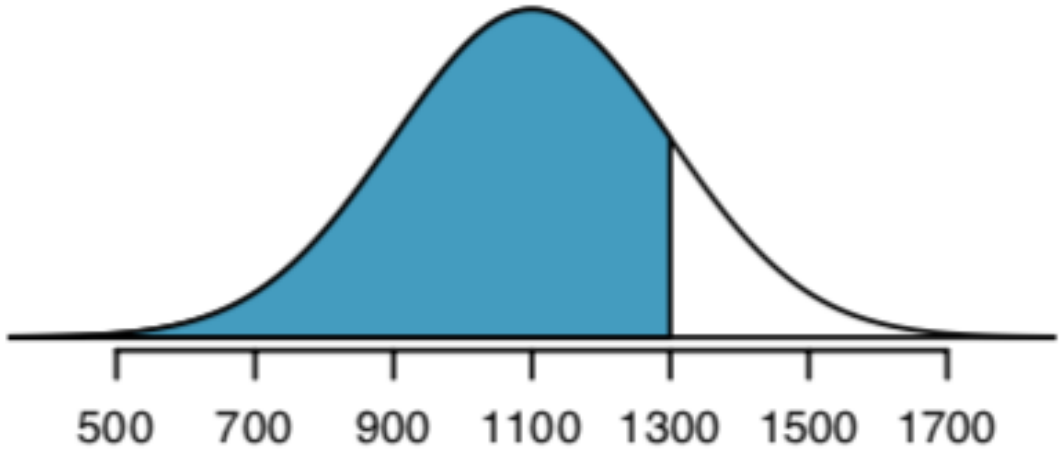
Finding Tail Areas

- We talked about using the area under a curve to think about proportions.
- Determining the area under the tail of a distribution is very useful in statistics!
- For example, your SAT percentile is the fraction of people who scored lower than you.

Finding Tail Areas

We can visualize a tail area as the curve and shading shown.



- This is the distribution for SAT scores with cutoff point $x = 1300$.
- The area to the left of x is the percentile.

Finding Tail Areas

There are several techniques for finding tail areas:

1. Integrate.
2. Use a probability table.
3. Use a statistical software.

Finding Tail Areas: Integration

The function that creates our normal distribution curve is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

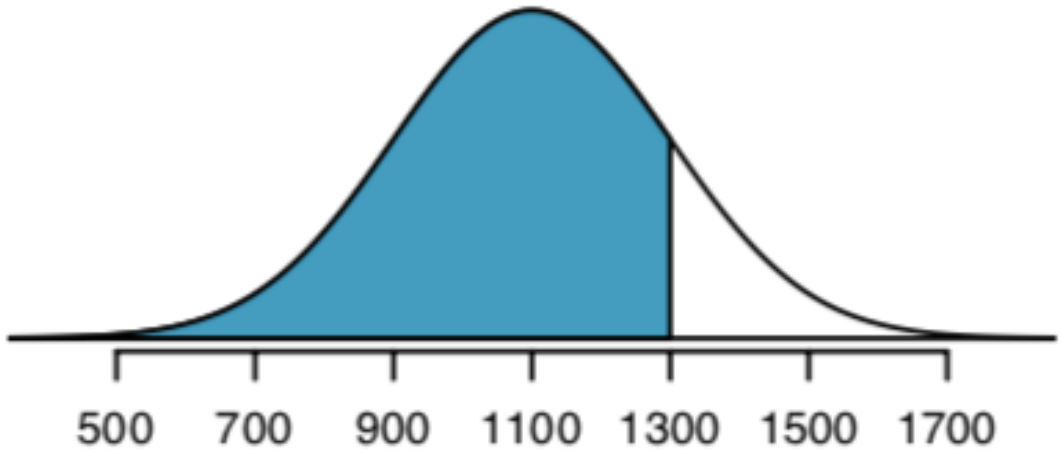
Don't write this down. We won't use it. In fact, it's impossible to integrate this by hand!

Finding Tail Areas: Probability Tables

Probability tables are often used in classrooms but they are rarely used in practice.

Finding Tail Areas: Software

Since we can't integrate by hand, we can have a computer integrate for us!



I will post a link in Canvas to an applet that does this.

Finding Tail Areas

- We will solve all normal distribution problems by first calculating z-scores.
- We do this because it will help us when we move on to Chapter 5.
- Therefore all tail area information should be provided in terms of z-scores.

Normal Probability

A crucial point:

- The area under a distribution curve is 1.
- This corresponds to the probabilities in a discrete probability distribution summing to 1!

Finding Areas to the Right

- Software programs usually return the area to the left (left tail) when given a Z-score.
- To get the area to the right
  1. Find the area to the left.
  2. Subtract this area from one.

Example: Normal Probability

Cumulative SAT scores are well-approximated by a normal model, $N(\mu = 1100, \sigma = 200)$.

Shannon is a randomly selected SAT taker, and nothing is known about her SAT aptitude. What is the probability Shannon scores at least 1190 on her SATs?

Recommendation

Draw a picture first; find the z-score second.

- Draw and label the normal curve and shade the area of interest.
- This helps to
  1. Provide a general estimate of the probability.
  2. Set up your problem correctly.
- *Then* you can identify the appropriate z-score and probabilities.