Exercises on Quantum-Computing Basics

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The goal of this exercise is to realize the digital quantum circuit that simulates the time evolution of the Schwinger model. Consider the Schwinger Hamiltonian in the purely fermionic formulation:

$$\begin{aligned} H =&\mathrm{x}\sum_{x=0}^{N-2}\left[\psi^\dagger(x)\psi(x+1)+\text{h.c.}\right]+\mu\sum_{x=0}^{N-1}(-1)^x\psi^\dagger(x)\psi(x)+ \\ &\sum_{x=0}^{N-2}\left\{\epsilon_0-\sum_{y=0}^{x}\left[\psi^\dagger(y)\psi(y)-\frac{1-(-1)^y}{2}\right]\right\}^2. \end{aligned} \tag{1}$$

After Jordan-Wigner transformation, we can map the Hamiltonian to qubits:

$$\begin{aligned} H =& \mathrm{x}\sum_{x=0}^{N-2}\left[\sigma^+(x)\sigma^-(x+1)+\text{h.c.}\right]+\frac{\mu}{2}\sum_{x=0}^{N-1}(-1)^{x+1}\sigma^z(x)+ \\ &\sum_{x=0}^{N-2}\left\{\epsilon_0+\frac{1}{2}\sum_{y=0}^{x}\left[\sigma^z(y)-(-1)^y\right]\right\}^2 \qquad (2) \\ =& H^{\mathrm{X}}+H^{ZZ}+H^{Z} \qquad (3) \end{aligned}$$

where H^{ZZ} and H^{Z} are diagonal in the electric basis.

Problem 1

We will use the following theory parameters:

- $N = 4$
- $\epsilon_0 = 0$
- $\mathrm{x} = 0.6$
- $\mu = 0.1$

We also set the total evolution time $t = 5$, which is separated into $N_T = 10$ Trotter steps. Hence, the evolution time for each step is $\delta t = 0.5$.

(a) Consider the following expression:

$$H^{ZZ} + H^{Z} = \theta_{x,y}\sigma^{z}(x)\sigma^{z}(y) + \phi_x \sigma^{z}(x) \tag{4}$$

Find the coefficients $\theta_{x,y}$ and ϕ_x, ideally as a function of μ, ϵ_0, x, y, N etc. To simplify, you can express them as numbers. These will be the rotation angles in your circuit.

(b) Prove the following equality for a two-qubit system:

$$R_Z\left(\frac{-\pi}{2}\right) \otimes R_Z\left(\frac{-\pi}{2}\right) \cdot R_{XX}(\theta) \cdot R_Z\left(\frac{\pi}{2}\right) \otimes R_Z\left(\frac{\pi}{2}\right) = R_{YY}(\theta), \tag{5}$$

This will be useful as IBM composer does not have R_{YY} gates.

(c) Construct the Trotterized time evolution on IBM composer. Here we will use the $V_1(t)$ ordering in the slides that preserves the charge symmetry in the Schwinger model.

$$V_1(t) = e^{-iH^{Z}t} e^{-iH^{ZZ}t} \prod_{i=0}^{N-4} e^{-iH^{X}_{i+1,i+2}t} \prod_{i=0}^{N-2} e^{-iH^{X}_{i,i+1}t} \tag{6}$$

The procedure therefore should be the following:

1. Starting from the bare vacuum $|1010\rangle$. (You just need to apply some X gates at the beginning.)

2. Evolve $H^{X}_{i,i+1}$ for even i

3. Evolve $H^{X}_{i,i+1}$ for odd i

4. Evolve $H^{ZZ}_{i,j}$ for all i, j pairs with the angles you obtained in (a)

5. Evolve H^{Z}_{i} for all i wit the angles you obtained in (a)

6. Repeat this s times with s ranges from 1 to N_T. Write down the survival probability for each s and compare these with the exact results. You should recover the upper panel of fig.6 of 2112.14262.

Note: the labeling in Qiskit is that the first (0^{th}) qubit represents the least significant digit.

Bonus

1. Modify the physical Hamiltonian in the first exercise with OBC. Check if the spectrum matches that of the full Hamiltonian.

2. Calculate the Trotterized time evolution in eq. 6 with $t = \delta t$ using exact exponentiation. Repeat the evolution N_T times. Check if this matches the circuit result.

3. The staggered density is defined as

$$\nu(t) = \frac{1}{N}\sum_n \nu_n(t) \tag{7}$$

$$\nu_n(t) = \langle\psi(t)|\frac{(-1)^{(n+1)}\sigma_n^{\mathrm{z}}+1}{2}|\psi(t)\rangle \tag{8}$$

Calculate this quantity using exact-diagonalization and Trotterized evolution. Compare your results to the lower panel in fig.6 of 2112.14262.