MET 4501 Fall 2024: In-Class Quiz 4 Name: SOLUTION

INSTRUCTIONS:

This quiz is open-book and open-note, and you may work with your classmates. Please answer all questions and show all of your work.

GIVEN:

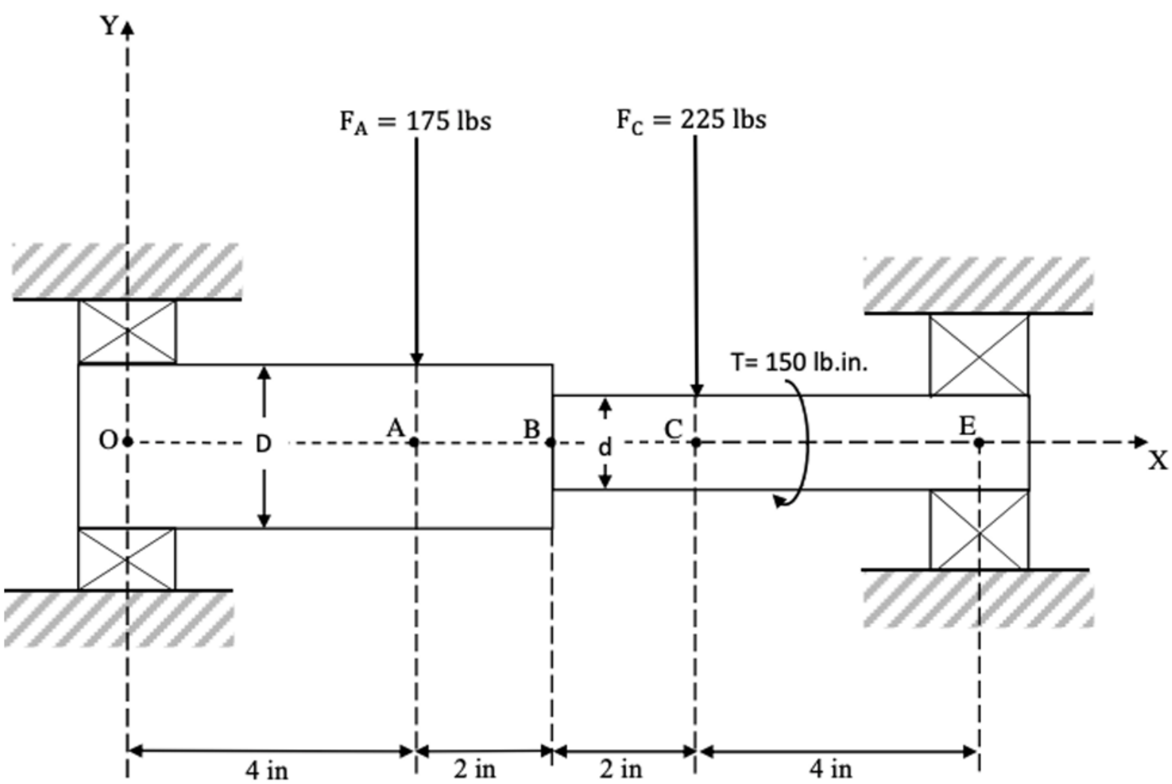
An AISI 1030 hot rolled steel shaft is rotating at a constant speed in the simply supported bearings at points *O* and *E*.

The shaft has a yield strength $S_y = 37.5$ ksi, ultimate tensile strength $S_{ut} = 68$ ksi, and a fully-corrected endurance limit of $S_e = 18.3$ ksi.

The two constant diameters of the stepped shaft are $D = 2$ in and $d = 1.2$ in.

The constant vertical loads at locations A and C are $F_A = 175$ lbf and $F_C = 225$ lbf and the shaft transmits a constant torque $T = 150$ lbf-in.

The fatigue stress concentration factors at the step are $K_f = 3$ for bending and $K_{fs} = 2.5$ for torsion.

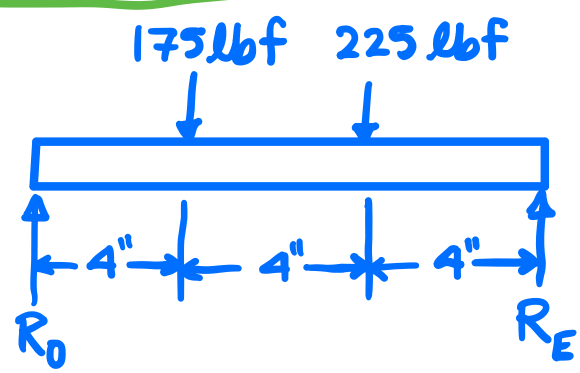


FIND:

1) Sketch diagrams showing the internal loads (bending and torsion) acting on the rotating shaft.
2) Identify the critical cross-section of the shaft.
3) For a point on the circumference of the shaft at the critical cross-section, sketch the bending stress as a function of time.
4) For a point on the circumference of the shaft at the critical cross-section, sketch the torsional shear stress as a function of time.
5) The factor of safety for infinite life using the Goodman criterion.
6) The factor of safety for yielding.

1) FOLLOWING THE STEPS OUTLINED IN OUR "IDENTIFICATION OF CRITICAL ELEMENTS WORKSHEET":

DRAW THE FREE BODY DIAGRAM



SOLVE FOR ALL REACTIONS

$$\Sigma F_y = 0: \quad R_O - 175\,\text{lbf} - 225\,\text{lbf} + R_E = 0$$

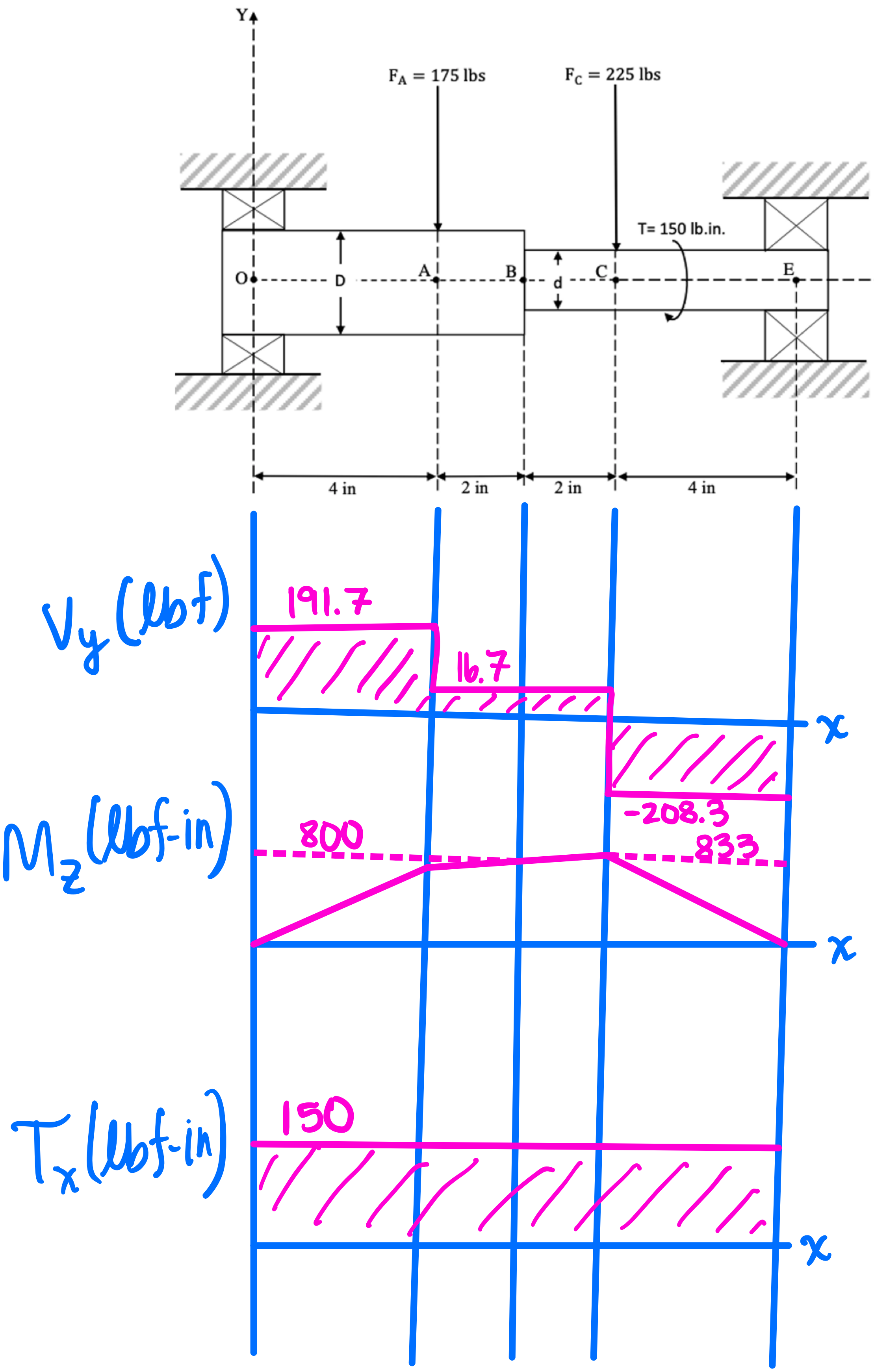
$$R_O + R_E = 400\,\text{lbf}$$

$$\Sigma M_O = 0: \quad R_E(12'') - (225\,\text{lbf})(8'') - (175\,\text{lbf})(4'') = 0$$

$$R_E = 208.3\,\text{lbf}$$

$$R_O = 400 - 208.3 = 191.7\,\text{lbf}$$

DETERMINE & SKETCH INTERNAL LOADS



$M_{B,z} = (208.3\ \text{lbf})(6'') - (225\ \text{lbf})(2'')$

$= 800\ \text{lbf-in}$

$M_{c,z} = (208.3\ \text{lbf})(4'')$

$= 833\ \text{lbf-in}$

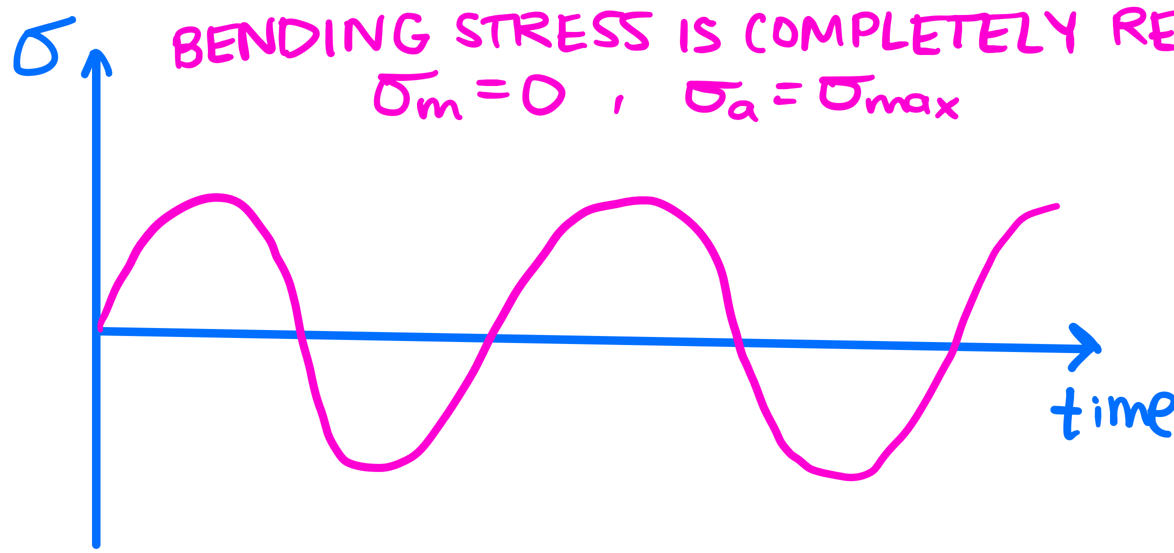
(THERE ARE NO AXIAL LOADS)

2) T_x IS CONSTANT AND V_y IS SMALL COMPARED TO M_z, SO WE WILL PICK A CRITICAL CROSS-SECTION BASED ON WHERE M_z IS GREATEST IN MAGNITUDE.

BECAUSE OF THE STRESS RAISER AND RELATIVELY HIGH M_z, THE CRITICAL CROSS-SECTION IS AT B.

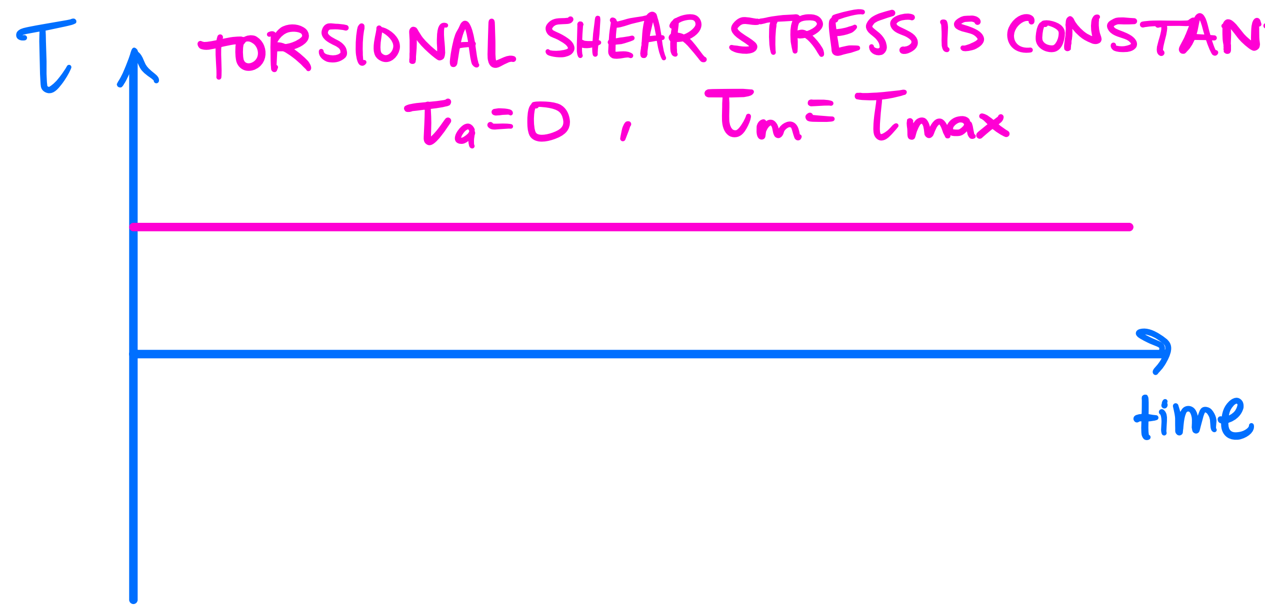
3) BENDING STRESS IS COMPLETELY REVERSED.

$\sigma_m = 0$, $\sigma_a = \sigma_{max}$



4) TORSIONAL SHEAR STRESS IS CONSTANT.

$\tau_a = 0$, $\tau_m = \tau_{max}$



5) WE HAVE A COMBINATION OF LOADING MODES, SO WE NEED TO CALCULATE THE ALTERNATING AND MEAN VON MISES STRESSES

NO AXIAL LOAD

$\tau_a = 0$ (CONSTANT TORQUE)

$$\sigma_a' = \sqrt{\left[K_{f,bend}\sigma_{a,bend} + K_{f,axial}\cancel{\sigma_{a,axial}}\right]^2 + 3\left[K_{fs}\cancel{\tau_a}\right]^2}$$

$$= \sqrt{\left[K_{f,bend}\sigma_{a,bend}\right]^2} = K_{f,bend}\,\sigma_{a,bend}$$

$K_f = 3$ (GIVEN) AND $\sigma_{a,bend} = \dfrac{M_z \cdot c}{I}$

$$\sigma_a' = 3 \cdot \frac{(800\text{ lbf-in})(0.6\text{ in})}{\frac{\pi}{64}(1.2\text{ in})^4}$$

$$= 14{,}150\text{ psi} = 14.15\text{ ksi}$$

$\sigma_m = 0$ (COMPLETELY REVERSED)

NO AXIAL LOAD

$$\sigma_m' = \sqrt{\left[K_{f,bend}\cancel{\sigma_{m,bend}} + K_{f,axial}\cancel{\sigma_{m,axial}}\right]^2 + 3\left[K_{fs}\tau_m\right]^2}$$

$$= \sqrt{3\left[K_{fs}\tau_m\right]^2} = \sqrt{3}\,K_{fs}\tau_m$$

$K_{fs} = 2.5$ (GIVEN) AND $\tau_m = \dfrac{T \cdot c}{J}$

$$\sigma_m' = \sqrt{3}(2.5)\frac{(150\ \text{lbf-in})(0.6\ \text{in})}{\frac{\pi}{32}(1.2\ \text{in})^4}$$

$$= 1914\ \text{psi} = 1.914\ \text{ksi}$$

APPLY GOODMAN CRITERIA:

$$n_f = \left[\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}\right]^{-1}$$

$$= \left[\frac{14.15\ \text{ksi}}{18.3\ \text{ksi}} + \frac{1.914\ \text{ksi}}{68\ \text{ksi}}\right]^{-1}$$

$$\boxed{n_f = 2.1}$$

6) $$n_y = \frac{S_y}{\sigma_a' + |\sigma_m'|} = \frac{37.5\ \text{ksi}}{14.15 + 1.914\ \text{ksi}}$$

$$\boxed{n_y = 2.3}$$

FATIGUE IS A GREATER THREAT THAN YIELDING (LOWER SAFETY FACTOR)