

EXTENDS *Naturals*

CONSTANTS *UNKNOWN*, *NONE*, *Participants*, *MaxAmount*

VARIABLES *initialMoney*, *lastBid*, *winner*

A1vars \triangleq $\langle \textit{initialMoney}, \textit{lastBid}, \textit{winner} \rangle$

A1Bid \triangleq

$\wedge \forall p \in \textit{Participants} : \textit{winner}[p] = \textit{UNKNOWN}$
 $\wedge \exists p \in \textit{Participants} :$
 $\quad \exists \textit{newBid} \in (\textit{lastBid}[p] + 1) .. \textit{initialMoney}[p] :$
 $\quad \quad \textit{lastBid}' = [\textit{lastBid} \text{ EXCEPT } ![p] = \textit{newBid}]$
 $\wedge \text{UNCHANGED } \langle \textit{winner}, \textit{initialMoney} \rangle$

A1FirstChooseWinner \triangleq

$\wedge \forall p \in \textit{Participants} : \textit{winner}[p] = \textit{UNKNOWN}$
 $\wedge \vee \exists p, p2 \in \textit{Participants} :$
 $\quad \wedge \forall p3 \in \textit{Participants} \setminus \{p\} : \textit{lastBid}[p] > \textit{lastBid}[p3]$
 $\quad \wedge \textit{winner}' = [\textit{winner} \text{ EXCEPT } ![p2] = p]$
 $\quad \vee \exists p \in \textit{Participants} : \textit{winner}' = [\textit{winner} \text{ EXCEPT } ![p] = \textit{NONE}]$
 $\wedge \text{UNCHANGED } \langle \textit{lastBid}, \textit{initialMoney} \rangle$

A1OthersChooseWinner \triangleq

$\wedge \exists p, p2 \in \textit{Participants} :$
 $\quad \wedge \textit{winner}[p] \neq \textit{UNKNOWN}$
 $\quad \wedge \textit{winner}[p2] = \textit{UNKNOWN}$
 $\quad \wedge \textit{winner}' = [\textit{winner} \text{ EXCEPT } ![p2] = \textit{winner}[p]]$
 $\wedge \text{UNCHANGED } \langle \textit{lastBid}, \textit{initialMoney} \rangle$

A1Init \triangleq

$\wedge \textit{initialMoney} \in [\textit{Participants} \rightarrow 0 .. \textit{MaxAmount}]$
 $\wedge \textit{lastBid} = [p \in \textit{Participants} \mapsto 0]$
 $\wedge \textit{winner} = [p \in \textit{Participants} \mapsto \textit{UNKNOWN}]$

A1Next \triangleq

$\vee \textit{A1Bid}$
 $\vee \textit{A1FirstChooseWinner}$
 $\vee \textit{A1OthersChooseWinner}$

A1TypeOK \triangleq

$\wedge \textit{initialMoney} \in [\textit{Participants} \rightarrow 0 .. \textit{MaxAmount}]$
 $\wedge \textit{lastBid} \in [\textit{Participants} \rightarrow \textit{Nat}]$
 $\wedge \textit{winner} \in [\textit{Participants} \rightarrow$
 $\quad \textit{Participants} \cup \{\textit{UNKNOWN}, \textit{NONE}\}]$

$terminated \triangleq \forall p \in Participants : winner[p] \neq UNKNOWN$

$termination \triangleq \Diamond \Box terminated$ liveness

$agreed \triangleq \forall p, p2 \in Participants :$
 $\quad \vee winner[p] = UNKNOWN$
 $\quad \vee winner[p2] = UNKNOWN$
 $\quad \vee winner[p] = winner[p2]$

$agreement \triangleq \Box agreed$ safety

$solvable \triangleq \forall p \in Participants :$
 $\quad lastBid[p] \in 0 \dots (initialMoney[p])$

$winWithHigherBid \triangleq \forall p, p2 \in Participants :$
 $\quad winner[p2] = p \Rightarrow lastBid[p] > lastBid[p2] \vee p = p2$

$valid \triangleq solvable \wedge winWithHigherBid$

$validity \triangleq \Box valid$ safety

$A1FairSpec \triangleq$
 $\quad \wedge A1Init$
 $\quad \wedge \Box [A1Next]_{A1vars}$
 $\quad \wedge WF_{A1vars}(A1OthersChooseWinner)$
 $\quad \wedge WF_{A1vars}(A1FirstChooseWinner)$

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