
MODULE *Auction2*

EXTENDS *Naturals, FiniteSets*

CONSTANTS *NULL, Participants, MaxAmount, UNKNOWN, NONE*

VARIABLES *initialMoney, lastBid, bid, round, passed, winner*

A2vars $\triangleq \langle \textit{initialMoney}, \textit{lastBid}, \textit{bid}, \textit{round}, \textit{passed}, \textit{winner} \rangle$

p is ready to act once all others have passed or caught up.

readyForAction(p) \triangleq

$\forall p2 \in \textit{Participants} :$

$\vee \textit{passed}[p2]$

$\vee \textit{round}[p] = \textit{round}[p2]$

A2Init \triangleq

$\wedge \textit{lastBid} = [p \in \textit{Participants} \mapsto 0]$

$\wedge \textit{bid} = [p \in \textit{Participants} \mapsto \textit{NULL}]$

$\wedge \textit{round} = [p \in \textit{Participants} \mapsto 1]$

$\wedge \textit{passed} = [p \in \textit{Participants} \mapsto \text{FALSE}]$

$\wedge \textit{initialMoney} \in [\textit{Participants} \rightarrow 0 \dots \textit{MaxAmount}]$

$\wedge \textit{winner} = [p \in \textit{Participants} \mapsto \textit{UNKNOWN}]$

A2Bid $\triangleq \exists p \in \textit{Participants} :$

$\wedge \textit{winner}[p] = \textit{UNKNOWN}$

$\wedge \neg \textit{passed}[p]$

$\wedge \textit{bid}[p] = \textit{NULL}$

$\wedge \exists p2 \in \textit{Participants} \setminus \{p\} : \textit{round}[p2] = \textit{round}[p]$

$\wedge \textit{readyForAction}(p)$

$\wedge \exists \textit{newBid} \in (\textit{lastBid}[p] + 1) \dots \textit{initialMoney}[p] :$

$\wedge \forall p2 \in \textit{Participants} : \textit{newBid} > \textit{lastBid}[p2]$

$\wedge \textit{bid}' = [\textit{bid} \text{ EXCEPT } ![p] = \textit{newBid}]$

$\wedge \text{UNCHANGED } \langle \textit{lastBid}, \textit{round}, \textit{passed}, \textit{initialMoney}, \textit{winner} \rangle$

A2Stand $\triangleq \exists p \in \textit{Participants} :$

$\wedge \textit{winner}[p] = \textit{UNKNOWN}$

$\wedge \neg \textit{passed}[p]$

$\wedge \textit{bid}[p] = \textit{NULL}$

$\wedge \exists p2 \in \textit{Participants} \setminus \{p\} : \textit{round}[p2] = \textit{round}[p]$

$\wedge \forall p2 \in \textit{Participants} \setminus \{p\} : \textit{lastBid}[p2] < \textit{lastBid}[p]$

$\wedge \textit{readyForAction}(p)$

$\wedge \textit{bid}' = [\textit{bid} \text{ EXCEPT } ![p] = \textit{lastBid}[p]]$

$\wedge \text{UNCHANGED } \langle \textit{lastBid}, \textit{round}, \textit{passed}, \textit{initialMoney}, \textit{winner} \rangle$

A2Pass $\triangleq \exists p \in \textit{Participants} :$

$\wedge \textit{winner}[p] = \textit{UNKNOWN}$

$\wedge \neg passed[p]$
 $\wedge readyForAction(p)$
 $\wedge bid[p] = NULL$
 $\wedge \exists p2 \in Participants \setminus \{p\} : round[p2] = round[p]$
 $\wedge passed' = [passed \text{ EXCEPT } ![p] = TRUE]$
 $\wedge UNCHANGED \langle bid, lastBid, round, initialMoney, winner \rangle$

$A2NextRound \triangleq \exists p \in Participants :$
 $\wedge winner[p] = UNKNOWN$
 $\wedge Cardinality(\{p2 \in Participants : \neg passed[p2]\}) \neq 0$
 $\wedge bid[p] \neq NULL$
 $\wedge \forall p2 \in Participants :$
 $\quad \vee passed[p2]$
 $\quad \vee \text{IF } round[p] = round[p2]$
 $\quad \quad \text{THEN } bid[p2] \neq NULL$
 $\quad \quad \text{ELSE } round[p2] > round[p]$
 $\wedge lastBid' = [lastBid \text{ EXCEPT } ![p] = bid[p]]$
 $\wedge bid' = [bid \text{ EXCEPT } ![p] = NULL]$
 $\wedge round' = [round \text{ EXCEPT } ![p] = @ + 1]$
 $\wedge UNCHANGED \langle passed, initialMoney, winner \rangle$

$A2ChooseWinner \triangleq \exists p \in Participants :$
 $\wedge winner[p] = UNKNOWN$
 $\wedge \vee \wedge \forall p2 \in Participants : passed[p2]$
 $\quad \wedge winner' = [winner \text{ EXCEPT } ![p] = NONE]$
 $\vee \exists p2 \in Participants :$
 $\quad \wedge \neg passed[p2]$
 $\quad \wedge \forall p3 \in (Participants \setminus \{p2\}) : passed[p3]$
 $\quad \wedge \forall p3 \in (Participants \setminus \{p2\}) : lastBid[p2] > lastBid[p3]$
 $\quad \wedge \forall p3 \in (Participants \setminus \{p2\}) : round[p2] > round[p3]$
 $\quad \wedge winner' = [winner \text{ EXCEPT } ![p] = p2]$
 $\wedge UNCHANGED \langle bid, lastBid, passed, round, initialMoney \rangle$

$A2Next \triangleq$
 $\vee A2Bid$
 $\vee A2Stand$
 $\vee A2Pass$
 $\vee A2NextRound$
 $\vee A2ChooseWinner$

$A2TypeOK \triangleq$
 $\wedge lastBid \in [Participants \rightarrow 0 .. MaxAmount]$
 $\wedge bid \in [Participants \rightarrow 1 .. MaxAmount \cup \{NULL\}]$
 $\wedge round \in [Participants \rightarrow Nat \setminus \{0\}]$
 $\wedge passed \in [Participants \rightarrow BOOLEAN]$
 $\wedge winner \in [Participants \rightarrow \{UNKNOWN, NONE\} \cup Participants]$

$$\wedge \text{initialMoney} \in [\text{Participants} \rightarrow 0 \dots \text{MaxAmount}]$$

$$\text{InvIncreasingBids} \triangleq \forall p \in \text{Participants} :$$

$$\vee \text{bid}[p] = \text{NULL}$$

$$\vee \wedge \forall p2 \in \text{Participants} \setminus \{p\} :$$

$$\text{round}[p] = \text{round}[p2] \Rightarrow \text{bid}[p] > \text{lastBid}[p2]$$

$$\wedge \text{bid}[p] \geq \text{lastBid}[p]$$

$$\text{A2FairSpec} \triangleq$$

$$\wedge \text{A2Init}$$

$$\wedge \Box[\text{A2Next}]_{\text{A2vars}}$$

$$\wedge \text{WF}_{\text{A2vars}}(\text{A2Pass})$$

$$\wedge \text{WF}_{\text{A2vars}}(\text{A2NextRound})$$

$$\wedge \text{WF}_{\text{A2vars}}(\text{A2ChooseWinner})$$

INSTANCE *Auction1*

THEOREM $\text{A2FairSpec} \Rightarrow \text{A1FairSpec}$

\ * Modification History
\ * Last modified Sat Jun 07 07:36:20 CEST 2025 by luca
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