On the symmetries of quark lagrangian in QCD:

$$\mathcal{L}_\psi = \bar{\psi}\, i\not{\partial}\, \psi \quad (1) \quad = \bar{\psi}_L i\not{\partial} \psi_L + \bar{\psi}_R i\not{\partial} \psi_R \quad (2)$$

with: $\begin{cases} \psi_L = P_L \psi = \frac{1-\gamma_5}{2}\psi \\ \psi_R = P_R \psi = \frac{1+\gamma_5}{2}\psi \end{cases}$

In 2 flavor QCD: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$. So lagrangian (2) has the following symmetries:

$$\begin{cases} \psi_L \to e^{i\alpha_a^L \tau_a} \psi_L \\ \psi_R \to e^{i\alpha_a^R \tau_a} \psi_R \end{cases}, \quad a = 0, 1, 2, 3$$

where τ_a are the generators of $U(2)$ lie algebra. this means that the 2-flavor quark lagrangian is invariant under transformations generated by generators of:

$$U(2)_L \otimes U(2)_R$$

one of the generators of $U(2)$ is $\tau_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ which translates into rephasing each of the quarks without any additional rotation in flavor space. So can decompose the symmetry group to:

$$U(2)_L \otimes U(2)_R = U(1)_L \otimes U(1)_R \otimes SU(2)_L \otimes SU(2)_R$$

where $SU(2)$ in now generated by three familiar pauli matrices.

Now note that $U(1)_L \otimes U(1)_R$ can be decomposed to a vector and an axial subgroup: $U(1)_V \otimes U(1)_A$, which are obviously the symmetries of lagrangian (1) [and therefor (2)]. To see this, note that under $U(1)_L \otimes U(1)_R$, ψ transforms as:

$$\psi = \psi_L + \psi_R \to e^{i\alpha^L}\psi_L + e^{i\alpha^R}\psi_R$$

$$= \frac{1}{2}\left[e^{i\alpha^L}(1-\gamma_5) + e^{i\alpha^R}(1+\gamma_5)\right]\psi$$

$$= \frac{1}{2}\left[(e^{i\alpha^L} + e^{i\alpha^R}) + (e^{i\alpha^L} - e^{i\alpha^R})\gamma_5\right]\psi$$

where α_L, α_R can be arbitrary and independent. The vector-subgroup arise when $\alpha^L = \alpha^R \equiv \alpha^V$, and the transformation above becomes:

$$\psi \to \frac{1}{2}\left[(2e^{i\alpha^V} + e^{i\alpha^V}) + 0\right]\psi = e^{i\alpha^V}\psi$$

which is a vector transformation.

The other choice that is orthogonal to the above choice is $\alpha^L = -\alpha^R \equiv \alpha^A$. This gives:

$$\psi \to \frac{1}{2}\left[(2e^{i\alpha^A} + e^{-i\alpha^A}) + (e^{i\alpha^A} - e^{-i\alpha^A})\gamma_5\right]\psi$$

$$= \left[\cos(\alpha^A) + i\gamma_5 \sin(\alpha^A)\right]\psi = e^{i\alpha^A\gamma_5}\psi$$

$$\gamma_5^2 = 1,\ \gamma_5^3 = \gamma_5$$

which is an axial transformation. Recall this although this is a symmetry of the action, it is broken by quantum effects. For the remaining $SU(2)_L \otimes SU(2)_R$, the same analysis can be done to break this down to $SU(2)_V \otimes SU(2)_A$ subgroup with transformations:

$$\begin{cases} \psi \to e^{i\alpha_j^V \tau_j}\psi \\ \psi \to e^{i\alpha_j^V \tau_j \gamma_5}\psi \end{cases},\quad i = 1,2,3$$

The next question is why the spontaneous symmetry breaking gets ride of $SU(2)_A$ and leaves $SU(2)_V$? It is not hard to see this. The condensate, "the vacuum expectation value of $\bar{\psi}\psi$", which breaks the chiral $SU(2)_L \otimes SU(2)_R$, has the following form:

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \rangle \neq 0$$

Going back to the two subgroups vector and axial, we already saw that: $\begin{cases} \alpha_j^L = \alpha_j^R \equiv \alpha_j^V \\ \alpha_j^L = -\alpha_j^R \equiv \alpha_j^A \end{cases}$, so the above condensate transforms as:

$$\langle \bar{\psi}\psi \rangle \longrightarrow \langle e^{-i\alpha_j^L \tau_j} e^{i\alpha_j^R \tau_j} \bar{\psi}_L \psi_R + e^{-i\alpha_j^R \tau_j} e^{i\alpha_j^L \tau_j} \bar{\psi}\psi \rangle$$

- $\to \langle \bar{\psi}\psi \rangle$: Invariant under vector transformation
- $\not\to \langle \bar{\psi}\psi \rangle$: Not invariant under axial transformation.

Again with the generators of the reduced symmetries being broken, we have lost thee generators (three previously conserved charges). These correspond to three massless Goldstone bosons, the pions.