Đề số 1-Probability and Statistics (MI2026-128433)-GK2021.1

Câu 1 - Level 1

Consider a binary code with 5 bits (0 or 1) in each code word. An example of a code word is 01010. How many code words have exactly two 1's?

Phương án 1:

10

$\binom{5}{2} = 10$

Phương án 2:

32

Phương án 3:

5

Phương án 4:

15

Phương án 5:

24

Câu 2 - Level 1

A statistics class for engineers consists of 15 industrial, 12 mechanical, 13 electrical, and 10 civil engineering students. If a person is randomly selected by the instructor to answer a question, what is the probability that the student chosen is a civil engineering or electrical engineering major?

Phương án 1:

0.46

$\frac{13}{50} + \frac{10}{50} = 0.46$

Phương án 2:

0.44

Phương án 3:

0.50

Phương án 4:

0.54

Phương án 5:

0.30

Câu 3 - Level 1

Suppose that A, B, and C are three independent events such that $P(A) = 1/5$, $P(B) = 1/4$, and $P(C) = 1/3$. What is the probability that none of these three events will occur?

Phương án 1:

2/5

$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A})P(\bar{B})P(\bar{C})$

$= \left(1 - \frac{1}{5}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{3}\right)$

$= \frac{2}{5}$

Phương án 2:

1/60

Phương án 3:

3/5

Phương án 4:

1/2

Phương án 5:

8/15

Câu 4 - Level 1

After being interviewed at two companies, Dung assesses that his probability of getting an offer from company A is 0.5, and his probability of getting an offer from company B is 0.45. If he believes that the probability that he will get offers from both companies is 0.2, what is the probability that he will get at least one offer from these two companies?

Phương án 1:

0.75

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.45 - 0.2$

$= 0.75$

Phương án 2:

0.95

Phương án 3:

0.65

Phương án 4:

0.70

Phương án 5:

0.60

Phương án 6:

0.80

Câu 5 - Level 1

Let A and B be two events such that $P(A) = 0.4$ and $P(B|\overline{A}) = 0.65$. What is the probability of $B\overline{A}$?

Phương án 1:
0.39

Phương án 2:
0.26

Phương án 3:
0.35

Phương án 4:
0.25

Phương án 5:
0.21

Phương án 6:
0.14

$P(B|\overline{A}) = \frac{P(B\overline{A})}{P(\overline{A})}$

$\Rightarrow P(B\overline{A}) = P(B|\overline{A}) \cdot P(\overline{A})$

$= (0.65)(1-0.4) = 0.39$

Câu 6 - Level 1

In a certain assembly plant, two machines, B1 and B2, make 70% and 30%, respectively, of the products. It is known from past experience that 90% and 80% of the products made by each machine, respectively, are non-defective. Now, suppose that a finished product is randomly selected. What is the probability that it is non-defective?

$P(B_1)$ $P(B_2)$

Phương án 1:
0.87

Phương án 2:
0.63

Phương án 3:
0.24

Phương án 4:
0.56

Phương án 5:
0.27

Phương án 6:
0.83

Total Prob: N = non-defective

$P(N) = P(N|B_1)P(B_1) + P(N|B_2)P(B_2)$

$= (0.9)(0.7) + (0.8)\cdot(0.3)$

$= 0.87$

Câu 7 - Level 1

In which of the following circumstances are the events A and B independent?

Phương án 1:
$P(A) = 0.3,\ P(B) = 0.7,\ P(AB) = 0.21$ ✓

Phương án 2:
$P(A) = 0.7,\ P(A|B) = 0.9$

Phương án 3:
$P(A) = 0.3,\ P(B) = 0.5,\ P(AB) = 0.2$

Phương án 4:
$P(A) = 0.35,\ P(B) = 0.65,\ P(AB) = 0.20$

Phương án 5:
$P(B) = 0.4,\ P(B|A) = 0.5$

$\rightarrow P(A \cap B) = P(A)\,P(B)$
$\rightarrow P(A|B) = P(A)$
$\rightarrow P(B|A) = P(B)$

Câu 8 - Level 1

The random variable X has the probability distribution

X	1	2	3	4	5
P	0.25	0.15	0.20	0.30	c

Find the probability that X is 2 or 5.

Phương án 1:
0.25

Phương án 2:
0.15

Phương án 3:
0.10

Phương án 4:
0.75

Phương án 5:
0.65

Phương án 6:
0.30

$c = 1 - (.25 + .15 + .20 + .30)$
$= 0.1$
$\rightarrow P(X=2 \text{ or } X=5) = P(X=2) + P(X=5)$
$= 0.15 + 0.1$
$= 0.25$

Câu 9 - Level 1

The random variable X has the probability distribution

X	-2	0	2	4
P	0.20	0.15	0.25	0.40

Find the expected value of the random variable $g(X) = 2X - 1$.

Phương án 1:
2.4

Phương án 2:
1.7

Phương án 3:
3.4

Phương án 4:
3.0

Phương án 5:
0.85

Phương án 6:
1.0

$$E[g(X)] = \sum_{k \in R_X} (2X-1) P_X(x)$$

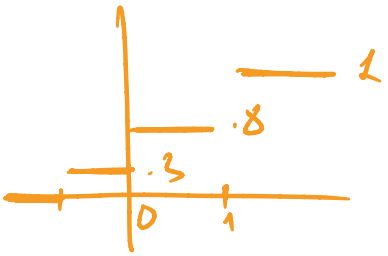
$$= (2(-2)-1)(0.20) + (2.0-1)(0.15)$$

$$+ (2.2-1)(0.25) + (2.4-1)(0.40)$$

$$= 2.4$$

Câu 10 - Level 1

Suppose the cumulative distribution function of the random variable X is



$$F_X(x) = \begin{cases} 0, & x \le -1, \\ 0.3, & -1 < x \le 0, \\ 0.8, & 0 < x \le 1, \\ 1.0, & x > 1. \end{cases}$$

Write $P_X(x)$, the probability mass function of X. Be sure to write the value of $P_X(x)$ for all x from $-\infty$ to $+\infty$.

$P_X(x=-1) = .3 - 0 = .3$

$P_X(x=0) = .8 - .3 = .5$

$P_X(x=1) = 1 - .8 = .2$

Phương án 1:

$$P_X(x) = \begin{cases} 0.3, & x = -1, \\ 0.5, & x = 0, \\ 0.2, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Phương án 2:

$$P_X(x) = \begin{cases} 0.3, & x = -1, \\ 0.8, & x = 0, \\ 1.0, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Phương án 3:

$$P_X(x) = \begin{cases} 0, & x = -1, \\ 0.3, & x = 0, \\ 0.5, & x = 1, \\ 0.2, & \text{otherwise.} \end{cases}$$

Phương án 4:

$$P_X(x) = \begin{cases} 0.3, & x = -1, \\ 0.2, & x = 0, \\ 0.5, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Phương án 5:

$$P_X(x) = \begin{cases} 0.5, & x = -1, \\ 0.2, & x = 0, \\ 0.3, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Phương án 6:

$$P_X(x) = \begin{cases} 0.5, & x = -1, \\ 0.3, & x = 0, \\ 0.2, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Câu 11 - Level 1

Suppose the cumulative distribution function of the random variable X is

$$F_X(x) = \begin{cases} 0, & x \leq -1, \\ \dfrac{x+1}{4}, & -1 < x \leq 3, \\ 1, & x > 3. \end{cases}$$

What is $P(0 < X < 1)$?

Phương án 1:

$1/4$

$F(1) - F(0) = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

Phương án 2:

$2/3$

Phương án 3:

$1/2$

Phương án 4:

$2/5$

Phương án 5:

$1/3$

Phương án 6:

$1/5$

Câu 12 - Level 1

The random variable X has the probability density function (PDF)

$$f_X(x) = \begin{cases} cx, & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the PDF to find c and $E(X)$, the expected value of the random variable X.

Phương án 1:

$c = 1/2,\ E(X) = 4/3$

Phương án 2:

$c = 1/2,\ E(X) = 3/4$

Phương án 3:

$c = 1/2,\ E(X) = 1/2$

Phương án 4:

$c = 2,\ E(X) = 4/3$

Phương án 5:

$c = 2,\ E(X) = 3/4$

Phương án 6:

$c = 2,\ E(X) = 1/2$

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$$

$$\Rightarrow \quad c\frac{x^2}{2}\Big|_0^2 = 1$$

$$2c = 1 \Rightarrow c = \frac{1}{2}$$

$$\Rightarrow E = \int_0^2 \frac{1}{2}x^2$$

$$= \frac{1}{2}\frac{x^3}{3}\Big|_0^2 = \frac{4}{3}$$

Câu 13 - Level 1

Suppose the cumulative distribution function of the random variable X is

$$F_X(x) = \begin{cases} 0, & x \leq 0, \\ 1 - ce^{-0.01x}, & x > 0, \end{cases}$$

where c is a constant. Determine c and the probability density function of X.

Phương án 1:

$$c = 1,\ f_X(x) = \begin{cases} 0, & x \leq 0, \\ 0.01e^{-0.01x}, & x > 0, \end{cases}$$

Phương án 2:

$$c = 1,\ f_X(x) = \begin{cases} 0, & x \leq 0, \\ e^{-0.01x}, & x > 0, \end{cases}$$

Phương án 3:

$$c = 1,\ f_X(x) = \begin{cases} 0, & x \leq 0, \\ -0.01e^{-0.01x}, & x > 0, \end{cases}$$

Phương án 4:

$$c = 1/2,\ f_X(x) = \begin{cases} 0, & x \leq 0, \\ 0.01e^{-0.01x}, & x > 0, \end{cases}$$

Phương án 5:

$$c = 1/2,\ f_X(x) = \begin{cases} 0, & x \leq 0, \\ e^{-0.01x}, & x > 0, \end{cases}$$

Phương án 6:

$$c = 1/2,\ f_X(x) = \begin{cases} 0, & x \leq 0, \\ -0.01e^{-0.01x}, & x > 0, \end{cases}$$

+ $f_X(x) = \dfrac{dF_X(x)}{x}$

$$= \begin{cases} 0 & , x \leq 0 \\ (0.01)\, c e^{-0.01x} & , x > 0 \end{cases}$$

→ only $c = 1$ satisfies

Câu 14 - Level 1

The random variable X has a binomial distribution with $n = 10$ and $p = 0.1$. What is $P(5 \leq X < 8)$?

Phương án 1:
0.0016346

$\binom{10}{5}(0.1)^5(0.9)^5 + \binom{10}{6}(0.1)^6(0.9)^4 + \binom{10}{7}(0.1)^7(0.9)^3$

$= 0.0016345638$

Phương án 2:
0.0016349

Phương án 3:
0.0014880

Phương án 4:
0.0016258

Phương án 5:
0.0001378

Phương án 6:
0.9998622

Câu 15 - Level 1

Suppose X has an exponential distribution with mean equal to 10. What is $P(X > 20)$?

$\lambda = 0.1$

Phương án 1:
0.1353

$P(X > 20) = 1 - P(X \leq 20)$

$= 1 - (1 - e^{-(0.1)\cdot 20})$

$= 0.1353$

Phương án 2:
0.9048

Phương án 3:
0.9512

Phương án 4:
0.9200

Phương án 5:
0.0952

Phương án 6:
0.3679

Câu 16 - Level 2

What is the probability of exactly one empty box when 3 balls are randomly placed in 3 boxes?

Phương án 1:
2/3

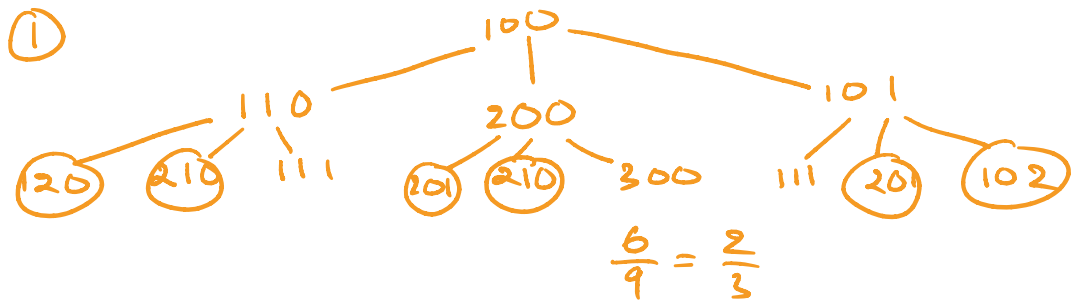
Phương án 2:
1/3

Phương án 3:
1/6

Phương án 4:
1/2

Phương án 5:
1/9

Phương án 6:
1/4



$\frac{6}{9} = \frac{2}{3}$

Or: (2) Possible permutation $= 3^3 = 27$

To have 1 empty box, we need to have a box containing 2 balls:

$\binom{3}{1} \cdot \binom{3}{2} \cdot \binom{2}{1} \; (3-2)!$

↓ empty box ↓ choose 2 balls ↘ box with 2 balls ↘ balls to other boxes

$\Rightarrow \frac{2}{3}$

Câu 17 - Level 2

Let A and B be two events such that $P(A) = 0.6$ and $P(A\overline{B}) = 0.25$. What is the probability of $\overline{A} + \overline{B}$?

Phương án 1:
0.65

Phương án 2:
0.45

Phương án 3:
0.85

Phương án 4:
0.40

Phương án 5:
0.75

Phương án 6:
0.70

$P(\overline{A} \cup \overline{B}) = 1 - P(\overline{A \cup \overline{B}})$

$= 1 - P(A \cap B)$ (∵ Demorgan)

$= 1 - P(A)P(B|A)$

$= 1 - (0.6) \cdot \frac{7}{12} = 0.65$

$P(A\overline{B}) = P(A) \cdot P(\overline{B}|A)$

$\Rightarrow P(\overline{B}|A) = \frac{0.25}{0.6} = \frac{5}{12}$

$\Rightarrow P(B|A) = 1 - \frac{5}{12} = \frac{7}{12}$

Câu 18 - Level 2

A certain electric system contains 10 components. Suppose that the probability that each individual component will fail is 0.2 and that the components fail independently of each other. What is the probability that at least two of the components have failed?

Phương án 1:

0.6242

$\Leftrightarrow$ 10 Bernoulli trials, with $p = 0.2$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - \left(\binom{10}{0}(0.2)^0 (0.8)^{10} + \binom{10}{1}(0.2)^1 (0.8)^9 \right)$

$= 1 - 0.376 = 0.6242$

Phương án 2:

0.3758

Phương án 3:

0.8926

Phương án 4:

0.2684

Phương án 5:

0.6424

Phương án 6:

0.3558

Câu 19 - Level 2

An urn contains 4 black and 6 brown balls. A ball is selected at random. If the ball drawn is brown, it is replaced and 4 additional brown balls are also put into the urn. If the ball drawn is black, it is not replaced in the urn and no additional balls are added. A ball is then drawn from the urn the second time. What is the probability that the ball selected at the second stage is brown?

Phương án 1:
0.6952

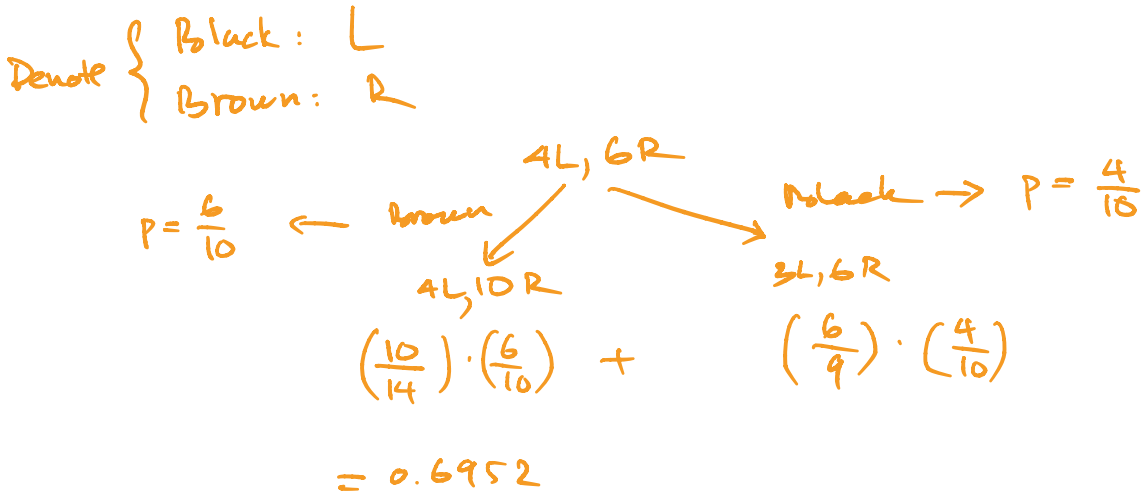
Phương án 2:
0.4286

Phương án 3:
0.2667

Phương án 4:
0.6000

Phương án 5:
0.6821

Phương án 6:
0.6925



Câu 20 - Level 2

Consider two urns. The first contains 2 white and 7 black balls, and the second contains 5 white and 6 black balls. We flip a fair coin and then draw a ball from the first urn or the second urn depending on whether the outcome was heads or tails. What is the conditional probability that the outcome of the toss was heads given that a white ball was selected?

Phương án 1:
22/67

Phương án 2:
45/67.

Phương án 3:
77/131

Phương án 4:
57/131

Phương án 5:
22/45.

Phương án 6:
57/77

W: white ball selected
H: heads received

Heads → first urn

Tails → second urn

$$\Rightarrow \text{Find } P(H|W) = \frac{P(H)\cdot P(W|H)}{P(W)}$$

$$= \frac{(0.5)\cdot(2/9)}{(\frac{2}{9})(0.5)+(\frac{5}{11})(0.5)} = \frac{22}{67}$$

LOTP

$$P(W) = P(W|H)P(H) + P(W|T)P(T)$$

Câu 21 - Level 2

Let X be a random variable denoting the proportion of students in a class who get a grade lower than C. Suppose X is a random variable with the following probability density function:

$$f_X(x) = \begin{cases} kx(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

A class is assumed to be unsuccessful if the proportion of students with a grade lower than C is greater than 0.5. What is the probability that a class is unsuccessful?

Phương án 1:
0.50

Phương án 2:
0.40

Phương án 3:
0.60

Phương án 4:
0.45

Phương án 5:
0.55

Phương án 6:
0.65

we have: $\int_0^1 (kx - kx^2)\,dx = 1$

$\Rightarrow \left(\frac{kx^2}{2} - \frac{kx^3}{3}\right)\Big|_0^1 \Leftrightarrow \frac{k}{2} - \frac{k}{3} = 1$

$\Rightarrow k = 6$

$\Rightarrow F_X(x) = \begin{cases} 3x^2 - 2x^3, & 0 \le x \le 1 \\ 0, & \text{ow} \end{cases}$

$P(X > 0.5) = 1 - P(X \le 0.5)$

$= 1 - (3.(0.5)^2 - 2.(0.5)^3)$

$= 0.5$

Câu 22 - Level 2

At a county fair, a ring toss game may be played for 25 cents. You are given three rings and then attempt to toss them individually onto a peg. If you successfully get one ring on a peg, you win a prize worth 50 cents. If you get two on, you get a prize worth 100 cents and if you get all three on, you win a prize worth 500 cents. Assuming the probability that you ring the peg is 0.1 each try, what is your expected gain if you play this game five times?

Phương án 1:
−48.25 cents

Phương án 2:
0.3 cents

Phương án 3:
1.5 cents

Phương án 4:
9.65 cents

Phương án 5:
−9.65 cents

Phương án 6:
48.25 cents

Consider one play, let X be the money gained

then:

$$P_X(x) = \begin{cases} \binom{3}{0}(.1)^0(.9)^3, & x = -25 \\ \binom{3}{1}(.1)^1\cdot(.9)^2, & x = 25 \\ \binom{3}{2}(.1)^2(.9)^1, & x = 75 \\ \binom{3}{3}(.1)^3\cdot(.9)^0, & x = 475 \end{cases}$$

$$\Rightarrow E[X] = \sum_{t \in \Omega_X} t \, P_X(x=t) = -9.65$$

LOE → Expected gain after 5 games $= (-9.65)\cdot 5 = -48.25$

Câu 23 - Level 2

Suppose that the number of customers that arrive at a retail store has a Poisson distribution. The past statistics show that 2% of the customers do not purchase anything. If 700 customers arrive within a day, what is the probability that there are 5 of them will not purchase anything? Choose two solution.

Phương án 1:
0.00373

Phương án 2:
0.00353

Phương án 3:
0.00451

Phương án 4:
0.00453

Phương án 5:
0.99627

Phương án 6:
0.99647

+ Suppose that the # customers that buy nothing follows a Binomial distribution

where $n = 700$, $p = 0.02$

$\Rightarrow B(700, 0.02) \approx P(700 \cdot 0.02)$

$= P(14)$

$\Rightarrow P_X(X = 5) = e^{-14} \cdot \frac{14^5}{5!} \approx 0.00373$

+ $p(X = 5) = \binom{700}{5} (0.02)^5 (0.98)^{695} \approx 0.00353$

Câu 24 - Level 2

with ⇒ Negative Binomial Distribution

A box contains 7 light bulbs, of which 2 are bad. The bulbs are tested one after another without replacement. Let X be the number of bulbs tested to locate the second bad bulb. What is $E(X)$, the expected value of the random variable X?

Phương án 1:
5.3333

Phương án 2:
6.3333

Phương án 3:
4.3333

Phương án 4:
3.3333

Phương án 5:
7.3333

Phương án 6:
2.3333

$\Omega_X = \{2,3,4,5,6,7\}$

$P(X=2) = \frac{2}{7}\cdot\frac{1}{6} = \frac{1}{21}$

$P(X=3) = \frac{2}{7}\cdot\frac{5}{6}\cdot\frac{1}{5} + \frac{5}{7}\cdot\frac{2}{6}\cdot\frac{1}{5} = \frac{2}{21}$

$P(X=4) = \frac{2}{7}\cdot\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{1}{4} + \frac{5}{7}\cdot\frac{2}{6}\cdot\frac{4}{5}\cdot\frac{1}{4} + \frac{5}{7}\cdot\frac{4}{6}\cdot\frac{2}{5}\cdot\frac{1}{4} = \frac{3}{21}$

$P(X=5) = \left(\frac{2}{7}\cdot\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{5}\right)\cdot 4 = \frac{4}{21}$

$P(X=6) = \left(\frac{2}{7}\cdot\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\right)\cdot 5 = \frac{5}{21}$

$P(X=7) = \left(\frac{2}{7}\cdot\frac{5}{6}\cdot\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}\cdot\frac{1}{2}\cdot 1\right)\cdot 6 = \frac{6}{21}$

$\Rightarrow E[X] = 16/3 \approx 5.33$

Câu 25 - Level 3

Let A and B be two events such that $P(AB) = 0.4$, $P(\overline{A}\,\overline{B}) = 0.3$. Furthermore $A\overline{B}$ and $\overline{A}B$ are equally likely. What is the probability of $\overline{A}$?

Phương án 1:
0.45

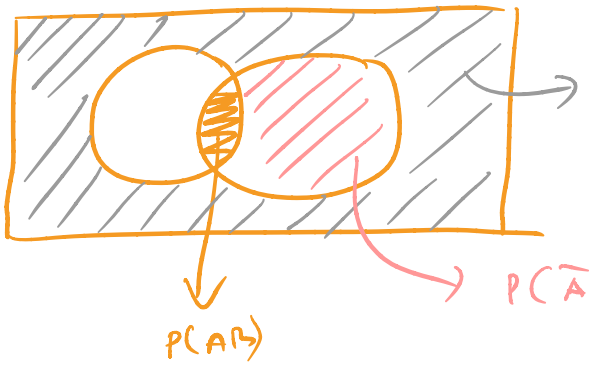
Phương án 2:
0.55

Phương án 3:
0.60

Phương án 4:
0.90

Phương án 5:
0.70

Phương án 6:
0.50



$P(\overline{A}B) = P(A\overline{B}) = \frac{(0.4-0.3)}{2} = 0.15$

$P(\overline{A}) = P(\overline{A}B) + P(\overline{A}\,\overline{B})$

$= 0.15 + 0.3 = 0.45$

Câu 26 - Level 3

A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 2 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

Phương án 1:
0.0716

Phương án 2:
0.4562

Phương án 3:
0.5438

Phương án 4:
0.0794

Phương án 5:
0.2081

Phương án 6:
0.1132

let x_1: # shipment chosen to test
x_2: # defective devices

$\Rightarrow P(x_2 = 2) = \binom{10}{2} p_1^2 (1-p_1)^8 = p_2$

where p_1 is the prob that there is at least 1 defective device

$\Rightarrow p_1 = P(x_2 \geq 1) = 1 - P(x < 1)$

$= 1 - \binom{20}{0}(0.03)^0(0.97)^{20} = 0.4562$

$\Rightarrow p_2 = 0.0716$

Câu 27 - Level 3

There are 3 boxes of balls: the first box contains 3 red balls, 2 white balls; the second box contains 2 red balls, 2 white balls; the third box has no balls. Draw randomly 1 ball from the first box and 1 ball from the second box and put them in the third box. Then, from the third box, 1 ball is drawn at random. Given that the ball drawn from the third box is red, what is the probability that the ball drawn from the first box is red?

Phương án 1:
9/11

Phương án 2:
11/20

Phương án 3:
6/11

Phương án 4:
3/11

Phương án 5:
3/5

Phương án 6:
3/20

R_3: Red is drawn from the third box

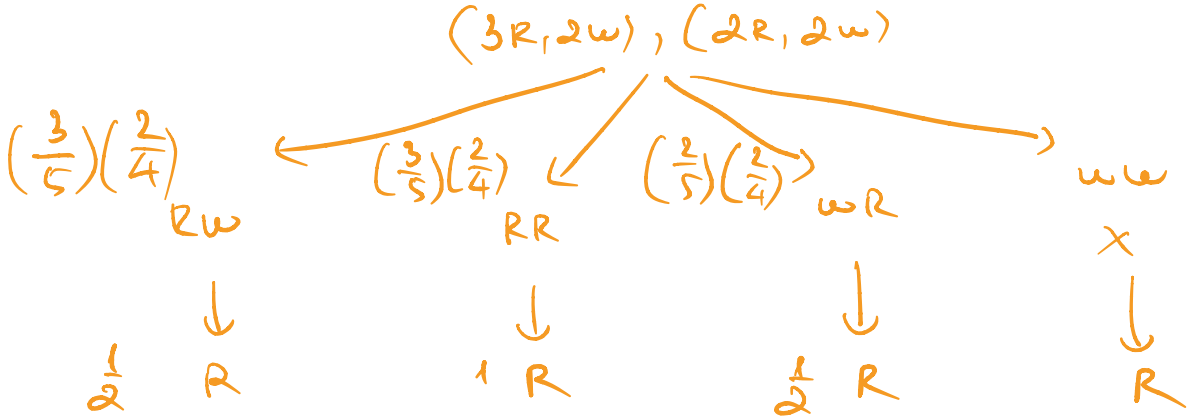
R_1: Red is drawn from the first box

→ Find $P(R_1|R_3)$

$$P(R_1|R_3) = \frac{P(R_3|R_1)\,P(R_1)}{P(R_3)}$$

$$= \frac{P(R_3R_1)\,P(R_1)}{P(R_1)\,P(R_3)}$$

$$= \frac{P(R_3R_1)}{P(R_3)} = \frac{9/20}{\frac{11}{20}} = \frac{9}{11}$$



$$P(R_3) = \underbrace{\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{2}\right) + \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)(1)}_{P(R_3R_1)} + \left(\frac{2}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{2}\right)$$

$$= \frac{11}{20}$$

Câu 28 - Level 3

A box contains 5 red balls, 10 white balls, and 15 blue balls. Suppose that 3 balls are selected at random one at a time and let X be the number of colors missing from the 3 selected balls. Find $E(Y)$, the expected value of the random variable Y, where $Y := X^2 + 5$.

Phương án 1:
6.2475

Phương án 2:
5.2475

Phương án 3:
7.3475

Phương án 4:
5.9594

Phương án 5:
6.9594

Phương án 6:
7.9594

PMF: $P_X(x=0) = \frac{5 \cdot 10 \cdot 15}{\binom{30}{3}} = \frac{75}{406}$

$P_X(x=1) = 1 - P(x=0) - P(x=2) = \frac{545}{812}$

$P_X(x=2) = \frac{\binom{5}{3} + \binom{10}{3} + \binom{15}{3}}{\binom{30}{3}} = \frac{117}{812}$

LOTUS: $E[Y] = (0^2+5)\frac{75}{406} + (1^2+5)\cdot\frac{545}{812} + (2^2+5)\cdot\frac{117}{812}$

≈ 6.2475

Câu 29 - Level 3

The random variable X has the probability density function

$$f_X(x) = \begin{cases} 0.5e^{-0.5x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Let $Y := \min\{2, X\}$. Find $E(Y)$, the expected value of the random variable Y.

Phương án 1:
1.2642

Phương án 2:
2.2642

Phương án 3:
3.6321

Phương án 4:
0.6321

Phương án 5:
1.7358

Phương án 6:
0.7358

$$E[Y] = \int_{-\infty}^{+\infty} y f_X(y)\, dy = \int_0^2 y f_X(y)\, dy + \int_2^{+\infty} y f_X(y) dy$$

$$= \int_0^2 y \cdot 0.5e^{-0.5y} dy + \int_2^{+\infty} e^{-0.5y} dy$$

$$\simeq 1.2642$$

Câu 30 - Level 3

Suppose that the number of customers that arrive at a retail store has a Poisson distribution with a rate of $\lambda = 36/\text{hour}$. Time working start from 9:00. What is the probability that the first customer arrives before 9:08 given that there were no customer before 9:03?

Phương án 1:
0.95021

Phương án 2:
0.16530

Phương án 3:
0.00823

Phương án 4:
0.54881

Phương án 5:
0.94015

Phương án 6:
0.83470

$\Leftrightarrow$ Prob that no customer waits more than 5 minutes

$\Rightarrow P\left(T < \frac{5}{60}\right) = 1 - e^{-36.5/60} \approx 0.95021$