CHAPTER 1: INFINITE SERIES

SERIES OF NUMBERS

INTRODUCTION

$\{u_n\}_{n=1}^{\infty}$ is a sequence of numbers, where $\sum_{n=1}^{\infty} u_n = u_1 + u_2 + \dots$

s is a n-th partial sum: $s_n = \sum_{i=1}^{n} u_i$

DIRICHLET'S CRITERION

↳ If $\begin{cases} \{s_n\} \text{ converges to } 0 \\ \lim_{n\to\infty} s_n = \mathcal{S} \in \mathbb{R} \end{cases} \Rightarrow \sum u_n$ is convergent and $\sum_{n=1}^{\infty} u_n = \mathcal{S}$

GEOMETRIC SERIES

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \dots$$
$(a \neq 0, r \neq 1)$

converges: $|r| < 1$, $\boxed{\sum = \frac{a}{1-r}}$

diverges: $|r| > 1$

! NOTE: A series is a number.

CONVERGENCE NECESSARY COND.

$$S = \sum_{n=1}^{\infty} u_n \Longrightarrow \lim_{n\to\infty} u_n = 0$$

(The converse is false)

↳ Test for Divergence: If $\nexists \lim_{n\to\infty} u_n$ or $\lim_{n\to\infty} u_n \neq 0$, S is div.

Arithmetic Operators

+ $\sum c.u_n = c.\sum u_n$, $c = \text{const}$
+ $\sum (a_n - b_n) = \sum a_n - \sum b_n$
+ $\sum (a_n + b_n) = \sum a_n + \sum b_n$

p-SERIES (Euler)

$$\zeta(p) = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \dots = \sum_{1}^{\infty} \frac{1}{n^p} \begin{cases} \text{conv: } p>1 \\ \text{div: } p \leq 1 \end{cases}$$

INTEGRAL TEST

Suppose $f(x)$ is a cont., positive, decreasing function on $[1, +\infty)$. Let $u_n = f(n)$

If $\int_1^\infty f(x)\,dx$ conv, $\sum_{n=1}^{\infty} u_n$ conv <Check hypo first>

If $\int_1^\infty f(x)\,dx$ div, $\sum u_n$ div

COMPARISON TEST 1

Let $\sum_1^\infty x_n$ and $\sum_1^\infty y_n$ be positive series

Suppose $x_n \leq y_n$

- If $\sum y_n$ conv $\Rightarrow \sum x_n$ conv
- If $\sum x_n$ div $\Rightarrow \sum y_n$ div

COMPARISON TEST 2

Suppose $\exists\, l = \lim \frac{x_n}{y_n} > 0 \rightarrow$ Both x_n, y_n div or conv.

USUALLY COMPARED WITH:

$$\begin{cases} \sum_{n=1}^{\infty} q^n \begin{cases} |q|<1 & \text{conv} \\ |q|\geq 1 & \text{div} \end{cases} \\ \sum_{n=1}^{\infty} \frac{1}{n^\alpha} \begin{cases} \alpha > 1 & \text{conv} \\ \alpha \leq 1 & \text{div} \end{cases} \end{cases}$$

RATIO TEST (d'Alambert)

let $\boxed{\exists L = \lim_{n\to\infty} \left| \frac{x_{n+1}}{x_n} \right|} \longrightarrow$

$L<1$: (abs) conv

$L>1$: div

$L=1$: inconclusive

COMMON:

$$\lim_{n\to\infty} \left(1+\frac{\alpha}{n}\right)^n = e^{\alpha}$$

ROOT TEST (Cauchy)

let $\boxed{\exists L = \lim \sqrt[n]{|x_n|}} \longrightarrow$

$L<1$: (abs) conv.

$L>1$: div

$L=1$: inconclusive

COMMON:

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} \sqrt[n]{a} = 1,\ a>0$$

ALTERNATING SERIES

$$\sum_{1}^{\infty} (-1)^n x_n,\ x_n > 0 \xrightarrow[\text{Criterion}]{\text{Leibniz's}} \text{. If } \begin{cases} x_{n+1} \le x_n \quad \forall n \\ \lim_{n\to\infty} x_n = 0 \end{cases} \Rightarrow \text{Conv.}$$

$\to$ Derivative

ABS. CONVERGENCE

$\sum |x_n|$ conv $\to$ $\sum x_n$ abs. conv

COND. CONVERGENCE

$$\begin{cases} \sum |x_n| \text{ div} \\ \sum x_n \text{ conv} \end{cases} \Rightarrow \sum x_n \text{ cond. conv.}$$

SERIES OF FUNCTIONS

DEFINITION

Suppose $(f_n(x))_{n=1}^{\infty}$ is a sequence of functions, $\sum_{n=1}^{\infty} f_n(x)$: formal sum

Then $\left(S_n(x) = \sum_{j=1}^{n} f_j(x)\right)_{n=1}^{\infty}$ is a sequence of partial sums

$\exists \lim_{n\to\infty} S_n(x) = f(x) \rightarrow \exists$ as a function $\Rightarrow$ The series $\sum_{n=1}^{\infty} f_n(x)$ conv.

- the set of values of x for which $\sum f_n(x)$ converges is called the domain of convergence.

↳ Use the same criteria and tests for series of numbers.

NOTE: At end points, we need to consider separately (Root and Ratio Test do not include those endpoints).

UNIFORM CONVERGENCE (PRIMITIVE DEFINITION)

Suppose $\sum f_n(x)$ is a series of functions and is in a domain $\mathcal{D}$.

Then: $\forall \varepsilon > 0, \exists N = N(\varepsilon)$: if $n > N$: $\left| \sum_{j=1}^{n} f_j(x) - f(x) \right| < \varepsilon$

ε : accuracy

N: how far to go to approach lim

↳ If $N = N(\varepsilon)$ depends on ε only, but not x, we say that the series uniformly converges to $f(x)$.

Weierstrass Criterion

For $\sum f_n(x)$, Suppose we can find a sequence $(T_n)_{n=1}^{\infty}$ s.t:

$$\begin{cases} |f_n(x)| \le T_n \\ \sum_{n=1}^{\infty} T_n \text{ converges} \end{cases} \longrightarrow \text{The series } \sum f_n(x) \text{ conv. uni.}$$

(SUFFICIENT CONDITION)

SUM OF THE SERIES

If $S(x) = \sum_{n=1}^{\infty} (\alpha n + 1) x^{\alpha n} \longrightarrow \int_0^x S(t)\,dt = ?$

as $\int_0^x (\alpha n + 1) t^{\alpha n}\,dt = x^{\alpha n + 1}$

If $S(x) = \sum_{n=1}^{\infty} \frac{x^{\alpha n}}{\alpha n} \longrightarrow S'(x) = ?$

as $\left(\frac{x^{\alpha n}}{\alpha n}\right)' = x^{\alpha n - 1}$

TAYLOR EXPANSION

Let $\boxed{t = x - x_0}$ $\rightarrow$ Find Maclaurin series

POWER SERIES

DEFINITION

$$\sum_{n=1}^{\infty} c_n (x - x_0)^n \quad (1)$$

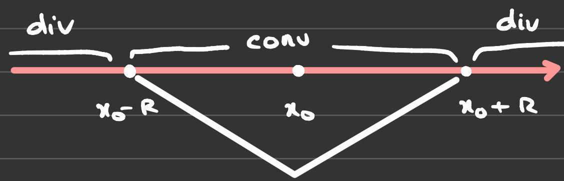
c_n : coefficient

x_0 : centre

ABEL'S THEOREM

Consider a power series (1), there are 3 possibilities

1) The series converges only when $x = x_0$.
2) The series converges when $|x - x_0| < R$ → R: Radius of convergence
   diverges when $|x - x_0| > R$
3) The series converges $\forall\, x \in \mathbb{R}$



CHECK FOR CONV !

THEOREM Suppose (1) has $R \in [0, \infty)$

→ (1) conv. uni. on $(x_0 - R,\ x_0 + R)$

$\sum_{n=1}^{\infty} u_n(x)$ is a power series ②

CONTINUITY	$\begin{cases} \text{② uni.conv on } [a,b] \\ u_n(x) \text{ continuous on } [a,b] \end{cases}$ → ② continuous on $[a.b]$
DIFFERENTIABILITY	$\begin{cases} \text{② uni.conv. on } [a,b] \\ u_n(x) \text{ has continuous derivative } u_n'(x) \text{ on } [a,b] \end{cases}$ → ② is differentia-ble $S'(x) = \sum_{n=1}^{\infty} u_n'(x)$
INTEGRABILITY	$\begin{cases} \text{② uni.conv on } [a,b] \\ u_n(x) \text{ continuous on } [a,b] \end{cases}$ → ② is integrable on $[a,b]$ $\int_a^b S(x) = \sum_{n=1}^{\infty} \int_a^b u_n(x)$
	② is infinitely many times differentiable, its derivatives has the same R.

TAYLOR, MACLAURIN SERIES	Let $f(x) = \sum_{n=0}^{\infty} c_n (x - x_0)^n$ be a power series with $R > 0$ Then $c_n = \frac{f^{(n)}(x_0)}{n!}$ $\Rightarrow$ TAYLOR'S FORMULAE: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$ If $x_0 = 0$ $\longrightarrow$ MACLAURIN'S EXPANSION $\longleftarrow$ $t = x - x_0$
CONVERGENCE OF POWER SERIES	Suppose $\lvert f^n(x) \rvert < L$ $(L > 0)$, $\forall n \in \mathbb{N}$ for $\lvert x - x_0 \rvert < d$ $(d > 0)$ Then $\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$ conv. to $\underline{\underline{f(x)}}$

FOURIER SERIES

DEFINITIONS	The TRIGONOMETRIC SERIES $S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ with the coefficients: $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} S(x)\, dx$ $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} S(x) \cos nx\, dx$ $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} S(x) \sin nx\, dx$ is called a FOURIER SERIES
DIRITCHLET'S THEOREM	DIRITCHLET'S CONDITION $f(x)$ $\begin{cases} \text{periodic}, \ p = 2\pi \\ \text{piecewise continuously differentiable} \\ \text{finitely many discontinuities} \end{cases}$ THEOREM $f(x)$ at x_0 is continuous : $S(x)$ conv. to $f(x)$ $S_f(x_0) = f(x)$ $f(x)$ at x_0 is discontinuous: $S(x)$ conv. $S_f(x_0) = \frac{f(x_0^-) + f(x_0^+)}{2}$

FOURIER EXPANSION

— ODD, EVEN FUNC

odd → Sine Series $\quad S_f(x) = \sum_{n=1}^{\infty} b_n \sin(nx) \quad a_n = 0$

Even → Cosine Series $\quad S_g(x) = \sum_{n=1}^{\infty} a_n \cos(nx) + \frac{a_0}{2} \quad b_n = 0$

DON'T FORGET ME ;)

— GENERAL PERIODIC

Period = 2L $\quad S_f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}$

where:

$$a_0 = \frac{2}{2L} \int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{2}{2L} \int_{-L}^{L} f(x) \cos\frac{n\pi}{L}x \,dx$$

$$b_n = \frac{2}{2L} \int_{-L}^{L} f(x) \sin\frac{n\pi}{L}x \,dx$$

— IN AN INTERVAL

Suppose $f(x)$ is a function defined on $[a,b]$

we may extend it to a periodic function $\tilde{f}(x)$ with $p \geq (b-a)$

→ FIND THE EXPANSION $S_{\tilde{f}}(x)$, then its sum $= f(x)$ at $x \in [a,b]$

CHAPTER 2: FIRST-ODER DEs

SEPARABLE EQUATIONS	$f(x)\,dx = g(y)\,dy$ → INTEGRATE TWO SIDES : $F(x) = G(y) + K$ is the solution
EQUATIONS WITHOUT y	$F(x, y') = 0$ → Solve for x : $x = g(y')$ SEPARABLE $t = y'(x)$ → SEPARABLE y and t Solve for y' : $y' = h(x)$! DOUBLE-CHECK THE FINAL SOLUTION
EQUATIONS WITHOUT x	$F(y, y') = 0$ → Solve for y : $y = g(y')$ SEPARABLE $t = \frac{dy}{dx}$ → SEPARABLE x and t Solve for y' : $y' = h(y)$

LINEAR DIFFERENTIAL EQUATIONS

GENERAL FORM: $y' + a(x)\,y = b(x)$

a, b constant	$y' + a.y = b \rightarrow$ multiply both sides by e^{ax}, integrate
a const	$y' + ay = b(x) \rightarrow$ multiply both sides by e^{ax}
a, b are functions	$y' + P(x)\,y = Q(x)$ (I)

INTEGRATING FACTOR: $\times\ I(x) = e^{\int P(x)\,dx}$

VARIATION OF PARAMETERS: (H): $y' + P(x)\,y = 0 \Rightarrow y = K\exp\left(-\int P(x)\,dx\right)$

Substitute to (I) $\rightarrow$ Solve for $K(x)$

! DOUBLE CHECK

NOTE: If $\frac{dy}{dx} = f\left(\frac{y}{x}\right) \rightarrow$ Let $u = \frac{y}{x} \Rightarrow \boxed{y' = u'x + u}$

BERNOULLI'S EQUATION

$$y' + p(x)\,y = q(x)\,y^{\alpha}$$

$\alpha = 0, 1$: Linear first-order

$\alpha \neq 0, 1$: Nonlinear

$$\xrightarrow[z = y^{1-\alpha}]{\times y^{-\alpha}} \quad \frac{1}{1-\alpha}\frac{dz}{dx} + p(x)\,z = q(x)$$

LINEAR IN TERMS OF $\underline{\underline{z}}$

TOTAL DIFFERENTIALS

$$P\,dx + Q\,dy = 0$$

$P_y \neq Q_x$: Exact DE

$\downarrow$ $P\,dx + Q\,dy = du$

$$u(x,y) = \int_{x_0}^{x} P(u, y_0)\,du + \int_{y_0}^{y} Q(x, v)\,dv$$

$$u(x,y) = \int_{x_0}^{x} P(u, y)\,du + \int_{y_0}^{y} Q(x_0, v)\,dv$$

$P_y \neq Q_x$: Non-exact

$$\boxed{\frac{P_y - Q_x}{Q} = \underline{\underline{g(x)}}} \quad \longrightarrow \quad I(x) = \exp\left(\int g(x)\,dx\right)$$

$$\boxed{\frac{P_y - Q_x}{-P} = h(\underline{\underline{y}})} \quad \longrightarrow \quad I(y) = \exp\left(\int h(y)\,dy\right)$$

CHAPTER 3: SECOND-ORDER DE

$$F(x, y, y', y'') = 0$$

LINEAR SECOND-ORDER DE

(I): $y'' + p(x)y' + q(x) = f(x)$ → Find solution y_c of (H), particular solution y_p of (I)
→ General solution: $y = y_c + y_p$

HOMOGENEOUS, CONST COEFFICIENTS

$ay'' + by' + cy = 0$ → Auxiliary equation: $ar^2 + br + c = 0$

SOLUTIONS OF AUX. EQ	GENERAL SOLUTION
2 distinct roots: r_1, r_2	$y = k_1 e^{r_1 x} + k_2 e^{r_2 x}$
Double root: $r = r_1 = r_2$	$y = k_1 e^{rx} + k_2 x e^{rx}$
2 complex roots: $\alpha \pm i\beta$	$y = k_1 e^{\alpha x} \cos \beta x + k_2 e^{\alpha x} \sin \beta x$

INHOMOGENEOUS, CONST COEFFICIENTS

$y'' + p(x)y' + q(x)y = f(x)$

FIND y_c: Using auxiliary equations

FIND y_p:
- → undetermined coeff
- → Variation of parameters

UNDETERMINED COEFFICIENTS

1. If $f(x) = e^{kx} \underbrace{P(x)}_{P_n}$ $\rightarrow$ FIND A sol: $y_p = e^{kx} \underbrace{Q(x)}_{P_n} \times \left[\begin{array}{l} x \\ x^2 \end{array} \right.$

If $f(x)$ is a solution to (H):

Single root: Multiply x

Double x^2

2. If $f(x) = e^{kx} P(x) \cos(mx)$ or $e^{kx} P(x) \sin(mx)$ $\rightarrow$ FIND $y_p = \left(e^{kx} Q(x) \cos(mx) + e^{kx} R(x) \sin(mx)\right) \times \left[\begin{array}{l} x \\ x^2 \end{array} \right.$

VARIATION OF PARAMETERS

$y_c = k_1 y_1(x) + k_2 y_2(x)$ is a sol. to (H)

$\rightarrow$ FIND $y_p = \underbrace{u_1(x)}_{\text{const}} y_1(x) + \underbrace{u_2(x)}_{\text{const}} y_2(x)$

$\rightarrow$ SOLVE $\begin{cases} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = f(x) \end{cases}$

SECOND ORDER ODE

— WITHOUT y, y'

$F(x, y'') = 0$

- $\rightarrow y'' = g(x) \rightarrow y' \rightarrow y$
- $\rightarrow x = h(y'') \rightarrow$ CONVERT TO FIRST ORDER DE
  $p = y' \rightarrow p' = y'' \rightarrow x = h(p')$

— WITHOUT y

$F(x, y', y'') = 0 \rightarrow p = y' \rightarrow p' = y'' \rightarrow F(x, p, p') = 0$

— WITHOUT x

$F(y, y', y'') = 0 \rightarrow p = y' \rightarrow p'_x = y'' \rightarrow y'' = \frac{dp}{dy} \cdot p$

$\rightarrow F\left(y, p, \frac{dp}{dy} \cdot p\right) = 0$

LINEAR SECOND-ORDER
GENERAL COEFFICIENTS

$$y'' + p(x)\,y' + q(x)\,y = f(x) \qquad (I)$$

Consider $y'' + p(x)\,y' + q(x)\,y = 0 \qquad (H)$

If y_1 and y_2 are linearly independent solutions of (H)
Then $y = k_1 y_1 + k_2 y_2$ is a general solution to (H) ; k_1, k_2: const

TEST FOR LINEARLY INDEPENDENCE ?
↳ WRONSKI'S DETERMINANT

$$w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0 \rightarrow \underline{\underline{LI}}$$

LIOUVILLE'S FORMULAE

$$y_2 = y_1 \int \frac{1}{y_1^2} \exp\left(-\int p(x)\,dx\right) dx$$

(y_1, y_2 are $\underline{\underline{LI}}$ and $\neq 0$)
→ we can always find LI solutions (thus general sol.) to (H)

SOLVE (I) LAGRANGE'S VARIATION OF PARAMETERS

$$\begin{cases} k_1' y_1 + k_2' y_2 = 0 \\ k_1' y_1' + k_2' y_2' = f(x) \end{cases} \rightarrow \text{particular solution}$$

General solution (I) = General solution (H) + Particular sol. (I)

SUPPOSITION

$$\left.\begin{aligned} y_1 &\xrightarrow{\text{sol.}} y'' + p(x)\, y' = f_1(x) \\ y_2 &\to y'' + p(x)\, y' = f_2(x) \end{aligned}\right\} \quad y = y_1 + y_2 \to y'' + p(x)\, y' = f_1(x) + f_2(x)$$

CHEBYSHEV

$$(1-x^2)\, y'' - xy' + n^2 y = 0 \to y = C_1 \cos(n \arccos x) + C_2 \sin(n \arccos x)$$

EULER

$$x^2 y'' + a x y' + b y = 0 \ , \ a, b \in \mathbb{R}$$

$$\Leftrightarrow \quad y'' + \frac{a}{x} y' + \frac{b}{x^2} y = 0 \quad \Rightarrow \quad \text{Put: } t = \ln|x|$$

$$\begin{cases} x y' = \dfrac{dy}{dt} \\ x^2 y'' = \dfrac{d^2 y}{dt^2} - \dfrac{dy}{dt} \end{cases}$$

CHAP 4: SYSTEM OF FIRST-ORDER ODES

SYSTEM OF FIRST-ORDER DIFF. EQUATIONS

Ex: $\begin{cases} y' = a \cdot y + b \cdot z & (1) \\ z' = c \cdot y + d \cdot z & (2) \end{cases}$

Differentiate (1): $y'' = a \cdot y' + b \cdot z' \xrightarrow{(2)} y''$: Linear ODE, 2-order

$\downarrow$

$z = ? \xleftarrow{(2)}$ Solve for general sol.

SPECIAL CASE: LINEAR

$$\begin{cases} y' = a \cdot y + b \cdot z \\ z' = c \cdot y + d \cdot z \end{cases} \rightarrow A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Characteristic equation: $\det(A - \lambda \cdot I) = 0 \rightarrow$ λ_1, λ_2 : eigen values

$\vec{v}_1, \vec{v}_2$: eigen vectors

Then $\vec{y}_1 = \vec{v}_1 \cdot e^{\lambda_1 x}$ $\qquad \vec{y}_2 = \vec{v}_2 \cdot e^{\lambda_2 x}$

General solution: $\vec{y} = k_1 \vec{y}_1 + k_2 \vec{y}_2$

CHAP 5: LAPLACE TRANSFORM

DEFINITIONS

$f(t): \mathbb{R}_{\geq 0}$ or $\mathbb{R}_{>0} \to F(s) = \underbrace{\mathcal{L}[f(t)]}_{\text{Laplace transform of } f(t)} = \int_0^{\infty} f(t)\, e^{-st}\, dt$

$$= \lim_{T \to \infty} \int_0^T f(t)\, e^{-s \cdot t}\, dt$$

LINEARITY $\mathcal{L}\left[k_1 \cdot f_1(t) + k_2 \cdot f_2(t) \right] = k_1 \mathcal{L}[f_1(t)] + k_2 \mathcal{L}[f_2(t)]$

SOME COMMON LAPLACE TRANSFORM

$f(t)$	$F(s)$	s
1	$1/s$	$s > 0$
t	$1/s^2$	$s > 0$
t^n $(n \in \mathbb{N})$	$\frac{n!}{s^{n+1}}$	$s > 0$
t^α $(\alpha > -1)$	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$	$s > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$
$\cos kt$	$\frac{s}{s^2+k^2}$	$s > 0$
$\sin kt$	$\frac{k}{s^2+k^2}$	$s > 0$

$\cosh(kt)$	$\frac{s}{s^2-k^2}$	$s > \lvert k \rvert$
$\sinh(kt)$	$\frac{k}{s^2-k^2}$	$s > \lvert k \rvert$
$u(t-a) \quad (a>0)$	$\frac{e^{-as}}{s}$	$s > 0$
$\frac{1}{2k^3}(\sin kt - kt\cos kt)$	$\frac{1}{(s^2+k^2)^2}$	$s > 0$

IVP WITH LAPLACE TRANSFORM

$f'(t)$ piecewise continuous on $[0, +\infty)$, $\mathcal{L}[f(t)] = F(s)$

$$\mathcal{L}[f'(t)] = s \cdot \mathcal{L}[f(t)] - f(0)$$

$$\mathcal{L}[f''(t)] = s^2 \cdot \mathcal{L}[f(t)] - s \cdot f(0) - f'(0)$$

$$\rightarrow \mathcal{L}[f^{(n)}(t)] = s^n \cdot \mathcal{L}[f(t)] - \left(f^{(n-1)}(0) + s \cdot f^{(n-2)}(0) + \dots + s^{n-1} f(0) \right)$$

↳ APPLICATION: SOLVE IVPS $\begin{cases} y' = F(y, t) \\ y(0) \end{cases}$

let $Y = \mathcal{L}[f(t)] \rightarrow \mathcal{L}[f'(t)] = s \cdot Y - y(0)$

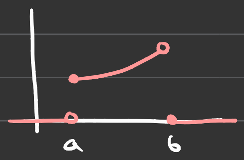
$\rightarrow Y = F(s)$ usually a rational fraction $\rightarrow$ Solve $Y \rightarrow y(t) = \mathcal{L}^{-1}[Y]$

↳ Similarly for high-order equations or system of ODEs

TRANSLATIONS W. LAPLACE TRANS.

$f(t)$	$F(s)$
$e^{kt} f(t)$	$F(s-k)$
$f(t-k)\, u(t-k)$	$F(s)\, e^{-s \cdot k}$
$f(t) \cdot u(t-k)$	$\mathcal{L}[f(t+k)]\, e^{-s \cdot k}$

NOTE :



$$f(t) = \begin{cases} 0, & t < a \\ (\cdot), & a \leq t < b \\ 0 & \end{cases}$$

$\rightarrow$ Can be written as: $f(t) = e^{at}\,[\,u(t-a) - u(t-b)\,]$

DERIVATIVES

$f(t)$	$F(s)$
$f'(t)$	$s\,F(s) - f(0)$
$-t \cdot f(t)$	$F'(s)$
$(-1)^k\, t^k f(t)$	$F^{(k)}(s)$

INTEGRALS

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(\tau)\, d\tau , \qquad s > c$$

$$\rightarrow f(t) = \mathcal{L}^{-1}[F(s)] = t \cdot \mathcal{L}^{-1}\left\{ \int_s^{\infty} F(\tau)\, d\tau \right\}$$

PERIODIC FUNCTIONS

$$f: \text{period} = T \rightarrow \quad F(s)\,(1 - e^{-s.T}) = \int_0^T f(t)\, e^{-s.t}\, dt$$

CONVOLUTION

$$f(t) * g(t) = \int_0^t f(x)\, g(t-x)\, dx$$

$\langle (f*g)(t) \rangle$

$$\rightarrow \mathcal{L}[f(t) * g(t)] = \mathcal{L}[f(t)] \cdot \mathcal{L}[g(t)]$$
$$= F(s) \cdot G(s)$$

$$\rightarrow \mathcal{L}^{-1}[f \cdot g] = \mathcal{L}^{-1}[f] * \mathcal{L}^{-1}[g]$$

PROPERTIES

1) $f * g = g * f$ (commutativity)
2) $(f * g) * h = f * (g * h)$ (associativity)
3) $f * (g + h) = f * g + f * h$ (distributivity)