

Success Failure

X	1	0
$P(X=x)$	$p$	$1-p$
$xP(X=x)$	$p$	$0$

Total:  $p$

$$\sum xP(X=x) = \mu$$

$x - \mu$	$1-p$	$-p$
$(x - \mu)^2$	$(1-p)^2$	$(-p)^2$
$P(X=x)(x - \mu)^2$	$p(1-p)^2$	$(1-p)p^2$

$$\sigma = \sqrt{p(1-p)}$$

$$\begin{aligned} \text{Total: } & p(1-2p+p^2) + p^2 - p^3 \\ & = p - 2p^2 + p^3 + p^2 - p^3 \\ & = p - p^2 \\ & = p(1-p) \end{aligned}$$

$P(\text{exactly one exceeds})$

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(A) (B) (C) (D)

$P(A = \text{exceeds}, B = \text{not}, C = \text{not}, D = \text{not})$

$$= P(A = \text{exceeds}) P(B = \text{not}) P(C = \text{not}) P(D = \text{not})$$

$$= 0.3 \times 0.7 \times 0.7 \times 0.7$$

$$= (0.3)^1 (0.7)^3$$

$$= 0.103$$

$P(\text{single scenario}) = P(k \text{ successes}) P(n-k \text{ failures})$

$$= \underbrace{p \times p \times \dots \times p}_k \times \underbrace{(1-p) \times (1-p) \times (1-p) \times \dots \times (1-p)}_{n-k}$$

$$= p^k (1-p)^{n-k}$$

$X = \#$  of students who own a car

Success = "own a car"

$$P(\text{success}) = 0.38$$

$$n = 20$$

$X$  is binomially distributed with  $n = 20$  and  $p = 0.38$

$$P(X=k) = \binom{20}{k} 0.38^k (0.62)^{20-k}$$

a) None own a car

$$P(X=0) = \binom{20}{0} 0.38^0 (0.62)^{20}$$

$$= 7.04 \times 10^{-5}$$

b) Mean & standard dev of  $X$

$$\mu = np = 20 \times 0.38 = 7.8$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{20 \times 0.38 \times 0.62} = 2.17$$

c) No more than 2 own a car

$$P(X \leq 2) = P(X=0 \text{ or } X=1 \text{ or } X=2)$$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{20}{0} 0.38^0 \times 0.62^{20} + \binom{20}{1} 0.38^1 \times 0.62^{19} + \binom{20}{2} 0.38^2 \times 0.62^{18}$$

$$= 1 \times 1 \times 0.62^{20} + \frac{20!}{1!19!} 0.38 \times 0.62^{19} + \frac{20!}{2!(18)!} 0.38^2 \times 0.62^{18}$$

$$= 0.0000704 + 20 \times 0.38 \times 0.62^{19} + \frac{20 \times 19}{2} 0.38^2 \times 0.62^{18}$$

$$= 0.0000704 + 0.00086 + 0.0051$$

$$= 0.00596$$

d) More than 2 own a car

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - 0.00596$$

$$= 0.994$$