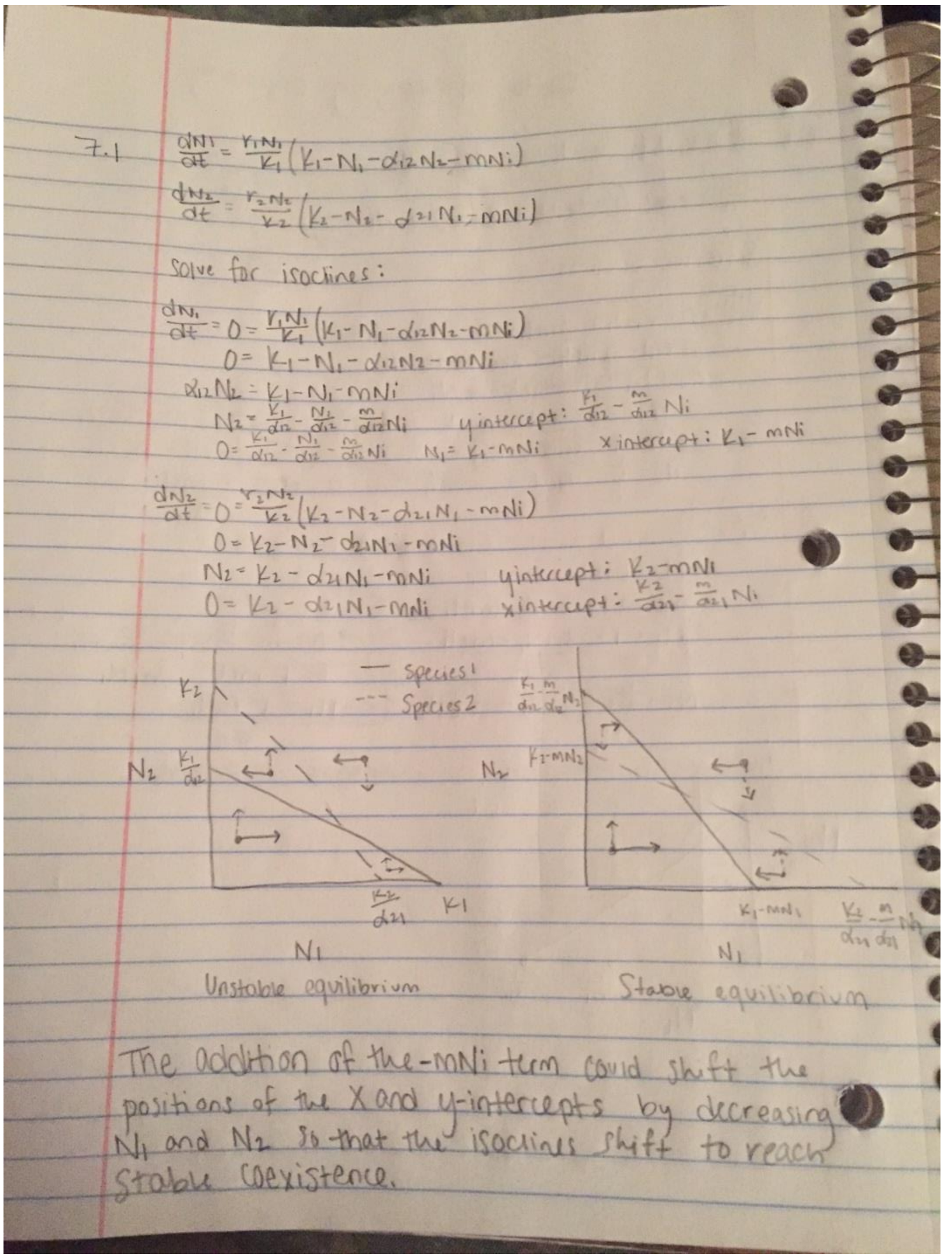
Lauren Goemaat

BI 471

HW 4

1. Hastings Problems 7.1, 7.2, 7.4

7.1



7.1 $\frac{dN_1}{dt} = \frac{r_1 N_1}{K_1}(K_1 - N_1 - \alpha_{12} N_2 - mN_i)$

$\frac{dN_2}{dt} = \frac{r_2 N_2}{K_2}(K_2 - N_2 - \alpha_{21} N_1 - mN_i)$

Solve for isoclines:

$\frac{dN_1}{dt} = 0 = \frac{r_1 N_1}{K_1}(K_1 - N_1 - \alpha_{12} N_2 - mN_i)$

$0 = K_1 - N_1 - \alpha_{12} N_2 - mN_i$

$\alpha_{12} N_2 = K_1 - N_1 - mN_i$

$N_2 = \frac{K_1}{\alpha_{12}} - \frac{N_1}{\alpha_{12}} - \frac{m}{\alpha_{12}} N_i$ y intercept: $\frac{K_1}{\alpha_{12}} - \frac{m}{\alpha_{12}} N_i$

$0 = \frac{K_1}{\alpha_{12}} - \frac{N_1}{\alpha_{12}} - \frac{m}{\alpha_{12}} N_i$ $N_1 = K_1 - mN_i$ x intercept: $K_1 - mN_i$

$\frac{dN_2}{dt} = 0 = \frac{r_2 N_2}{K_2}(K_2 - N_2 - \alpha_{21} N_1 - mN_i)$

$0 = K_2 - N_2 - \alpha_{21} N_1 - mN_i$

$N_2 = K_2 - \alpha_{21} N_1 - mN_i$ y intercept: $K_2 - mN_i$

$0 = K_2 - \alpha_{21} N_1 - mN_i$ x intercept: $\frac{K_2}{\alpha_{21}} - \frac{m}{\alpha_{21}} N_i$

The addition of the $-mN_i$ term could shift the positions of the x and y-intercepts by decreasing N_1 and N_2 so that the isoclines shift to reach stable coexistence.

7.2

7.2 (a) $\frac{dp_1}{dt} = m_1 p_1 (1-p_1) - ep_1$

$$0 = m_1 p_1 (1-p_1) - ep_1$$
$$ep_1 = m_1 p_1 (1-p_1)$$
$$\frac{e\cancel{p_1}}{m_1 \cancel{p_1}} = 1 - p_1$$
$$\frac{e}{m_1} = 1 - p_1 \qquad p_1 = 1 - \frac{e}{m_1}$$

For p_1 to be positive, extinction rate must be smaller than colonization rate. More patches need to be created for species 1 than are being lost.

(b) $\frac{dp_2}{dt} = m_2 p_2 (1 - p_1 - p_2) - m_1 p_1 p_2 - ep_2$

$$\frac{dp_2}{dt} = m_2 p_2 \left(1 - \left(1 - \frac{e}{m_1}\right) - p_2\right) - m_1 \left(1 - \frac{e}{m_1}\right) p_2 - ep_2$$

$$0 = m_2 p_2 \left(1 - \left(1 - \frac{e}{m_1}\right) - p_2\right) - m_1 \left(1 - \frac{e}{m_1}\right) p_2 - ep_2$$
$$0 = m_2 p_2 \left(\frac{e}{m_1} - p_2\right) - m_1 p_2 \cancel{+ ep_2 - ep_2}$$
$$m_1 p_2 = m_2 p_2 \left(\frac{e}{m_1} - p_2\right)$$
$$\frac{m_1 \cancel{p_2}}{m_2 \cancel{p_2}} = \frac{e}{m_1} - p_2$$
$$\frac{m_1}{m_2} = \frac{e}{m_1} - p_2 \qquad p_2 = \frac{e}{m_1} - \frac{m_1}{m_2}$$

For both species to survive, the colonization rate of m_2 must be greater than the colonization rate of m_1.

7.2

(c) $p_1 = 1 - \frac{e}{m_1}$

$p_2 = \frac{e}{m_1} - \frac{m_1}{m_2}$

$m_1 = 3 \quad m_2 = 5 \quad e = 2$

$p_1 = 1 - \frac{2}{3} = 0.33$

$p_2 = \frac{2}{3} - \frac{3}{5} = 0.07$

Increase e to 2.5

$p_1 = 1 - \frac{2.5}{3} = 0.16$

$p_2 = \frac{2.5}{3} - \frac{3}{5} = 0.23$

Increase e to 3

$p_1 = 1 - \frac{3}{3} = 0$

$p_2 = \frac{3}{3} - \frac{3}{5} = 0.4$

The equilibrium for species 1 became negative as e increased while the equilibrium for species 2 increased.

Increase e to 3.5

$p_1 = 1 - \frac{3.5}{3} = -0.16$

$p_2 = \frac{3.5}{3} - \frac{3}{5} = 0.56$

(d) The equilibrium of species 2 increased because there was less competition with species 1.

7.4

(a) In the laboratory setting, the Lotka-Volterra equation can be controlled so that the only variables acting on the competition system are the ones that work into the equation: r, N, K, and α. In the field setting, there are more parameters to take into account in than the Lotka-Volterra equation accounts for. These could include environmental stress, food availability, other predators/competitors, or human effects such as hunting.

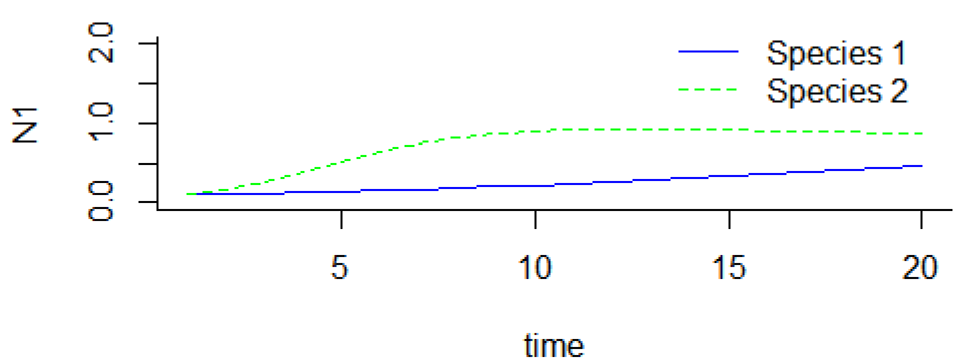
(b) In the laboratory, the numbers of the two species grown together are readily observable since they are contained in the lab. In the field, the species will have to be located and accounted for in order to count them and it cannot be guaranteed that they grew together.

(c) In the laboratory, the use of resources by the same competitors can be directly observed and quantified. In the field, the use of resources by the same competitors may not be directly observable and may have to be extrapolated by observing each species using the resource.

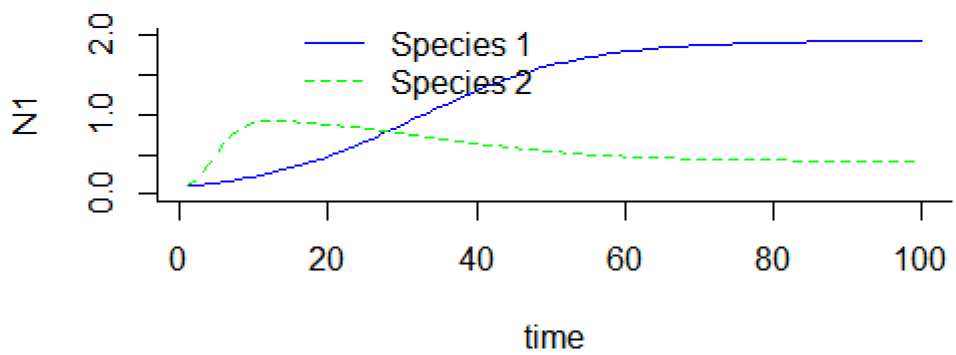
(d) In the field, species can be manipulated so that they occur in the same place at the same time. This could occur through fencing or habitat compartmentalizing that forces the species to co-occur. In the field, resources can be manipulated to force species to occur in the same place at the same time.

2. Suppose you are awarded research funding to study the outcomes of interactions between two species for 20 days, `t = 1:20`, whose dynamics are well-described by a Lotka-Volterra competition model. Species 1 changes according to `r1 = 0.1, K1 = 2` and Species 2 changes according to `r2 = 0.6, K2 = 1`. Moreover, the effect of Species 2 on the growth rate of Species 1 `a12 = 0.15` and the effect of 1 on 2 `a21 = 0.3`. Assume initial conditions for both species are identical and equal to `0.1`. Plot the system's dynamics, including `ylim = c(0, 2)` as an argument in your `plot()` function so that both species fit on a single plot. What would be the conclusion from this experiment? Suppose you got funding to perform the same study for 100 days `t = 1:100`. Would your conclusions be the same? Why should short- and longer-term ecological experiments be interpereted in different ways?

The initial experiment would conclude that species 2 was closer to its carrying capacity than species 1 in this environment and that species 2 may have a bigger negative impact on species 1 than 1 has on 2.



The conclusion for the experiment over the course of 100 days is different than the conclusion drawn over 20 days. This experiment shows that species 1 has a higher population size in the long run and is closer to its carrying capacity. This would suggest that species 1 exhibits a larger negative effect on species 2 than 2 exerts on 1.



A short term ecological study provides a snapshot of the ecosystem at the time of sampling but it does not provide information about what the stable system looks like. A long term ecological study would better display the patterns of the ecosystem and the relationship between the different species. Since these two lengths of study provide different information they should be interpreted in different ways.

```r
library(deSolve)

comp<-function(t,y,p){
  N1<-y[1]
  N2<-y[2]
  with(as.list(p),{
    dN1.dt<-(r1*N1/K1)*(K1-N1-a12*N2)
    dN2.dt<-(r2*N2/K2)*(K2-N2-a21*N1)
    return(list(c(dN1.dt,dN2.dt)))
  })
}

##define model parameters
t<-1:100
y0<-c('N1'=0.1,'N2'=0.1)
p<-c('r1'=0.1,'r2'=0.6,
     'K1'=2,'K2'=1,
     'a12'=0.15, 'a21'=0.3)

##simulate model
sim<-ode(y=y0,times=t,func=comp,parms=p,method='lsoda')
sim<-as.data.frame(sim)

plot(N1~time,data=sim,type='l',col='blue',bty='l',ylim=c(0,2))
points(N2~time,data=sim,type='l',lty=2,col='green')

legend(13, 2.4, c('Species 1', 'Species 2'), lty = c(1, 2), col = c('blue', 'green'), bty = 'n')

```

3. Review the ideas fellow classmates submitted in HW3 for team teaching and the wiki-a-thon. Choose a topic you'd like to contribute to. If you did not submit a topic last week, or

if you have a new idea, you can submit it now (include a link to google docs that you make open so that everyone with the edit can view).

I would like to contribute to the topic of Amphibians and B. dendrobatidis or Plant pollinator networks.