## Reversible Data Hiding Using Additive Prediction-Error Expansion

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#### **ABSTRACT**

Reversible data hiding is a technique that embeds secret data into cover media through an invertible process. In this paper, we propose a reversible data hiding scheme that can embed a large amount of secret data into image with imperceptible modifications. The prediction-error, difference between pixel value and its predicted value, is used to embed a bit '1' or '0' by expanding it additively or leaving it unchanged. Low distortion is guaranteed by limiting pixel change to 1 and averting possible pixel over/underflow; high pure capacity is achieved by adopting effective predictors to greatly exploit pixel correlation and avoiding large overhead like location map. Experimental results demonstrate that the proposed scheme provides competitive performances compared with other state-of-the-art schemes.

## **Categories and Subject Descriptors**

I.4.m [Image Processing and Computer Vision]: Miscellaneous; K.6.5 [Security and Protection]: Authentication

#### **General Terms**

Algorithms, Experimentation, Performance

## Keywords

Additive prediction-error expansion, difference expansion, reversible data hiding, reversible watermarking

#### 1. INTRODUCTION

Data hiding is a technique that imperceptibly hides secret data into cover media contents such as audios, images, and videos. Being the essential of watermarking and steganography, data hiding is widely adopted in applications including copyright protection, content authentication, and media asset management. For many data hiding schemes, there exists a drawback that the cover media is permanently distorted

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due to irreversible operations such as replacement, quantization, roundoff and truncation. This makes data hiding prohibited in applications dictating high fidelity, e.g. military and medical imaging. In these scenarios, reversible data hiding, which can recover the original cover media, is desired and required.

Recently, many reversible data hiding schemes were proposed. A well known category is reversible data hiding using difference expansion (DE), which was first proposed by Tian [15]. By expanding the differences between the two neighboring pixels within pixel pairs, image redundancy was largely exploited in his scheme. Tseng et al. [16] improved Tian's scheme by shifting the histogram to form a new type of pixel pair, which contributes to increase the embedding capacity while keeping the distortion low. DE is also generalized and applied to triplets and quads by Alattar [1], who proposed several DE schemes of better performance with enlarged overall capacity and lowered overhead cost. Recently, Kim [8] proposed a novel scheme devoted to further reduce the size of location map. Furthermore, Lin et al. [12] proposed another DE scheme, where the location map is removed completely. Instead of expanding interpixel differences, Thodi et al. [14] and Kuribayashi et al. [10] applied the DE method to prediction-errors, which exploit image redundancy to greater extent than interpixel differences. Thodi et al. also utilized a histogram operation to disambiguate expanded pixels and non-expanded pixels instead of using a location map, and only recorded those overflow pixels with an overflow map which is much more compressible than a location map. Hu et al. [5] advanced Thodi et al.'s strategy and proposed a scheme with improved overflow map.

Another well known category of reversible data hiding schemes uses histogram to embed data. Ni et al. [13] proposed a novel reversible data hiding scheme of very high image quality but limited capacity. They utilized the peak/zero points of histogram for data embedding. Lin and Hsueh [11] expanded Ni et al.'s approach and applied similar operation to histogram of three-pixel differences, by which much better performance was achieved. By properly selecting the peak/zero points via dynamic programming, Chung et al. [3] proposed a scheme with larger embedding capacity.

However, the delimitation between the above two categories is somehow indefinite, since DE can also be interpreted in the perspective of histogram, and that is what scheme [9] means essentially. Conversely, histogram operation can also be employed to improve the performance of DE [16], or help compress location map [2] or even eliminate it [5,9,14].

This paper presents a reversible data hiding scheme using additive prediction-error expansion. The proposed scheme embeds bits by expanding prediction-errors, which are actually differences between pixel values and their predicted values, and it expands prediction-errors by addition, which is similar to the peak/zero histogram operation. The resulting additive prediction-error expansion strategy is efficient, because prediction-errors are good at decorrelating pixels and enlarging capacity, while additive expansion is free of large distortion and expensive overhead information. Another important merit of this paper is that a study involving many predictors varying from simple to complex is conducted to examine their performances in reversible data hiding. From the study, we find the predictor frequently used in previous schemes [5, 10, 14] is actually not a good one in most cases. Instead, we find another predictor which is better and simpler.

The rest of the paper is organized as follows. Details of the proposed scheme including additive expansion, prediction, over/underflow prevention and overhead information, and capacity and visual quality are described in Section 2. Then experimental results are presented and evaluated in Section 3. Finally, conclusions are drawn in Section 4.

## 2. THE PROPOSED SCHEME

## 2.1 Additive Expansion

For most DE schemes [1, 5, 8, 10, 12, 14-16], as well as some histogram operation schemes like [9], the essential of reversible data hiding is the bit-shifting expansion, which can be expressed as

$$d' = 2d + b, (1)$$

where d is a difference and b is a binary bit to be embedded. The bit-shifting expansion (1) is reversible because the embedded bit can be extracted via  $b = d' \mod 2$  and the original difference can be recovered via d = d'/2.

Being different, the proposed additive expansion is based on the statistical characters of prediction-error, and it quite resembles the peak/zero points operation of histogram. Firstly, predicted values of pixels are calculated utilizing a predictor, which works by guessing a pixel value from its previous pixels. Then prediction-errors are obtained via

$$e = x - \hat{x},\tag{2}$$

where  $\hat{x}$  is the predicted value of pixel x. The prediction-errors can be positive or negative, and the positive part and the negative part should be considered separately in additive expansion. However, to ease the discussion, we consider them uniformly by dealing with absolute prediction-error. The histogram of absolute prediction-error is exemplified in Fig. 1, where the peak point and the left-most zero point are marked and identified with ME and LE respectively. The additive expansion is formulated as

$$e' = \begin{cases} e + sign(e) \times b, & |e| = ME \\ e + sign(e) \times 1, & |e| \in [ME + 1, LE) \end{cases}$$
 (3)

where

$$sign(e) = \begin{cases} 1, & e \ge 0 \\ -1, & e < 0. \end{cases}$$
 (4)

In Fig. 1, we can see that prediction-errors at ME (expandable prediction-errors) are expanded to hide bits by

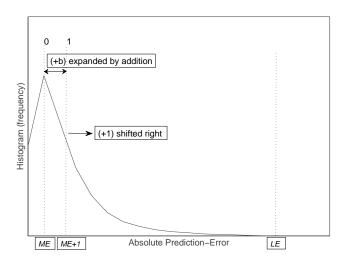


Figure 1: Illustration of additive prediction-error expansion.

addition, whereas prediction-errors between ME+1 and LE-1 (shiftable prediction-errors) are shifted right by 1 to differentiate them from expanded prediction-errors. After that, we finish the embedding by replacing x with  $x'=\hat{x}+e'$ .

During the extracting, the same prediction process is undertaken to recalculate the same predicted value  $\hat{x}$  and the corresponding prediction-error  $e' = x' - \hat{x}$ . Once the same ME and LE are known, we can tell where bits are embedded and extract them via

$$b = \begin{cases} 0, & |e'| = ME \\ 1, & |e'| = ME + 1. \end{cases}$$
 (5)

Then original prediction-errors can be recovered through

$$e = \begin{cases} e' - sign(e') \times b, & |e'| \in [ME, ME + 1] \\ e' - sign(e') \times 1, & |e'| \in (ME + 1, LE] \end{cases}$$
 (6)

Finally, we restore the original pixel by  $x = \hat{x} + e$ .

Compared with the bit-shifting expansion, the additive expansion is advantageous in two aspects: 1) the distortion caused by additive expansion is less, since pixels are changed at most by 1; 2) no location map is needed to tell between expanded prediction-errors and non-expanded ones since they are distinguishable with ME and LE.

## 2.2 Prediction

In the proposed additive expansion, the expanded differences are prediction-errors, differences between pixel values and their predicted values. There are two reasons for preferring prediction-errors to interpixel differences.

The first reason is that prediction requires no blocking, which is a process of dividing the cover image into non-overlapped pixels blocks required in many DE schemes. By blocking, it takes more than one pixel to find a difference, which significantly reduces the amount of differences and in turn lessens the potential embedding capacity. Let  $I = \{x[i,j], 1 \le i \le H, 1 \le j \le W\}$  be an image of width W and height H. For a pixel block sized X, the number of differences is just WH(X-1)/X, which means that 1/X pixels are spent to find differences. The cost is considerably expensive because X cannot be large, or else the decorrelation capability of differences degrades sharply due to absence of

locality. Nevertheless, replacing interpixel differences with prediction-errors, we get WH differences not considering some negligible marginal pixels. This means there are more candidates for data embedding, which promises a larger capacity.

The second reason for preferring prediction-errors to interpixel differences is credited to the superior decorrelation ability of prediction-error. Fundamentally, the feasibility of reversible data hiding is due to high redundancy or correlation within images, which means that "the value of any given pixel can be reasonably predicted from the value of its neighbors" [4]. To maximize the capacity of data hiding, we should exploit the correlation to the largest extent. As reversible data hiding is a relatively new discipline, lossless image compression has undergone decades of active research to exploit the redundancy of image and has achieved fruitful results. So, intuitively, it may be rewarding to introduce effective tools therein into reversible data hiding. Prediction is such a tool proved of excellent decorrelation capability.

As a matter of fact, prediction-error has already been preferred in schemes [5, 10, 14] as improvements of Tian's approach. However, in schemes [5, 10, 14], prediction-errors were still expanded by bit-shifting, which tends to cause larger distortion since differences are doubled. What is more, the predictor they adopted is not a good one in most cases, and that is the very issue we address in this section.

To find a good predictor, an intensive study of different predictors has been conducted and we finally obtain five choices. They are 1) the Median Edge Detector predictor (MED) [17], 2) an enhanced predictor for JPEG-LS (EJP) [6], 3) the Gradient-Adjusted Predictor (GAP) [18], 4) a simplified version of the GAP (SGAP), and 5) a leastsquare based predictor proposed in [7] (ELA). Being an important component of LOCO-I [17], MED is a context adaptive predictor combining three linear predictors by detecting edges and adjusting coefficients correspondingly. It is also the predictor used in previous reversible data hiding schemes [5, 10, 14]. The enhanced predictor for JPEG-LS was proposed in [6] where the performance of LOCO-I was reported to be enhanced by replacing MED with EJP. The GAP predictor is also an adaptive predictor which is adjusted according to seven different contexts around the predicted pixel. Its simplified version, namely SGAP, gets rid of the complicated adaption and unexceptionally adopts the most common case. The ELA is a least-square based predictor, coefficients of which are adjusted to minimize the least-square error via an autoregression process.

The MED predictor is

$$\hat{x} = \begin{cases} \min(x_w, x_n) &, x_{nw} \ge \max(x_w, x_n) \\ \max(x_w, x_n) &, x_{nw} \le \min(x_w, x_n) \\ x_n + x_w - x_{nw}, & otherwise, \end{cases}$$
 (7)

where  $x_n$ ,  $x_w$ ,  $x_{nw}$  are causal pixels illustrated in Fig. 2. The SGAP predictor is

$$\hat{x} = \frac{x_n + x_w}{2} + \frac{x_{ne} - x_{nw}}{4}.$$
 (8)

The formulas of EJP, GAP and ELA are quite complex, so we refer to [6,7,18] for the details.

As the capacity of additive prediction-error expansion approximates the number of expandable prediction-errors, we have implemented the five predictors and have worked out those numbers of ten test images. The results are shown

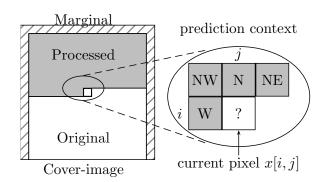


Figure 2: Prediction. Image is processed in the raster scan order. Marginal pixels are left unchanged to correctly initialize the same prediction during extracting. Causal pixels are identified with their relative position to the current pixel.

Table 1: Prediction in additive expansion.

| Image MED EJP        |       |       |       |
|----------------------|-------|-------|-------|
| Image MED EJP        | GAP   | SGAP  | ELA   |
| Lena 62205 56170     | 67475 | 67669 | 62540 |
| Babbon 20495 17603   | 22076 | 21411 | 22083 |
| Plane 80123 72513 8  | 83084 | 81625 | 63138 |
| Man 55269 47626      | 56773 | 55625 | 49617 |
| Boat 33123 28715 3   | 36089 | 34763 | 35673 |
| Goldhill 40037 32747 | 41195 | 39831 | 40419 |
| Tiffany 50950 44388  | 45943 | 45089 | 35910 |
| Peppers 32666 32394  | 38822 | 38822 | 38678 |
| Sailboat 30062 28411 | 34104 | 33846 | 32230 |
| Barbara 43376 37510  | 46876 | 45358 | 45252 |
| Average 44831 39808  | 47244 | 46404 | 42554 |

in Table 1. Among the five predictors, the GAP predictor stands out as the best choice in average, followed by the SGAP predictor and the MED predictor. For all images except Tiffany, the GAP predictor performs better than the MED predictor, which is the one adopted in [5,10,14]. Moreover, the simplified GAP predictor (SGAP) is still better than the MED predictor for eight of the ten images.

However, it is suspicious of unfair comparison because the MED predictor adopted in [5, 10, 14] was intended for bitshifting expansion while we are considering additive expansion in Table 1. To justify our discussion, we also compare the performances of those predictors in bit-shifting expansion. For DE scheme using bit-shifting expansion, the overall capacity grows when more differences are expanded, and simultaneously the location map becomes more compressible. That is, the pure capacity grows as there are more small expandable differences. Moreover, smaller differences introduce less distortion ( $\Delta x = x' - x = e + b$ ) when they are expanded. So the number of small prediction-errors indicates the performance of predictors for reversible data hiding. By setting the threshold to 10, we have worked out the number of prediction-errors less than the threshold. The results are tabulated in Table 2, where GAP is still the best predictor in average, followed by ELA, SGAP, MED and EJP.

 $<sup>^1 \</sup>text{All}$  test images considered in this paper are  $512 \times 512$  8-bit grayscale standard images.

| Table 2: Pre | diction in | ${f n}$ bit-shif | ting expansion. |
|--------------|------------|------------------|-----------------|
|--------------|------------|------------------|-----------------|

| Image    | MED    | EJP    | GAP    | $\operatorname{SGAP}$ | ELA    |
|----------|--------|--------|--------|-----------------------|--------|
| Lena     | 240357 | 226750 | 242376 | 241781                | 242688 |
| Baboon   | 147889 | 127679 | 152611 | 148356                | 156782 |
| Plane    | 238042 | 221261 | 237872 | 233049                | 234180 |
| Man      | 220604 | 199933 | 222190 | 219158                | 217115 |
| Boat     | 207326 | 180770 | 214078 | 207876                | 209746 |
| Goldhill | 219422 | 190300 | 219867 | 214332                | 216413 |
| Tiffany  | 217093 | 197924 | 220063 | 217085                | 209195 |
| Peppers  | 217345 | 200156 | 228056 | 227303                | 222929 |
| Sailboat | 187909 | 167977 | 196453 | 195122                | 189724 |
| Barbara  | 196765 | 173820 | 197847 | 191863                | 222283 |
| Average  | 209275 | 188657 | 213141 | 209593                | 212106 |

It is worth notice that the SGAP predictor is rather competitive considering its great simplicity which requires much less computation than the other four predictors. On our platform, the SGAP predictor takes 6.7ms to finish the prediction of a test image in average, whereas the time is 36ms for MED under the same circumstance, and the same times for EJP, GAP and ELA are still longer.

# 2.3 Over/underflow Prevention and Overhead Information

Up to now, two issues remain not addressed for the scheme to achieve reversibility. The first issue is the over/underflow problem, which happens when pixels are changed from 255 to 256 or from 0 to -1. Our solution is to apply additive expansion only to pixels valued from 1 to 254. However, it causes ambiguities when pixels valued 1 are changed to 0, or pixels valued 254 are changed to 255. So these pixels need be recorded as overhead information to tell whether boundary pixels (pixels valued 0 or 255) are changed or not during the extracting process. We refer to [11] for further details of this technique. Moreover, the overflow map techniques in [5, 14] are also applicable to address this issue.

The second issue concerns the recording of overhead information. Beside the data for disambiguating boundary pixels, ME and LE should also be recorded to identify expanded and shifted prediction-errors. Being different, ME and LEcannot be directly recorded as overhead information, since they are the keys to initialize the extracting. Instead, they can be embedded into the same image using other data hiding techniques like LSB. Note in Fig. 2 that some marginal pixels are not predicted because of lack of prediction contexts, and we can embed the overhead like ME and LE right into these pixels using LSB replacement. An illustration of this strategy is given in Fig. 3. Those pixels accommodating ME and LE will not be used at the very beginning of the extracting process since the image is processed in raster scan order. Therefore, this strategy has no impact on the reversibility of the proposed scheme as long as we record the original LSB bits of these pixels as payload and place them back once they are extracted.

### 2.4 Capacity and Visual Quality

The proposed scheme guarantees high image quality since pixels are changed at most by 1 and the over/underflow problem is prevented. When lower image quality is acceptable, it is also capable of providing larger capacity through two strategies.

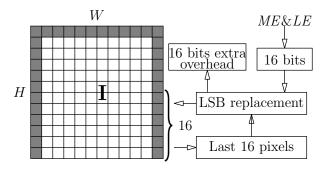


Figure 3: LSB replacement of the overhead information of *ME* and *LE*.

One strategy is to apply the proposed scheme recursively, namely through multi-layer embedding. To alleviate the accumulation of distortion in multi-layer embedding, we seek an optimal predictor from the five predictors for each embedding layer instead of employing a fixed predictor for all layers. Moreover, we also seek an optimal scanning order by rotating the image by  $90^{\circ}$ ,  $180^{\circ}$  or  $270^{\circ}$  to maximize the capacity. This strategy performs well in pursuing high performance, however, it is time-consuming due to the multi-layer embedding.

The other strategy is to generalize the additive predictionerror expansion to

$$e' = \begin{cases} e + sign(e) \times (b)_n, & |e| = ME \\ e + sign(e) \times (n-1), & |e| \in [ME+1, LE) \end{cases}$$
(9)

where  $(b)_n$  is a base n digit to be embedded and LE identifies n-1 sequent zero points. This strategy achieves capacity modulation in single-layer embedding and does not need the adaption in the first one.

Note that both strategies require some adjustments to ensure reversibility. For example, the code of the chosen predictor in the first strategy and the adopted n in the second strategy should also be recorded like ME and LE.

#### 3. EXPERIMENTAL RESULTS

The proposed reversible data hiding scheme is implemented and tested using MATLAB. In our experiments, a random bit sequence is taken as the hidden data, and image quality and capacity are measured with PSNR and bpp (bit per pixel) respectively. The results are presented in Fig. 4. For multi-layer embedding, the adopted predictor and the scanning order are dynamically chosen to adapt to each embedding layer, and for single-layer embedding, the adopted predictor is SGAP and larger capacities are achieved by enlarging the base n. The test images and their marked counterparts are also presented in Fig. 6, where the visual quality of the marked images is satisfactory.

To obtain an objective evaluation of the proposed scheme, several other schemes are compared in Fig. 4. Schemes [8, 9, 12] achieve reversible data hiding through bit-shifting expansion, and their capacities are controlled using threshold, whereas scheme [11] achieves larger capacities through multi-layer embedding. The single-layer embedding of the proposed scheme performs well when the PSNR is above 45dB, however, its curves slump quickly when it is pursuing larger capacity. For single-layer embedding, the proposed

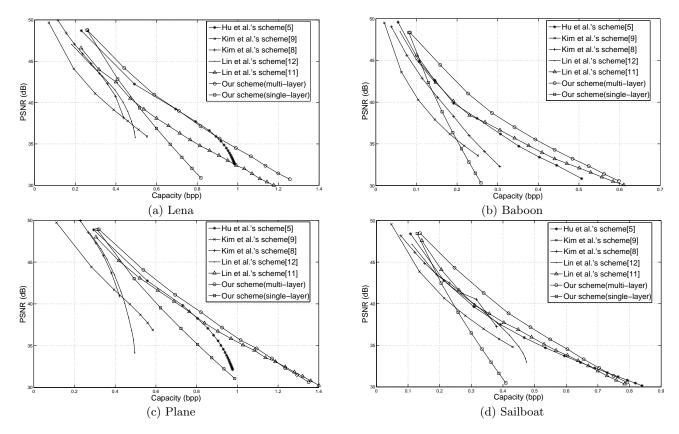


Figure 4: The performance evaluation of the proposed scheme over standard test images.

scheme is better than [8,9,12] for smooth images (Lena and Plane), but it is not as good as the three schemes at low PSNR standards for the other two images. For multi-layer embedding, the proposed scheme performs better than schemes [8,9,12] for all the four images, as well as scheme [11], which is also a multi-layer embedding scheme.

Other schemes using prediction-error [5, 10, 14] are also considered, and the latest one proposed by Hu et al. [5] is chosen for evaluation as it provides the best performance. The comparison is also presented in Fig. 4, where our single-layer embedding is not as good as their scheme, while our multi-layer embedding is generally better. The advantage of the proposed scheme is apparent at high PSNR standards (>40 dB), whereas its ascendancy becomes marginal when PSNR becomes lower (Lena). However, once the limit of single-layer embedding (1 bpp) is approached, the multi-layer embedding becomes competitive again (Lena and Plane)

One important merit of this paper is that a predictor (SGAP) simpler but better than the widely adopted predictor (MED) is identified for reversible data hiding. To validate our study, the two predictors are compared considering the normal bit-shifting expansion (scheme [5] is adopted here). The results are presented in Fig. 5, where better performance is obtained by simply replacing MED with SGAP.

#### 4. CONCLUSIONS

This paper presents a reversible data hiding scheme using additive prediction-error expansion, which significantly exploits the image redundancy through prediction and completely disposes of the location map through additive expan-

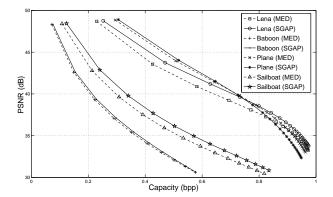


Figure 5: MED predictor vs. SGAP predictor for bit-shifting expansion.

sion. Several predictors are also studied considering their performances in reversible data hiding, and a predictor (SGAP) better than the normal MED predictor is identified. Compared with other state-of-the-art schemes, the proposed one provides high image quality in single-layer embedding and achieves large capacity through multi-layer embedding.

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Figure 6: Original and marked grayscale test images.

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