Lauren Goemaat

BI 471

HW 2

1. Hastings Problem 4.1

(a)

$$\frac{dN}{dt} = rN\left[1 - (N/K)^{\theta}\right]$$

$$0 = r\widehat{N}\left[1 - \left(\frac{\widehat{N}}{K}\right)^{\theta}\right]$$

$$0 = 1 - \left(\frac{\widehat{N}}{K}\right)^{\theta} \rightarrow \left(\frac{\widehat{N}}{K}\right)^{\theta} = 1 \rightarrow \theta\left(ln(\widehat{N}) - ln(K)\right) = 0 \rightarrow \ln(\widehat{N}) = \ln(K) \rightarrow \widehat{N} = K$$

$$0 = r\widehat{N} \rightarrow \widehat{N} = 0$$

$$\frac{dN}{dt} = \frac{dn}{dt} \rightarrow \frac{dn}{dt} = F(N) \rightarrow F(N) = rN\left[1 - \left(\frac{N}{K}\right)^{\theta}\right]$$

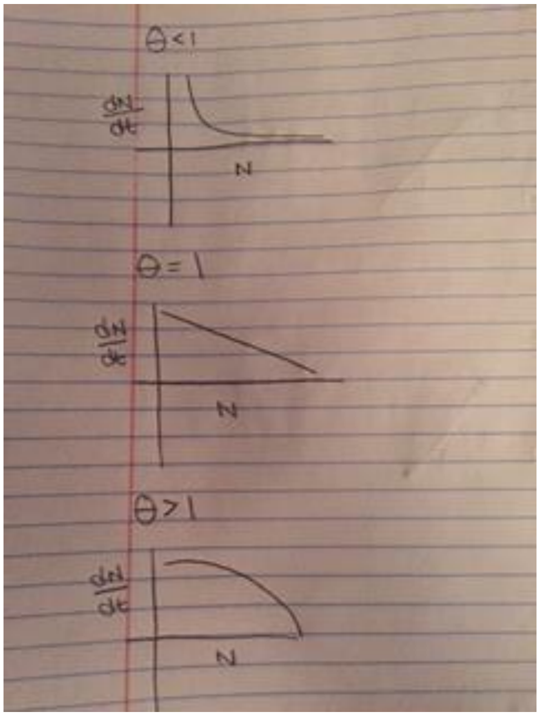
$$F(N) = rN\left[1 - \left(\frac{N}{K}\right)^{\theta}\right] = rN - rN\left(\frac{N}{K}\right)^{\theta}$$

$$\frac{dF}{dN} = r - \left(rN * \theta\left(\frac{N}{K}\right)^{\theta-1} + r\left(\frac{N}{K}\right)^{\theta}\right)$$

$$\frac{dn}{dt} \approx n\left(r - \left(rN * \theta\left(\frac{N}{K}\right)^{\theta-1} + r\left(\frac{N}{K}\right)^{\theta}\right)\right)\Bigg|_{N=0} \rightarrow \frac{dn}{dt} = rn$$

$$\frac{dn}{dt} \approx n\left(r - \left(rN * \theta\left(\frac{N}{K}\right)^{\theta-1} + r\left(\frac{N}{K}\right)^{\theta}\right)\right)\Bigg|_{N=K} \rightarrow \frac{dn}{dt} = rn - rnK + rn$$

(b)




When theta is below 1, the per capita growth rate starts off large and goes to zero at the carrying capacity. When theta is above 1, the per capita growth rate remains large for a while and then rapidly drops to zero at the carrying capacity. When theta is equal to 1, the per capita growth rate linearly declines to zero at the carrying capacity.

(c) This model is superior to the logistic growth model because it models non-linear growth. This model takes the exponent theta into account which allows for different species to be modelled under the same conditions. Some taxa would have a theta that is below zero and would show a slower rate of increase, while others would have a theta that is above zero and would show a faster rate of increase.

Hastings Problem 4.3

(a)

$$\frac{dN}{dt} = rN(N-a)[1-(N/K)]$$

$$0 = r\hat{N}\left(\hat{N}-a\right)\left[1-\left(\hat{N}/K\right)\right]$$

$$0 = r\hat{N} \rightarrow \hat{N} = 0$$

$$0 = \hat{N} - a \rightarrow \hat{N} = a$$

$$0 = 1-\left(\hat{N}/K\right) \rightarrow \hat{N}/K = 1 \rightarrow \hat{N} = K$$

(b)

$$\frac{dN}{dt} = \frac{dn}{dt} \rightarrow \frac{dn}{dt} = F(N) \rightarrow F(N) = rN(N-a)[1-(N/K)]$$

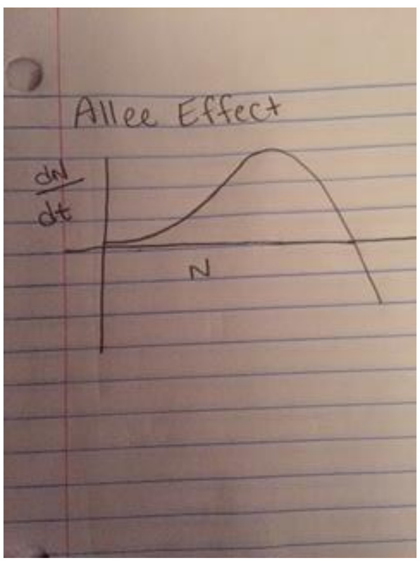
$$F(N) = (rN^2 - arN)\left(rN - \frac{rN^2}{K}\right)$$

$$\frac{dF}{dN} = (2rN - ar)\left(rN - \frac{rN^2}{K}\right) + (rN^2 - arN)\left(r - \frac{2rN}{K}\right)$$

$$\frac{dn}{dt} \approx n\left((2rN - ar)\left(rN - \frac{rN^2}{K}\right) + (rN^2 - arN)\left(r - \frac{2rN}{K}\right)\right)\Bigg|_{N=0} \rightarrow \frac{dn}{dt} = 0$$

$$\frac{dn}{dt} \approx n\left((2rN - ar)\left(rN - \frac{rN^2}{K}\right) + (rN^2 - arN)\left(r - \frac{2rN}{K}\right)\right)\Bigg|_{N=K} \rightarrow \frac{dn}{dt}$$
$$= n\big((rK^2 - arK)(-r)\big) = -nr^2K^2 + nar^2K$$

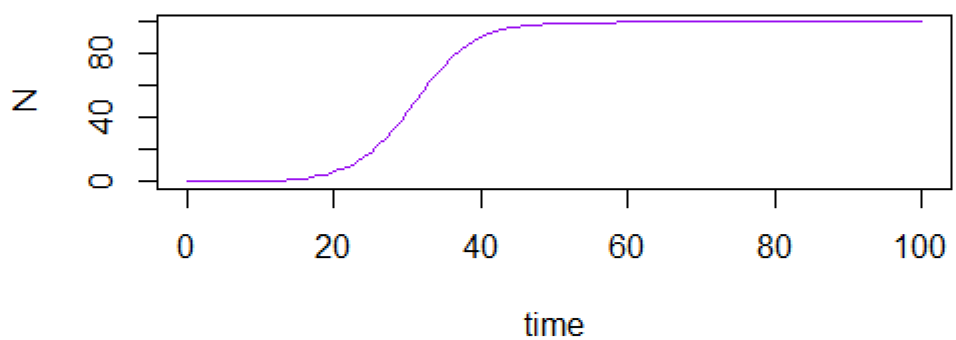
(c)




(d) This model shows a slow rate of per capita growth rate at low population sizes which then increases and decreases in a similar manner to the simple logistic model. This explains how the population declines at low population sizes because there is a low population growth rate.

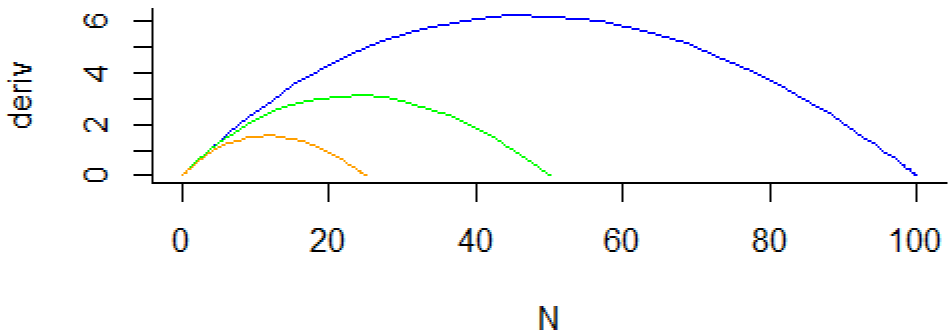
2. (a) Write a function to simulate continuous time logistic growth in R. Using the values r = 0.25 and K = 100, simulate and plot this model over the time range t = 0 to t = 100, using runif() to randomly draw the initial conditions 0.01 < N0 < 0.1 from a uniform distribution. Include any code and figures.

```r
library(deSolve)

log.growth<-function(t,y,p){
  N<-y[1]
  with(as.list(p),{
    dN.dt<-r*N*(1-(N/K))
    return(list(dN.dt))
  })
}

p<-c('r'=0.25,'K'=100)
y0<-c('N'=runif(1,min=0.01,max=0.1))
t<-0:100

y0

sim<-ode(y=y0,times=t,func=log.growth,parms=p,method='lsoda')
sim<-as.data.frame(sim)

plot(N~time,data=sim,type='l',col='purple')

```




(b) Simulate this model two additional times for the same parameter set, but with K = 50 and K = 25. Plot the population level growth rate (hint: ?diff) vs. population abundance for all three simulations (on the same plot). Include any code and figures.

```r
library(deSolve)

log.growth<-function(t,y,p){
  N<-y[1]
  with(as.list(p),{
    dN.dt<-r*N*(1-(N/K))
    return(list(dN.dt))
  })
}

p<-c('r'=0.25,'K'=100)
y0<-c('N'=runif(1,min=0.01,max=0.1))
t<-0:100

y0

sim<-ode(y=y0,times=t,func=log.growth,parms=p,method='lsoda')
sim<-as.data.frame(sim)

plot(N~time,data=sim,type='l',col='purple')

p.2<-c('r'=0.25,'K'=50)

sim.2<-ode(y=y0,times=t,func=log.growth,parms=p.2,method='lsoda')
sim.2<-as.data.frame(sim.2)

p.3<-c('r'=0.25,'K'=25)

sim.3<-ode(y=y0,times=t,func=log.growth,parms=p.3,method='lsoda')
sim.3<-as.data.frame(sim.3)

sim$deriv<-c(diff(sim$N),NA)
sim.2$deriv<-c(diff(sim.2$N),NA)
sim.3$deriv<-c(diff(sim.3$N),NA)

plot(deriv~N,data=sim,type='l',col='blue',bty='l')
points(deriv~N, data=sim.2, type='l', col='green')
points(deriv~N,data=sim.3,type='l',col='orange')

```



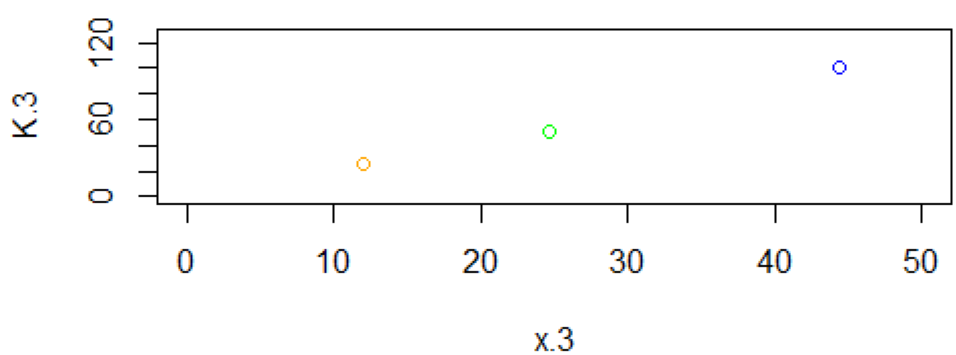

(c) Find the population abundance that yields the maximum population growth rate for each of the above three simulations. Visualize the effect of carrying capacity on population size at maximum growth rate by plotting these values against their corresponding K parameter. Include any code and figures.

Simulation 1: N=44.40

Simulation 2: N=24.61

Simulation 3: N=11.96

```r
max(sim$deriv, na.rm = TRUE)

which(sim$deriv == max(sim$deriv, na.rm = TRUE))

sim$N[which(sim$deriv == max(sim$deriv, na.rm = TRUE))]

x.1<-sim$N[which(sim$deriv == max(sim$deriv, na.rm = TRUE))]

K.1<-100

max(sim.2$deriv,na.rm=TRUE)

which(sim.2$deriv==max(sim.2$deriv,na.rm=TRUE))

sim.2$N[which(sim.2$deriv==max(sim.2$deriv,na.rm=TRUE))]

x.2<-sim.2$N[which(sim.2$deriv==max(sim.2$deriv,na.rm=TRUE))]

K.2<-50

max(sim.3$deriv,na.rm=TRUE)

which(sim.3$deriv==max(sim.3$deriv,na.rm=TRUE))

sim.3$N[which(sim.3$deriv==max(sim.3$deriv,na.rm=TRUE))]

x.3<-sim.3$N[which(sim.3$deriv==max(sim.3$deriv,na.rm=TRUE))]

K.3<-25

plot(x.3,K.3,type='p',col='orange',ylim=c(0,125),xlim=c(0,50))
points(x.2,K.2,type='p',col='green')
points(x.1,K.1,type='p',col='blue')
```

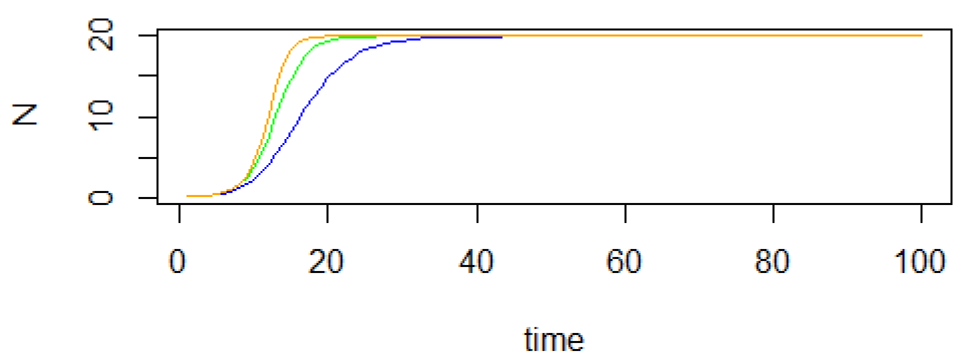



3. Suppose you manage a fishery and are tasked with maximizing the fishery's yield by managing the populations of three fish species that grow according to the theta logistic growth

model (see [Hastings Ch. 4]()). A scientist visited the fishery and determined the theta value for each fish: 0.5 for species A, 1 for species B and 1.8 for species C. Which species will be maintained at the highest population abundance in your fishery? Include any code and figures.

---

```
library(deSolve)

log.growth.theta<-function(t,y,p){
  N<-y[1]
  with(as.list(p),{
    dN.dt<-r*N*(1-(N/K)^theta)
    return(list(dN.dt))
  })
}

p.A<-c('r'=0.5,'K'=20,'theta'=0.5)
y0<-c('N'=0.05)
t<-1:100

p.B<-c('r'=0.5,'K'=20,'theta'=1)

p.C<-c('r'=0.5,'K'=20,'theta'=1.8)

sim.A <- ode(y = y0, times = t, func = log.growth.theta, parms = p.A, method = 'lsoda')
sim.A <- as.data.frame(sim.A)

sim.B <- ode(y = y0, times = t, func = log.growth.theta, parms = p.B, method = 'lsoda')
sim.B <- as.data.frame(sim.B)

sim.C <- ode(y = y0, times = t, func = log.growth.theta, parms = p.C, method = 'lsoda')
sim.C <- as.data.frame(sim.C)

plot(N ~ time, data = sim.A, type = 'l', col = 'blue')
points(N ~ time, data = sim.B, type = 'l', col = 'green')
points(N ~ time, data = sim.C, type = 'l', col = 'orange')

```




Species C with the highest theta value will be maintained at the highest population size.

4. Extra credit. In class we reviewed logistic population growth, starting with the model dN/dt = rN(1-N/K). Go through the steps to derive the solution for this model at time t = T, or N(T).

$$\frac{dN}{dt} = rN(1 - N/K) \Rightarrow$$

$$\int_{N(0)}^{N(T)} \frac{dN}{N(1-N/K)} = \int_{N(0)}^{N(T)} r\,dt \Rightarrow$$

$$\int_{N(0)}^{N(T)} \frac{1}{N} + \frac{\frac{1}{K}}{1-N/K}\, dN = \int_{N(0)}^{N(T)} r\,dt \Rightarrow$$

$$\ln N(T) - \ln(1 - N(T)/K) - \ln N(0) + \ln(1 - N(0)/K) = rT$$

$$\frac{N(T)(1 - N(0)/K)}{(1 - N(T)/K)N(0)} = e^{rt}$$

$$N(T) = \frac{N(0)e^{rt}}{1 + N(0)(e^{rt} - 1)/K}$$