MODULE *Auction3*

EXTENDS *Naturals*, *FiniteSets*, *Sequences*, *TLC*

CONSTANTS *NULL*, *Participants*, *MaxAmount*, *UNKNOWN*, *NONE*, *PASS*, *CHANGE*

VARIABLES *msgs*, *frontiers*, *initialMoney*, *lastBid*, *bid*, *round*, *passed*, *winner*

$A3vars \triangleq \langle msgs, frontiers, initialMoney, lastBid, bid, round, passed, winner \rangle$

$max(n1, n2) \triangleq$ IF $n1 > n2$ THEN $n1$ ELSE $n2$

$knowsHasPassed(p, p2) \triangleq$
  $\land$ LET $lastMsgIdx \triangleq frontiers[p][p2]$
    IN IF $lastMsgIdx = 0$ THEN FALSE
       ELSE $msgs[p2][lastMsgIdx] = PASS$

$knownLastBid(p, p2) \triangleq$
  LET $frontier \triangleq frontiers[p]$
  IN IF $frontier[p2] = 0$ THEN 0
     ELSE LET $lastKnownMsg \triangleq msgs[p2][frontier[p2]]$
          IN IF $lastKnownMsg = PASS$
             THEN IF $frontier[p2] = 1$ THEN 0
                  ELSE $msgs[p2][frontier[p2] - 2]$
             ELSE IF $lastKnownMsg \in Nat$
                  THEN $lastKnownMsg$
                  ELSE $msgs[p2][frontier[p2] - 1]$

$count(el, seq) \triangleq$
  LET RECURSIVE $helper(\_)$
      $helper(s) \triangleq$
        IF $s = \langle\rangle$ THEN 0
        ELSE IF $Head(s) = el$
             THEN $1 + helper(Tail(s))$
             ELSE $helper(Tail(s))$
  IN $helper(seq)$

$knownRound(p, p2) \triangleq$
  LET $lastMsgIdx \triangleq frontiers[p][p2]$
  IN IF $lastMsgIdx = 0$ THEN 1
     ELSE $count(CHANGE, SubSeq(msgs[p2], 1, lastMsgIdx)) + 1$

$noActionInCurrentRound(p) \triangleq$
  LET $lastMsgIdx \triangleq frontiers[p][p]$
  IN IF $lastMsgIdx = 0$ THEN TRUE
     ELSE $msgs[p][lastMsgIdx] = CHANGE$

$A3readyForAction(p) \triangleq$

$\forall\, p2 \in Participants :$
$\lor\ knowsHasPassed(p,\ p2)$
$\lor\ round[p] = knownRound(p,\ p2)$

$addMsg(p,\ msg) \triangleq$
$\land\ frontiers' = [frontiers \text{ EXCEPT } ![p][p] = @ + 1]$
$\land\ msgs' = [msgs \text{ EXCEPT } ![p] = @ \circ \langle msg \rangle]$

$A3Init \triangleq$
$\land\ msgs = [p \in Participants \mapsto \langle\rangle]$
$\land\ frontiers\ \ = [p \in Participants \mapsto [pa \in Participants \mapsto 0]]$
$\land\ lastBid = [p \in Participants \mapsto 0]$
$\land\ bid = [p \in Participants \mapsto NULL]$
$\land\ round = [p \in Participants \mapsto 1]$
$\land\ passed = [p\ \ \in Participants \mapsto \text{FALSE}]$
$\land\ initialMoney \in [Participants \to 0\ ..\ MaxAmount]$
$\land\ winner = [p\ \in Participants \mapsto UNKNOWN]$

$A3Bid \triangleq \exists\, p \in Participants :$
$\land\ winner[p] = UNKNOWN$
$\land\ noActionInCurrentRound(p)$
$\land\ A3readyForAction(p)$
$\land \exists\, p2 \in Participants \setminus \{p\} : knownRound(p,\ p2) = round[p]$
$\land \exists\, newBid \in (lastBid[p] + 1)\ ..\ initialMoney[p] :$
$\land \forall\, p2\ \ \in Participants : newBid > knownLastBid(p,\ p2)$
$\land\ bid' = [bid \text{ EXCEPT } ![p] = newBid]$
$\land\ addMsg(p,\ newBid)$
$\land \text{UNCHANGED } \langle lastBid,\ round,\ passed,\ initialMoney,\ winner\rangle$

$A3Stand \triangleq \exists\, p \in Participants :$
$\land\ winner[p] = UNKNOWN$
$\land\ noActionInCurrentRound(p)$
$\land \exists\, p2 \in Participants \setminus \{p\} : knownRound(p,\ p2) = round[p]$
$\land \forall\, p2 \in Participants \setminus \{p\} : knownLastBid(p,\ p2) < lastBid[p]$
$\land\ A3readyForAction(p)$
$\land\ bid' = [bid \text{ EXCEPT } ![p] = lastBid[p]]$
$\land\ addMsg(p,\ lastBid[p])$
$\land \text{UNCHANGED } \langle lastBid,\ round,\ passed,\ initialMoney,\ winner\rangle$

$A3Pass \triangleq \exists\, p \in Participants :$
$\land\ \ winner[p] = UNKNOWN$
$\land\ \ noActionInCurrentRound(p)$
$\land\ \ \exists\, p2 \in Participants \setminus \{p\} : knownRound(p,\ p2) = round[p]$
$\land\ \ A3readyForAction(p)$
$\land\ \ passed' = [passed \text{ EXCEPT } ![p] = \text{TRUE}]$

$\land$ $addMsg(p,\ PASS)$
$\land$ UNCHANGED $\langle bid,\ lastBid,\ round,\ initialMoney,\ winner\rangle$

$A3NextRound \triangleq \exists\, p \in Participants :$
$\land\ winner[p] = UNKNOWN$
$\land\ Cardinality(\{p2 \in Participants : \neg knowsHasPassed(p,\ p2)\}) \neq 0$
$\land\ bid[p] \neq NULL$
$\land \forall\, p2 \in Participants :$
$\lor\ knowsHasPassed(p,\ p2)$
$\lor$ IF $round[p] = knownRound(p,\ p2)$
THEN IF $frontiers[p][p2] = 0$ THEN FALSE
ELSE $msgs[p2][frontiers[p][p2]] \in Nat$
ELSE $knownRound(p,\ p2) > round[p]$
$\land\ lastBid' = [lastBid$ EXCEPT $![p] = bid[p]]$
$\land\ bid' = [bid$ EXCEPT $![p] = NULL]$
$\land\ round' = [round$ EXCEPT $![p] = @ + 1]$
$\land\ addMsg(p,\ CHANGE)$
$\land$ UNCHANGED $\langle passed,\ initialMoney,\ winner\rangle$

$A3Merge \triangleq \exists\, sender,\ receiver \in Participants :$
$\land$ LET $sFrontier \triangleq frontiers[sender]$
$rFrontier \triangleq frontiers[receiver]$
$newRFrontier \triangleq [p \in Participants \mapsto max(sFrontier[p],\ rFrontier[p])]$
IN $frontiers' = [frontiers$ EXCEPT $![receiver] = newRFrontier]$
$\land$ UNCHANGED $\langle initialMoney,\ msgs,\ bid,\ lastBid,\ passed,\ round,\ winner\rangle$

$A3ChooseWinner \triangleq \exists\, p \in Participants :$
$\land\ winner[p] = UNKNOWN$
$\land\ \lor\ \land \forall\, p2 \in Participants : knowsHasPassed(p,\ p2)$
$\land\ winner' = [winner$ EXCEPT $![p] = NONE]$
$\lor \exists\, p2 \in Participants :$
$\land \neg knowsHasPassed(p,\ p2)$
$\land \forall\, p3 \in (Participants \setminus \{p2\}) :$
$\land\ knowsHasPassed(p,\ p3)$
$\land\ knownLastBid(p,\ p2) > knownLastBid(p,\ p3)$
$\land\ knownRound(p,\ p2) > knownRound(p,\ p3)$
$\land\ winner' = [winner$ EXCEPT $![p] = p2]$
$\land$ UNCHANGED $\langle msgs,\ frontiers,\ bid,\ lastBid,\ passed,\ round,\ initialMoney\rangle$

$A3Next \triangleq$
$\lor\ A3Bid$
$\lor\ A3Stand$
$\lor\ A3Pass$
$\lor\ A3NextRound$
$\lor\ A3Merge$
$\lor\ A3ChooseWinner$

$A3FairSpec \triangleq$
$\quad \land\ A3Init$
$\quad \land\ \Box[A3Next]_{A3vars}$
$\quad \land\ \mathrm{WF}_{A3vars}(A3Pass)$
$\quad \land\ \mathrm{WF}_{A3vars}(A3NextRound)$
$\quad \land\ \mathrm{WF}_{A3vars}(A3Merge)$
$\quad \land\ \mathrm{WF}_{A3vars}(A3ChooseWinner)$

INSTANCE $Auction2$

THEOREM $A3FairSpec \Rightarrow A2FairSpec$