Poisson GLM of Wafer Imperfections

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8/9/2017

```r
dat311 <- data.frame(treatment = rep(c("A", "B"), each = 10), n_im = c(8, 7,
    6, 6, 3, 4, 7, 2, 3, 4, 9, 9, 8, 14, 8, 13, 11, 5, 7, 6))
```

Let the two treatments have counts i.i.d. as Poisson with means μ_A and μ_B, respectively. Consider the model: $\log\mu = \alpha + \beta x$ where x is an indicator variable for treatment B (vs. treatment A).

a) Show that $\beta = \log(\mu_B/\mu_A)$ and $e^\beta = \mu_B/\mu_A$.

$$\log\mu_B = \alpha + \beta$$
$$\log\mu_A = \alpha$$
$$\beta = \log\mu_B - \alpha = \log\mu_B - \log\mu_A = \log(\mu_B/\mu_A)$$
$$e^\beta = e^{\log(\mu_B/\mu_A)} = \mu_B/\mu_A$$

b) Fit the model. Report the prediction equation and interpret $\hat{\beta}$.

```r
fit311<-glm(n_im~treatment,family=poisson,data=dat311)
```

Our prediction equation is:

$$\log\mu = \beta_0 + \beta_1 x = 1.61 + 0.59x$$
$$\mu = e^{\beta_0+\beta_1 x} = e^{\beta_0} \cdot e^{\beta_1 x} = 5 \cdot 1.8^x$$

We interpret our model as predicting an average count of 5 imperfections for treatment A and an average count of 9 imperfections for treatment B, an 80% increase, an increase by a factor of $e^{\beta_1} = 1.8$.

c) Test $H_0 : \mu_A = \mu_B$ by testing $H_0 : \beta_1 = 0$.

We'll conduct a Wald test.

```r
par<-summary(fit311)$coefficients[2,1:2]
pchisq((par[1]/par[2])^2,1,.05,lower.tail=FALSE)
```

```
##   Estimate
## 0.00112737
```

$$\left[\frac{\hat{\beta} - \beta_0}{SE(\hat{\beta})}\right]^2 = \left[\frac{0.587}{0.176}\right]^2 = 11.1 > \chi^2(0.05) = 3.84$$

Since our $p < 0.05$, our effect of treatment is statistically significant.

d) Construct a 95% confidence interval for μ_A/μ_B.

$$\beta : (\beta \pm 1.96 \cdot SE(\beta)) = (0.24, 0.93)$$
$$\mu_B/\mu_A = e^\beta : (e^{\beta \pm 1.96 \cdot SE(\beta)}) = (1.27, 2.54)$$