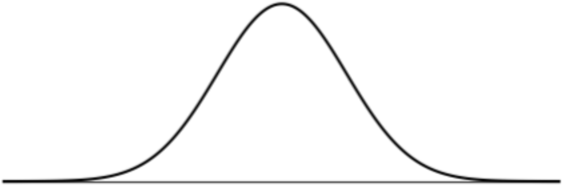
Density Curves

Density curves represent probabilities.
Basic Properties:

1. A density curve is always on or above the horizontal axis.
2. The total area under a density curve equals one.
3. The probability of falling within a specified interval equals the corresponding area under the density curve.

The Normal Distribution



Normal distributions have density curves which are always...

- Symmetric.
- Unimodal.
- "Bell curves".

Variables such as SAT scores closely follow the normal distribution.

The Normal Distribution

The normal distribution has most measurements falling somewhere near the middle - or average - and values get less likely as we move further into the tails.

Variables such as SAT scores closely follow the normal distribution.

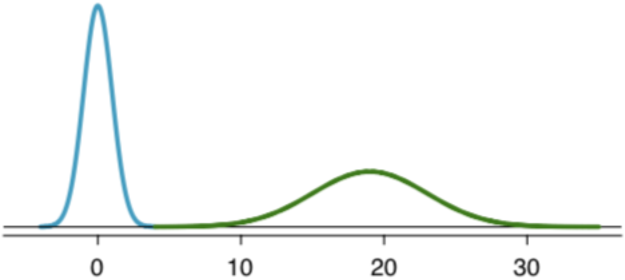
Normal Distributions

- Many variables are nearly normal, but none are exactly normal.
- While not perfect for any single problem, the normal distribution is very *useful* for a variety of problems.
- We will use it in data exploration and to solve important problems in statistics.

The Normal Distribution Model

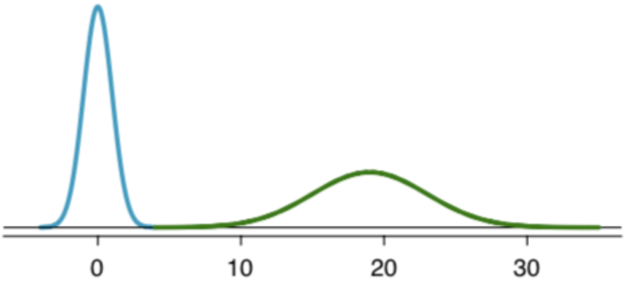
- The symmetric, unimodal, bell-shaped curve of the normal distribution can vary based on:
  - Mean (μ)
  - Standard deviation (σ)
- These adjustable details are called **model parameters**.

Parameters: Normal Distribution



- Changing the mean shifts the curve to the left or right.
- Changing the standard deviation stretches or constricts the curve.
  - (This can make the peak appear narrower or flatter.)

Parameters: Normal Distribution



- The distribution on the left has $\mu = 0$ and $\sigma = 1$.
- The distribution on the right has $\mu = 19$ and $\sigma = 4$.
- The change in μ from 0 to 19 moves the distribution to the right.
- The change in σ from 1 to 4 flattens the distribution.

Standard Normal Distribution

The **standard normal distribution** is a normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.

$$N(\mu = 0, \sigma = 1)$$

Standardizing with Z-Scores

We often want to put data onto a standardized scale, which can make comparisons more reasonable.

We can standardize a normal random variable using z-scores!

$$z = \frac{x - \mu}{\sigma}$$

Standardizing with Z-Scores