

Chapter 1: Functions, Graphs, and Limits

1.1 The Cartesian Plane and the Distance Formula

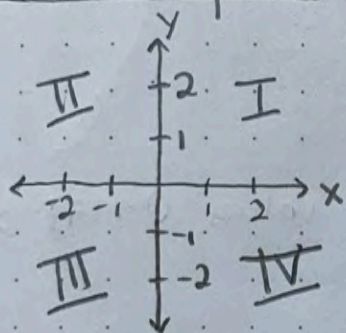
Learning Outcomes:

- Plot points in a coordinate plane and read data presented graphically.
- Find the distance between two points in a coordinate plane.
- Find the midpoints of segments connecting two points.
- Translate points in a coordinate plane.

> The Cartesian Plane

We can extend the real number line to represent ordered pairs of real numbers in a plane.

This is called the rectangular coordinate system or Cartesian plane.



Two number lines intersect at right angles.
The center is 0 for each line.

Typically, the horizontal line is the x-axis.

and the vertical line is the y-axis.

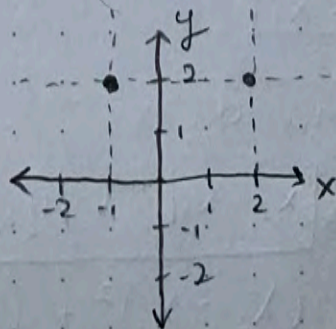
The point of intersection is the origin.

These axes divide the plane into four quadrants.

Each point corresponds to an ordered pair (x, y) where x and y are the coordinates. The x -coordinate is the directed distance from the y -axis to that point. Similarly, the y -axis is the directed distance from the x -axis to that point.

The notation (x, y) can be a point in a plane or an interval. Context will tell us which is meant.

Ex Plot $(-1, 2)$ and $(2, 2)$

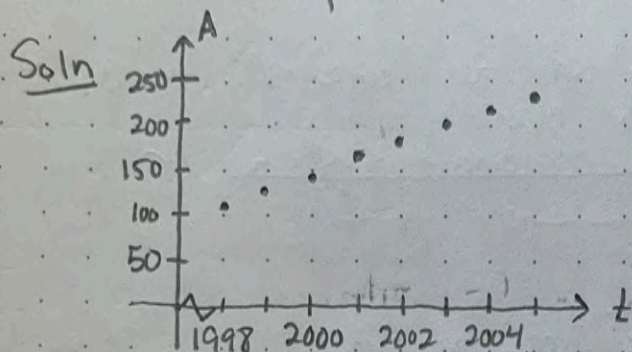


We can use this to visualize the relationship b/w two variables.

Ex A = amount (billions of \$) spent on prescription drugs in the United States
 t = year

t	1998	1999	2000	2001	2002	2003	2004	2005
A	108.7	125.8	145.6	164.1	182.7	203.1	221.0	230.3

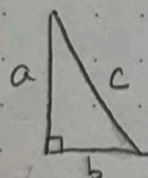
Sketch a scatterplot of the data.



break indicates that this axis has values 0-1997 omitted.

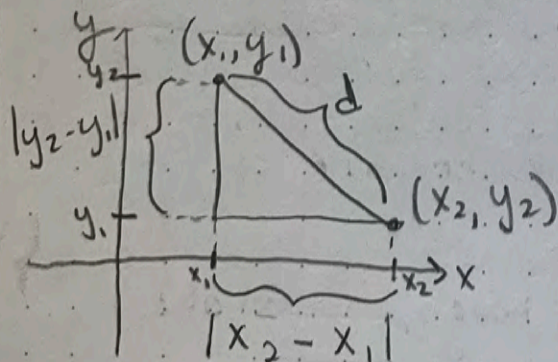
> The Distance Formula

Consider the Pythagorean Theorem



$$a^2 + b^2 = c^2$$

We can use this concept to develop a distance formula!



The length of the horiz. side is $|x_2 - x_1|$

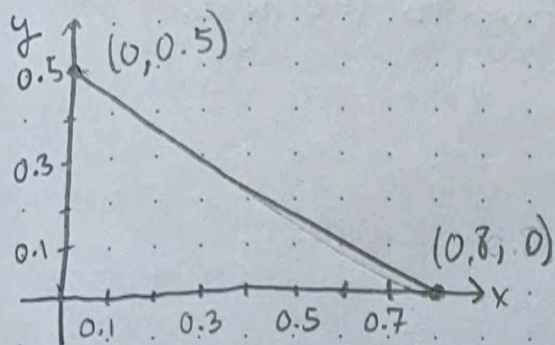
The length of the vertical side is $|y_2 - y_1|$

So the distance d is

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (\text{distance formula})$$

Ex You take a walk across a park. Your map app won't measure across the park, but gives you the distance for the roads around it.



How much distance did your route save you?

Soln Following the road, you walk

$$|0.8 - 0| + |0.5 - 0| = 1.3 \text{ miles}$$

On your shortcut, you walk

$$\sqrt{(0.8 - 0)^2 + (0.5 - 0)^2} = \sqrt{0.64 + 0.25} = 0.94 \text{ miles}$$

So you saved yourself $1.3 - 0.94 = 0.36$ miles.

> The Midpoint Formula

The midpoint of the segment joining (x_1, y_1) and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

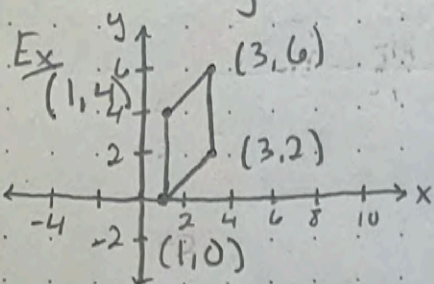
This is the average values of the respective coordinates of the two endpoints!

Ex Find the midpoint of the segment joining $(-5, -3)$ and $(9, 3)$

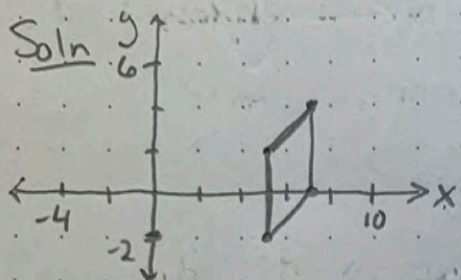
Soln

$$\text{Midpoint} = \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2} \right) = \left(\frac{4}{2}, \frac{0}{2} \right) = (2, 0)$$

> Translating Points in the Plane



Translate the parallelogram two units down and four to the right.

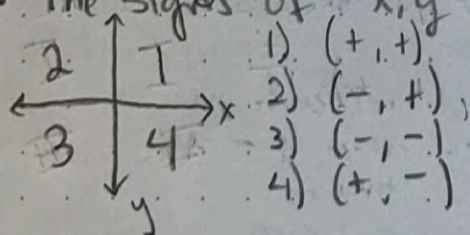


Two units down = -2 on y-axis
Four units right = +4 on x-axis

$$\begin{aligned} (1, 4) &\rightarrow (1+4, 4-2) = (5, 2) \\ (1, 0) &\rightarrow (1+4, 0-2) = (5, -2) \\ (3, 6) &\rightarrow (3+4, 6-2) = (7, 4) \\ (3, 2) &\rightarrow (3+4, 2-2) = (7, 0) \end{aligned}$$

> Concept Check

- The x (y) coordinate of a point on the y (x) axis is zero.
- The signs of x, y in the four quadrants are



- 1) (+, +)
- 2) (-, +)
- 3) (-, -)
- 4) (+, -)

- To divide a line segment into four equal parts, how many times is the midpoint formula used?

- 1.
- 2.
- 3.

- To find distance, it does NOT matter which point is chosen as (x, y) because we square the differences.