Lattice BSM in-class exercises: Lecture 1

Exercise 1: Consider the Standard Model decay of a neutral ρ meson into an electron-positron pair. Suppose that we would like to use lattice simulations to predict the decay rate $\Gamma(\rho^0 \to e^+ e^-)$. [1] The relevant matrix element can be given in terms of the **vector decay constant**, for which you should use the convention

$$\langle 0|\bar{q}\gamma^i q|\rho\rangle = M_\rho^2 f_\rho \epsilon^i, \tag{1}$$

where ϵ^i is a unit polarization vector. The neutral ρ meson is an isospin-triplet state, which means its quark composition is $\rho^i \sim \frac{1}{\sqrt{2}}(\bar{u}\gamma^i u - \bar{d}\gamma^i d)$. Note that f_ρ is dimensionless in these conventions.

a) At a microscopic level, the relevant process for this decay is $\bar{q}q \to e^+ e^-$. Draw the leading-order Feynman diagram for this process. Write the amplitude $\mathcal{M}$ for the decay process, using factorization to include the vector decay constant as defined above.

b) Show that the decay rate is equal to

$$\Gamma(\rho^0 \to e^+ e^-) = \frac{4\pi}{3}\langle q\rangle^2 \alpha^2 M_\rho f_\rho^2 \tag{2}$$

where α is the fine-structure constant $e^2/4\pi$, and

$$\langle q\rangle \equiv \frac{1}{\sqrt{2}}\left[\left(\frac{2}{3}\right) - \left(-\frac{1}{3}\right)\right] \tag{3}$$

is the "average charge" of the ρ^0 state, capturing the relative contributions of the up and down quarks. The differential decay rate is related to the squared amplitude by

$$\frac{d\Gamma}{d\Omega} = \frac{1}{64\pi^2 M_\rho}|\mathcal{M}|^2, \tag{4}$$

and I have ignored the electron mass, i.e. you should take $m_e = 0$ to simplify the calculation. If your QFT is rusty, **show as much as you can**; e.g. explain the functional dependence on $\alpha^2, \langle q\rangle^2, M_\rho, f_\rho$.

[1] You might wonder why we're doing SM decays in a BSM lecture. The same derivation we go through here will also apply for decays of analogues of the rho meson in composite models.

Exercise 2: As mentioned in lecture, baryon-like composite dark matter is constrained in a novel way by collider experiments. Let's assume the dark sector is very similar to QCD: based on an SU(3) gauge group, with dark quarks U and D that have electric charges $q_U = +2/3$ and $q_D = -1/3$. The dark matter candidate is thus the "dark neutron" N, which is a UDD bound state. With this setup, the theory will also contain charged "dark pions" $\Pi^\pm \sim \bar{U}D, \bar{D}U$ which will be the lightest mesons in the spectrum.

a) Consider the ratio M_Π/M_N as a function of the dark quark mass m_Q. Over the full range of possible masses $0 \leq m_Q \leq \infty$, what are the minimum and maximum values of this ratio?

b) The charged dark pions can be produced in particle colliders. In particular, they would have been copiously produced by the LEP electron-positron collider, which had a center of mass energy of approximately $\sqrt{s} \sim 200$ GeV. (The relevant process is $e^+e^- \rightarrow \gamma \rightarrow \Pi^+\Pi^-$; the dark pions are always produced in pairs due to charge conservation.) Based on this observation and your answer to part A, what is the minimum possible dark matter mass M_N in this model?

c) We can consider a variant of this model where the gauge group is changed to SU(N_c). With N_c colors, the baryon becomes a bound state of N_c dark quarks. What is the minimum dark matter mass M_N due to LEP charged-pion searches as a function of N_c?

d) **Bonus challenge:** LEP gives a very strong constraint on composite dark matter sectors with electric charge. Using the dark pions themselves as a dark matter candidate would weaken the bound somewhat, but it does not eliminate it. Are there other variations on composite dark matter for which the LEP bound can be eliminated, allowing GeV-scale or below composite dark matter, while still having dark quarks that carry electric charge?