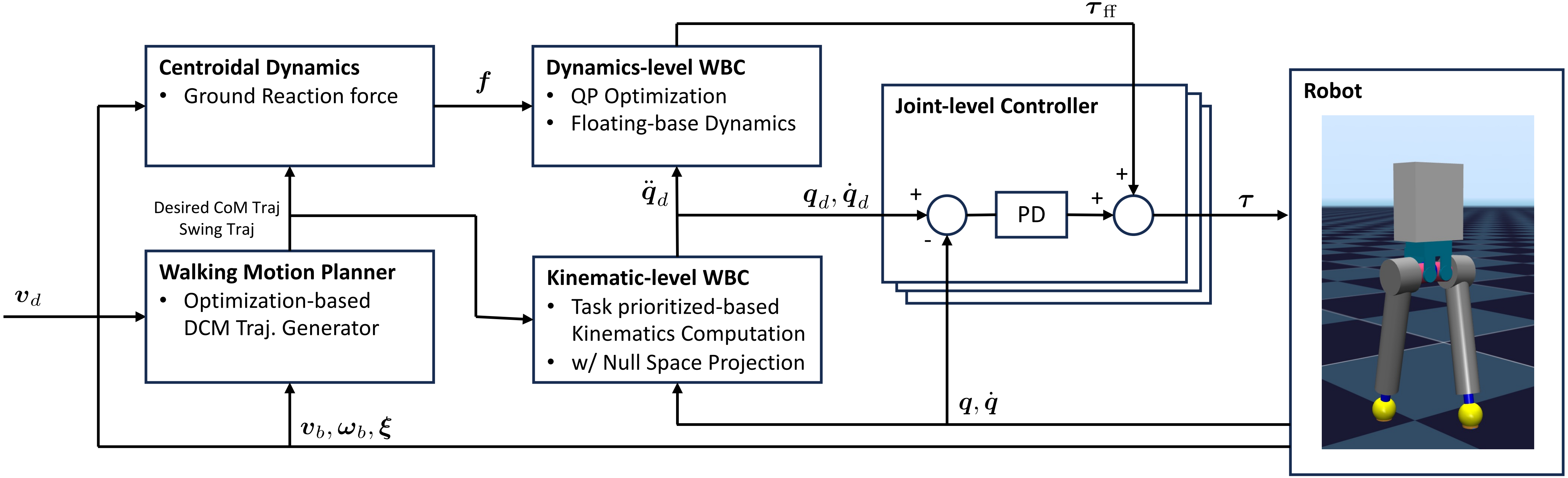
Ongoing Dynamic Locomotion Research

JunYoung Kim

Locomotion control diagram



Centroidal Dynamics
- Ground Reaction force

Dynamics-level WBC
- QP Optimization
- Floating-base Dynamics

Joint-level Controller

Robot

Walking Motion Planner
- Optimization-based DCM Traj. Generator

Kinematic-level WBC
- Task prioritized-based Kinematics Computation
- w/ Null Space Projection

Desired CoM Traj
Swing Traj

Walking Motion Planner - Step Timing Adjustment (1)

- Propose an approach that combines both step location and timing adjustment for generating robust gaits.

Step length   Step Duration   Upper bound

$$B_l = \max\left(B_l^1, B_l^2, B_l^3\right) \le T_{\text{nom}} \le \min\left(B_u^1, B_u^2, B_u^3\right) = B_u$$

Lower bound   Step Width

$$T_{\text{nom}} = \frac{B_l + B_u}{2}$$

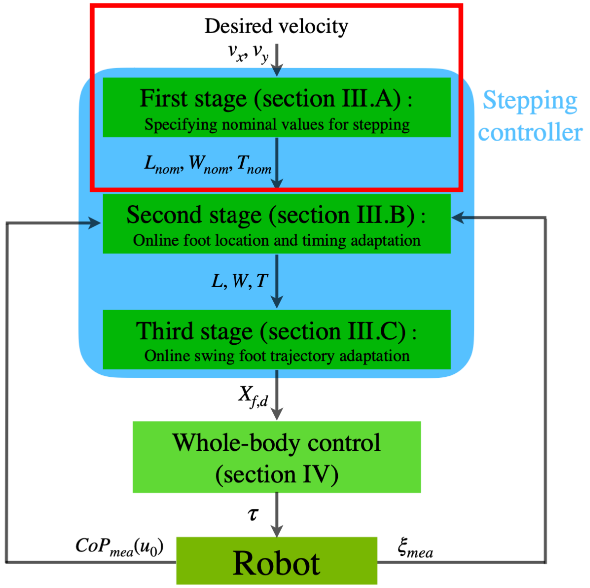
$$L_{\text{nom}} = v_x\left(\frac{B_l + B_u}{2}\right)$$

$$W_{\text{nom}} = v_y\left(\frac{B_l + B_u}{2}\right)$$

$$\boxed{b_{x,\text{ nom}}} = \frac{L_{\text{nom}}}{e^{\omega_0 T_{\text{nom}}} - 1}$$

$$\boxed{b_{y,\text{ nom}}} = (-1)^n \frac{l_p}{1 + e^{\omega_0 T_{\text{nom}}}} - \frac{W_{\text{nom}}}{1 - e^{\omega_0 T_{\text{nom}}}}$$

Nominal DCM offset : By achieving this value at the end of a step, we make sure that the CoM travels a desired distance during a specified time in the next step (without perturbation) which realizes the desired average velocity during T.



Desired velocity

v_x, v_y

Stepping controller

First stage (section III.A) : Specifying nominal values for stepping

$L_{nom}, W_{nom}, T_{nom}$

Second stage (section III.B) : Online foot location and timing adaptation

L, W, T

Third stage (section III.C) : Online swing foot trajectory adaptation

$X_{f,d}$

Whole-body control (section IV)

τ

$CoP_{mea}(u_0)$

Robot

ξ_{mea}

Fig. 2. Block diagram of the walking algorithm.

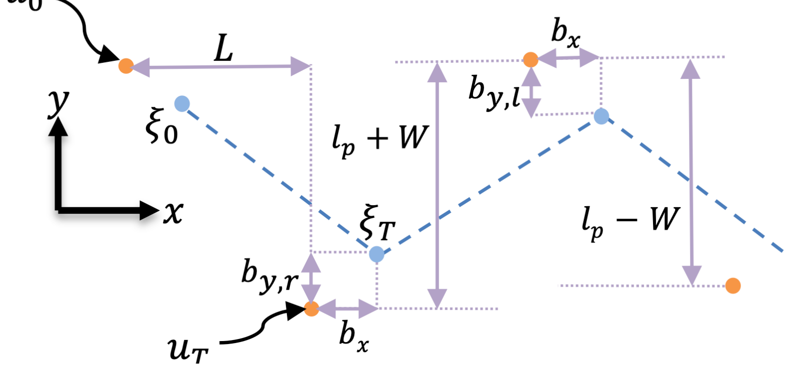


Fig. 1. Schematic view of walking with footprints, DCM, and DCM offset.

Ref: Step Timing Adjustment: A Step toward Generating Robust Gaits, Walking Control Based on Step Timing Adaptation

Walking Motion Planner - Step Timing Adjustment (2)

$$\min_{u_{T,x},u_{T,y},\tau,b_x,b_y} \alpha_1 \left\|u_{T,x}-L_{nom}\right\| + \alpha_2 \left\|u_{T,y}-W_{nom}\right\| + \alpha_3 \left\|\tau-\tau_{nom}\right\| + \alpha_4 \left\|b_x-b_{x,\text{ nom}}\right\| + \alpha_5 \left\|b_y-b_{y,nom}\right\|$$

$$\text{s.t.} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{T,x} \\ u_{T,y} \\ \tau \\ b_x \\ b_y \end{bmatrix} \leq \begin{bmatrix} L_{\max} \\ -L_{\min} \\ W_{\max} \\ -W_{\min} \\ e^{\omega_0 T_{\max}} \\ -e^{\omega_0 T_{\min}} \end{bmatrix}$$

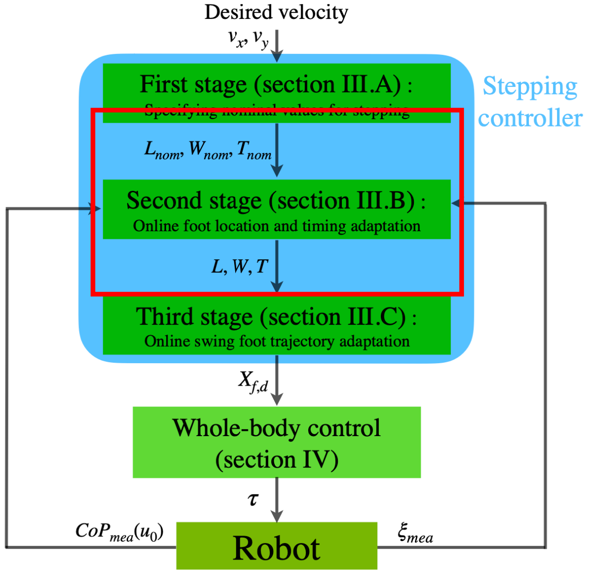
$$\begin{bmatrix} 1 & 0 & -\left(\xi_{\text{mea}}-u_0\right)e^{-\omega_0 t} & 1 & 0 \\ 0 & 1 & -\left(\xi_{\text{mea}}-u_0\right)e^{-\omega_0 t} & 0 & 1 \end{bmatrix} \boxed{\begin{bmatrix} u_{T,x} \\ u_{T,y} \\ \tau \\ b_x \\ b_y \end{bmatrix}} = \begin{bmatrix} u_{0,x} \\ u_{0,y} \end{bmatrix}$$

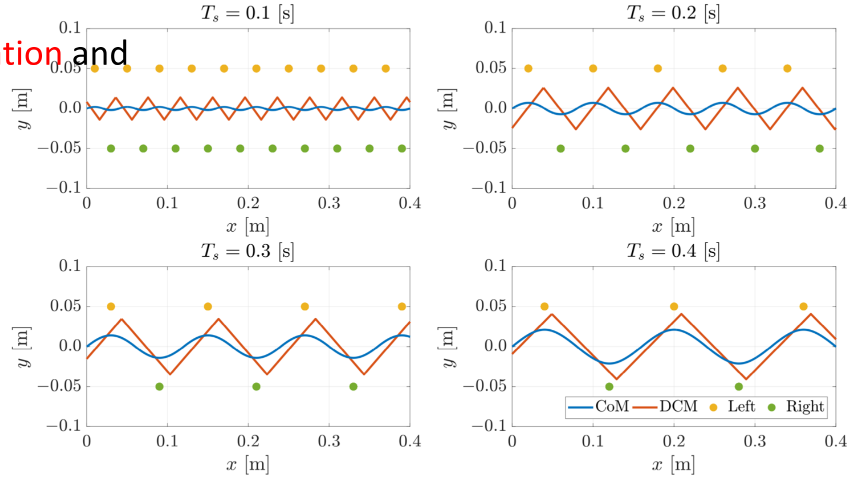
Landing location

outputs of the optimization problem are the landing location and time of the swing foot as well as the DCM offset.

$$\tau = e^{\omega_0 T} \Longrightarrow \underline{T} = \frac{1}{\omega_0}\log(\tau)$$

Step duration

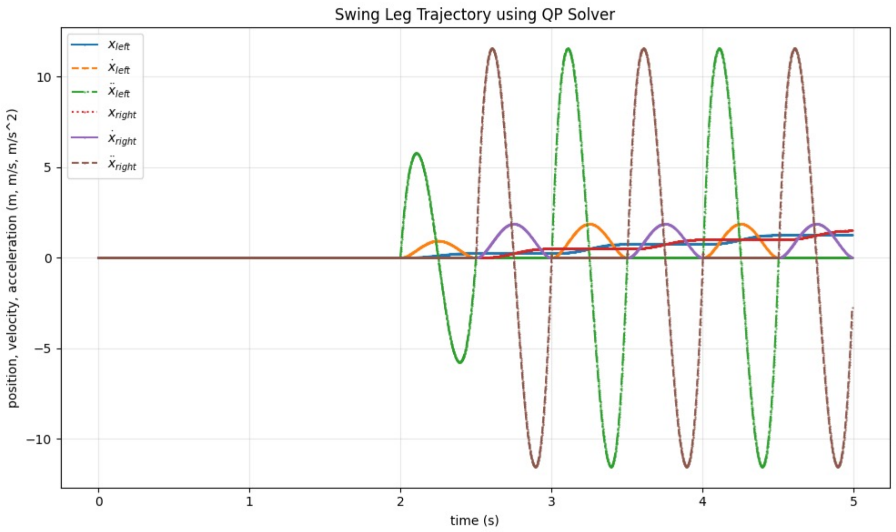


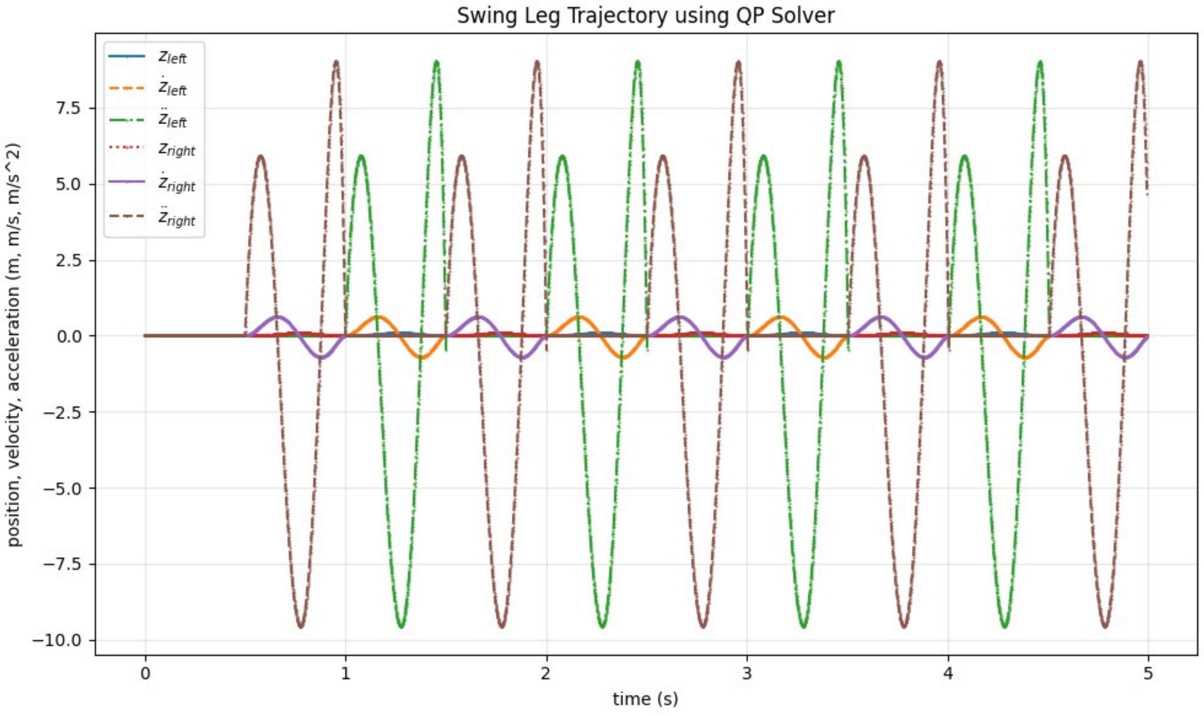


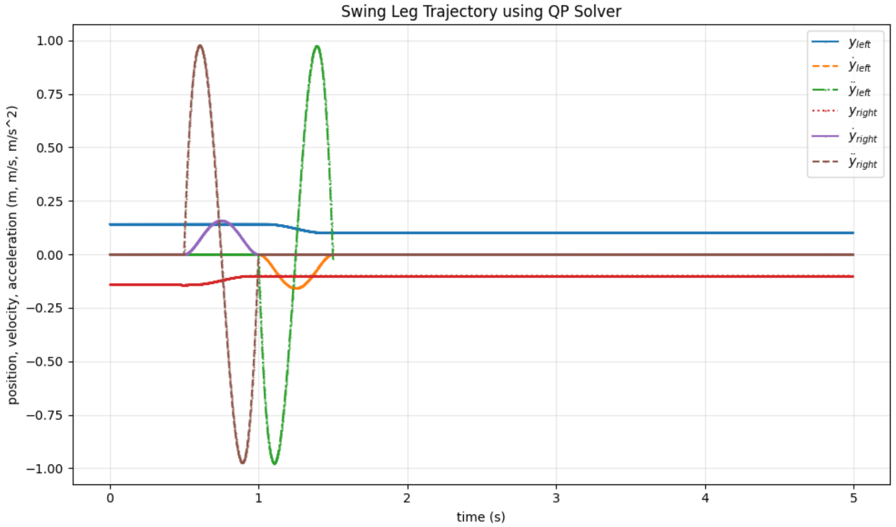
Ref: [Step Timing Adjustment: A Step toward Generating Robust Gaits](), [Walking Control Based on Step Timing Adaptation]()

Walking Motion Planner - Step Timing Adjustment (3)

- Scenario Description
  - Stand (<0.5s)
  - walking on the spot (<1.5s)
  - Walk (> 1.5s) w/ linear Velocity 1.0m/s
- Reference Footstep Trajectory Generation using QuadProg Solver
  - 5th (x,y) + 9th (z) polynomial constraints







Controller Model

- Simplified control model templates to optimize ground reaction forces at the footstep location.
- Linear relationship between <u>CoM translational</u> and <u>body angular acceleration</u> and <u>the forces</u>.

$$m(\ddot{\boldsymbol{p}}_c + \boldsymbol{g}) = \sum_{i=1}^{n_c} \boldsymbol{f}_i$$

$$\left(\frac{d}{dt}(\boldsymbol{I}\boldsymbol{\omega}) = \boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega}) \approx \boldsymbol{I}\dot{\boldsymbol{\omega}} \right)$$

$$\boldsymbol{I}\dot{\boldsymbol{\omega}}_b = \sum_{i=1}^{n_c} \boldsymbol{r}_i \times \boldsymbol{f}_i$$

$$\underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{I}_{3\times 3} \\ [\boldsymbol{r}_1]\times & [\boldsymbol{r}_2]\times \end{bmatrix}}_{\boldsymbol{A}} \boldsymbol{f} = \underbrace{\begin{bmatrix} m(\ddot{\boldsymbol{p}}_c + \boldsymbol{g}) \\ \boldsymbol{I}\dot{\boldsymbol{\omega}}_b \end{bmatrix}}_{\boldsymbol{b}}$$

$$\underbrace{\begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{I}_{3\times 3} \\ [\boldsymbol{p}_1 - \boldsymbol{p}_c]\times & [\boldsymbol{p}_2 - \boldsymbol{p}_c]\times \end{bmatrix}}_{\boldsymbol{A}} \boldsymbol{f} = \underbrace{\begin{bmatrix} m(\ddot{\boldsymbol{p}}_c + \boldsymbol{g}) \\ \boldsymbol{I}\dot{\boldsymbol{\omega}}_b \end{bmatrix}}_{\boldsymbol{b}}$$

Centroidal Dynamics (1)

- Balance control enforces <u>PD control</u> on the CoM and body orientation
- The goal of balance controller is to resolve an optimal distribution of leg forces $\boldsymbol{f}$ that drive the approximate centroidal dynamics to the corresponding desired dynamics

$$\begin{bmatrix} \ddot{\boldsymbol{p}}_{c,d} \\ \dot{\boldsymbol{\omega}}_{b,d} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_{p,p}(\boldsymbol{p}_{c,d} - \boldsymbol{p}_c) + \boldsymbol{K}_{d,p}(\dot{\boldsymbol{p}}_{c,d} - \dot{\boldsymbol{p}}_c) \\ \boldsymbol{K}_{p,\omega} \log(\underset{\text{Orientation error}}{\underline{\boldsymbol{R}_d \boldsymbol{R}^T}}) + \boldsymbol{K}_{d,\omega}(\boldsymbol{\omega}_{b,d} - \boldsymbol{\omega}_b) \end{bmatrix}$$

$$\boldsymbol{b}_d = \begin{bmatrix} m(\ddot{\boldsymbol{p}}_{c,d} + \boldsymbol{g}) \\ \boldsymbol{I}\dot{\boldsymbol{\omega}}_{b,d} \end{bmatrix}$$

- QP formulation

$$\min_{\boldsymbol{f} \in \mathbb{R}^6} \underset{\text{Deriving the centroidal dynamics to desired value}}{\underline{(\boldsymbol{A}\boldsymbol{f} - \boldsymbol{b}_d)^\top \boldsymbol{S} (\boldsymbol{A}\boldsymbol{f} - \boldsymbol{b}_d)}} + \underset{\text{Minimizing the forces used}}{\underline{\alpha \|\boldsymbol{f}\|^2}}$$

$$\text{s.t. } \underset{\text{Enforce to ensure optimized forces lie in the friction cone}}{\underline{\boldsymbol{C}\boldsymbol{f} \leq \boldsymbol{d}}}$$

Centroidal Dynamics (2)

QP Formulation

$$\min_{\boldsymbol{\rho}\in\mathbb{R}^{12}} \underbrace{(\boldsymbol{A}\boldsymbol{f}'\boldsymbol{\rho}-\boldsymbol{b}_d)^\top \boldsymbol{S}\,(\boldsymbol{A}\boldsymbol{f}'\boldsymbol{\rho}-\boldsymbol{b}_d)}_{\text{Deriving the centroidal dynamics to desired value}} + \underbrace{\alpha\|\boldsymbol{\rho}\|^2}_{\text{Minimizing the forces used}}$$

$$\text{s.t.}\ \ \boldsymbol{\rho}\geq 0$$

$$\boldsymbol{G}=\boldsymbol{A}\boldsymbol{f}'^T\boldsymbol{S}\boldsymbol{A}\boldsymbol{f}'+\alpha\boldsymbol{I}$$

$$\boldsymbol{g0}=-\boldsymbol{A}\boldsymbol{f}'^T\boldsymbol{S}\boldsymbol{b}_d$$

QuadProgpp Standard from

$$\boxed{\begin{aligned}&\min_{\boldsymbol{x}}\ \frac{1}{2}\boldsymbol{x}^T\boldsymbol{G}\boldsymbol{x}+\boldsymbol{g0}\boldsymbol{x}\\ &\text{s.t.}\ \ \boldsymbol{C}_{\mathrm{e}}^T x+\boldsymbol{c}_{\mathrm{e}}=0\\ &\qquad\ \boldsymbol{C}_{\mathrm{i}}^T x+\boldsymbol{c}_{\mathrm{i}}\geq 0\end{aligned}}$$

Objective Function

$$\underbrace{\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{I}_{3\times3}\\ [\boldsymbol{p}_1-\boldsymbol{p}_c]\times & [\boldsymbol{p}_2-\boldsymbol{p}_c]\times\end{bmatrix}}_{\boldsymbol{A}}\boldsymbol{f}=\underbrace{\begin{bmatrix}m\,(\ddot{\boldsymbol{p}}_c+\boldsymbol{g})\\ \boldsymbol{I}\dot{\boldsymbol{\omega}}_b\end{bmatrix}}_{\boldsymbol{b}}$$

$$\underbrace{\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{I}_{3\times3}\\ [\boldsymbol{p}_1-\boldsymbol{p}_c]\times & [\boldsymbol{p}_2-\boldsymbol{p}_c]\times\end{bmatrix}}_{\boldsymbol{A}}\underbrace{\underbrace{\begin{bmatrix}\boldsymbol{R}_{C,left} & \mathbf{0}_{3\times3}\\ \mathbf{0}_{3\times3} & \boldsymbol{R}_{C,right}\end{bmatrix}}_{\text{Rotation Mat w.r.t. Contact Frame}\quad \boldsymbol{f}'}\underset{\text{Approximation Cone}}{\begin{bmatrix}\boldsymbol{U}_{3\times6} & \mathbf{0}_{3\times6}\\ \mathbf{0}_{3\times6} & \boldsymbol{U}_{3\times6}\end{bmatrix}}}_{\boldsymbol{f}}\boldsymbol{\rho}_{12\times1}=\underbrace{\begin{bmatrix}m\,(\ddot{\boldsymbol{p}}_c+\boldsymbol{g})\\ \boldsymbol{I}\dot{\boldsymbol{\omega}}_b\end{bmatrix}}_{\boldsymbol{b}_d}$$

$$\boldsymbol{G}=\boldsymbol{A}\boldsymbol{f}'^T\boldsymbol{S}\boldsymbol{A}\boldsymbol{f}'+\alpha\boldsymbol{I}$$

$$\boldsymbol{g0}=-\boldsymbol{A}\boldsymbol{f}'^T\boldsymbol{S}\boldsymbol{b}_d$$

$$\boldsymbol{U}=\begin{bmatrix}\boldsymbol{u}_1 & \boldsymbol{u}_2 & \cdots & \boldsymbol{u}_m\end{bmatrix}\quad \boldsymbol{u}_i=\begin{bmatrix}cos(\frac{2\pi(i-1)}{m})\\ sin(\frac{2\pi(i-1)}{m})\\ 1\end{bmatrix}$$

[MBM.2017]

Equality Constraints

$$\boldsymbol{C}_{\mathrm{e}}=\mathbf{0}$$

$$\boldsymbol{c}_{\mathrm{e}}=\mathbf{0}$$

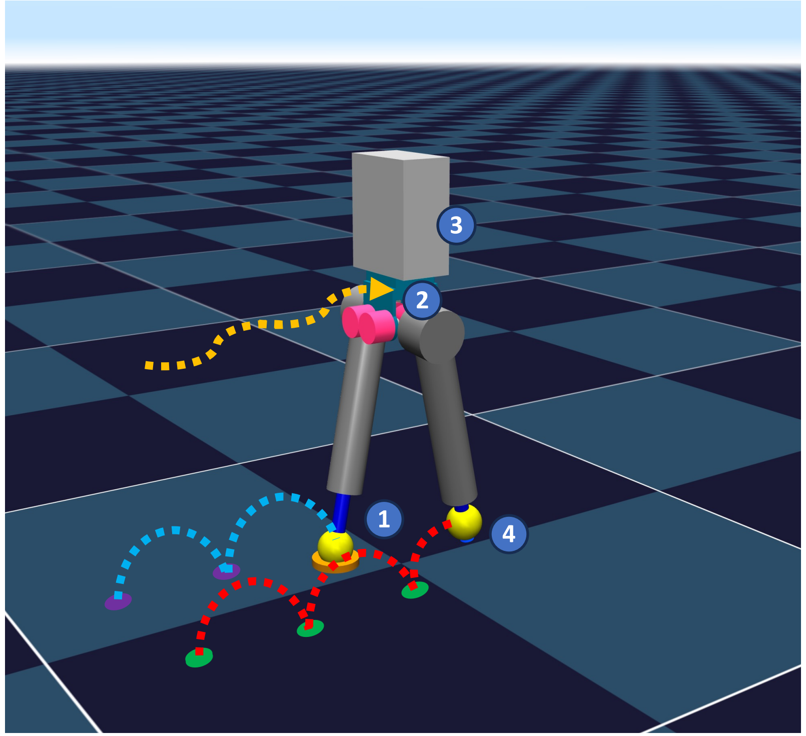
Inequality Constraints

$$\boldsymbol{c}_{\mathrm{i}}=\mathbf{0}_{12\times1}$$

$$\underbrace{\boldsymbol{I}_{12\times12}}_{\boldsymbol{C}_{\mathrm{i}}}\boldsymbol{\rho}_{12\times1}\geq\mathbf{0}_{12\times1}$$

Kinematic-level Whole-Body Control (1)

- Managing multiple tasks concurrently when performing dynamic motion.
- Define four tasks for WBC
  1. Supporting leg in fixed position (task1)
  2. Maintaining the posture (task2)
  3. Maintaining the CoM position of the body (task3)
  4. Controls the swing leg to follow predetermined trajectory (task4)
- Task prioritized-based WBC
  - Basic idea is to compute incremental joint positions based on operational space position errors and add them to current joint position



Figure#, Multiple task control during dynamic locomotion, numbers demonstrates task priority

$$\boldsymbol{\Delta q}_i = \boldsymbol{\Delta q}_{i-1} + \boldsymbol{J}^{\dagger}_{i|pre}\left(\boldsymbol{x}_i^{\text{des}} - \boldsymbol{x}_i - \boldsymbol{J}_i \boldsymbol{\Delta q}_{i-1}\right)$$

$$\boldsymbol{q^d} = \boldsymbol{q} + \boldsymbol{\Delta q}$$

$$\dot{\boldsymbol{q}}_i^d = \dot{\boldsymbol{q}}_{i-1}^d + \boldsymbol{J}^{\dagger}_{i|pre}\left(\dot{\boldsymbol{x}}_i^{\text{des}} - \boldsymbol{J}_i \dot{\boldsymbol{q}}_{i-1}^d\right),$$

$$\ddot{\boldsymbol{q}}_i^d = \ddot{\boldsymbol{q}}_{i-1}^d + \boldsymbol{J}^{\dagger}_{i|pre}\left(\ddot{\boldsymbol{x}}_i^{\text{des}} - \dot{\boldsymbol{J}}_i \dot{\boldsymbol{q}} - \boldsymbol{J}_i \ddot{\boldsymbol{q}}_{i-1}^d\right).$$

i = Task #

$$\boldsymbol{J}_{i|pre} = \boldsymbol{J}_i \boldsymbol{N}_{i-1}$$

$$\boldsymbol{N}_{i-1} = \boldsymbol{N}_{1|0} \cdots \boldsymbol{N}_{i-1|i-2}$$

$$\boldsymbol{N}_{i-1|i-2} = \boldsymbol{I} - \boldsymbol{J}^{\dagger}_{i-1|pre} \boldsymbol{J}_{i-1|pre}$$

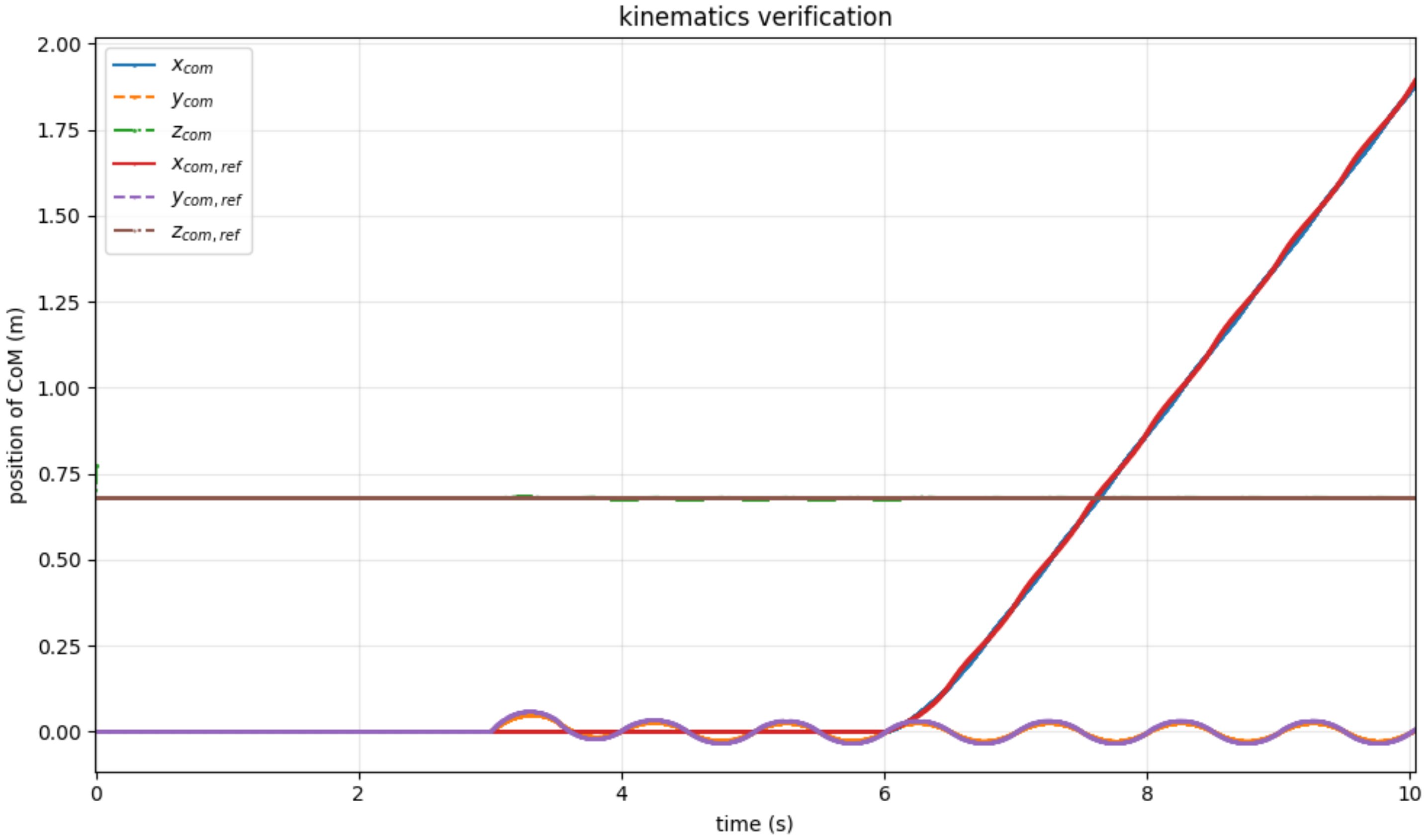
$$\boldsymbol{N}_0 = \boldsymbol{I}$$

Compute nullspace projection matrix

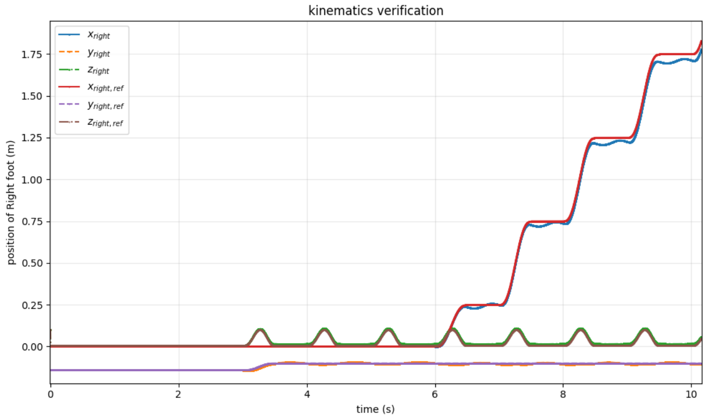
Ref: Dynamic locomotion for passive-ankle biped robots and humanoids using whole-body locomotion control

Kinematic-level Whole-Body Control (2)

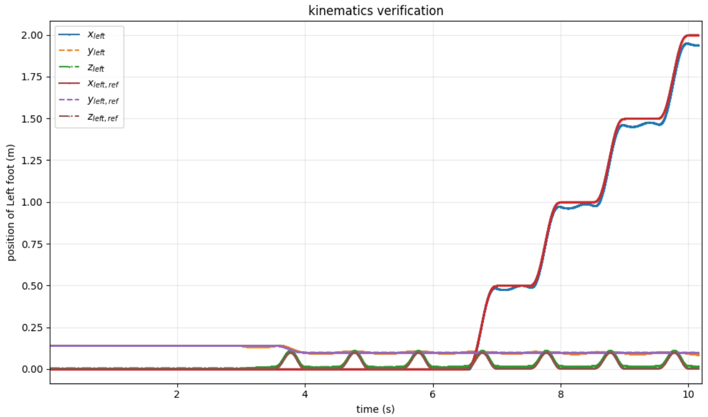
- Verification of recursive prioritized inverse kinematics.



Figure#, Kinematic computation of Center of Mass and Reference Task Trajectory



Figure#, Kinematic computation of right foot



Figure#, Kinematic computation of left foot

Dynamic-level Whole-Body Control

QP Formulation

$$\ddot{\boldsymbol{q}}^{\text{cmd}} = \ddot{\boldsymbol{q}}^{d} + k_d(\dot{\boldsymbol{q}}^{d} - \dot{\boldsymbol{q}}) + k_p(\boldsymbol{q}^{d} - \boldsymbol{q})$$

$$\min_{\boldsymbol{\delta}_{\rho}, \boldsymbol{\delta}_{\ddot{q}}} \quad \frac{1}{2}\begin{bmatrix}\boldsymbol{\delta}_{\ddot{q}} & \boldsymbol{\delta}_{\rho}\end{bmatrix}^T \boldsymbol{G} \begin{bmatrix}\boldsymbol{\delta}_{\ddot{q}} & \boldsymbol{\delta}_{\rho}\end{bmatrix} + \boldsymbol{g}_0 \begin{bmatrix}\boldsymbol{\delta}_{\ddot{q}} & \boldsymbol{\delta}_{\rho}\end{bmatrix}$$

$$\text{subject to} \quad \underbrace{\overset{\text{(6x6)(6x6)(6x12)}}{\begin{bmatrix}\boldsymbol{M}_{6\times 12} & -\boldsymbol{J}_c^T \boldsymbol{R}_c \boldsymbol{U}\end{bmatrix}}}_{\boldsymbol{C}_e} \underset{\text{(24x1)}}{\begin{bmatrix}\boldsymbol{\delta}_{\ddot{q}} \\ \boldsymbol{\delta}_{\rho}\end{bmatrix}} + \underbrace{\boldsymbol{M}\ddot{\boldsymbol{q}}^{\text{cmd}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) - \overset{\text{(6x6)(6x1)}}{\boldsymbol{J}_c^T \boldsymbol{f}_{\text{qp}}}}_{\boldsymbol{c}_e} = \boldsymbol{0} \qquad \text{(floating-base dynamics)}$$

$$\underbrace{\overset{\text{(6x12)}}{\begin{bmatrix}\boldsymbol{J}_c & \boldsymbol{0}_{6\times 12}\end{bmatrix}}}_{\boldsymbol{C}_e} \begin{bmatrix}\boldsymbol{\delta}_{\ddot{q}} \\ \boldsymbol{\delta}_{\rho}\end{bmatrix} + \boldsymbol{J}_c\ddot{\boldsymbol{q}}^{\text{cmd}} + \dot{\boldsymbol{J}}_c\dot{\boldsymbol{q}} = \boldsymbol{0} \qquad \text{(contact constraints)}$$

$$\underbrace{\begin{bmatrix}\boldsymbol{0}_{12\times 12} & \boldsymbol{I}_{12\times 12}\end{bmatrix}}_{\boldsymbol{C}_i} \begin{bmatrix}\boldsymbol{\delta}_{\ddot{q}} \\ \boldsymbol{\delta}_{\rho}\end{bmatrix} + \underbrace{\boldsymbol{\rho}_{\text{qp}}}_{\boldsymbol{c}_i} \geq \boldsymbol{0}_{12\times 1} \qquad \text{(reaction force)}$$

QuadProgpp Standard from

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \frac{1}{2}\boldsymbol{x}^T\boldsymbol{G}\boldsymbol{x} + \boldsymbol{g0x} \\ \text{s.t.} \quad & \boldsymbol{C}_{\text{e}}^T x + \boldsymbol{c}_{\text{e}} = 0 \\ & \boldsymbol{C}_{\text{i}}^T x + \boldsymbol{c}_{\text{i}} \geq 0 \end{aligned}$$

$$\boldsymbol{U} = \begin{bmatrix}\boldsymbol{u}_1 & \boldsymbol{u}_2 & \cdots & \boldsymbol{u}_m\end{bmatrix}$$

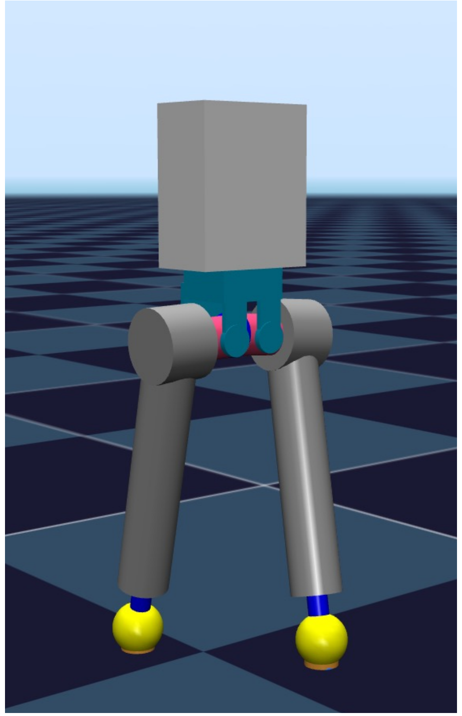
$$\boldsymbol{u}_i = \begin{bmatrix} cos(\frac{2\pi(i-1)}{m}) \\ sin(\frac{2\pi(i-1)}{m}) \\ 1 \end{bmatrix}$$

$$\underset{\text{(24x24)}}{\boldsymbol{G}} = \begin{bmatrix}\boldsymbol{W}_{\boldsymbol{\delta}_{\ddot{q}}} & \boldsymbol{0}_{12\times 12} \\ \boldsymbol{0}_{12\times 12} & \boldsymbol{W}_{\boldsymbol{\delta}_{\rho}}\end{bmatrix} \qquad \underset{\text{(12x24)}}{\boldsymbol{C}_i} = \begin{bmatrix}\boldsymbol{0}_{12\times 12} & \boldsymbol{I}_{12\times 12}\end{bmatrix} \qquad \underset{\text{(12x24)}}{\boldsymbol{C}_e} = \begin{bmatrix}\boldsymbol{M}_{6\times 12} & -\boldsymbol{J}_c^T\boldsymbol{R}_c\boldsymbol{U} \\ \boldsymbol{J}_c & \boldsymbol{0}_{6\times 12}\end{bmatrix}$$

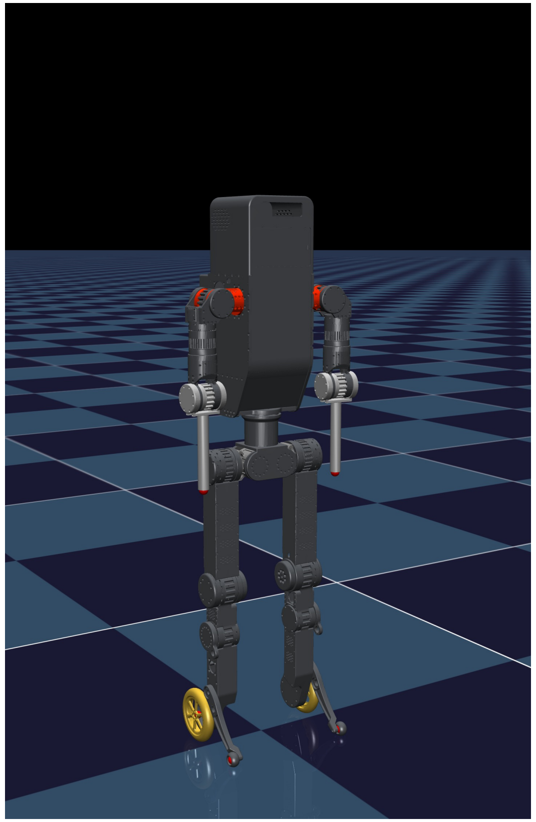
$$\underset{\text{(24x1)}}{\boldsymbol{g}_0} = \boldsymbol{0}_{24\times 1} \qquad \underset{\text{(12x1)}}{\boldsymbol{c}_i} = \boldsymbol{\rho}_{\text{qp}} \qquad \underset{\text{(12x1)}}{\boldsymbol{c}_e} = \begin{bmatrix}\boldsymbol{M}\ddot{\boldsymbol{q}}^{\text{cmd}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{J}_c^T\boldsymbol{f}_{\text{qp}} \\ \boldsymbol{J}_c\ddot{\boldsymbol{q}}^{\text{cmd}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{q}}\end{bmatrix}$$

Future Work to do:

- Implements Dynamic Locomotion Controller on Wheel-legged humanoid to validate the Architecture, in MuJoCo.
- Implements to Physical Wheel-legged Humanoid.



Figure#, PointFoot Template Model



Figure#, Wheel-legged Humanoid model in MuJoCo

Figure#, Wheel-legged Humanoid

Reference Papers

[1] Step Timing Adjustment: A Step toward Generating Robust Gaits

[2] Walking Control Based on Step Timing Adaptation

[3] Momentum Control with Hierarchical Inverse Dynamics on a Torque-Controlled Humanoid

[4] Design and control of BRAVER: a bipedal robot actuated via proprioceptive electric motors (Task Prioritized - WBC)

[5] Dynamic Locomotion For Passive-Ankle Biped Robots And Humanoids Using Whole-Body Locomotion Control

[6] Highly Dynamic Quadruped Locomotion via Whole-Body Impulse Control and Model Predictive Control (WBIC)

[7] MIT Cheetah 3: Design and Control of a Robust, Dynamic Quadruped Robot (MPC + WBC)

[8] A general framework for managing multiple tasks in highly redundant robotic systems (Task-Priority)

[9] Dynamic locomotion for passive-ankle biped robots and humanoids using whole-body locomotion control