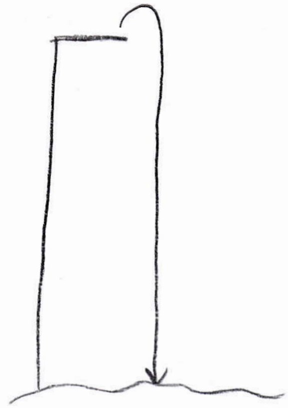
Math 26

Exam 1 Key

1a) $v_0 = 6 \qquad h_0 = 66$

$h = -16t^2 + 6t + 66$



b) $v = h'$

$= -32t + 6$

c) $0 = -32t + 6$

$t = \frac{6}{32}$

$= 0.1875$

$h = -16(0.1875)^2 + 6(0.1875) + 66$

$= 66.5625$

So she jumps up 0.5625 feet before starting her descent.

d) $a = v'$

$= -32$

e) New velocity: $7t^2 - 32t + 6$.

Stops moving downward when velocity $= 0$

$0 = 7t^2 - 32t + 6$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{32 \pm \sqrt{32^2 - 4 \times 7 \times 6}}{2(7)}$$

$$= \frac{32 \pm 29.25748}{14}$$

$t = 0.196, \quad t = 4.376$

Which one?

Reaches surface at $h = 0 = -16t^2 + 6t + 66$

$$\frac{-6 \pm \sqrt{6^2 - 4 \times (-16) \times 66}}{2(-16)} \rightarrow t = -1.852, \quad t = 2.227$$

Must touch pool bottom after surface of water $\rightarrow$ use $t = 4.376$

At $t = 4.376$, $h = -214.134$ so 214.13 feet deep!

[Note, technically, if we change velocity, the pos. function will also change! We will talk about this in Ch 6. If you got $h = -214.13$ OR noticed the issue with the position function, you got credit for this problem.>

2. $f(x) = \dfrac{1}{x^2+2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x+\Delta x)^2+2} - \dfrac{1}{x^2+2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x^2+2) - [(x+\Delta x)^2+2]}{\Delta x (x^2+2)[(x+\Delta x)^2+2]}$$

$$= \lim_{\Delta x \to 0} \frac{x^2+2 - (x^2+2x\Delta x+(\Delta x)^2+2)}{\Delta x (x^2+2)(x^2+2x\Delta x+(\Delta x)^2+2)}$$

$$= \lim_{\Delta x \to 0} \frac{-2x\Delta x - (\Delta x)^2}{\Delta x (x^2+2)(x^2+2x\Delta x+(\Delta x)^2+2)}$$

$$= \lim_{\Delta x \to 0} \frac{-2x - (\Delta x)^2}{(x^2+2)(x^2+2x\Delta x+(\Delta x)^2+2)}$$

$$= \frac{\lim_{\Delta x \to 0}[-2x - (\Delta x)^2]}{\lim_{\Delta x \to 0}[(x^2+2)(x^2+2x\Delta x+(\Delta x)^2+2]}$$

$$= \frac{\lim_{\Delta x \to 0}[-2x - (\Delta x)^2]}{\lim_{\Delta x \to 0}[x^2+2]\lim_{\Delta x \to 0}[x^2+2x\Delta x+(\Delta x)^2+2]}$$

$$= \frac{-2x}{(x^2+2)(x^2+2)}$$

$$= \frac{-2x}{(x^2+2)^2}$$

3. $h(x) = \dfrac{(x^2+1)(\sqrt{3x-2})}{x^3+2}$

$$h'(x) = \frac{(x^3+2)\frac{d}{dx}\left[(x^2+1)\sqrt{3x-2}\right] - (x^2+1)\sqrt{3x-2}\,\frac{d}{dx}\left[x^3+2\right]}{(x^3+2)^2}$$

Then $\frac{d}{dx}\left[x^3+2\right] = 3x^2+0 = 3x^2$

And $\frac{d}{dx}\left[(x^2+1)\sqrt{3x-2}\right] = (x^2+1)\frac{d}{dx}\left[(3x-2)^{1/2}\right] + (3x-2)^{1/2}\frac{d}{dx}\left[x^2+1\right]$

$$= (x^2+1)\left[\tfrac{1}{2}(3x-2)^{-1/2}(3)\right] + (3x-2)^{1/2}(2x)$$

$$= \frac{3(x^2+1)}{2(3x-2)^{1/2}} + 2x(3x-2)^{1/2}$$

$$= \frac{3(x^2+1) + 4x(3x-2)^{1/2}(3x-2)^{1/2}}{2(3x-2)^{1/2}}$$

$$= \frac{3(x^2+1) + 4x(3x-2)}{2\sqrt{3x-2}}$$

$$= \frac{3x^2+3+12x^2-8x}{2\sqrt{3x-2}}$$

$$= \frac{15x^2-8x+3}{2\sqrt{3x-2}}$$

Then $h'(x) = \dfrac{(x^3+2)\left(\dfrac{15x^2-8x+3}{2\sqrt{3x-2}}\right) - (x^2+1)\sqrt{3x-2}\,(3x^2)}{(x^3+2)^2}$

$$h'(x) = \frac{(x^3+2)(15x^2-8x+3) - (x^2+1)(3x-2)^{1/2}(3x^2)\left[2(3x-2)^{1/2}\right]}{2(3x-2)^{1/2}(x^3+2)^2}$$

$$= \frac{(x^3+2)(15x^2-8x+3) - 6x^2(x^2+1)(3x-2)}{2(x^3+2)^2\sqrt{3x-2}}$$