• **Questão 1:**

ⓐ $$f'(x) = 0 + \frac{(4-x^2)\cdot(0) - 1\cdot(-2x)}{(4-x^2)^2} = \frac{2x}{(4-x^2)^2}$$

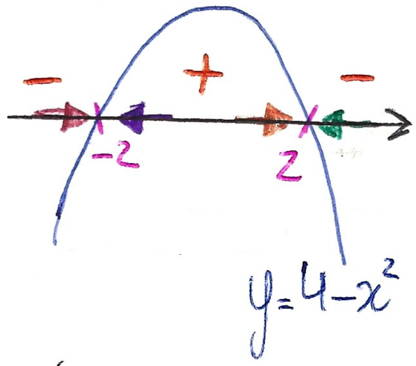
* Domínio de f: $4 - x^2 = 0 \therefore x = 2$ ou $x = -2$

$$Dom(f) = \mathbb{R} \setminus \{-2, 2\} = (-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$$

$$f'(x) > 0 \underset{\substack{\uparrow \\ (4-x^2)^2 > 0}}{\therefore} 2x > 0 \therefore x > 0$$

Então, f é crescente em $[0, 2)$ e em $(2, +\infty)$

ⓑ
$$\begin{cases} \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} 2 + \dfrac{1}{\underbrace{4-x^2}_{0^-}} = -\infty \\ \text{ou} \\ \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} 2 + \dfrac{1}{\underbrace{4-x^2}_{0^+}} = +\infty \end{cases}$$



Logo, $x = 2$ é uma assíntota vertical

$$\begin{cases} \lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} 2 + \dfrac{1}{\underbrace{4-x^2}_{0^+}} = +\infty \\ \text{ou} \\ \lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} 2 + \dfrac{1}{\underbrace{4-x^2}_{0^-}} = -\infty \end{cases}$$

Logo, $x = -2$ é uma assíntota vertical

c) $\lim_{x \to +\infty} 2 + \underbrace{\frac{1}{4-x^2}}_{\to 0} = 2 + 0 = 2$

Então, $y=2$ é uma assíntota horizontal

$\lim_{x \to -\infty} 2 + \underbrace{\frac{1}{4-x^2}}_{\to 0} = 2 + 0 = 2$

Logo, $y=2$ é a única assíntota horizontal

• Questão 2:

(a) $\int x\cos^2(\overset{u}{x^2})\,dx \overset{(*)}{=} \int \cos^2(u)\,\frac{du}{2} = \frac{1}{2}\int \frac{1+\cos(2u)}{2}\,du$

$(*)\ u = x^2 \therefore \frac{du}{dx} = 2x \therefore \frac{du}{2} = x\,dx$

$= \frac{1}{4}\int 1+\cos(2u)\,du = \frac{1}{4}\left(u + \frac{\text{sen}(2u)}{2}\right) + C$

$= \frac{1}{4}x^2 + \frac{1}{8}\text{sen}(2x^2) + C.$

$uv - \int v\,du$

(b) $\int \overbrace{x^2 \underbrace{\ln(3x)}_{u}\, dx}^{dv} \overset{(*)}{=} \frac{x^3}{3}\ln(3x) - \int \frac{x^{\cancel{3}\,2}}{3}\cdot\frac{1}{\cancel{x}}\,dx$

$(*)\begin{cases} u = \ln(3x) \therefore \frac{du}{dx} = \frac{1}{\cancel{3}x}\cdot\cancel{3} \therefore du = \frac{1}{x}\,dx \\ \int dv = \int x^2\,dx \therefore v = \frac{x^3}{3} \end{cases}$

$= \frac{x^3}{3}\ln(3x) - \frac{1}{3}\int x^2\,dx = \frac{x^3}{3}\ln(3x) - \frac{1}{3}\cdot\frac{x^3}{3} + C$

$= \frac{x^3}{3}\ln(3x) - \frac{x^3}{9} + C.$

• Questão 3:

ⓐ $f'(x) = e^{2(x-1)^2-1} \cdot (4(x-1)) = 4(x-1)\underbrace{e^{2(x-1)^2-1}}_{>0}$

$$\begin{cases} f'(x) > 0 \therefore x-1 > 0 \therefore x > 1 \\ f'(x) < 0 \therefore x-1 < 0 \therefore \boxed{x < 1} \end{cases}$$

$P = (e, \underline{0}) \in graf(f^{-1}) \therefore 0 \in Dom(f)$

$Im(f^{-1}) = Dom(f)$

Então, $Dom(f) = (-\infty, 1]$

$x = f(y) \therefore x = e^{2(y-1)^2-1} \therefore \ln(x) = \ln\left(e^{2(y-1)^2-1}\right)$

$\therefore \ln(x) = 2(y-1)^2 - 1 \therefore \sqrt{(y-1)^2} = \sqrt{\dfrac{1+\ln(x)}{2}}$

$\therefore |y-1| = \sqrt{\dfrac{1+\ln(x)}{2}} \therefore \begin{cases} y = 1 + \sqrt{\dfrac{1+\ln(x)}{2}} \\ \boxed{y = 1 - \sqrt{\dfrac{1+\ln(x)}{2}}} \end{cases}$

Em $P = (e, 0)$: $\begin{cases} 0 \neq 1 + \sqrt{\dfrac{1+\ln(e)}{2}} = 1 + \sqrt{\dfrac{2}{2}} = 1 + 1 = 2 \\ \boxed{\ln(e) = 1} \\ 0 = 1 - \sqrt{\dfrac{1+\ln(e)}{2}} = 1 - \sqrt{\dfrac{2}{2}} = 1 - 1 = 0 \end{cases}$

Logo, $f^{-1}(x) = 1 - \sqrt{\dfrac{1+\ln(x)}{2}}$

(b) $P=(e,0) \therefore f^{-1}(e)=0$

(i) <u>TFI</u>: $(f^{-1})'(e) = \dfrac{1}{f'(f^{-1}(e))} = \dfrac{1}{f'(0)} = \dfrac{1}{4(-1)e^{2(-1)^2-1}} = \dfrac{1}{-4e}$

ou

(ii) $f^{-1}(x) = 1 - \left(\dfrac{1+\ln(x)}{2}\right)^{\frac{1}{2}} \therefore (f^{-1})'(x) = -\dfrac{1}{2}\left(\dfrac{1+\ln(x)}{2}\right)^{-\frac{1}{2}} \cdot \left(\dfrac{\frac{1}{x}}{2}\right)$

$\therefore (f^{-1})'(e) = -\dfrac{1}{2}\left(\dfrac{1+\overset{=1}{\ln(e)}}{2}\right)^{-\frac{1}{2}} \cdot \left(\dfrac{1}{2e}\right) = -\dfrac{1}{4e} \cdot \left(\overset{=1}{\dfrac{2}{2}}\right)^{-\frac{1}{2}} = -\dfrac{1}{4e}$

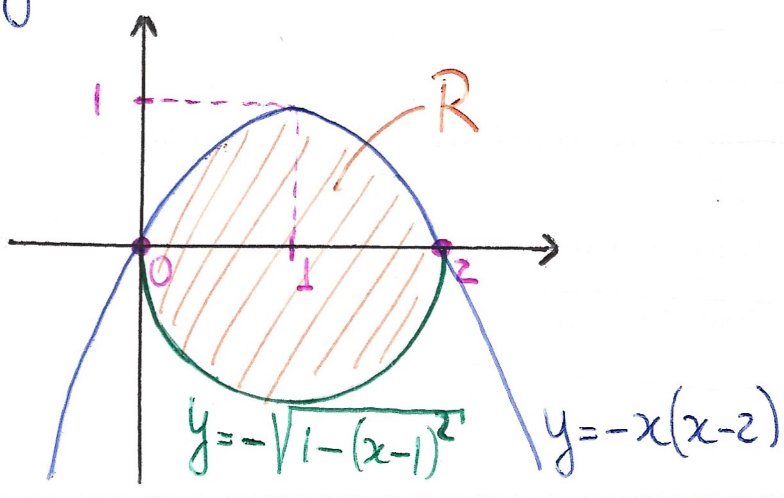
Logo, $y = \underbrace{(f^{-1})'(e)}_{-\frac{1}{4e}}(x-e) + \underbrace{f^{-1}(e)}_{=0} \therefore y = -\dfrac{1}{4e}(x-e)$

• <u>Questão 4</u>:

(a) $\underbrace{y = -\sqrt{1-(x-1)^2}}_{} \therefore y^2 = 1-(x-1)^2 \therefore (x-1)^2 + y^2 = 1$

semicírculo inferior: $C=(1,0)$ e $r=1$

$y = -x(x-2) = -x^2+2x$ (Parábola)



(b) * Área da região acima do eixo x:

$$\int_0^2 -x(x-2)\,dx = \int_0^2 -x^2+2x\,dx = \left(-\frac{x^3}{3}+x^2\right)\Big|_0^2$$

$$= \left(-\frac{8}{3}+4\right)-(0) = \frac{4}{3}$$

* Área da região abaixo do eixo x:

$$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$$

Então: $\text{Área}(R) = \frac{4}{3} + \frac{\pi}{2}$