}|$ $\Rightarrow$ $\boxed{W = Fs\cos\alpha}$



Work done by gravitational force $W_g = mgd\cos\alpha$

Work done by a spring $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

POWER

Power := the rate at which work is done

$$\boxed{P_{avg} = \frac{W}{\Delta t}}$$

Instantaneous: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

$= Fv \cos \alpha$

NOTE: $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

ENERGY

LAW OF CONSERVATION OF ENERGY

"The total energy is neither decreased or increased in any process. Energy can be transformed from one form to another, and transferred from one object to another. But the total amount remains constant."

KINETIC

$KE = \frac{1}{2} m v^2$

WORK - KINETIC ENERGY THEOREM

"The net work done on an object is equal to the change in the object's kinetic energy."

$$\boxed{W_{net} = \Delta KE} = KE_f - KE_i$$

CONSERVATIVE FORCE := a force with the property that the total work done in moving a particle between 2 points is independent of the taken path.

$$\int_{\widehat{AB}} \vec{F} \cdot d\vec{r} = \int_A^B \vec{F} \cdot d\vec{r} \qquad \oint \vec{F} \cdot d\vec{r} = 0$$

POTENTIAL

$PE = mgz$

THEOREM "Work done by conservative force is equal to the decrease in potential energy associated with the force."

$$\boxed{W = -\Delta PE = PE_i - PE_f}$$

MECHANICAL

$E = KE + PE = \text{const}$ [conservative forces only]

GRAVITATION

NEWTON'S LAW OF GRAVITATION "Two point particles affect each other with a force that is proportional to the product of their masses and inversely proportional to the square of their seperation."

$$\boxed{F = G\frac{m_1 m_2}{r^2}}, \quad G = 6.67 \times 10^{-11}\ \mathrm{Nm^2\,kg^{-2}}$$

Some terminologies

Gravitational field is a region of space where a mass experience a force.

Gravitational PE $\quad PE = -G\dfrac{mM}{r} = -\displaystyle\int_{\infty}^{r} -\frac{GMm}{r^2}\,dr$

G. field strength $\quad \vec{g} = -G\dfrac{M}{r^2}\hat{r}$ $\qquad \dfrac{\text{force}}{\text{mass}} :=$ forced experienced per unit mass by a small test mass at a point in g.field

G. potential $\quad \phi = -G\dfrac{M}{r}$ $\qquad \dfrac{PE}{\text{mass}} :=$ work done per unit mass to move a mass from infinity to a point.

Law of period $\quad T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3$

Orbital velocity := the v that is sufficient for a body to remain in orbit around a g. centre $\boxed{v_o = \sqrt{\frac{GM}{R}}}$

$\langle F_c = F_G \rangle$

Escape velocity := the velocity that is sufficient for a body to escape from a G. centre.

$\boxed{v_e = \sqrt{\frac{2GM}{R}}}$ $\langle F_G = \frac{1}{2} m v_e^2 \rangle$

MECHANICS

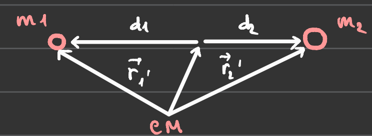
DYNAMICS OF RIGID BODIES

Rigid body

A rigid body is a solid body in which deformation is zero or negligibly small.

Centre of gravity (CG)

:= the point through which the force of gravity is assumed to act on the body.



$\underbrace{\sum_{i=1}^{n} m_i \vec{r}_i'}_{\text{weighted pos.}} = 0$ (system of n particles)

Centre of mass (CM)

:= the point where the weighted relative position of the distributed mass sums to zero.

Continuous distribution of mass

1D: $dm = \lambda\, dx$ (λ: mass per unit length)

2D: $dm = \sigma\, dA$ (σ: mass / area)

3D: $dm = \rho\, dV$ (ρ: mass / volume)

APPENDIX: TRANSLATION, ROTATION

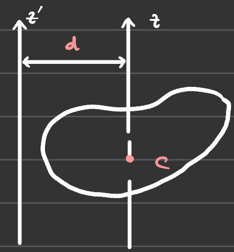
	Translation	Rotation	Connection
POSITION	x	θ	$x = r\theta$
VELOCITY	$v = dx/dt$	$\omega = d\theta/dt$	$v = r\omega$ (rolling w/o slipping)
ACCELE.	$a = dv/dt$	$\alpha = d\omega/dt$	$a_T = r\alpha$
MASS	m	I	$I = \sum mr^2$
Newton II (1)	$\sum F = ma$	$\sum \tau = I\alpha$	$\tau = rF\sin\varphi$
WORK	$W = Fd$	$W = \tau\theta$	
KE	$KE = \frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$	
POWER	$P = Fv$	$P = \tau\omega$	
WORK-KE	$W = \Delta KE$	$W = \Delta KE$	
LIN. MOMENTUM	$p = m\vec{v}$	ANG. MMT $L = I\omega$	arbitrary object: $L = \overbrace{mv}^{\text{linear momentum}}\underbrace{r}_{\text{distance to rot. axis}}\sin\theta$
Newton II (2)	$\sum\vec{F} = d\vec{p}/dt$	$\sum\vec{\tau} = d\vec{L}/dt$	
CONSERVATION (isolated system)	$\vec{p}$ = const	$\vec{L}$ = const	

<References: Halliday & Resnick – Table 11-1 and 10-3; Giancoli, P. 219>

MOMENT OF INERTIA

Parallel-axis Theorem (Huygens-Steiner Theorem)

- The moment of inertia I of a system about a given axis is calculated as:
  $I = I_{cm} + md^2$ where I_{cm} is the m.o.i of the system about the axis passing through the cm of the system, is parallel to and is at distance d from the given axis.



Proof: Without loss of generality, assume that the perpendicular distance b.w 2 axes lies along the x-axis. Thus the moment of inertia along the z-axis is:

$$I_{cm} = \int (x^2 + y^2)\, dm$$

z'-axis:

$$I = \int (x+d)^2 + y^2 \, dm$$

$$= \int \underbrace{x^2 + y^2 \, dm}_{I_{cm}} + \underbrace{d^2 \int dm}_{md^2} + 2\underbrace{d \int x \, dm}_{0}$$

$$I = I_{cm} + md^2 \qquad \square$$

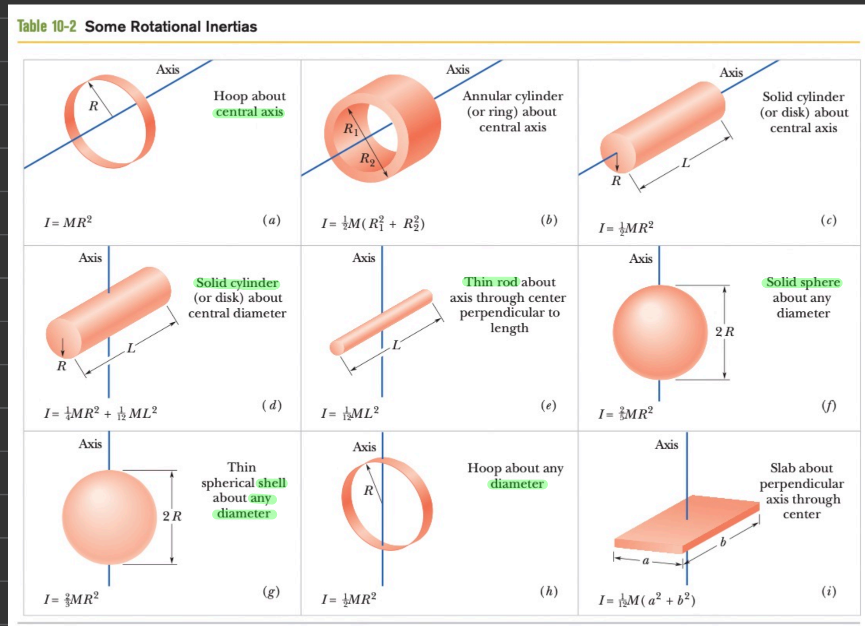


Table 10-2 Some Rotational Inertias

Hoop about central axis $I = MR^2$ (a)	Annular cylinder (or ring) about central axis $I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)	Solid cylinder (or disk) about central axis $I = \frac{1}{2}MR^2$ (c)
Solid cylinder (or disk) about central diameter $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)	Thin rod about axis through center perpendicular to length $I = \frac{1}{12}ML^2$ (e)	Solid sphere about any diameter $I = \frac{2}{5}MR^2$ (f)
Thin spherical shell about any diameter $I = \frac{2}{3}MR^2$ (g)	Hoop about any diameter $I = \frac{1}{2}MR^2$ (h)	Slab about perpendicular axis through center $I = \frac{1}{12}M(a^2 + b^2)$ (i)

! FULL PROOF HERE :

http://hyperphysics.phy-astr.gsu.edu/hbase/inecon.html

<Reference : Halliday - P 274>

COLLISIONS

Energy → Total momentum = const if the system is closed and isolated.
→ Total energy is always conserved.
→ Total KE = const if no heat or other energy produced in collision.

ΣKE

→ = const → Elastic collision:

$$\begin{cases} KE_i = KE_f \\ E_i = E_f \end{cases} \rightarrow \begin{cases} v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i} \\ v_{2f} = \dfrac{2m_1}{m_1 + m_2} v_{1i} \end{cases}$$

→ ≠ const → Inelastic collision

$$\vec{P}_i = \vec{P}_f$$

↓ 2 objects stick together

Completely inelastic collision
(Greatest loss in kinetic energy)

$$m_1 v_{1i} = (m_1 + m_2) V$$

$$\rightarrow V = \frac{m_1}{m_1 + m_2} v_{1i}$$

MECHANICAL OSCILLATION & WAVES

Oscillation := a repetitive variation in time of some quantity about a central value.

Mechanical oscillation := a repeated back and forth movement of an object about a point of equilibrium.

Oscillation $\xrightarrow[\text{regular cycle}]{\text{repeated}}$ Periodic motion $\xrightarrow{\text{sinusoidal}}$ Harmonic motion

CONDITIONS:

1. There exists a point of equilibrium (equil. for short) ($\Sigma \vec{F} = \Sigma \vec{\tau} = 0$) at which the net force acting on the object is zero.
2. There exists a restoring force acting on the object whenever it is out of the point of equil.
3. When the object goes to the point of equil., in order for it to pass this position, it needs to have an inertia.

SIMPLE HARMONIC MOTION (SHM)

"A SHM is the motion about a fixed point such that the acceleration is proportional to the displacement from, and is directed towards, the point."

FORMULATIONS

Frequency f of periodic, or oscillatory, motion is the number of oscillations per second.

Period T is the time required for one complete oscillation (cycle).

Linear oscillator A particle with mass m that moves under the influence of a Hooke's law restoring force given by $F = -kx$ exhibits SHM:

$$\omega = \sqrt{\frac{k}{m}} \qquad a = -\omega^2 x \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

SHM In a SHM, the displacement $x(t)$ from the equil. point is described by:

$$\boxed{\underbrace{x}_{\text{displacement}} = \underbrace{A}_{\text{amplitude}} \sin(\underbrace{\underbrace{\omega}_{\text{angular frequency}} t + \phi}_{\text{phase}})}$$

$$\Rightarrow \quad v = \omega A \cos(\omega t + \phi)$$
$$a = -\omega^2 A \sin(\omega t + \phi)$$

MATHEMATICAL PROOF

We have: $\ddot{x}(t) = -\omega^2 x(t)$, with $x(0) = x_0$ and $v(0) = v_0$

Characteristic equation: $\lambda = -\omega^2 \rightarrow \lambda = \pm i\omega$

Then $x = c_1 . e^{-i\omega t} + c_2 . e^{+i\omega t} = c_1(\cos\omega t - i.\sin\omega t) + c_2(\cos\omega t + i\sin\omega t)$

$x = (c_1 + c_2)\cos\omega t - i(c_1 - c_2)\sin\omega t \quad (1)$

$\xrightarrow{x(0) = x_0} x_0 = c_1 + c_2 \quad (*)$

Differentiate (1): $v = -(c_1 + c_2)\omega\sin\omega t - i(c_1 - c_2)\omega\cos\omega t$

$\xrightarrow{v(0) = v_0} v_0 = -i(c_1 - c_2)\omega \quad (**)$

From (*) and (**) we yield that: $x = x_0\cos\omega t + \frac{v_0}{\omega}\sin\omega t$

Put $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$, $\sin\phi = \frac{x_0}{A}$, $\cos\phi = \frac{v_0}{A\omega}$

$\Rightarrow x = A\sin(\omega t + \phi)$

ENERGY

$$KE = \frac{1}{2}mv^2 = \boxed{\frac{1}{2}m\omega^2A^2\cos^2(\omega t)}$$

$$PE = \frac{1}{2}kx^2 = \boxed{\frac{1}{2}m\omega^2A^2\sin^2(\omega t)}$$

$$E = KE + PE = \boxed{\frac{1}{2}m\omega^2A^2}$$

$$\rightarrow \begin{cases} KE = \frac{1}{2}E(1+\cos 2\omega t) \\ PE = \frac{1}{2}E(1-\cos 2\omega t) \end{cases}$$

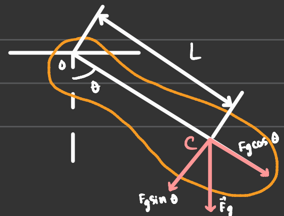
THEOREM: "In a SHM, the kinetic energy and potential energy oscillate at a frequency which is 2 times greater than the frequency of the motion, and the mechanical energy is conserved.
The kinetic and potential energy varies in opposite phase with a period that is half of the period of the oscillation."

PHYSICAL PENDULUMS

$$T = 2\pi\sqrt{\frac{L}{g}}$$ (Simple pendulum) or $$\boxed{T = 2\pi\sqrt{\frac{I}{mgL}}}$$ (physical pendulum - complicated distribution of mass)

L: distance from CM → axis

$$I = ml^2 \rightarrow \omega = \sqrt{\frac{g}{L}}$$ (small angle)



DAMPED OSCILLATIONS

:= Oscillation in which energy is dissipated and, therefore, amplitude is decreased due to friction or resistive force are called damped oscillations.

$$F_{restoring} + F_{viscous} = ma$$

$$-kx - cv = ma$$

c ↳ viscous coefficient

FORMULATION:

$\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$:= natural angular frequency

$\boxed{\beta = \frac{c}{2m}}$:= damping coefficient

$\boxed{\zeta = \frac{\beta}{\omega_0}}$:= damping ratio

$\zeta = 1$: critically damped → x decreased to 0 in the shortest time

$\zeta > 1$: overdamped → oscillator returns to equil. position w/o oscillating

$\zeta < 1$: underdamped → oscillating w/ decaying amplitude, $\omega = \omega_0\sqrt{1-\zeta^2}$

Eq:

$$\boxed{x = A\sin(\omega t + \phi)}$$

$$\begin{cases} A = A_0 e^{-\beta t} \\ \omega = \sqrt{\omega_0^2 - \beta^2} \end{cases}$$

$$\delta = \ln \frac{A(t)}{A(t+T)} :=$$ the **logarithmic decrement** = the natural log of the ratio of any 2 successive amplitudes.

$$\to \quad \zeta = \frac{1}{\sqrt{1 + \left(2\pi / \delta\right)^2}}$$

FORCED OSCILLATION, RESONANCE

A **forced oscillation** is an oscillation under the action of external periodic forces.

In a forced oscillation, **resonance** is the phenomenon that the amplitude is maximum at certain values of driving frequency.

$$\omega_d = \omega = \sqrt{\omega_0^2 - 2\beta^2} = \omega_0 \sqrt{1 - 2\zeta^2}$$

$$\langle u_{tt}(x,t) = c^2 u_{xx}(x,t) \rangle$$

$$\sqrt{\frac{E}{\rho}}$$

E → elastic modulus

ρ → density

Wave eq, Wave function

General:

$$\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}, \quad c = \sqrt{\frac{k\ell^2}{m}} = \sqrt{\frac{E}{\rho}}, \quad k := \text{spring stiffness}$$

c → speed

! Different quantities

Monochromatic wave:

$$\boxed{\underbrace{u(x,t)}_{\text{displacement}} = \underbrace{u_m}_{\text{amplitude}} \overbrace{\sin\underbrace{(kx - \omega t)}_{\text{phase}}}^{\text{oscillating term}}}, \quad k = \frac{2\pi}{\lambda}$$

k → angular wave number

x → position

ω → angular frequency

λ → wavelength

Wave speed: $v = \lambda f$

Period: $T = \frac{2\pi}{\omega}$

POWER

$$\boxed{P_{avg} = \frac{1}{2}\mu c \omega^2 u_m^2}$$

:= average rate at which energy is transmitted by a wave on a stretched string

μ → String linear density

NOTE:
- μ, c not depend on material, tension of string
- ω, u_m not depend on wave generation process

ENERGY

E. flux $\boxed{S = \frac{P_{avg}}{A}}$ $\left(\frac{\text{energy/unit time}}{\text{unit area}}\right)$

A → area, not amplitude

E. density $\boxed{e = \frac{S}{c}}$

THERMODYNAMICS

PRELIMINARIES

States of matter

Matters = atoms + molecules $\xrightarrow[\text{forces b/w them}]{\text{Spacing of the molecules}}$ determine the state

Gases
- → No fixed volume, no shape
- → Seperation b/w molecules >> interaction
- → Ideal gases
  - → collision = perfectly elastic
  - → no intermolecular attractive interaction

Experimental Laws

Boyle's law: "The volume of a gas is inversely proportional (denoted $\propto^{-1}$ in this notebook) to its pressure, provided that the temperature is held constant."

$T = \text{const}, \quad V \propto \frac{1}{P}$

Charles' Law: "The volume of a gas is directly proportional ($\propto$) to its thermodynamic temperature, provided that the pressure is const."

$p = \text{const}, \quad V \propto T$

Gay-Lussac's Law: "The pressure of a gas is $\propto$ to its thermodynamic temp, provided that the volume is const."

$V = \text{const}, \quad p \propto T$

$\approx$ for real gas with $\begin{cases} \text{low } p \\ \text{high } T \end{cases}$

EQUATION OF STATE

$$\boxed{pV = nRT}$$

n: number of moles

$$pV = Nk_BT$$

N: total number of particles $= n \cdot N_A$

$R = 8.31 \text{ J K}^{-1} \text{mol}^{-1}$:= molar gas const

$k_B = \dfrac{R}{N_A} = 1.38 \times 10^{-23} \text{ J K}^{-1}$:= Boltzmann const

IDEAL GAS

:= is the one for which obeys the eq. of state, $pV = nRT$, $\forall p, V, T$

KINETIC THEORY OF GASES

1. A gas contains a very large number of particles (atoms, molecules).
2. The forces b/w particles are negligible, except during collisions.
3. The volume of the particle is neglibile compared to the volume occupied by gas.
4. The collisions of the particles with each other and the containers are perfectly elastic.
5. The particles mostly move in straight line at constant speed. The collision time is negligible compared with the time b/w collisions.

Molecular motion

p and V relation

mean - square - speed

$$\boxed{pV = \frac{1}{3} N m \langle c^2 \rangle}$$

molecular mass

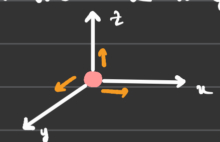
Mean kinetic energy $\langle E_k \rangle = \frac{1}{2} m \langle c^2 \rangle \rightarrow \langle E_k \rangle = \frac{3}{2} k_B T$ (1 molecule)

Deegrees of freedom

Degrees of freedom (d.o.f)

The number of d.o.f i of a gas is the number of independent parameters used to determine the position of each molecule in gas.

Monoatomic gases $i = 3$

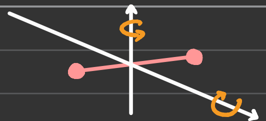


→ 3 translational freedom

all types has 3 trans.

< D.o.f can be defined as the independent ways that a molecule can store energy, each such d.o.f has an energy of $\frac{1}{2}kT$ per molecule >

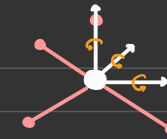
Diatomic gases $i = 5$



3 trans. + 2 rotational

< Ref: Halliday, P569 >

Polyatomic gases $i = 6$
(≥ 3 atoms)



max = 3 trans.
max = 3 rot.

$$\boxed{\langle E_k \rangle = \frac{i}{2} k_B T = \frac{i}{2} RT}$$

$c_{rms} = \sqrt{\langle c^2 \rangle} = \sqrt{\frac{3RT}{M}}$:= root-mean square speed

$\simeq 1.1 \langle c \rangle$

Molar mass $= \frac{m}{n}$

INTERNAL ENERGY

Definition: Internal energy of a thermodynamic system is defined by the state of the system and can be expressed as the sum of a random distribution of kinetic and potential energy associated w/ the molecules of the system

$$U = \sum E_k + \underset{0}{\underset{\shortparallel}{\sum E_p}} \rightarrow \boxed{U = N.\langle E_k \rangle} \rightarrow U = \frac{m}{M} \frac{i}{2} RT$$

Maxwell's distribution

$$F(c) = \left(\frac{m}{2\pi k_B T} \right)^{3/2} 4\pi c^2 \exp\left(\frac{-mc^2}{2 k_B T} \right)$$

Mean speed $\langle c \rangle = \sqrt{\frac{8RT}{\pi M}}$

Root-mean-square speed $c_{rms} = \sqrt{\frac{3RT}{M}}$

Most probable speed $c_p = \sqrt{\frac{2RT}{M}}$

THERMODYNAMICS

FIRST LAW OF THERMODYNAMICS

Law of Rudolf Clausis

- "The change in internal energy of a system is equal to the heat added to the system plus the work done on the system."

$$\boxed{\Delta U = Q + W}$$

Heat: added Q; lost Q'

Work done: on system W; by system W'

Molar heat capacity

$$C = \frac{M}{m}\frac{\partial Q}{\partial T}$$

$$\boxed{C_p = \left(\frac{i+2}{2}R\right)}$$

$$\boxed{C_v = \frac{i}{2}R}$$

Heat capacity ratio (adiabatic index) $\boxed{\gamma = \frac{C_p}{C_V} = \frac{i+2}{i}}$

Resistance

FOUR GAS PROCESSES

work done on the system

Line	Const	Type	Formulae				
	p	isobaric	$W = -p\Delta V$	<0	$Q = n\overbrace{\frac{i+2}{2}}^{c_p} R\Delta T$	>0	
	T	isothermal	$W = nRT\ln\left(\frac{V_i}{V_f}\right)$	<0	$Q = nRT\ln\left(\frac{V_f}{V_i}\right)$	>0	$\Delta U = 0$
<sloper>	pV^{γ}	adiabatic	$W = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$	<0	$Q = 0$		$\Delta U = W$
			$\xrightarrow{T\nearrow} W = n\underbrace{\frac{i}{2}}_{c_v} R\Delta T$				
	V	isochoric	$W = 0$		$Q = n\underbrace{\frac{i}{2}}_{c_v} R\Delta T$		$\Delta U = Q$

THERMODYNAMICS

SECOND LAW OF THERMODYNAMICS

WHY ?

- All processes obey the first law, but some do not occur naturally. Ex:
- New quantity entropy to deal with one-way, irreversible processes.

Heat engine

:= a system that converts thermal energy to mechanical energy, which can be used to do work.

Process

Working substance takes a heat Q_1 from heat source

↓

does work W'

↓

transfer heat Q_2' to colder sink

Efficiency

$$\boxed{\eta = 1 - \frac{Q_2'}{Q_1} = 1 - \frac{\text{heat transferred}}{\text{heat taken}}}$$

Carnot's engine

= ideal engine, all processes are reversible, no wasteful energy.

It efficiency is :

$$\eta = 1 - \frac{T_{low}}{T_{high}}$$

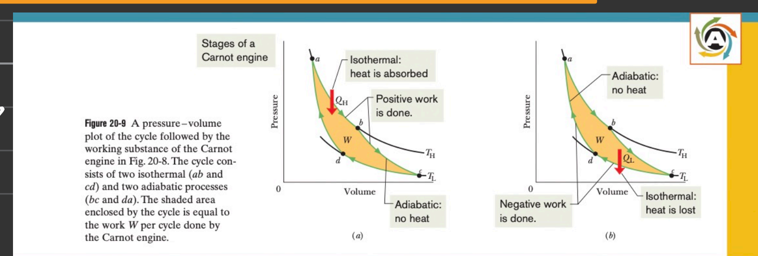


Figure 20-9 A pressure–volume plot of the cycle followed by the working substance of the Carnot engine in Fig. 20-8. The cycle consists of two isothermal (*ab* and *cd*) and two adiabatic processes (*bc* and *da*). The shaded area enclosed by the cycle is equal to the work *W* per cycle done by the Carnot engine.

Carnot's cycle

Entropy

:= a quantitive measure of the disorder of a system. By the 2nd law of Thermodynamics, as time goes on, energy is degraded to less useful forms (less available to do useful work). That is, lower order.

Ex:

PHYSICS APPLIED
Biological development

Biological Development

An interesting example of the increase in entropy relates to the biological development and growth of organisms. Clearly, a human being is a highly ordered organism. The development of an individual from a single cell to a grown person is a process of increasing order. Evolution too might be seen as an increase in order.

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Do these processes violate the second law of thermodynamics? No, they do not. In the processes of growth and evolution, and even during the mature life of an individual, waste products are eliminated. These small molecules that remain as a result of metabolism are simple molecules without much order. Thus they represent relatively higher disorder or entropy. Indeed, the total entropy of the molecules cast aside by organisms during the processes of development and growth is greater than the decrease in entropy associated with the order of the growing individual or evolving species.

In a reversible process: $\boxed{\Delta S = \frac{Q}{T}}$

SECOND LAW OF THERMODYNAMICS

"The total entropy of an isolated system can never decrease over time, and is const if and only if all processes within the system are reversible. Isolated systems spontaneously evolve towards thermodynamic equilibrium, the state with maximum entropy."

Equations form:
- isolated system : $\Delta S \geq 0$
- system evolving : $\Delta S > 0$
- all processes are reversible : $\Delta S = 0$

Ideal gas : $\Delta S = \frac{m}{M} C_v \ln \frac{P_i}{P_f} + \frac{m}{M} C_p \ln \frac{V_f}{V_i}$