
MODULE *Monopoly*

EXTENDS *Integers, Sequences, FiniteSets*

CONSTANTS *NULL*, Model value because TLA+ does not have built in support for null
NumPlayers, Number of players participating
StartingMoney, Amount of money that each player starts with
TotalMoney, Total money available
DiceMax, Highest number one die can show
JailFine, Fine for getting out of jail
BaseRailRent, Rent when owning 1 railroad
PassGoReward Reward for passing Go

VARIABLES *positions*, Board position of each player
money, Cash amount of each player
inJail, Jail status of each player
jailTime, Amount of rounds that each player is already in jail for
isBankrupt, Bankruptcy status of each player
board, Current status of the board (*e.g.* owner of properties, current level of properties etc.)
turnPlayer, Player taking turn at the moment
phase, Current phase of the game (determines the possible actions)
bankMoney, Amount of money is left in the bank
goojfChOwner, Owner of the “Get out of Jail free” card of the Chance deck
goojfCcOwner, Owner of the “Get out of Jail free” card of the Community Chest deck
doublesCount, Number of consecutive doubles rolled by the current player
free4AllOrder, Order of players in free-4-all phase
debt, Debt of a player in bankruptcy-prevention phase
chanceCards, Will never change, more transparent than constant
communityChestCards, Will never change
jailIndex Index of jail square on board, will never change

vars \triangleq $\langle positions, money, inJail, jailTime, isBankrupt, board, turnPlayer, phase, bankMoney, goojfChOwner, goojfCcOwner, doublesCount, free4AllOrder, debt, chanceCards, communityChestCards, jailIndex \rangle$

abs(*n*) \triangleq IF *n* < 0 THEN - *n* ELSE *n*

RECURSIVE *SeqSum*(-)

SeqSum(*sq*) \triangleq IF *sq* = $\langle \rangle$ THEN 0
ELSE *Head*(*sq*) + *SeqSum*(*Tail*(*sq*))

incrCirc(*initial*, *amount*, *maxIdx*) \triangleq ((*initial* + *amount* - 1) % *maxIdx*) + 1

currentSquare \triangleq *board*[*positions*[*turnPlayer*]]

isProperty(*field*) \triangleq *field.type* \in { “street”, “rail”, “util” }

$$\begin{aligned}
& \text{PayBank}(\text{player}, \text{amount}) \triangleq \\
& \quad \wedge \text{amount} \in 0 \dots \text{money}[\text{player}] \\
& \quad \wedge \text{money}' = [\text{money} \text{ EXCEPT } ![\text{player}] = @ - \text{amount}] \\
& \quad \wedge \text{bankMoney}' = \text{bankMoney} + \text{amount} \\
& \text{CollectFromBank}(\text{player}, \text{amount}) \triangleq \\
& \quad \wedge \text{amount} > 0 \\
& \quad \wedge \text{IF } \text{bankMoney} \geq \text{amount} \\
& \quad \quad \text{THEN } \wedge \text{money}' = [\text{money} \text{ EXCEPT } ![\text{player}] = @ + \text{amount}] \\
& \quad \quad \quad \wedge \text{bankMoney}' = \text{bankMoney} - \text{amount} \\
& \quad \quad \text{ELSE } \wedge \text{money}' = [\text{money} \text{ EXCEPT } ![\text{player}] = @ + \text{bankMoney}] \\
& \quad \quad \quad \wedge \text{bankMoney}' = 0 \\
& \text{ownedPropertyIdxs}(\text{player}) \triangleq \\
& \quad \{i \in 1 \dots \text{Len}(\text{board}) : \\
& \quad \quad \text{IF } \neg \text{isProperty}(\text{board}[i]) \\
& \quad \quad \quad \text{THEN FALSE} \\
& \quad \quad \quad \text{ELSE } \text{board}[i].\text{owner} = \text{player}\} \\
& \text{noStreetFromSameSetHasBuildings}(\text{strIdx}) \triangleq \\
& \quad \text{LET } p_set \triangleq \text{board}[\text{strIdx}].\text{set} \\
& \quad \text{IN } \text{Cardinality}(\\
& \quad \quad \{i \in 1 \dots \text{Len}(\text{board}) : \\
& \quad \quad \quad \text{IF } \neg \text{board}[i].\text{type} = \text{"street"} \\
& \quad \quad \quad \quad \text{THEN FALSE} \\
& \quad \quad \quad \quad \text{ELSE } \wedge \text{board}[i].\text{level} > 1 \\
& \quad \quad \quad \quad \quad \wedge \text{board}[i].\text{set} = p_set \\
& \quad \quad \}) = 0 \\
& \text{ownsAllOfSet}(\text{owner}, \text{set}) \triangleq \\
& \quad \forall \text{idx} \in \text{DOMAIN } \text{board} : \\
& \quad \quad \text{IF } \text{board}[\text{idx}].\text{type} \neq \text{"street"} \\
& \quad \quad \quad \text{THEN TRUE} \\
& \quad \quad \quad \text{ELSE } \text{board}[\text{idx}].\text{set} = \text{set} \Rightarrow \text{board}[\text{idx}].\text{owner} = \text{owner} \\
& \text{permutationSequences}(S) \triangleq \\
& \quad \{p \in \text{UNION } \{[1 \dots \text{Cardinality}(S) \rightarrow S]\} : \\
& \quad \quad \forall i1, i2 \in \text{DOMAIN } p : \\
& \quad \quad \quad i1 \neq i2 \Rightarrow p[i1] \neq p[i2]\} \\
& \text{initializeFree4All} \triangleq \\
& \quad \wedge \text{phase}' = \text{"free-4-all"} \\
& \quad \wedge \exists \text{order} \in \text{permutationSequences}(\{p \in 1 \dots \text{NumPlayers} : \neg \text{isBankrupt}[p]\}) : \\
& \quad \quad \text{free4AllOrder}' = \text{order} \\
& \text{terminated} \triangleq \\
& \quad \text{Cardinality}(\{i \in 1 \dots \text{NumPlayers} : \neg \text{isBankrupt}[i]\}) = 1
\end{aligned}$$

$EndPreRoll \triangleq$
 $\wedge \neg terminated$
 $\wedge phase = \text{"pre-roll"}$
 $\wedge phase' = \text{"roll"}$
 $\wedge UNCHANGED \langle positions, money, inJail, isBankrupt, board, turnPlayer,$
 $bankMoney, goojfCcOwner, goojfChOwner, doublesCount, jailTime,$
 $chanceCards, communityChestCards, debt, free4AllOrder, jailIndex \rangle$

$PlayGoojfCh \triangleq$
 $\wedge \neg terminated$
 $\wedge phase = \text{"pre-roll"}$
 $\wedge goojfChOwner = turnPlayer$
 $\wedge inJail[turnPlayer]$
 $\wedge inJail' = [inJail \text{ EXCEPT } ![turnPlayer] = FALSE]$
 $\wedge jailTime' = [jailTime \text{ EXCEPT } ![turnPlayer] = 0]$
 $\wedge goojfChOwner' = NULL$
 $\wedge UNCHANGED \langle bankMoney, board, chanceCards, communityChestCards, debt,$
 $doublesCount, free4AllOrder, goojfCcOwner, isBankrupt,$
 $jailIndex, money, phase, positions, turnPlayer \rangle$

$PlayGoojfCc \triangleq$
 $\wedge \neg terminated$
 $\wedge phase = \text{"pre-roll"}$
 $\wedge goojfCcOwner = turnPlayer$
 $\wedge inJail[turnPlayer]$
 $\wedge inJail' = [inJail \text{ EXCEPT } ![turnPlayer] = FALSE]$
 $\wedge jailTime' = [jailTime \text{ EXCEPT } ![turnPlayer] = 0]$
 $\wedge goojfCcOwner' = NULL$
 $\wedge UNCHANGED \langle bankMoney, board, chanceCards, communityChestCards, debt,$
 $doublesCount, free4AllOrder, goojfChOwner, isBankrupt,$
 $jailIndex, money, phase, positions, turnPlayer \rangle$

$PayJailFine \triangleq$
 $\wedge \neg terminated$
 $\wedge inJail[turnPlayer]$
 $\wedge money[turnPlayer] \geq JailFine$
 $\wedge PayBank(turnPlayer, JailFine)$
 $\wedge inJail' = [inJail \text{ EXCEPT } ![turnPlayer] = FALSE]$
 $\wedge jailTime' = [jailTime \text{ EXCEPT } ![turnPlayer] = 0]$
 $\wedge UNCHANGED \langle board, chanceCards, communityChestCards, debt,$
 $doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner,$
 $isBankrupt, jailIndex, phase, positions, turnPlayer \rangle$

$UnmortgageProperty(player) \triangleq \exists idx \in ownedPropertyIdxs(player) :$

$$\begin{aligned}
& \wedge \text{board}[idx].\text{mortgaged} \\
& \wedge \text{LET } \text{mortgageValue} \triangleq \text{board}[idx].\text{value} \div 2 \\
& \quad \text{unmortgageCost} \triangleq \text{mortgageValue} + (\text{mortgageValue} \div 10) \\
& \text{IN } \wedge \text{money}[\text{player}] \geq \text{unmortgageCost} \\
& \quad \wedge \text{board}' = [\text{board} \text{ EXCEPT } ![idx].\text{mortgaged} = \text{FALSE}] \\
& \quad \wedge \text{PayBank}(\text{player}, \text{unmortgageCost}) \\
\\
& \text{MortgageProperty}(\text{player}) \triangleq \exists idx \in \text{ownedPropertyIdxs}(\text{player}) : \\
& \quad \wedge \neg \text{board}[idx].\text{mortgaged} \\
& \quad \wedge \text{board}[idx].\text{type} = \text{"street"} \Rightarrow \text{noStreetFromSameSetHasBuildings}(idx) \\
& \quad \wedge \text{LET } \text{mortgageValue} \triangleq \text{board}[idx].\text{value} \div 2 \\
& \quad \text{IN } \wedge \text{board}' = [\text{board} \text{ EXCEPT } ![idx].\text{mortgaged} = \text{TRUE}] \\
& \quad \wedge \text{CollectFromBank}(\text{player}, \text{mortgageValue}) \\
\\
& \text{allFromSetAreHigherOrEqualLevel}(\text{set}, \text{level}) \triangleq \\
& \quad \forall idx \in \text{DOMAIN } \text{board} : \\
& \quad \text{IF } \text{board}[idx].\text{type} \neq \text{"street"} \\
& \quad \text{THEN TRUE} \\
& \quad \text{ELSE } \text{board}[idx].\text{set} = \text{set} \Rightarrow \text{board}[idx].\text{level} \geq \text{level} \\
\\
& \text{UpgradeStreet}(\text{player}) \triangleq \exists idx \in \text{ownedPropertyIdxs}(\text{player}) : \\
& \quad \wedge \text{IF } \neg \text{board}[idx].\text{type} = \text{"street"} \\
& \quad \text{THEN FALSE} \\
& \quad \text{ELSE LET } \text{street} \triangleq \text{board}[idx] \\
& \quad \quad \text{IN } \wedge \neg \text{street}.\text{mortgaged} \\
& \quad \quad \wedge \text{ownsAllOfSet}(\text{player}, \text{street}.\text{set}) \\
& \quad \quad \wedge \text{street}.\text{level} < \text{Len}(\text{board}[idx].\text{rent}) \\
& \quad \quad \wedge \text{allFromSetAreHigherOrEqualLevel}(\text{street}.\text{set}, \text{street}.\text{level}) \\
& \quad \quad \wedge \text{money}[\text{player}] \geq \text{street}.\text{houseCost} \\
& \quad \quad \wedge \text{board}' = [\text{board} \text{ EXCEPT } ![idx].\text{level} = @ + 1] \\
& \quad \quad \wedge \text{PayBank}(\text{player}, \text{street}.\text{houseCost}) \\
\\
& \text{allFromSetAreLowerOrEqualLevel}(\text{set}, \text{level}) \triangleq \\
& \quad \forall idx \in \text{DOMAIN } \text{board} : \\
& \quad \text{IF } \text{board}[idx].\text{type} \neq \text{"street"} \\
& \quad \text{THEN TRUE} \\
& \quad \text{ELSE } \text{board}[idx].\text{set} = \text{set} \Rightarrow \text{board}[idx].\text{level} \leq \text{level} \\
\\
& \text{DowngradeStreet}(\text{player}) \triangleq \exists idx \in \text{ownedPropertyIdxs}(\text{player}) : \\
& \quad \wedge \text{IF } \neg \text{board}[idx].\text{type} = \text{"street"} \\
& \quad \text{THEN FALSE} \\
& \quad \text{ELSE LET } \text{street} \triangleq \text{board}[idx] \\
& \quad \quad \text{IN } \wedge \text{street}.\text{level} > 1 \\
& \quad \quad \wedge \text{allFromSetAreLowerOrEqualLevel}(\text{street}.\text{set}, \text{street}.\text{level}) \\
& \quad \quad \wedge \text{board}' = [\text{board} \text{ EXCEPT } ![idx].\text{level} = @ - 1] \\
& \quad \quad \wedge \text{CollectFromBank}(\text{player}, \text{street}.\text{houseCost} \div 2)
\end{aligned}$$

$$\begin{aligned}
PreRollUnmortgage &\triangleq \\
&\wedge \neg terminated \\
&\wedge phase = \text{"pre-roll"} \\
&\wedge UnmortgageProperty(turnPlayer) \\
&\wedge UNCHANGED \langle chanceCards, communityChestCards, debt, \\
&\quad doublesCount, free4AllOrder, goojfCcOwner, \\
&\quad goojfChOwner, inJail, isBankrupt, jailIndex, \\
&\quad jailTime, phase, positions, turnPlayer \rangle \\
\\
PreRollMortgage &\triangleq \\
&\wedge \neg terminated \\
&\wedge phase = \text{"pre-roll"} \\
&\wedge MortgageProperty(turnPlayer) \\
&\wedge UNCHANGED \langle chanceCards, communityChestCards, debt, \\
&\quad doublesCount, free4AllOrder, goojfCcOwner, \\
&\quad goojfChOwner, inJail, isBankrupt, jailIndex, \\
&\quad jailTime, phase, positions, turnPlayer \rangle \\
\\
PreRollUpgrade &\triangleq \\
&\wedge \neg terminated \\
&\wedge phase = \text{"pre-roll"} \\
&\wedge UpgradeStreet(turnPlayer) \\
&\wedge UNCHANGED \langle chanceCards, communityChestCards, debt, doublesCount, \\
&\quad free4AllOrder, goojfCcOwner, goojfChOwner, inJail, \\
&\quad isBankrupt, jailIndex, jailTime, phase, positions, turnPlayer \rangle \\
\\
PreRollDowngrade &\triangleq \\
&\wedge \neg terminated \\
&\wedge phase = \text{"pre-roll"} \\
&\wedge DowngradeStreet(turnPlayer) \\
&\wedge UNCHANGED \langle chanceCards, communityChestCards, debt, doublesCount, \\
&\quad free4AllOrder, goojfCcOwner, goojfChOwner, inJail, \\
&\quad isBankrupt, jailIndex, jailTime, phase, positions, turnPlayer \rangle \\
\\
TakePreRollAction &\triangleq \\
&\vee EndPreRoll \\
&\vee PlayGoojfCh \\
&\vee PlayGoojfCc \\
&\vee PayJailFine \\
&\vee PreRollUnmortgage \\
&\vee PreRollMortgage \\
&\vee PreRollUpgrade \\
&\vee PreRollDowngrade \\
\\
ChangeOwnerOfProperties(from, to) &\triangleq \\
&\wedge board' = [i \in \text{DOMAIN } board \mapsto
\end{aligned}$$

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    LET  $field \triangleq board[i]$ 
    IN IF  $isProperty(field)$ 
        THEN IF  $field.owner = from$ 
            THEN  $[field \text{ EXCEPT } !.owner = to]$ 
            ELSE  $field$ 
        ELSE  $field]$ 
     $\wedge$  IF  $goojfCcOwner = from$ 
        THEN  $goojfCcOwner' = to$ 
        ELSE UNCHANGED  $\langle goojfCcOwner \rangle$ 
     $\wedge$  IF  $goojfChOwner = from$ 
        THEN  $goojfChOwner' = to$ 
        ELSE UNCHANGED  $\langle goojfChOwner \rangle$ 

CollectIfPassGo  $\triangleq$ 
    IF  $positions'[turnPlayer] < positions[turnPlayer]$ 
    THEN CollectFromBank( $turnPlayer$ ,  $PassGoReward$ )
    ELSE UNCHANGED  $\langle money, bankMoney \rangle$ 

MoveAfterRoll( $amount$ )  $\triangleq$ 
     $\wedge positions' = [positions \text{ EXCEPT } ![turnPlayer] = incrCirc(@, amount, Len(board))]$ 
     $\wedge$  CollectIfPassGo

GoToJail  $\triangleq$ 
     $\wedge inJail' = [inJail \text{ EXCEPT } ![turnPlayer] = TRUE]$ 
     $\wedge doublesCount' = 0$ 
     $\wedge positions' = [positions \text{ EXCEPT } ![turnPlayer] = jailIndex]$ 
     $\wedge initializeFree4All$ 

RollAndMove  $\triangleq$ 
     $\exists d1, d2 \in 1 \dots DiceMax :$ 
     $\wedge \neg terminated$ 
     $\wedge phase = \text{"roll"}$ 
     $\wedge inJail[turnPlayer] = FALSE$ 
     $\wedge$  IF  $d1 \neq d2$ 
        THEN  $\wedge MoveAfterRoll(d1 + d2)$ 
         $\wedge doublesCount' = 0$ 
         $\wedge phase' = \text{"post-roll"}$ 
         $\wedge$  UNCHANGED  $\langle inJail, free4AllOrder \rangle$ 
    ELSE IF  $doublesCount = 2$  Current throw is 3rd consecutive doubles
        THEN  $\wedge GoToJail$ 
         $\wedge$  UNCHANGED  $\langle bankMoney, money \rangle$ 
    ELSE  $\wedge MoveAfterRoll(d1 + d2)$ 
         $\wedge doublesCount' = doublesCount + 1$ 
         $\wedge phase' = \text{"post-roll"}$ 
         $\wedge$  UNCHANGED  $\langle inJail, free4AllOrder \rangle$ 
     $\wedge$  UNCHANGED  $\langle board, chanceCards, communityChestCards, debt, goojfCcOwner,$ 

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goojfChOwner, isBankrupt, jailIndex, jailTime, turnPlayer

RollInJail \triangleq

$\exists d1, d2 \in 1 \dots DiceMax :$

$\wedge \neg terminated$

$\wedge phase = \text{"roll"}$

$\wedge inJail[turnPlayer] = \text{TRUE}$

$\wedge \text{IF } d1 \neq d2$

THEN IF *jailTime*[*turnPlayer*] = 2 has missed doubles for the 3rd time

THEN $\wedge MoveAfterRoll(d1 + d2)$

$\wedge jailTime' = [jailTime \text{ EXCEPT } ![turnPlayer] = 0]$

$\wedge inJail' = [inJail \text{ EXCEPT } ![turnPlayer] = \text{FALSE}]$

$\wedge \text{IF } money[turnPlayer] \geq JailFine$

THEN $\wedge PayBank(turnPlayer, JailFine)$

$\wedge phase' = \text{"post-roll"}$

$\wedge \text{UNCHANGED } \langle debt \rangle$

ELSE $\wedge phase' = \text{"bankruptcy-prevention"}$

$\wedge debt' = [creditor \mapsto NULL,$

$amount \mapsto JailFine,$

$nextPhase \mapsto \text{"post-roll"}]$

$\wedge \text{UNCHANGED } \langle free4AllOrder \rangle$

ELSE $\wedge jailTime' = [jailTime \text{ EXCEPT } ![turnPlayer] = @ + 1]$

$\wedge initializeFree4All$

$\wedge \text{UNCHANGED } \langle money, bankMoney, positions, inJail, debt \rangle$

ELSE $\wedge MoveAfterRoll(d1 + d2)$ Player will not get to roll again even if they rolled doubles.

$\wedge jailTime' = [jailTime \text{ EXCEPT } ![turnPlayer] = 0]$

$\wedge inJail' = [inJail \text{ EXCEPT } ![turnPlayer] = \text{FALSE}]$

$\wedge phase' = \text{"post-roll"}$

$\wedge \text{UNCHANGED } \langle free4AllOrder, debt \rangle$

$\wedge \text{UNCHANGED } \langle board, chanceCards, communityChestCards, doublesCount,$

$goojfCcOwner, goojfChOwner, isBankrupt, jailIndex, turnPlayer \rangle$

TakeRollAction $\triangleq \vee RollAndMove$

$\vee RollInJail$

BuyProperty \triangleq

IF $\neg isProperty(currentSquare)$

THEN FALSE

ELSE $\wedge \neg terminated$

$\wedge phase = \text{"post-roll"}$

$\wedge currentSquare.owner = NULL$

$\wedge money[turnPlayer] \geq currentSquare.value$

$\wedge PayBank(turnPlayer, currentSquare.value)$

$\wedge board' = [board \text{ EXCEPT } ![positions[turnPlayer]].owner = turnPlayer]$

$\wedge phase' = \text{"doubles-check"}$

$\wedge \text{UNCHANGED } \langle inJail, positions, turnPlayer, doublesCount, jailTime,$

isBankrupt, goojfCcOwner, goojfChOwner, chanceCards, communityChestCards, debt, free4AllOrder, jailIndex

$PayPlayer(from, to, amount) \triangleq$
 $money' = [money \text{ EXCEPT } ![from] = @ - amount,$
 $![to] = @ + amount]$

$getStreetRent(street) \triangleq$
 IF *street.level* > 1
 THEN *street.rent*[*street.level*]
 ELSE IF *ownsAllOfSet*(*street.owner*, *street.set*)
 THEN *street.rent*[1] * 2
 ELSE *street.rent*[1]

$PayStreetRent \triangleq$
 IF *currentSquare.type* \neq "street"
 THEN FALSE
 ELSE $\wedge \neg terminated$
 $\wedge phase = \text{"post-roll"}$
 $\wedge \text{LET } rentCost \triangleq getStreetRent(currentSquare)$
 $owner \triangleq currentSquare.owner$
 IN $\wedge owner \notin \{NULL, turnPlayer\}$
 $\wedge \neg currentSquare.mortgaged$
 $\wedge money[turnPlayer] \geq rentCost$
 $\wedge PayPlayer(turnPlayer, owner, rentCost)$
 $\wedge phase' = \text{"doubles-check"}$
 $\wedge \text{UNCHANGED } \langle bankMoney, board, chanceCards, communityChestCards,$
 $debt, doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner,$
 $inJail, isBankrupt, jailIndex, jailTime, positions, turnPlayer \rangle$

$PreventBankruptcyOnStreetRent \triangleq$
 IF *currentSquare.type* \neq "street"
 THEN FALSE
 ELSE $\wedge \neg terminated$
 $\wedge phase = \text{"post-roll"}$
 $\wedge \text{LET } rentCost \triangleq getStreetRent(currentSquare)$
 $owner \triangleq currentSquare.owner$
 IN $\wedge owner \notin \{NULL, turnPlayer\}$
 $\wedge \neg currentSquare.mortgaged$
 $\wedge money[turnPlayer] < rentCost$
 $\wedge debt = NULL$
 $\wedge debt' = [creditor \mapsto owner,$
 $amount \mapsto rentCost,$
 $nextPhase \mapsto \text{"doubles-check"}]$
 $\wedge phase' = \text{"bankruptcy-prevention"}$
 $\wedge \text{UNCHANGED } \langle bankMoney, board, chanceCards, communityChestCards, doublesCount,$

*free4AllOrder, goojfCcOwner, goojfChOwner, inJail, isBankrupt,
jailIndex, jailTime, money, positions, turnPlayer*

getRailRent(owner) \triangleq
 LET *ownedRails* \triangleq *Cardinality*(
 i \in DOMAIN *board* :
 IF \neg *board*[*i*].*type* = "rail"
 THEN FALSE
 ELSE *board*[*i*].*owner* = *owner*
 }
 IN *BaseRailRent* * $2^{(\text{ownedRails}-1)}$

PayRailRent \triangleq
 IF *currentSquare.type* \neq "rail"
 THEN FALSE
 ELSE $\wedge \neg$ *terminated*
 \wedge *phase* = "post-roll"
 \wedge LET *owner* \triangleq *currentSquare.owner*
 rentCost \triangleq *getRailRent(owner)*
 IN \wedge *owner* \notin {*NULL*, *turnPlayer*}
 $\wedge \neg$ *currentSquare.mortgaged*
 \wedge *money*[*turnPlayer*] \geq *rentCost*
 \wedge *PayPlayer*(*turnPlayer*, *owner*, *rentCost*)
 \wedge *phase'* = "doubles-check"
 \wedge UNCHANGED (*bankMoney*, *board*, *chanceCards*, *communityChestCards*, *debt*,
 doublesCount, *free4AllOrder*, *goojfCcOwner*, *goojfChOwner*, *inJail*,
 isBankrupt, *jailIndex*, *jailTime*, *positions*, *turnPlayer*)

PreventBankruptcyOnRailRent \triangleq
 IF *currentSquare.type* \neq "rail"
 THEN FALSE
 ELSE $\wedge \neg$ *terminated*
 \wedge *phase* = "post-roll"
 \wedge LET *owner* \triangleq *currentSquare.owner*
 rentCost \triangleq *getRailRent(owner)*
 IN \wedge *owner* \notin {*NULL*, *turnPlayer*}
 $\wedge \neg$ *currentSquare.mortgaged*
 \wedge *money*[*turnPlayer*] < *rentCost*
 \wedge *debt* = *NULL*
 \wedge *debt'* = [*creditor* \mapsto *owner*,
 amount \mapsto *rentCost*,
 nextPhase \mapsto "doubles-check"]
 \wedge *phase'* = "bankruptcy-prevention"
 \wedge UNCHANGED (*bankMoney*, *board*, *chanceCards*, *communityChestCards*, *doublesCount*,
 free4AllOrder, *goojfCcOwner*, *goojfChOwner*, *inJail*, *isBankrupt*, *jailIndex*,
 jailTime, *money*, *positions*, *turnPlayer*)

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ownsBothUtilities(owner)  $\triangleq$ 
  LET ownedUtils  $\triangleq$  Cardinality({
    i  $\in$  DOMAIN board :
      IF board[i].type  $\neq$  "util"
      THEN FALSE
      ELSE board[i].owner = owner
  })
  IN ownedUtils = 2

TryPayUtilRent  $\triangleq$ 
  IF currentSquare.type  $\neq$  "util"
  THEN FALSE
  ELSE  $\exists d1, d2 \in 1 \dots DiceMax$  :
     $\wedge \neg terminated$ 
     $\wedge phase = \text{"post-roll"}$ 
     $\wedge$  LET owner  $\triangleq$  currentSquare.owner
      multiplier  $\triangleq$  IF ownsBothUtilities(owner) THEN 10 ELSE 4
      rentCost  $\triangleq$  (d1 + d2) * multiplier
    IN  $\wedge owner \notin \{NULL, turnPlayer\}$ 
       $\wedge$  IF money[turnPlayer]  $\geq$  rentCost
        THEN  $\wedge PayPlayer(turnPlayer, owner, rentCost)$ 
           $\wedge phase' = \text{"doubles-check"}$ 
           $\wedge$  UNCHANGED  $\langle debt \rangle$ 
        ELSE  $\wedge debt = NULL$ 
           $\wedge debt' = [creditor \mapsto owner,$ 
             $amount \mapsto rentCost,$ 
             $nextPhase \mapsto \text{"doubles-check"}]$ 
           $\wedge phase' = \text{"bankruptcy-prevention"}$ 
           $\wedge$  UNCHANGED  $\langle money \rangle$ 
       $\wedge$  UNCHANGED  $\langle bankMoney, board, chanceCards, communityChestCards, doublesCount,$ 
         $free4AllOrder, goojfCcOwner, goojfChOwner, inJail, isBankrupt,$ 
         $jailIndex, jailTime, positions, turnPlayer \rangle$ 

PayTax  $\triangleq$ 
  IF currentSquare.type  $\neq$  "tax"
  THEN FALSE
  ELSE  $\wedge \neg terminated$ 
     $\wedge phase = \text{"post-roll"}$ 
     $\wedge money[turnPlayer] \geq currentSquare.value$ 
     $\wedge PayBank(turnPlayer, currentSquare.value)$ 
     $\wedge phase' = \text{"doubles-check"}$ 
     $\wedge$  UNCHANGED  $\langle board, chanceCards, communityChestCards, debt,$ 
       $doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner,$ 
       $inJail, isBankrupt, jailIndex, jailTime, positions, turnPlayer \rangle$ 

PreventBankruptcyOnTax  $\triangleq$ 

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IF currentSquare.type ≠ "tax"
THEN FALSE
ELSE ∧ ¬terminated
      ∧ phase = "post-roll"
      ∧ money[turnPlayer] < currentSquare.value
      ∧ debt' = [creditor ↦ NULL,
                  amount ↦ currentSquare.value,
                  nextPhase ↦ "doubles-check"]
      ∧ phase' = "bankruptcy-prevention"
      ∧ UNCHANGED ⟨bankMoney, board, chanceCards, communityChestCards,
                    doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner,
                    inJail, isBankrupt, jailIndex, jailTime, money, positions, turnPlayer⟩

AuctionProperty ≜
  IF ¬isProperty(currentSquare)
  THEN FALSE
  ELSE ∧ ¬terminated
        ∧ phase = "post-roll"
        ∧ currentSquare.owner = NULL
        ∧ ∨ ∃ winner ∈ 1 .. NumPlayers :
          ∃ bid ∈ {1, 5, 10} : should theoretically be 1 .. money[winner] but this makes state space explode
          ∧ money[winner] ≥ bid would be unnecessary with 1 .. money[winner]
          ∧ ¬isBankrupt[winner]
          ∧ PayBank(winner, bid)
          ∧ board' = [board EXCEPT ![positions[turnPlayer]].owner = winner]
          ∨ UNCHANGED ⟨board, bankMoney, money⟩
        ∧ phase' = "doubles-check"
        ∧ UNCHANGED ⟨chanceCards, communityChestCards, debt, doublesCount,
                      free4AllOrder, goojfCcOwner, goojfChOwner, inJail, isBankrupt,
                      jailIndex, jailTime, positions, turnPlayer⟩

LandOnGoToJail ≜
  ∧ ¬terminated
  ∧ currentSquare.type = "go-to-jail"
  ∧ GoToJail
  ∧ UNCHANGED ⟨bankMoney, board, chanceCards, communityChestCards,
                debt, goojfCcOwner, goojfChOwner, isBankrupt, jailIndex,
                jailTime, money, turnPlayer⟩

AdvanceTo(destinationIdx) ≜
  ∧ positions' = [positions EXCEPT ![turnPlayer] = destinationIdx]
  ∧ CollectIfPassGo

ExecuteCard(card) ≜
  LET type ≜ card.type

```

```

IN CASE  $type = \text{"collect"} \rightarrow \wedge CollectFromBank(turnPlayer, card.amount)$ 
 $\wedge phase' = \text{"doubles-check"}$ 
 $\wedge \text{UNCHANGED } \langle debt, positions, goojfCcOwner, goojfChOwner,$ 
 $doubleCount, inJail, free4AllOrder \rangle$ 
 $\square type = \text{"pay"} \rightarrow \text{IF } money[turnPlayer] \geq card.amount$ 
 $\text{THEN } \wedge PayBank(turnPlayer, card.amount)$ 
 $\wedge phase' = \text{"doubles-check"}$ 
 $\wedge \text{UNCHANGED } \langle debt, positions, goojfCcOwner, goojfChOwner,$ 
 $doubleCount, inJail, free4AllOrder \rangle$ 
 $\text{ELSE } \wedge debt' = [creditor \mapsto NULL,$ 
 $amount \mapsto card.amount,$ 
 $nextPhase \mapsto \text{"doubles-check"}]$ 
 $\wedge phase' = \text{"bankruptcy-prevention"}$ 
 $\wedge \text{UNCHANGED } \langle money, bankMoney, goojfCcOwner,$ 
 $goojfChOwner, positions, doubleCount,$ 
 $inJail, free4AllOrder \rangle$ 
 $\square type = \text{"advance"} \rightarrow \wedge AdvanceTo(card.square)$ 
 $\wedge \text{UNCHANGED } \langle debt, goojfCcOwner, goojfChOwner, phase,$ 
 $doubleCount, inJail, free4AllOrder \rangle$ 
 $\square type = \text{"go-to-jail"} \rightarrow \wedge GoToJail$ 
 $\wedge \text{UNCHANGED } \langle debt, goojfCcOwner, goojfChOwner,$ 
 $money, bankMoney \rangle$ 
 $\square type = \text{"goojf-cc"} \rightarrow \wedge goojfCcOwner' = turnPlayer$ 
 $\wedge phase' = \text{"doubles-check"}$ 
 $\wedge \text{UNCHANGED } \langle money, bankMoney, positions, goojfChOwner,$ 
 $debt, doubleCount, inJail, free4AllOrder \rangle$ 
 $\square type = \text{"goojf-ch"} \rightarrow \wedge goojfChOwner' = turnPlayer$ 
 $\wedge phase' = \text{"doubles-check"}$ 
 $\wedge \text{UNCHANGED } \langle money, bankMoney, positions, goojfCcOwner,$ 
 $debt, doubleCount, inJail, free4AllOrder \rangle$ 

```

```

DrawAndExecuteChanceCard  $\triangleq$ 
IF  $currentSquare.type \neq \text{"chance"}$ 
THEN FALSE
ELSE  $\wedge \neg terminated$ 
 $\wedge phase = \text{"post-roll"}$ 
 $\wedge \exists cardIdx \in \text{IF } goojfChOwner = NULL$ 
 $\text{THEN } 1 \dots Len(chanceCards)$ 
 $\text{ELSE } 1 \dots (Len(chanceCards) - 1) :$ 
 $\text{LET } card \triangleq chanceCards[cardIdx]$ 
 $\text{IN } ExecuteCard(card)$ 
 $\wedge \text{UNCHANGED } \langle board, chanceCards, communityChestCards, isBankrupt,$ 
 $jailIndex, jailTime, turnPlayer \rangle$ 

```

```

DrawAndExecuteCommunityChestCard  $\triangleq$ 

```

```

IF currentSquare.type ≠ "community-chest"
  THEN FALSE
  ELSE ∧ ¬terminated
        ∧ phase = "post-roll"
        ∧ ∃ cardIdx ∈ IF goojfCcOwner = NULL
          THEN 1 .. Len(communityChestCards)
          ELSE 1 .. (Len(communityChestCards) - 1) :
          LET card ≜ communityChestCards[cardIdx]
          IN ExecuteCard(card)
        ∧ UNCHANGED ⟨board, chanceCards, communityChestCards, isBankrupt,
                     jailIndex, jailTime, turnPlayer⟩

EndPostRoll ≜
  ∧ phase = "post-roll"
  ∧ phase' = "doubles-check"

DoNothingOnOwnProperty ≜
  IF ¬isProperty(currentSquare)
  THEN FALSE
  ELSE ∧ ¬terminated
        ∧ currentSquare.owner = turnPlayer
        ∧ EndPostRoll
        ∧ UNCHANGED ⟨bankMoney, board, chanceCards, communityChestCards, debt,
                     doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner, inJail,
                     isBankrupt, jailIndex, jailTime, money, positions, turnPlayer⟩

DoNothingOnJailSquare ≜
  ∧ ¬terminated
  ∧ currentSquare.type = "jail"
  ∧ EndPostRoll
  ∧ UNCHANGED ⟨bankMoney, board, chanceCards, communityChestCards, debt,
               doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner, inJail,
               isBankrupt, jailIndex, jailTime, money, positions, turnPlayer⟩

DoNothingOnGo ≜
  ∧ ¬terminated
  ∧ currentSquare.type = "go"
  ∧ EndPostRoll
  ∧ UNCHANGED ⟨bankMoney, board, chanceCards, communityChestCards, debt,
               doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner, inJail,
               isBankrupt, jailIndex, jailTime, money, positions, turnPlayer⟩

DoNothingOnFreeParking ≜
  ∧ ¬terminated

```

$$\begin{aligned}
& \wedge \text{currentSquare.type} = \text{"free-parking"} \\
& \wedge \text{EndPostRoll} \\
& \wedge \text{UNCHANGED } \langle \text{bankMoney, board, chanceCards, communityChestCards, debt,} \\
& \quad \text{doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner, inJail,} \\
& \quad \text{isBankrupt, jailIndex, jailTime, money, positions, turnPlayer} \rangle \\
\\
\text{DoNothingOnMortgagedProperty} & \triangleq \\
& \text{IF } \neg \text{isProperty}(\text{currentSquare}) \\
& \quad \text{THEN FALSE} \\
& \quad \text{ELSE } \wedge \neg \text{terminated} \\
& \quad \quad \wedge \text{currentSquare.owner} \neq \text{turnPlayer} \\
& \quad \quad \wedge \text{currentSquare.mortgaged} \\
& \quad \quad \wedge \text{EndPostRoll} \\
& \quad \quad \wedge \text{UNCHANGED } \langle \text{bankMoney, board, chanceCards, communityChestCards, debt,} \\
& \quad \quad \text{doublesCount, free4AllOrder, goojfCcOwner, goojfChOwner, inJail,} \\
& \quad \quad \text{isBankrupt, jailIndex, jailTime, money, positions, turnPlayer} \rangle \\
\\
\text{TakePostRollAction} & \triangleq \\
& \vee \text{DoNothingOnOwnProperty} \\
& \vee \text{DoNothingOnJailSquare} \\
& \vee \text{DoNothingOnGo} \\
& \vee \text{DoNothingOnFreeParking} \\
& \vee \text{DoNothingOnMortgagedProperty} \\
& \vee \text{PayStreetRent} \\
& \vee \text{PayRailRent} \\
& \vee \text{PreventBankruptcyOnStreetRent} \\
& \vee \text{PreventBankruptcyOnRailRent} \\
& \vee \text{TryPayUtilRent} \\
& \vee \text{BuyProperty} \\
& \vee \text{AuctionProperty} \\
& \vee \text{PayTax} \\
& \vee \text{PreventBankruptcyOnTax} \\
& \vee \text{LandOnGoToJail} \\
& \vee \text{DrawAndExecuteChanceCard} \\
& \vee \text{DrawAndExecuteCommunityChestCard} \\
\\
\text{DoublesCheck} & \triangleq \\
& \wedge \neg \text{terminated} \\
& \wedge \text{phase} = \text{"doubles-check"} \\
& \wedge \text{IF } \text{doublesCount} > 0 \\
& \quad \text{THEN } \wedge \text{phase}' = \text{"pre-roll"} \\
& \quad \quad \wedge \text{UNCHANGED } \langle \text{free4AllOrder} \rangle \\
& \quad \text{ELSE } \text{initializeFree4All} \\
& \wedge \text{UNCHANGED } \langle \text{bankMoney, board, chanceCards, communityChestCards, debt,} \\
& \quad \text{doublesCount, goojfCcOwner, goojfChOwner, inJail, isBankrupt,}
\end{aligned}$$

jailIndex, jailTime, money, positions, turnPlayer⟩

PayOffDebt \triangleq

IF *phase* \neq “bankruptcy-prevention”

THEN FALSE

ELSE $\wedge \neg terminated$

$\wedge money[turnPlayer] \geq debt.amount$

\wedge IF *debt.creditor* = NULL

THEN *PayBank*(*turnPlayer*, *debt.amount*)

ELSE \wedge *PayPlayer*(*turnPlayer*, *debt.creditor*, *debt.amount*)

\wedge UNCHANGED ⟨*bankMoney*⟩

$\wedge phase' = debt.nextPhase$

$\wedge debt' = NULL$

\wedge UNCHANGED ⟨*board*, *chanceCards*, *communityChestCards*, *doublesCount*,
free4AllOrder, *goofCcOwner*, *goofChOwner*, *inJail*, *isBankrupt*,
jailIndex, *jailTime*, *positions*, *turnPlayer*⟩

BankruptcyPreventionMortgage \triangleq

IF *phase* \neq “bankruptcy-prevention”

THEN FALSE

ELSE $\wedge \neg terminated$

$\wedge money[turnPlayer] < debt.amount$

$\wedge MortgageProperty(turnPlayer)$

\wedge UNCHANGED ⟨*chanceCards*, *communityChestCards*, *debt*,
doublesCount, *free4AllOrder*, *goofCcOwner*,
goofChOwner, *inJail*, *isBankrupt*, *jailIndex*,
jailTime, *phase*, *positions*, *turnPlayer*⟩

BankruptcyPreventionDowngrade \triangleq

IF *phase* \neq “bankruptcy-prevention”

THEN FALSE

ELSE $\wedge \neg terminated$

$\wedge money[turnPlayer] < debt.amount$

$\wedge DowngradeStreet(turnPlayer)$

\wedge UNCHANGED ⟨*chanceCards*, *communityChestCards*, *debt*,
doublesCount, *free4AllOrder*, *goofCcOwner*,
goofChOwner, *inJail*, *isBankrupt*, *jailIndex*,
jailTime, *phase*, *positions*, *turnPlayer*⟩

RECURSIVE *BoardAfterBankruptcyToBank*($_$)

BoardAfterBankruptcyToBank(*currentBoard*) \triangleq

IF *currentBoard* = ⟨⟩ THEN ⟨⟩

ELSE LET *field* \triangleq *Head*(*currentBoard*)

IN IF $\neg isProperty(field)$

THEN ⟨*field*⟩ \circ *BoardAfterBankruptcyToBank*(*Tail*(*currentBoard*))

ELSE IF *field.owner* = *turnPlayer*

```

THEN LET newField  $\triangleq$  [field EXCEPT !.owner = NULL,
                                !.mortgaged = FALSE]
      IN   $\langle$ newField $\rangle \circ$  BoardAfterBankruptcyToBank(Tail(currentBoard))
ELSE   $\langle$ field $\rangle \circ$  BoardAfterBankruptcyToBank(Tail(currentBoard))

TransferAllAssetsToBank  $\triangleq$ 
   $\wedge$  PayBank(turnPlayer, money[turnPlayer])
   $\wedge$  IF goojfChOwner = turnPlayer
      THEN goojfChOwner' = NULL
      ELSE UNCHANGED  $\langle$ goojfChOwner $\rangle$ 
   $\wedge$  IF goojfCcOwner = turnPlayer
      THEN goojfCcOwner' = NULL
      ELSE UNCHANGED  $\langle$ goojfCcOwner $\rangle$ 
   $\wedge$  board' = BoardAfterBankruptcyToBank(board)

RECURSIVE BoardAfterBankruptcyToPlayer(-, -)
BoardAfterBankruptcyToPlayer(creditor, currentBoard)  $\triangleq$ 
  IF currentBoard =  $\langle$  $\rangle$  THEN  $\langle$  $\rangle$ 
  ELSE LET field  $\triangleq$  Head(currentBoard)
      IN  IF  $\neg$ isProperty(field)
          THEN  $\langle$ field $\rangle \circ$  BoardAfterBankruptcyToPlayer(creditor, Tail(currentBoard))
          ELSE IF field.owner = turnPlayer
              THEN LET newField  $\triangleq$  [field EXCEPT !.owner = creditor]
                  IN   $\langle$ newField $\rangle \circ$  BoardAfterBankruptcyToPlayer(creditor, Tail(currentBoard))
              ELSE  $\langle$ field $\rangle \circ$  BoardAfterBankruptcyToPlayer(creditor, Tail(currentBoard))

TransferAllAssetsToPlayer(creditor)  $\triangleq$ 
   $\wedge$  PayPlayer(turnPlayer, creditor, money[turnPlayer])
   $\wedge$  IF goojfChOwner = turnPlayer
      THEN goojfChOwner' = creditor
      ELSE UNCHANGED  $\langle$ goojfChOwner $\rangle$ 
   $\wedge$  IF goojfCcOwner = turnPlayer
      THEN goojfCcOwner' = creditor
      ELSE UNCHANGED  $\langle$ goojfCcOwner $\rangle$ 
   $\wedge$  board' = BoardAfterBankruptcyToPlayer(creditor, board)

RECURSIVE GiveTurnToNextLivePlayer(-)
GiveTurnToNextLivePlayer(curr)  $\triangleq$ 
   $\wedge$  LET next  $\triangleq$  incrCirc(curr, 1, NumPlayers)
      IN  IF isBankrupt[next]
          THEN GiveTurnToNextLivePlayer(next)
          ELSE  $\wedge$  turnPlayer' = next
               $\wedge$  doublesCount' = 0
               $\wedge$  free4AllOrder' = NULL
               $\wedge$  phase' = "pre-roll"

```


$GoBankrupt \triangleq$
 IF $phase \neq \text{"bankruptcy-prevention"}$
 THEN FALSE
 ELSE $\wedge \neg terminated$
 $\wedge money[turnPlayer] < debt.amount$
 $\wedge \forall idx \in \text{DOMAIN } board :$
 IF $\neg isProperty(board[idx])$ THEN TRUE
 ELSE $board[idx].owner = turnPlayer \Rightarrow board[idx].mortgaged$
 \wedge IF $debt.creditor = NULL$
 THEN $TransferAllAssetsToBank$
 ELSE $\wedge TransferAllAssetsToPlayer(debt.creditor)$
 $\wedge \text{UNCHANGED } \langle bankMoney \rangle$
 $\wedge isBankrupt' = [isBankrupt \text{ EXCEPT } ![turnPlayer] = \text{TRUE}]$
 $\wedge GiveTurnToNextLivePlayer(turnPlayer)$
 $\wedge \text{UNCHANGED } \langle chanceCards, communityChestCards, debt, inJail,$
 $jailIndex, jailTime, positions \rangle$

$TakeBankruptcyPreventionAction \triangleq$
 $\vee PayOffDebt$
 $\vee BankruptcyPreventionMortgage$
 $\vee BankruptcyPreventionDowngrade$
 $\vee GoBankrupt$

$ConcludeFree4AllActions \triangleq$
 $\wedge \neg terminated$
 $\wedge phase = \text{"free-4-all"}$
 \wedge IF $free4AllOrder = \langle \rangle$
 THEN FALSE
 ELSE $\wedge free4AllOrder' = Tail(free4AllOrder)$
 $\wedge \text{UNCHANGED } \langle bankMoney, board, chanceCards, communityChestCards,$
 $debt, doublesCount, goojfCcOwner, goojfChOwner, inJail,$
 $isBankrupt, jailIndex, jailTime, money, phase,$
 $positions, turnPlayer \rangle$

$F4AUnmortgage \triangleq$
 $\wedge \neg terminated$
 $\wedge phase = \text{"free-4-all"}$
 \wedge IF $free4AllOrder = \langle \rangle$
 THEN FALSE
 ELSE LET $player \triangleq Head(free4AllOrder)$
 IN $UnmortgageProperty(player)$
 $\wedge \text{UNCHANGED } \langle chanceCards, communityChestCards, debt, doublesCount,$
 $free4AllOrder, goojfCcOwner, goojfChOwner, inJail, isBankrupt,$
 $jailIndex, jailTime, phase, positions, turnPlayer \rangle$

$F4AMortgage \triangleq$

$$\begin{aligned}
& \wedge \neg \textit{terminated} \\
& \wedge \textit{phase} = \text{"free-4-all"} \\
& \wedge \text{IF } \textit{free4AllOrder} = \langle \rangle \\
& \quad \text{THEN FALSE} \\
& \quad \text{ELSE LET } \textit{player} \triangleq \textit{Head}(\textit{free4AllOrder}) \\
& \quad \quad \text{IN } \textit{MortgageProperty}(\textit{player}) \\
& \wedge \text{UNCHANGED } \langle \textit{chanceCards}, \textit{communityChestCards}, \textit{debt}, \textit{doublesCount}, \\
& \quad \quad \textit{free4AllOrder}, \textit{goojfCcOwner}, \textit{goojfChOwner}, \textit{inJail}, \textit{isBankrupt}, \\
& \quad \quad \textit{jailIndex}, \textit{jailTime}, \textit{phase}, \textit{positions}, \textit{turnPlayer} \rangle \\
\\
\textit{F4AUpgrade} & \triangleq \\
& \wedge \neg \textit{terminated} \\
& \wedge \textit{phase} = \text{"free-4-all"} \\
& \wedge \text{IF } \textit{free4AllOrder} = \langle \rangle \\
& \quad \text{THEN FALSE} \\
& \quad \text{ELSE LET } \textit{player} \triangleq \textit{Head}(\textit{free4AllOrder}) \\
& \quad \quad \text{IN } \textit{UpgradeStreet}(\textit{player}) \\
& \wedge \text{UNCHANGED } \langle \textit{chanceCards}, \textit{communityChestCards}, \textit{debt}, \textit{doublesCount}, \\
& \quad \quad \textit{free4AllOrder}, \textit{goojfCcOwner}, \textit{goojfChOwner}, \textit{inJail}, \textit{isBankrupt}, \\
& \quad \quad \textit{jailIndex}, \textit{jailTime}, \textit{phase}, \textit{positions}, \textit{turnPlayer} \rangle \\
\\
\textit{F4ADowngrade} & \triangleq \\
& \wedge \neg \textit{terminated} \\
& \wedge \textit{phase} = \text{"free-4-all"} \\
& \wedge \text{IF } \textit{free4AllOrder} = \langle \rangle \\
& \quad \text{THEN FALSE} \\
& \quad \text{ELSE LET } \textit{player} \triangleq \textit{Head}(\textit{free4AllOrder}) \\
& \quad \quad \text{IN } \textit{DowngradeStreet}(\textit{player}) \\
& \wedge \text{UNCHANGED } \langle \textit{chanceCards}, \textit{communityChestCards}, \textit{debt}, \textit{doublesCount}, \\
& \quad \quad \textit{free4AllOrder}, \textit{goojfCcOwner}, \textit{goojfChOwner}, \textit{inJail}, \textit{isBankrupt}, \\
& \quad \quad \textit{jailIndex}, \textit{jailTime}, \textit{phase}, \textit{positions}, \textit{turnPlayer} \rangle \\
\\
\textit{EndTurn} & \triangleq \\
& \wedge \neg \textit{terminated} \\
& \wedge \textit{phase} = \text{"free-4-all"} \\
& \wedge \textit{free4AllOrder} = \langle \rangle \\
& \wedge \textit{GiveTurnToNextLivePlayer}(\textit{turnPlayer}) \\
& \wedge \text{UNCHANGED } \langle \textit{bankMoney}, \textit{board}, \textit{chanceCards}, \textit{communityChestCards}, \\
& \quad \quad \textit{debt}, \textit{goojfCcOwner}, \textit{goojfChOwner}, \textit{inJail}, \textit{isBankrupt}, \\
& \quad \quad \textit{jailIndex}, \textit{jailTime}, \textit{money}, \textit{positions} \rangle \\
\\
\textit{TakeFree4AllAction} & \triangleq \\
& \vee \textit{ConcludeFree4AllActions} \\
& \vee \textit{F4AUnmortgage} \\
& \vee \textit{F4AMortgage} \\
& \vee \textit{F4AUpgrade}
\end{aligned}$$

$\vee F4ADowngrade$
 $\vee EndTurn$

$Init \triangleq \wedge turnPlayer = 1$
 $\wedge positions = [i \in 1 \dots NumPlayers \mapsto 1]$
 $\wedge money = [i \in 1 \dots NumPlayers \mapsto StartingMoney]$
 $\wedge inJail = [i \in 1 \dots NumPlayers \mapsto FALSE]$
 $\wedge jailTime = [i \in 1 \dots NumPlayers \mapsto 0]$
 $\wedge isBankrupt = [i \in 1 \dots NumPlayers \mapsto FALSE]$
 $\wedge phase = \text{"pre-roll"}$
 $\wedge bankMoney = TotalMoney - (NumPlayers * StartingMoney)$
 $\wedge goojfChOwner = NULL$
 $\wedge goojfCcOwner = NULL$
 $\wedge doublesCount = 0$
 $\wedge board = \langle$
 $\quad [type \mapsto \text{"go"}],$
 $\quad [type \mapsto \text{"street"}, value \mapsto 20, owner \mapsto NULL, set \mapsto 1, level \mapsto 1,$
 $\quad \quad rent \mapsto \langle 1, 4, 10 \rangle, houseCost \mapsto 10, mortgaged \mapsto FALSE],$
 $\quad [type \mapsto \text{"street"}, value \mapsto 22, owner \mapsto NULL, set \mapsto 1, level \mapsto 1,$
 $\quad \quad rent \mapsto \langle 2, 8, 20 \rangle, houseCost \mapsto 12, mortgaged \mapsto FALSE],$
 $\quad [type \mapsto \text{"community-chest"}],$
 $\quad [type \mapsto \text{"chance"}],$
 $\quad [type \mapsto \text{"tax"}, value \mapsto 20],$
 $\quad [type \mapsto \text{"rail"}, value \mapsto 25, owner \mapsto NULL, mortgaged \mapsto FALSE],$
 $\quad [type \mapsto \text{"jail"}],$
 $\quad [type \mapsto \text{"rail"}, value \mapsto 25, owner \mapsto NULL, mortgaged \mapsto FALSE],$
 $\quad [type \mapsto \text{"free-parking"}],$
 $\quad [type \mapsto \text{"util"}, value \mapsto 21, owner \mapsto NULL, mortgaged \mapsto FALSE],$
 $\quad [type \mapsto \text{"util"}, value \mapsto 21, owner \mapsto NULL, mortgaged \mapsto FALSE],$
 $\quad [type \mapsto \text{"go-to-jail"}]\rangle$
 $\wedge jailIndex = 8$
 $\wedge free4AllOrder = NULL$
 $\wedge debt = NULL$
 $\wedge chanceCards = \langle$
 $\quad [type \mapsto \text{"collect"}, amount \mapsto 10],$
 $\quad [type \mapsto \text{"pay"}, amount \mapsto 30],$
 $\quad [type \mapsto \text{"advance"}, square \mapsto 7],$
 $\quad [type \mapsto \text{"go-to-jail"}],$
 $\quad [type \mapsto \text{"goojf-ch"}]\rangle$
 $\wedge communityChestCards = \langle$
 $\quad [type \mapsto \text{"collect"}, amount \mapsto 20],$
 $\quad [type \mapsto \text{"pay"}, amount \mapsto 20],$
 $\quad [type \mapsto \text{"advance"}, square \mapsto 1],$
 $\quad [type \mapsto \text{"go-to-jail"}],$
 $\quad [type \mapsto \text{"goojf-cc"}]\rangle$

$$\begin{aligned}
Next &\triangleq \vee TakePreRollAction \\
&\vee TakeRollAction \\
&\vee TakePostRollAction \\
&\vee DoublesCheck \\
&\vee TakeBankruptcyPreventionAction \\
&\vee TakeFree4AllAction \\
\\
FairSpec &\triangleq \\
&\wedge Init \\
&\wedge \Box[Next]_{vars} \\
&\wedge WF_{vars}(Next) \\
\\
TypeOK &\triangleq \wedge turnPlayer \in 1 \dots NumPlayers \\
&\wedge \forall p \in 1 \dots NumPlayers : \\
&\quad \wedge positions[p] \in 1 \dots Len(board) \\
&\quad \wedge money[p] \in 0 \dots TotalMoney \\
&\quad \wedge inJail[p] \in \text{BOOLEAN} \\
&\quad \wedge isBankrupt[p] \in \text{BOOLEAN} \\
&\quad \wedge jailTime[p] \in 0 \dots 2 \\
&\wedge phase \in \{ \text{"pre-roll"}, \text{"roll"}, \text{"post-roll"}, \text{"bankruptcy-prevention"}, \\
&\quad \text{"doubles-check"}, \text{"free-4-all"} \} \\
&\wedge bankMoney \in 0 \dots TotalMoney \\
&\wedge \forall i \in \text{DOMAIN } board : \\
&\quad \wedge board[i].type \in \{ \text{"go"}, \text{"street"}, \text{"community-chest"}, \\
&\quad \text{"chance"}, \text{"tax"}, \text{"rail"}, \text{"jail"}, \\
&\quad \text{"free-parking"}, \text{"util"}, \text{"go-to-jail"} \} \\
&\quad \wedge isProperty(board[i]) \Rightarrow \wedge board[i].value \in Nat \\
&\quad \wedge board[i].owner \in 1 \dots NumPlayers \cup \{ NULL \} \\
&\quad \wedge board[i].mortgaged \in \text{BOOLEAN} \\
&\quad \wedge board[i].type = \text{"street"} \Rightarrow \wedge board[i].set \in Nat \\
&\quad \wedge \forall j \in \text{DOMAIN } board[i].rent : j \in Nat \\
&\quad \wedge board[i].level \in \text{DOMAIN } board[i].rent \\
&\quad \wedge board[i].houseCost \in Nat \\
&\quad \wedge board[i].type = \text{"tax"} \Rightarrow board[i].value \in Nat \\
&\wedge goojfChOwner \in 1 \dots NumPlayers \cup \{ NULL \} \\
&\wedge goojfCcOwner \in 1 \dots NumPlayers \cup \{ NULL \} \\
&\wedge doublesCount \in 0 \dots 2 \\
&\wedge free4AllOrder \in \{ NULL \} \cup Seq(1 \dots NumPlayers) \\
&\wedge free4AllOrder \neq NULL \Rightarrow \\
&\quad \forall i1, i2 \in \text{DOMAIN } free4AllOrder : \\
&\quad \vee i1 = i2 \\
&\quad \vee free4AllOrder[i1] \neq free4AllOrder[i2] \\
&\wedge debt \in \{ NULL \} \cup [creditor : \{ NULL \} \cup 1 \dots NumPlayers, \\
&\quad amount : Nat, \\
&\quad nextPhase : \{ \text{"pre-roll"}, \text{"roll"}, \text{"post-roll"},
\end{aligned}$$

$$\begin{aligned}
& \text{"doubles-check", "free-4-all"} \}] \\
& \wedge \forall i \in \text{DOMAIN } \text{chanceCards} : \\
& \quad \wedge \text{chanceCards}[i].\text{type} \in \{ \text{"collect", "pay", "advance",} \\
& \quad \quad \text{"go-to-jail", "goojf-ch"} \} \\
& \quad \wedge \text{chanceCards}[i].\text{type} \in \{ \text{"collect", "pay"} \} \\
& \quad \quad \Rightarrow \text{chanceCards}[i].\text{amount} \in \text{Nat} \\
& \quad \wedge \text{chanceCards}[i].\text{type} = \text{"advance"} \Rightarrow \text{chanceCards}[i].\text{square} \in \text{DOMAIN } \text{board} \\
& \wedge \forall i \in \text{DOMAIN } \text{communityChestCards} : \\
& \quad \wedge \text{communityChestCards}[i].\text{type} \in \{ \text{"collect", "pay", "advance",} \\
& \quad \quad \text{"go-to-jail", "goojf-cc"} \} \\
& \quad \wedge \text{communityChestCards}[i].\text{type} \in \{ \text{"collect", "pay"} \} \\
& \quad \quad \Rightarrow \text{communityChestCards}[i].\text{amount} \in \text{Nat} \\
& \quad \wedge \text{communityChestCards}[i].\text{type} = \text{"advance"} \\
& \quad \quad \Rightarrow \text{chanceCards}[i].\text{square} \in \text{DOMAIN } \text{board} \\
& \wedge \text{jailIndex} \in \text{DOMAIN } \text{board} \\
\\
& \text{InvNoPossessionsIfBankrupt} \triangleq \\
& \quad \forall p \in 1 \dots \text{NumPlayers} : \\
& \quad \quad \text{isBankrupt}[p] \Rightarrow \wedge \text{Cardinality}(\text{ownedPropertyIdxs}(p)) = 0 \\
& \quad \quad \quad \wedge \text{money}[p] = 0 \\
& \quad \quad \quad \wedge \text{goojfChOwner} \neq p \\
& \quad \quad \quad \wedge \text{goojfCcOwner} \neq p \\
\\
& \text{InvNoActionsPossibleIfBankrupt} \triangleq \\
& \quad \forall p \in 1 \dots \text{NumPlayers} : \\
& \quad \quad \text{isBankrupt}[p] \Rightarrow \wedge \text{turnPlayer} \neq p \\
& \quad \quad \quad \wedge \text{free4AllOrder} \neq \text{NULL} \\
& \quad \quad \quad \Rightarrow \forall i \in \text{DOMAIN } \text{free4AllOrder} : \text{free4AllOrder}[i] \neq p \\
\\
& \text{InvNoDebtToBankruptPlayer} \triangleq \\
& \quad \text{IF } \text{debt} = \text{NULL} \\
& \quad \quad \text{THEN TRUE} \\
& \quad \quad \text{ELSE } \text{debt.creditor} \neq \text{NULL} \Rightarrow \neg \text{isBankrupt}[\text{debt.creditor}] \\
\\
& \text{InvConservationOfMoney} \triangleq \\
& \quad \text{bankMoney} + \text{SeqSum}(\text{money}) = \text{TotalMoney} \\
\\
& \text{InvStreetLevelRange} \triangleq \\
& \quad \forall i1, i2 \in \text{DOMAIN } \text{board} : \\
& \quad \quad \text{IF } \text{board}[i1].\text{type} \neq \text{"street"} \vee \text{board}[i2].\text{type} \neq \text{"street"} \\
& \quad \quad \quad \text{THEN TRUE} \\
& \quad \quad \text{ELSE } \text{board}[i1].\text{set} = \text{board}[i2].\text{set} \\
& \quad \quad \quad \Rightarrow \text{abs}(\text{board}[i1].\text{level} - \text{board}[i2].\text{level}) \leq 1
\end{aligned}$$
