The role of (lattice) QCD in Flavor Physics

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Outline

- Motivation: Open Questions
- The role of (lattice) QCD in flavor physics
  - History
  - Challenges
- Examples
  - Leptonic kaon, pion decay
  - Semileptonic kaon decay
  - First row CKM unitarity
  - Semileptonic D-meson decay

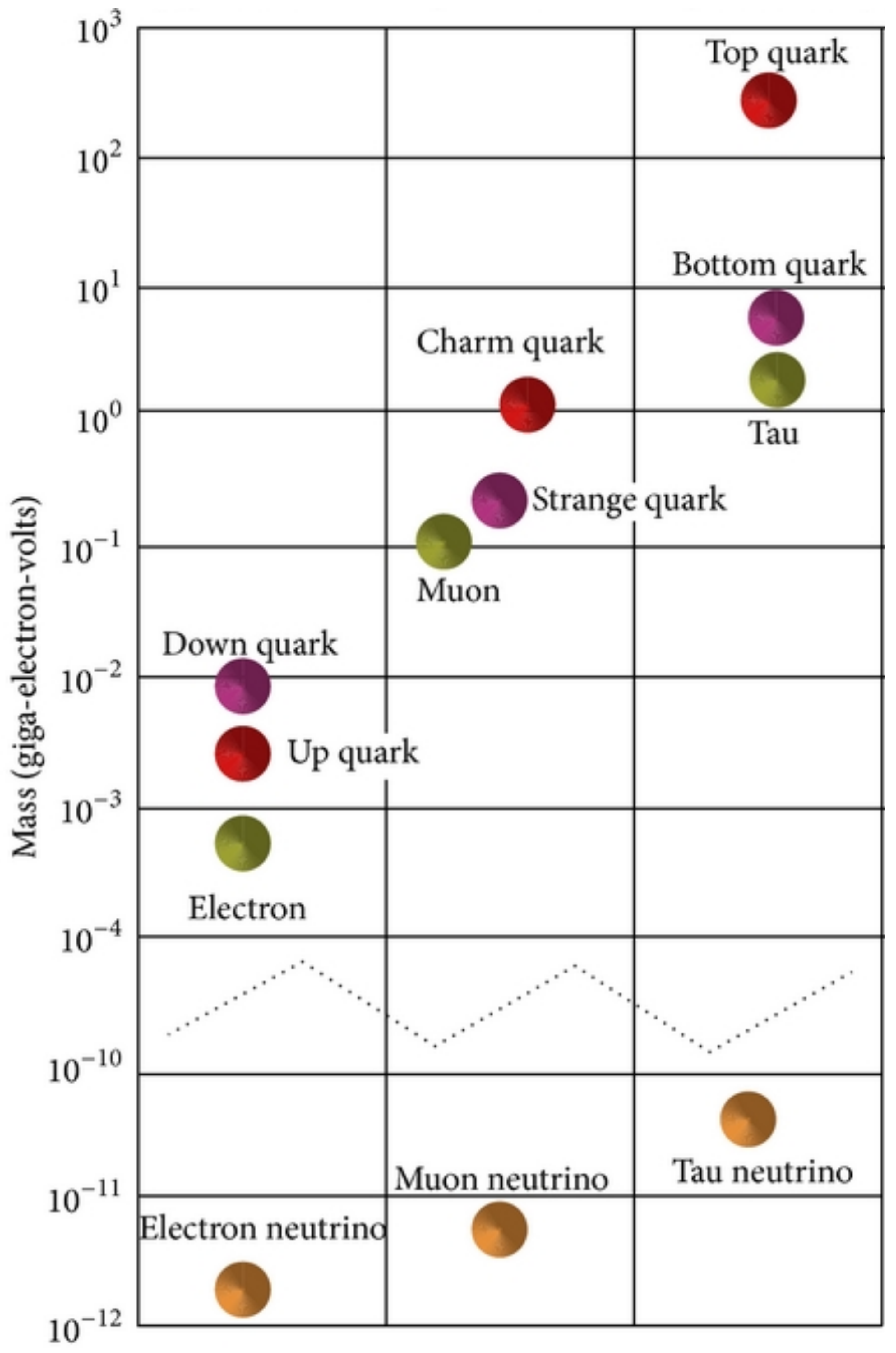
Open Questions in Flavor Physics

- ✦ Who ordered that? (Why three generations?)
- ✦ origins of fermion masses: Higgs-Yukawa?
  -same for quarks, charged leptons and neutrinos?
- ✦ origin of mass hierarchy
- ✦ structure of CKM and PNNS matrices
- ✦ CP violation?
- ✦ strong CP problem: $\bar{\theta} = \theta - \arg\det m_q$

  - neutron EDM: $|d_n| \lesssim 3\,10^{-26}\,e\,\mathrm{cm}$ ➠ $\bar{\theta} \lesssim 10^{-10}$
  - but $m_u > 0$ (lattice QCD) ➠ origin of cancellation?

⋮

Possible answers to these questions generally give rise to new particles and/or interactions.



[G. Giacomelli et al, AHEP 2013 (2013)]

Origin of fermion masses in the Standard Model

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \; + \; i\bar{\psi} \not{D} \psi \; + \; \bar{\psi}_i Y_{ij} \psi_j \phi + \mathrm{h.c.} \; + \; |D_\mu \phi|^2 - V(\phi)$$

gauge field terms for $SU(3)_c \times SU(2)_L \times U(1)_Y$

fermion-gauge boson interaction terms

Higgs Yukawa

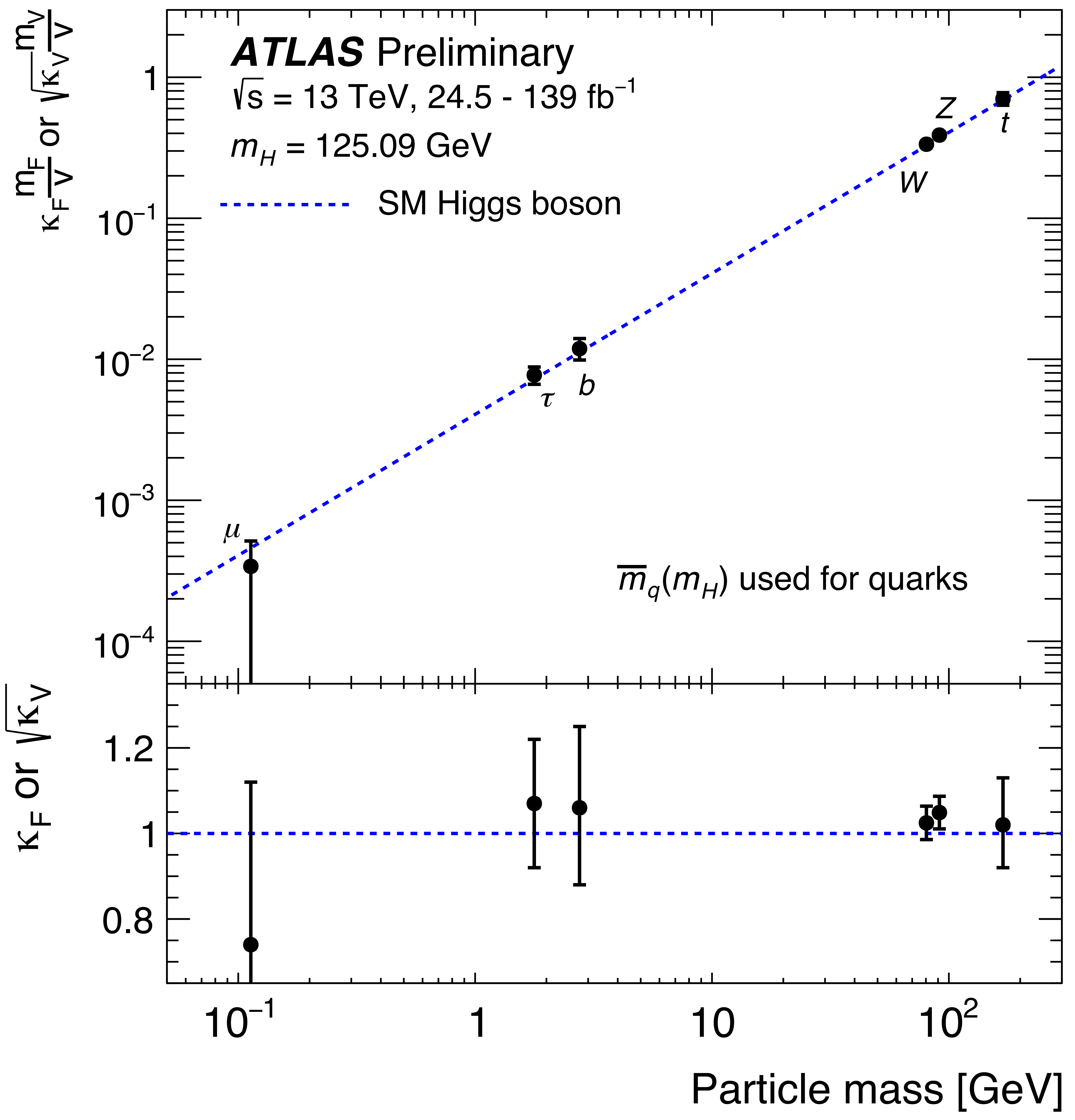
Higgs-gauge boson interactions

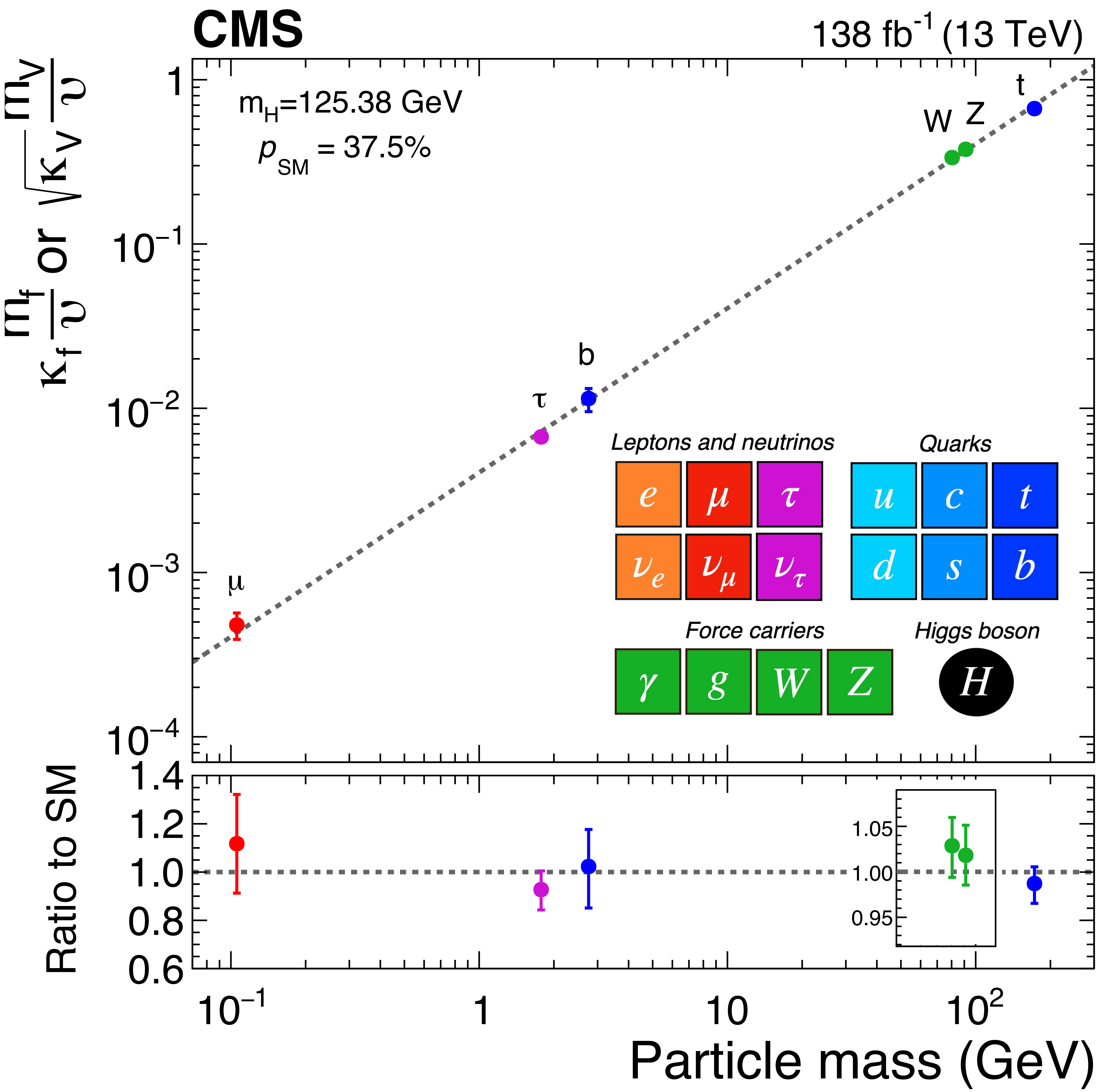
Higgs potential: self interactions

mass terms + fermion-Higgs interactions:

$$\sum_f m_f \bar{\psi}_f \psi_f \, [1 + H/v] \qquad m_f = Y_f \, v/\sqrt{2}$$

Higgs physics @ LHC



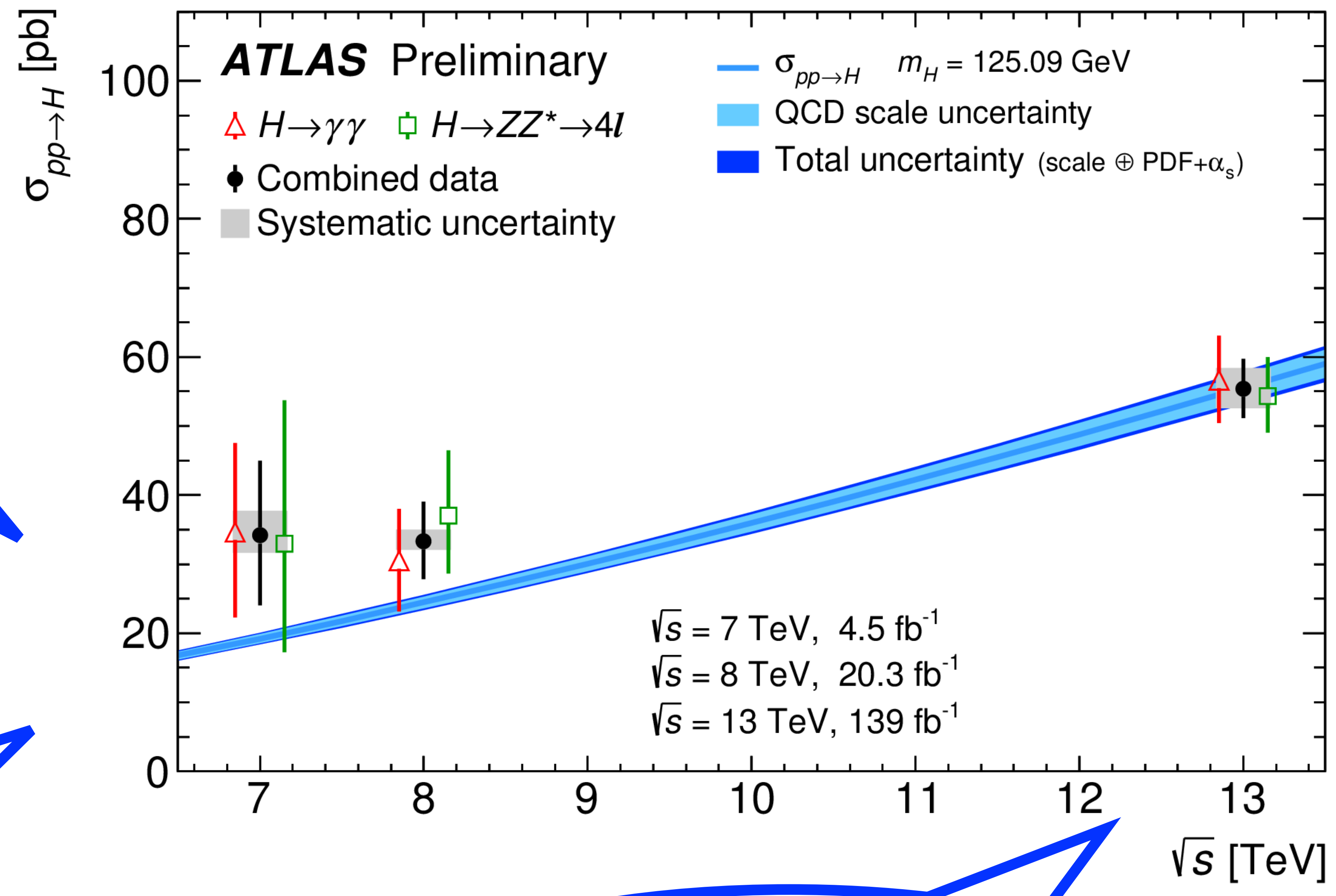


Higgs production and decay

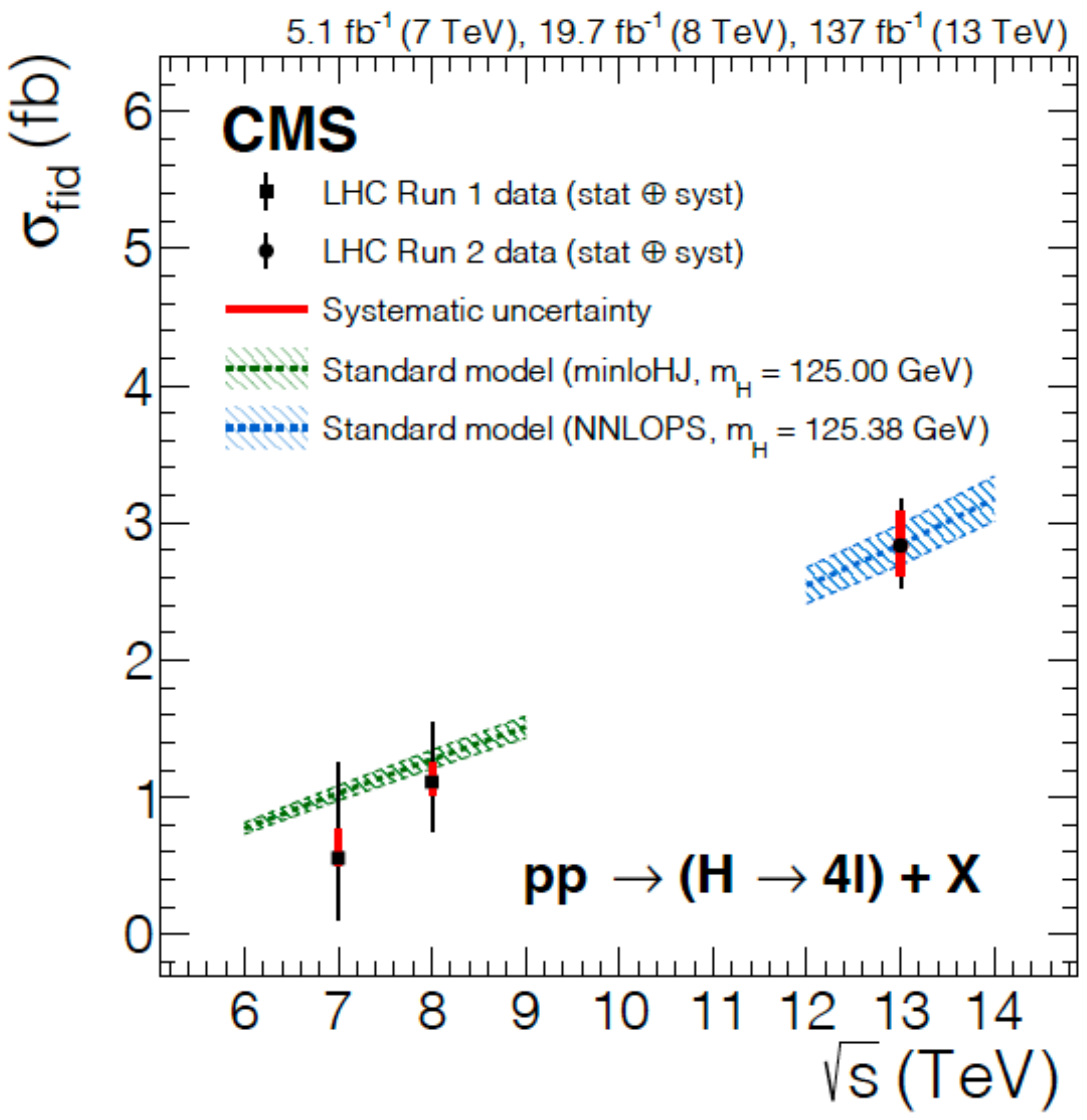
Radja Boughezal @ P5 SLAC town hall

Electroweak corrections at 2 loops

PDFs@NNLO



Precision determination of quark masses



QCD@N3LO

Precision extraction of α_S

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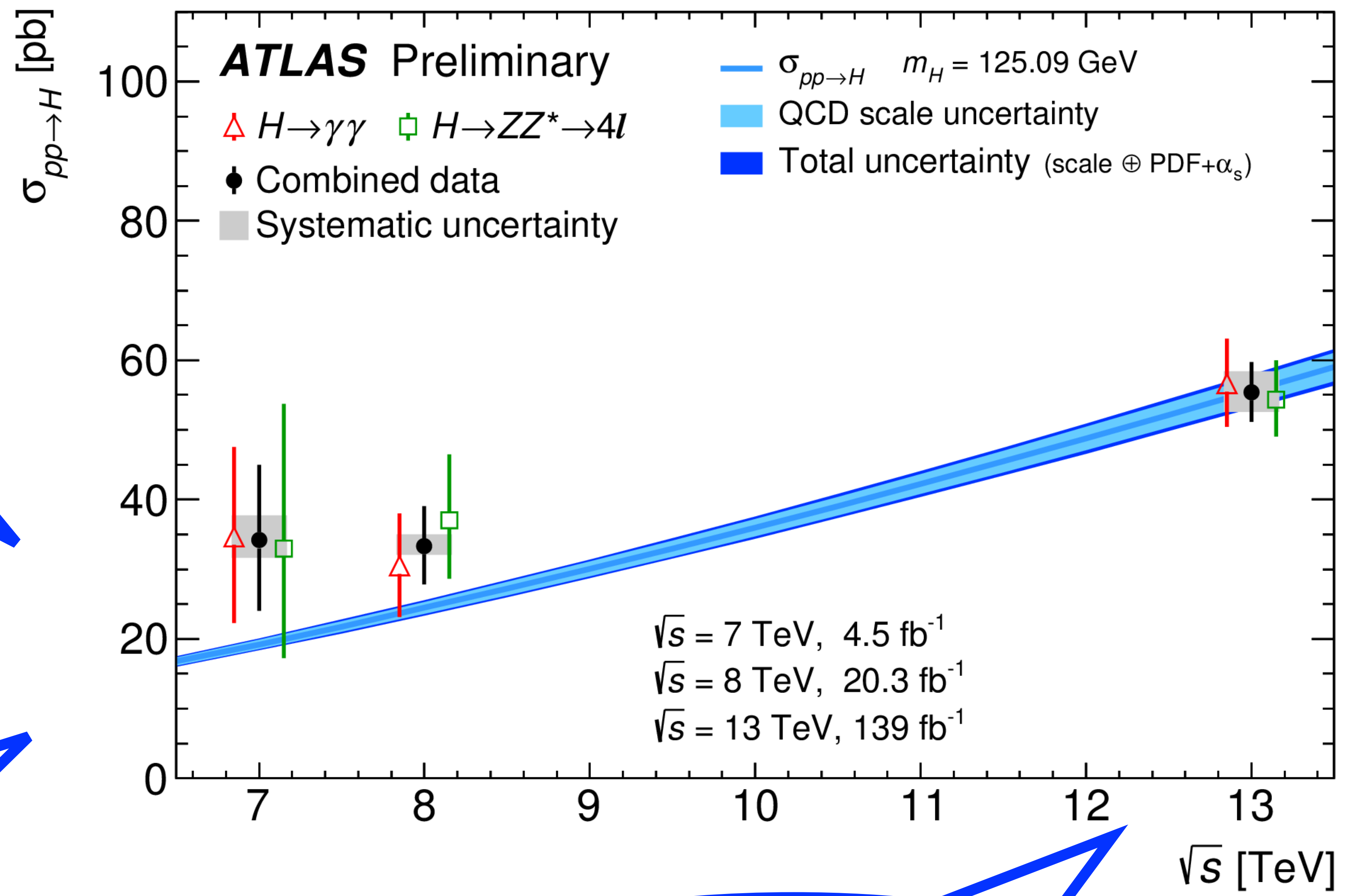

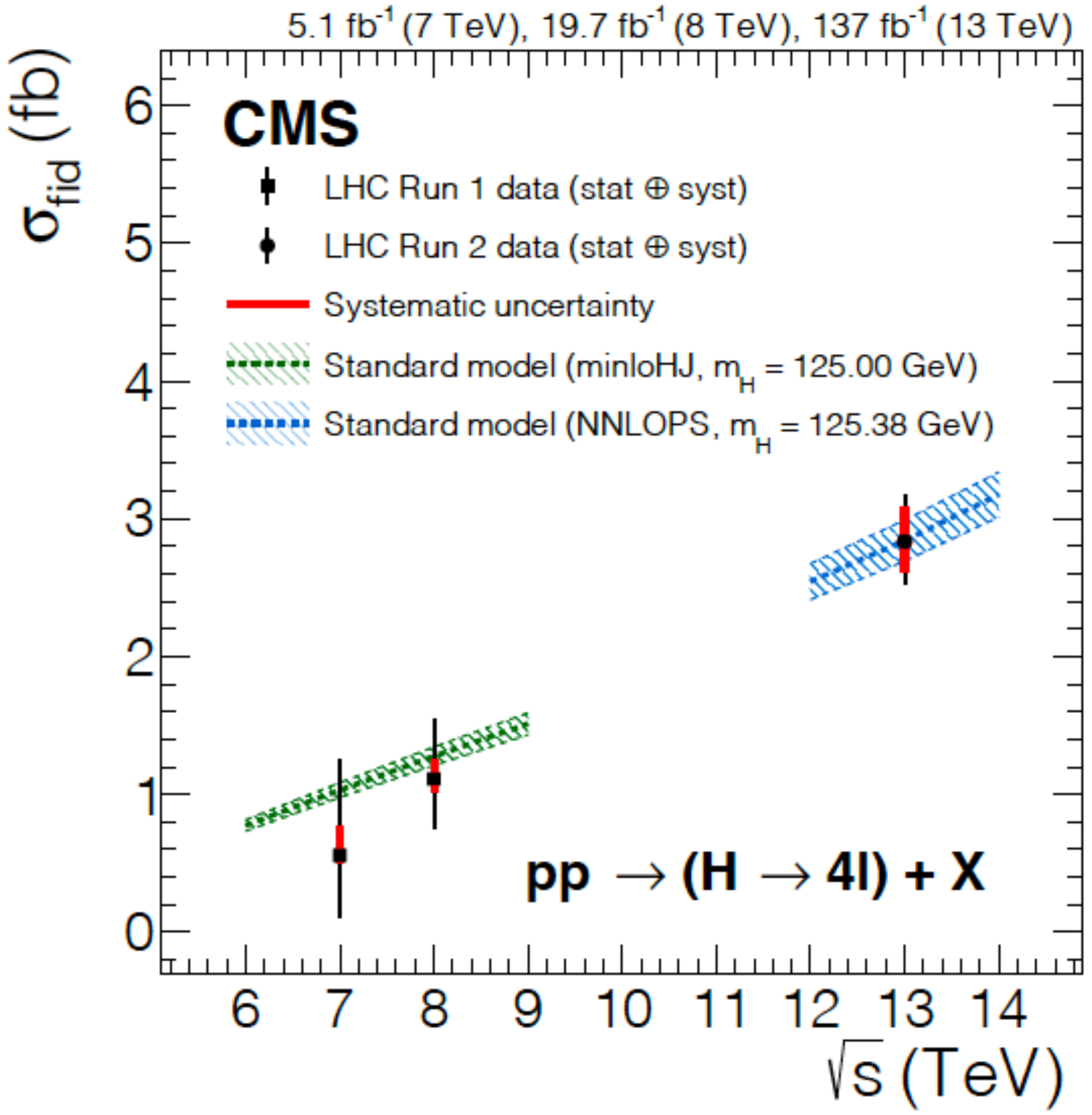

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Precision determination of quark masses

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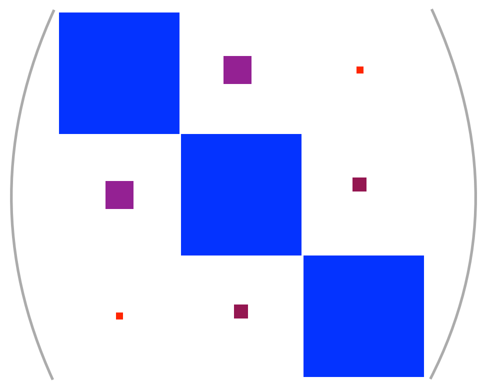
Future lattice QCD inputs

Lattice QCD inputs

Open Questions in Flavor Physics

- quark flavor violation caused in SM by weak interactions (mediated by charged W boson), described by CKM (Cabibbo, Kobayashi, Maskawa) matrix:

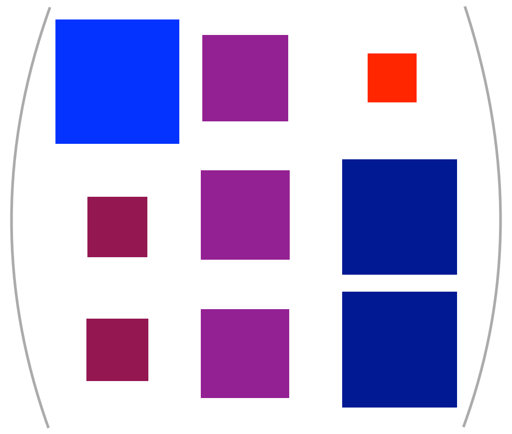
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} =$$



- The CKM matrix is unitary in the SM ➠ testable relations between CKM elements
- CP violation, described by 1 complex phase: probably not enough to account for matter—anti-matter asymmetry
- Origin of hierarchy (cf PMNS matrix)?

  neutrino mixing:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\rm PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad U_{\rm PMNS} =$$



- no charged lepton flavor changing decays in SM ➠ searches for processes such as $\mu \to e$
- lepton flavor universality, except for effects due to lepton mass differences $m_e \ll m_\mu \ll m_\tau$

Indirect new physics searches

- look for forbidden (in the SM) processes:
  - ★ proton decay
  - ★ Lepton flavor violating decays, for example:

    $\mu \to e\gamma,\, B \to \tau\mu, \ldots$
  - ★ ….

Observation ➠ discovery of new physics

- study rare processes — loop-suppressed in the SM:
  - ★ neutral meson mixing
  - ★ flavor changing neutral current decays
  - ★ muon magnetic moment
  - ★….
- check lepton flavor universality: $\tau/\mu,\, \mu/e$
- (over)determine SM parameters with high precision
  - ★ test unitarity of CKM matrix (quark-W couplings)

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Desired information on

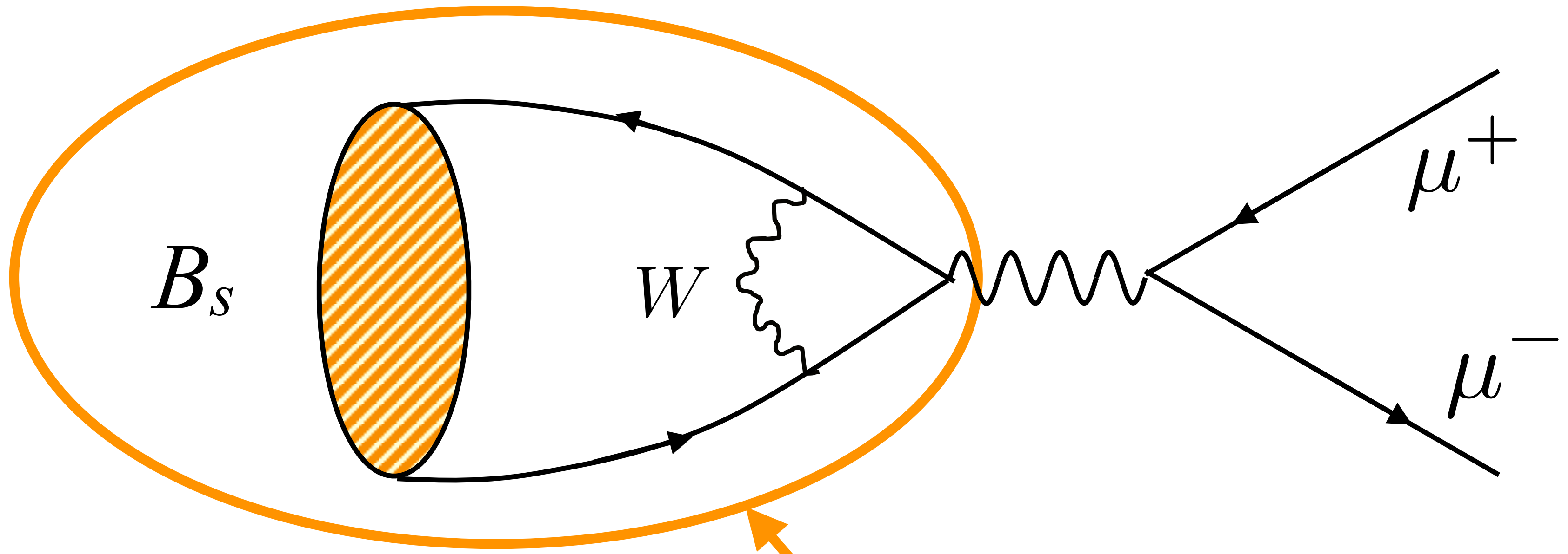
short-distance physics (BSM particles, DM, …) or fundamental parameters (CKM, m_q, α_s, θ_{23},,…)

is hidden by hadronic/nuclear effects (nonperturbative QCD).

➠ need precise QCD calculations to complement experimental measurements: Lattice QCD

The role of (lattice) QCD in flavor physics

example: $B_s \to \mu^+\mu^-$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

$\Gamma\left(K^+ \to \ell^+\nu_\ell(\gamma)\right)$

$d\Gamma(B^0 \to D^{*-}\mu^+\nu_\mu), \ldots$

$B(B_s \to \mu\mu), \ldots$

$\Delta m_{d(s)}$ $\cdots$

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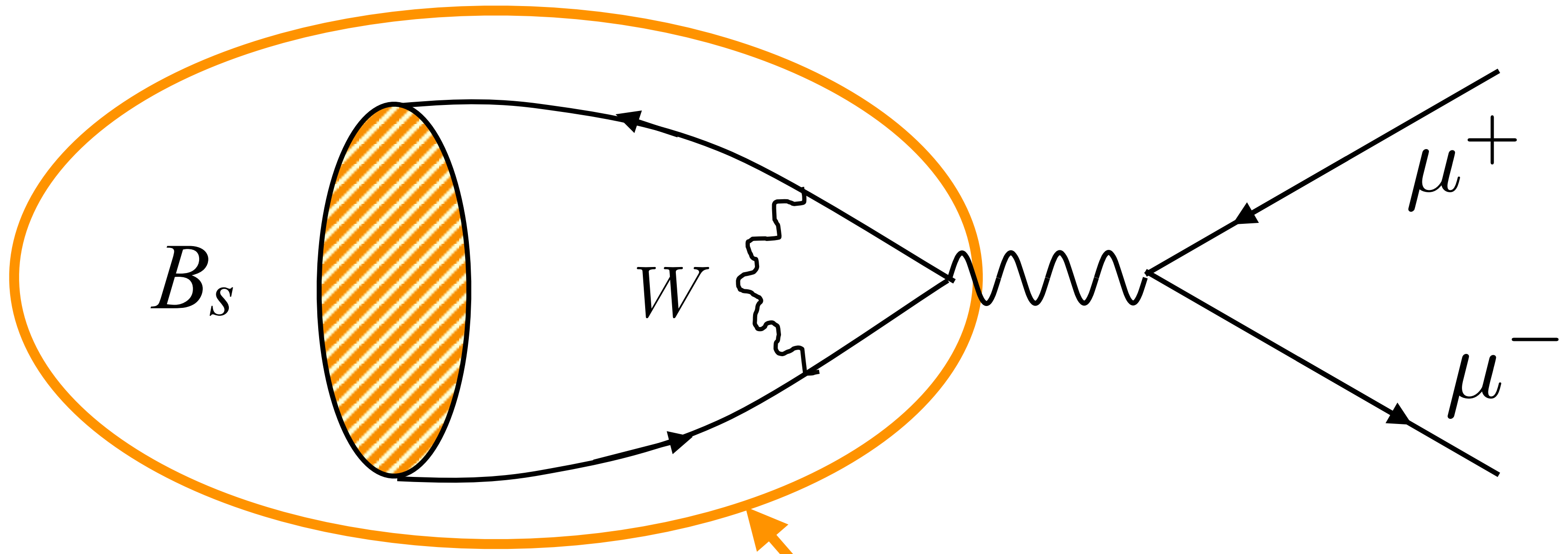

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Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

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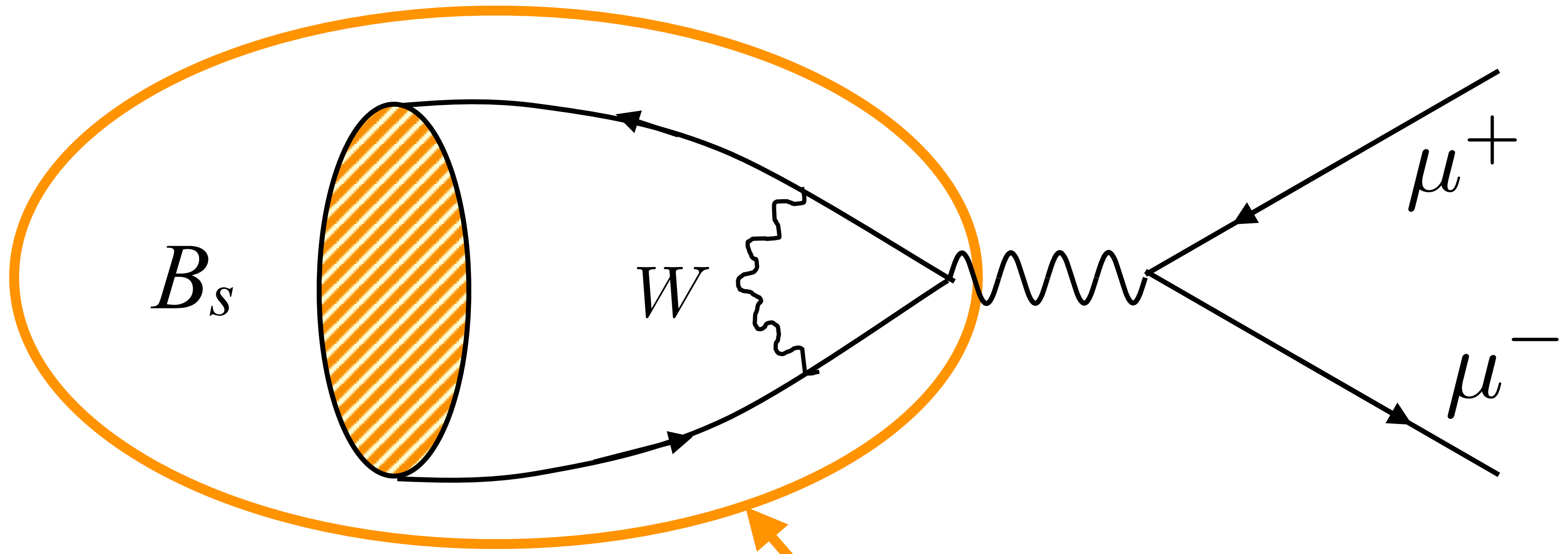

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$\Delta m_{d(s)} \quad \cdots$

Two main purposes:

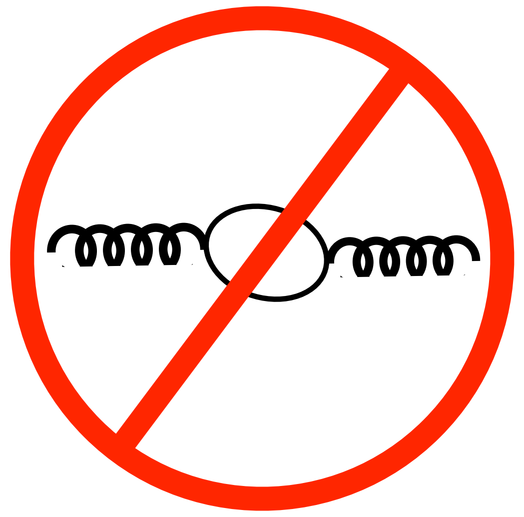
- ✦ combine experimental measurements with LQCD results to determine SM parameters.
- ✦ confront experimental measurements with SM theory using LQCD inputs.

Lattice QCD

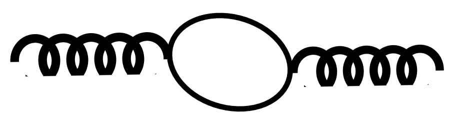
parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

a (selective) view of the history: LQCD for Flavor

- 1971 and 1974 — Discovery of charm (J/ψ)
- 1973 — **Kobayashi** & **Maskawa** — Gross, Politzer, Wilczek *Asymptotic Freedom*
- 1974 — Wilson's *Confinement of quarks:* gauge theory on space-time lattice
- 1977 — Discovery of beauty (Υ)
- 1979 — Creutz's *Monte Carlo study of quantized SU(2) gauge theory*
- 1981 — Hamber & Parisi, Weingarten: first quenched LQCD calculations of hadron masses



- 1984-1985 — Cabbibo, Martinelli, Petronzio; Brower et al; Bernard et al: Weak Matrix Elements
- 1989 — Sharpe review at Lattice 1989 conference on Weak Matrix Elements
- 2003 — First lattice QCD simulations that include realistic sea quark effects



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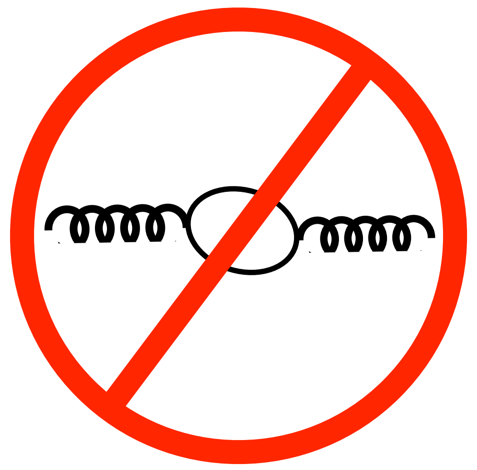
Status 1989

Sharpe @ Lattice 1989 [Nuc. Phys. B (Proc. Suppl.) 17 (1990)]

What?	Why?	Who? [6]	Level
	Nucleon matrix elements		
f_π/m_N, f_K/f_π	check	MANY	2
Axial vector matrix elements: g_A ...	check	Sömmer [7]	2
EM form factors: $G_M(q^2)$, ...	check	Wilcox, Draper/Liu [8]	2
Structure functions	check	Rossi [9]	1
Neutron Electric Dipole Moment	measure θ_{QCD}	Goksch [10]	1
	Heavy-light mesons		
f_D, f_B, B_D, B_B	$\overline{D}D$ and $\overline{B}B$ mixing	Eichten, Martinelli [11]	1-2
$D \to Ke\nu$, $(B \to \pi e\nu)$, ...	measure V_{cs}, V_{ub}	El Khadra, [12] Sachrajda [13]	1-2
$D \to K\pi$	check	Sachrajda, Simone	1
	K decay and mixing amplitudes		
B_K	extract δ from ϵ	Bernard, Kilcup, [14] Martinelli	3
$K \to \pi\pi$ ($\Delta I = 1/2$ rule)	check	Bernard, Kilcup, Martinelli	2
ϵ'	over-determine δ	Kilcup, Bernard	2

Table 1: Work done on weak matrix elements in the year preceding September 1989

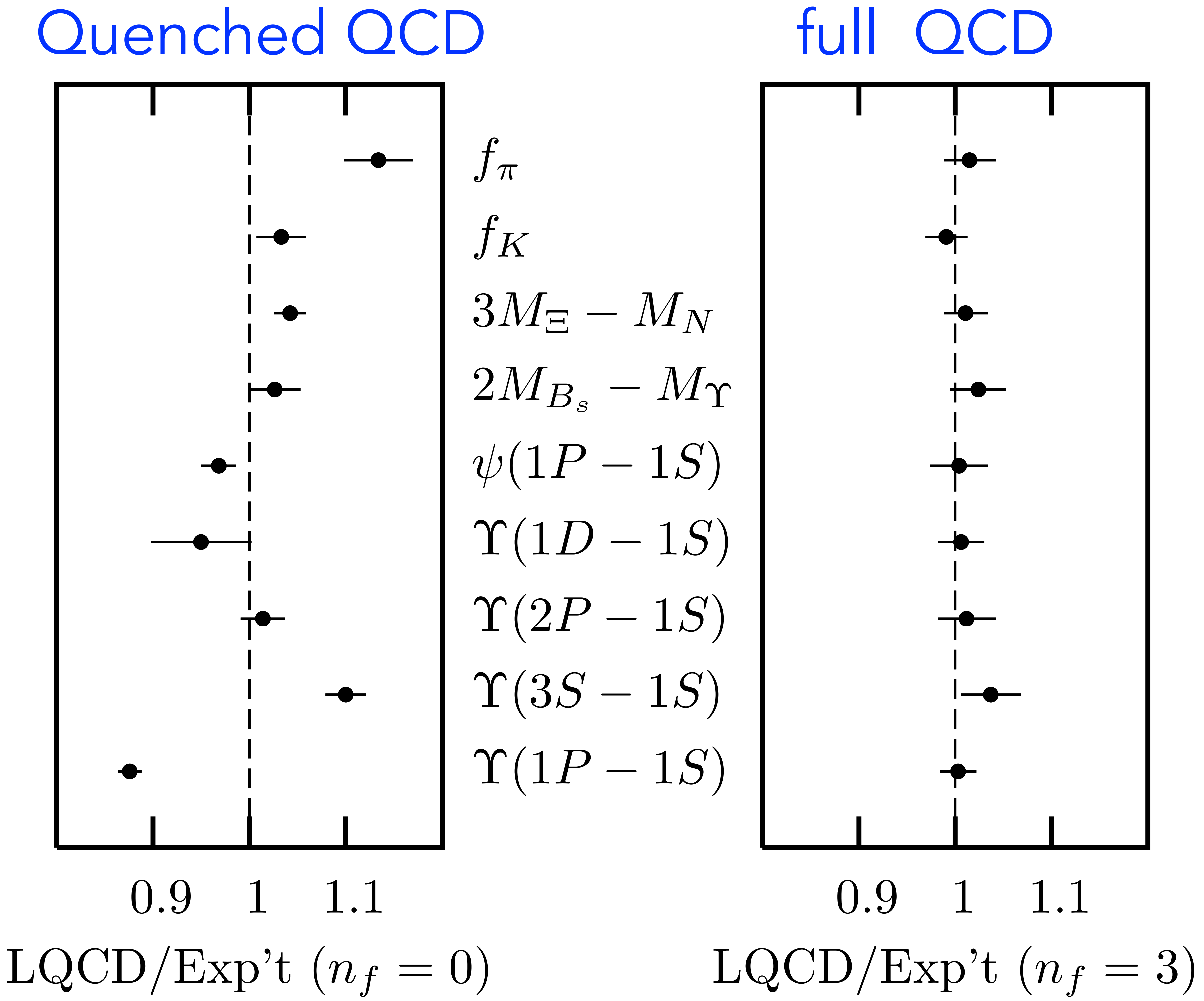
All lattice QCD simulations use the quenched approximation: $n_f = 0$

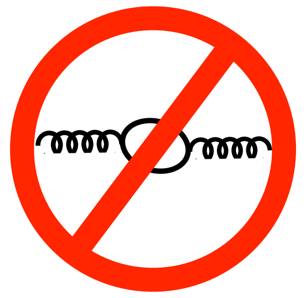


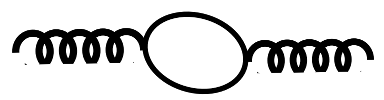
2003-2005: first ``realistic" lattice QCD results

based on simulations with three flavors of sea quarks ($n_f = 2 + 1$):

C. Davies et al [HPQCD, MILC, Fermilab Lattice, hep-lat/0304004, 2004 PRL]







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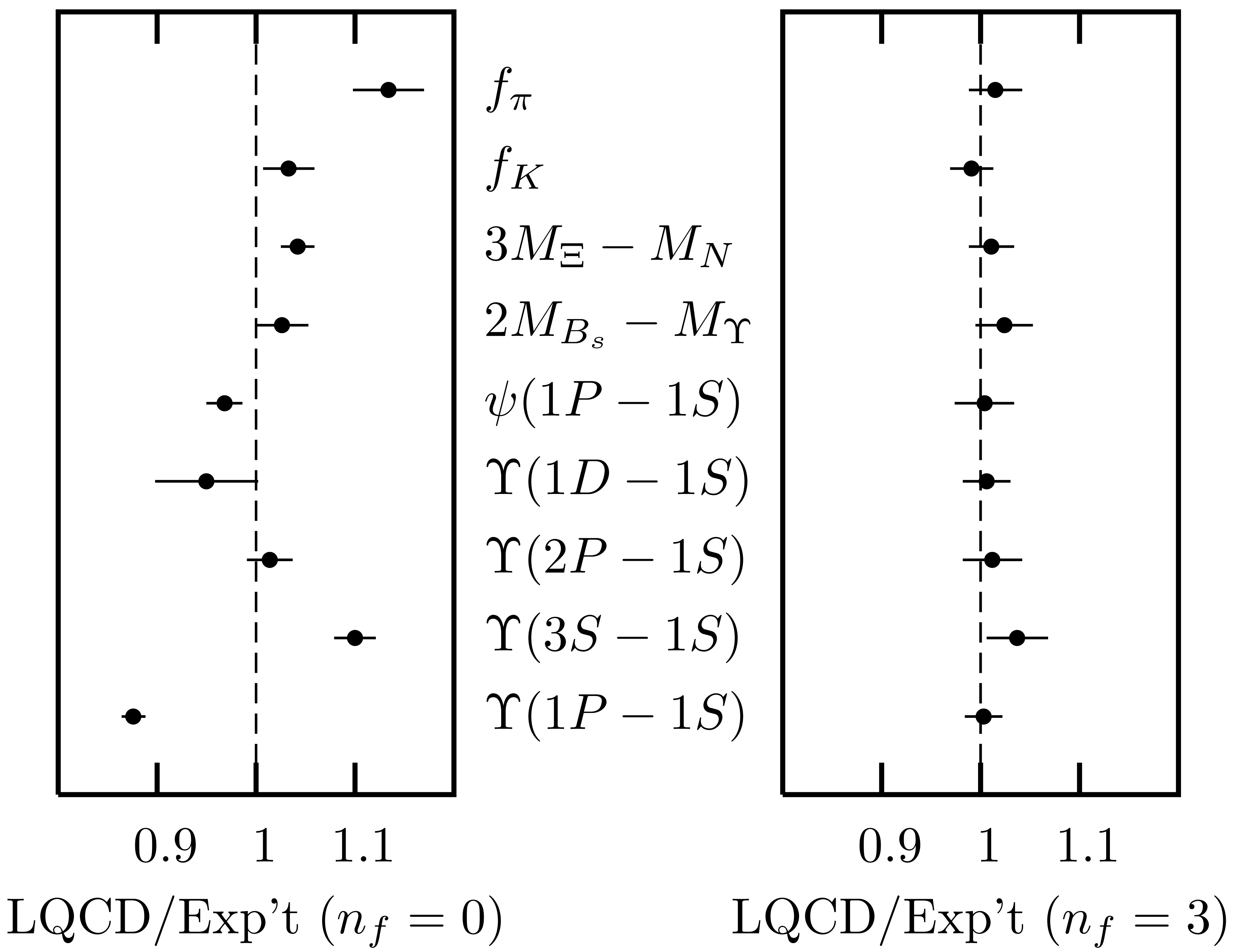
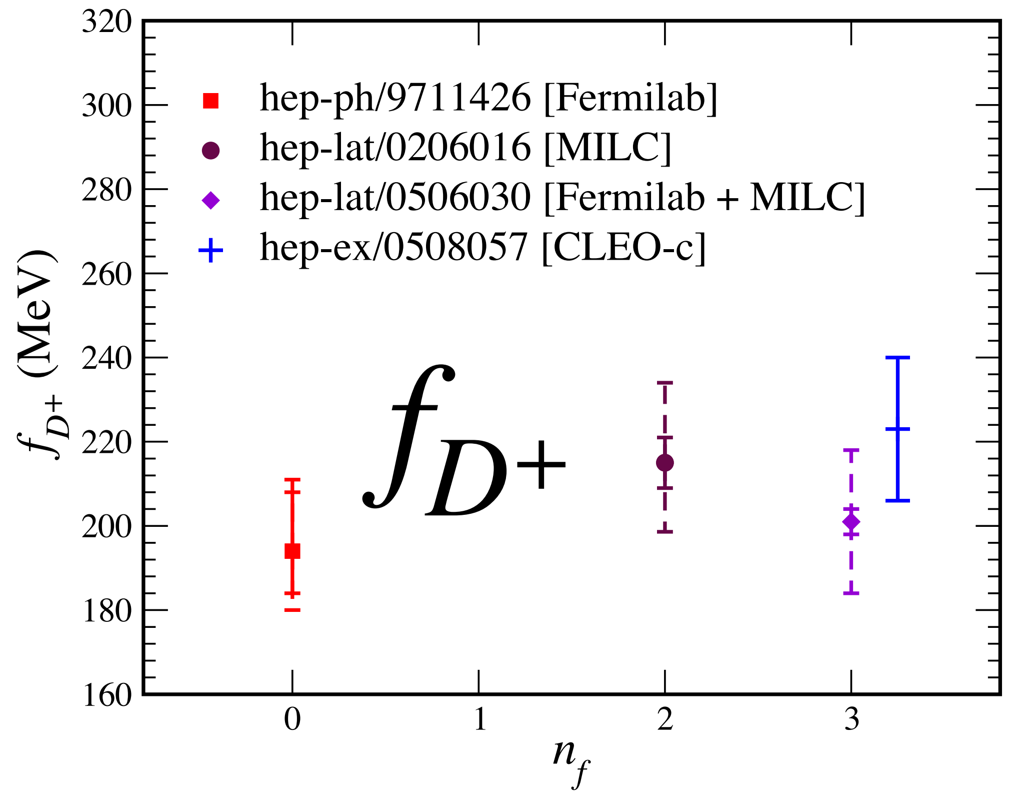
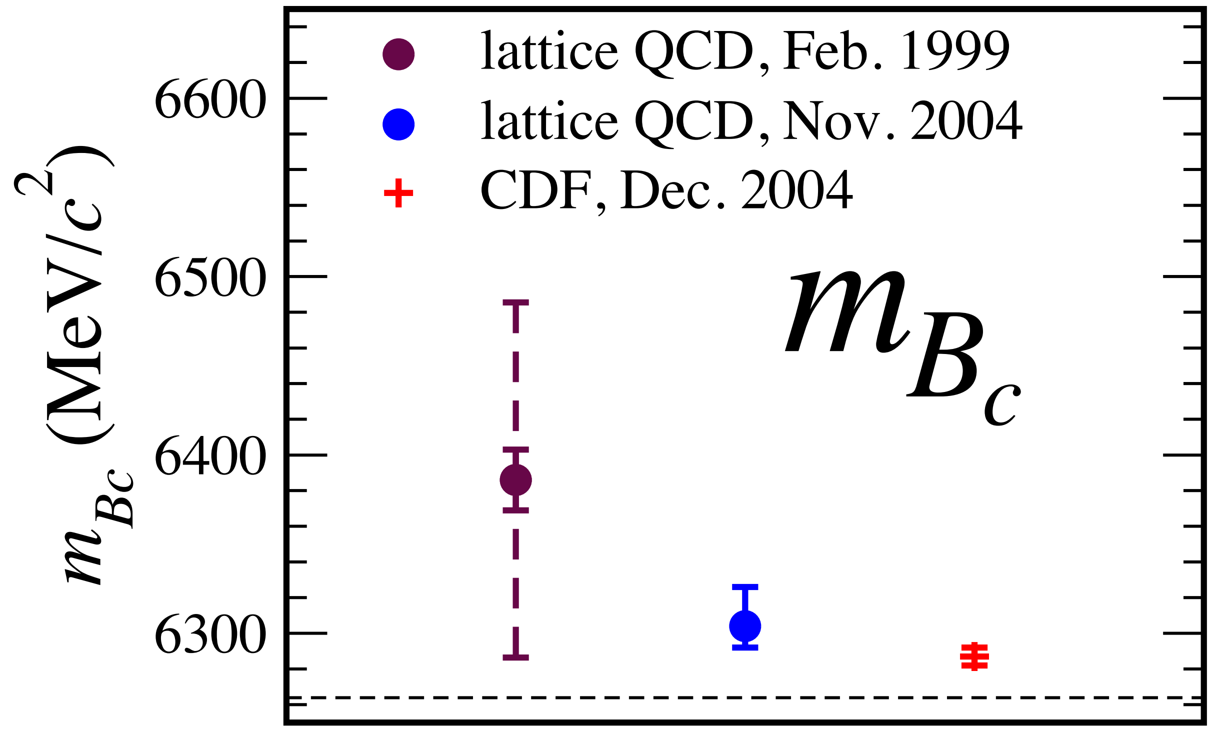
A. Kronfeld et al [Fermilab Lattice, MILC, HPQCD, hep-lat/0509169,Int.J.Mod.Phys 2006]

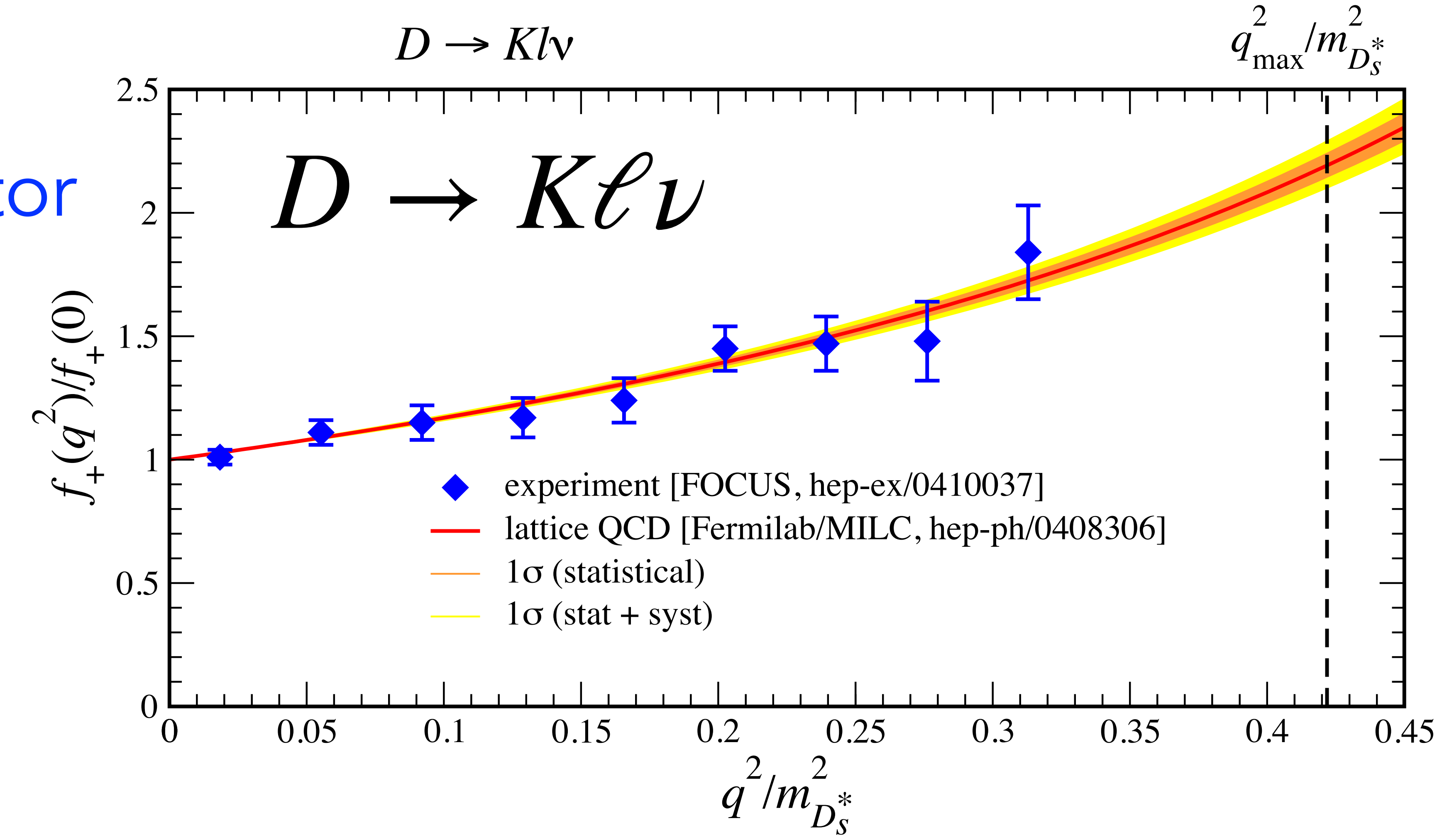


First lattice QCD *predictions,* confirmed by experiment:





shape of form factor

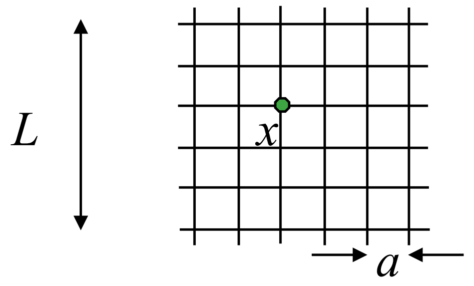


Snowmass 2013 ➠ present

https://www.usqcd.org/documents/13flavor.pdf and [J. Butler et al, arXiv:1311.1076]

Quantity	CKM element	2013 expt. error	2007 forecast lattice error	2013 lattice error	2018 forecast lattice error	2021 FLAG Average
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	–	0.4%	0.2%	0.18 %
f_D	$\lvert V_{cd}\rvert$	4.3%	5%	2%	$< 1\%$	0.3 %
f_{D_s}	$\lvert V_{cs}\rvert$	2.1%	5%	2%	$< 1\%$	0.2 %
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	–	4.4%	2%	0.7 % [from 2212.12648]
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	–	2.5%	1%	0.6 %
$B \to D^*\ell\nu$	$\lvert V_{cb}\rvert$	1.3%	–	1.8%	$< 1\%$	~1.5 % [from 2105.14019, 2304.03137, 2306.05657]
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	–	8.7%	2%	~3 %
f_B	$\lvert V_{ub}\rvert$	9%	–	2.5%	$< 1\%$	0.7 % (0.6 % for f_{B_s})
ξ	$\lvert V_{ts}/V_{td}\rvert$	0.4%	2–4%	4%	$< 1\%$	1.3 %
Δm_s	$\lvert V_{ts}V_{tb}\rvert^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$	1.3 %

QED corrections dominant source of theory error



Lattice QCD Introduction

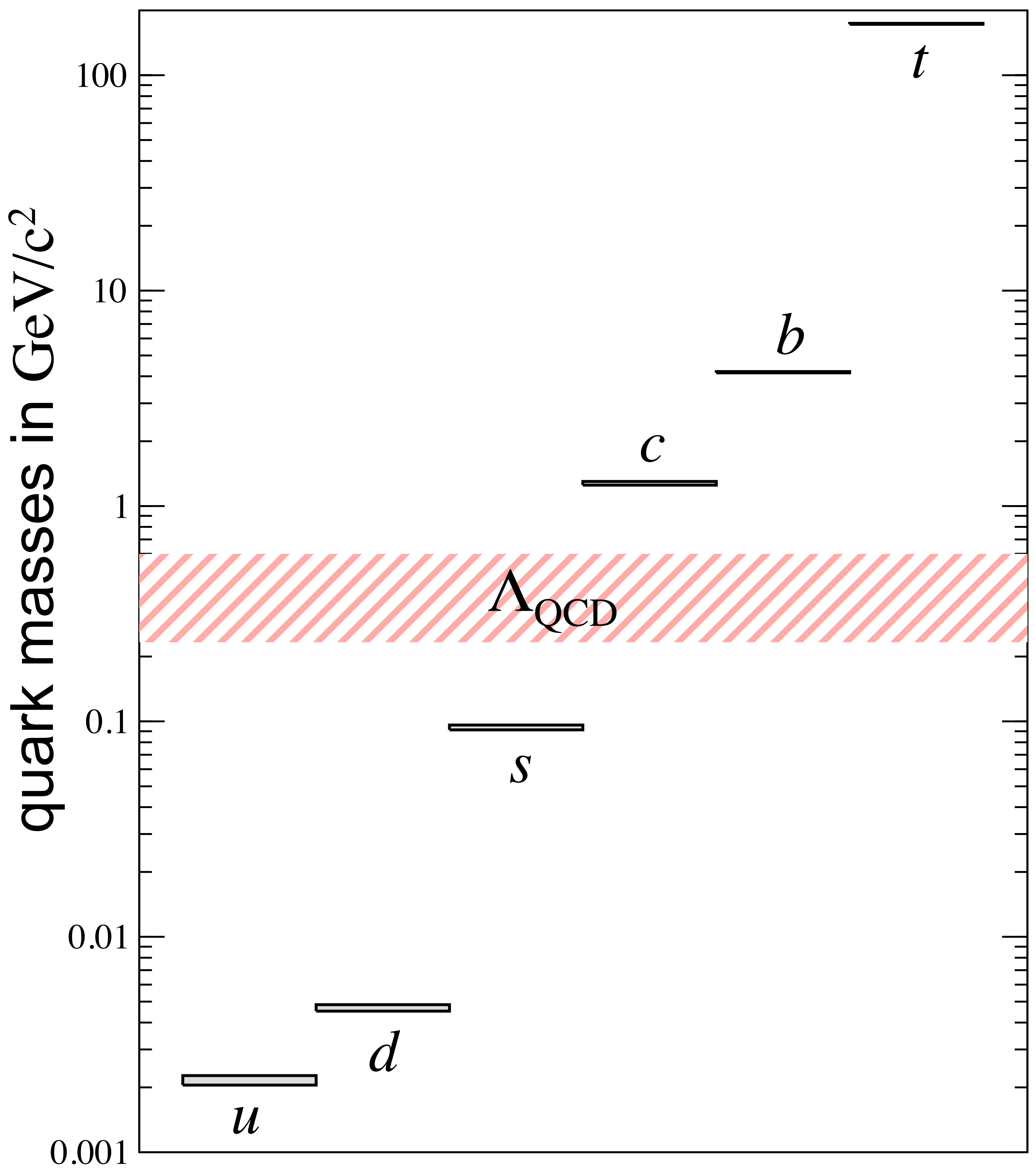
systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on Effective Field Theory (EFT) descriptions of QCD ➙ ab initio

- finite a:
  Symanzik EFT ⇐ Asymptotic Freedom & Renormalizability
- light quark masses:
  Chiral Perturbation Theory ⇐ Chiral Symmetry & Spontaneous Symmetry Breaking
- heavy quarks:
  HQET ⇐ Heavy Quark symmetry
- finite L:
  finite volume EFT ⇐ Confinement & S-matrix

Flavor symmetries in QCD

Chiral Symmetry



- ⭐ spontaneously broken by chiral condensate: $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$

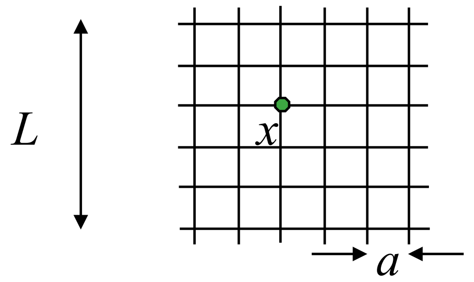
  Lattice QCD: $\langle \bar{\psi}\psi \rangle^{\overline{MS}}(2\,\mathrm{GeV}) = \left(272 \pm 5\right)^3 \mathrm{MeV}^3$ $\quad m_u = m_d \to 0$

  [Y. Aoki et al, FLAG review 2021, 2111.09849]

- ⭐ explicitly broken by $m_q \neq 0$: small corrections
  - ➠ Chiral Perturbation Theory
    - ➢ Isospin symmetry - SU(2) $\quad m_u, m_d \ll \Lambda_{QCD} \quad \Rightarrow \quad u \approx d \quad \Rightarrow \quad n \approx p$
    - ➢ SU(3) flavor symmetry $\quad m_u, m_d < m_s \ll \Lambda_{QCD} \quad \Rightarrow \quad s \approx u,\, d \quad \Rightarrow \quad K \approx \pi$

Heavy Quark Symmetry

- ⭐ heavy quarks: $\quad 1/m_Q \to 0$
  - ➢ Heavy Quark Symmetry (HQS) $\quad m_b, m_c \gg \Lambda_{QCD} \quad \Rightarrow \quad c \approx b \quad \Rightarrow \quad D \approx B$
  - ➠ Heavy Quark Effective Theory (HQET)



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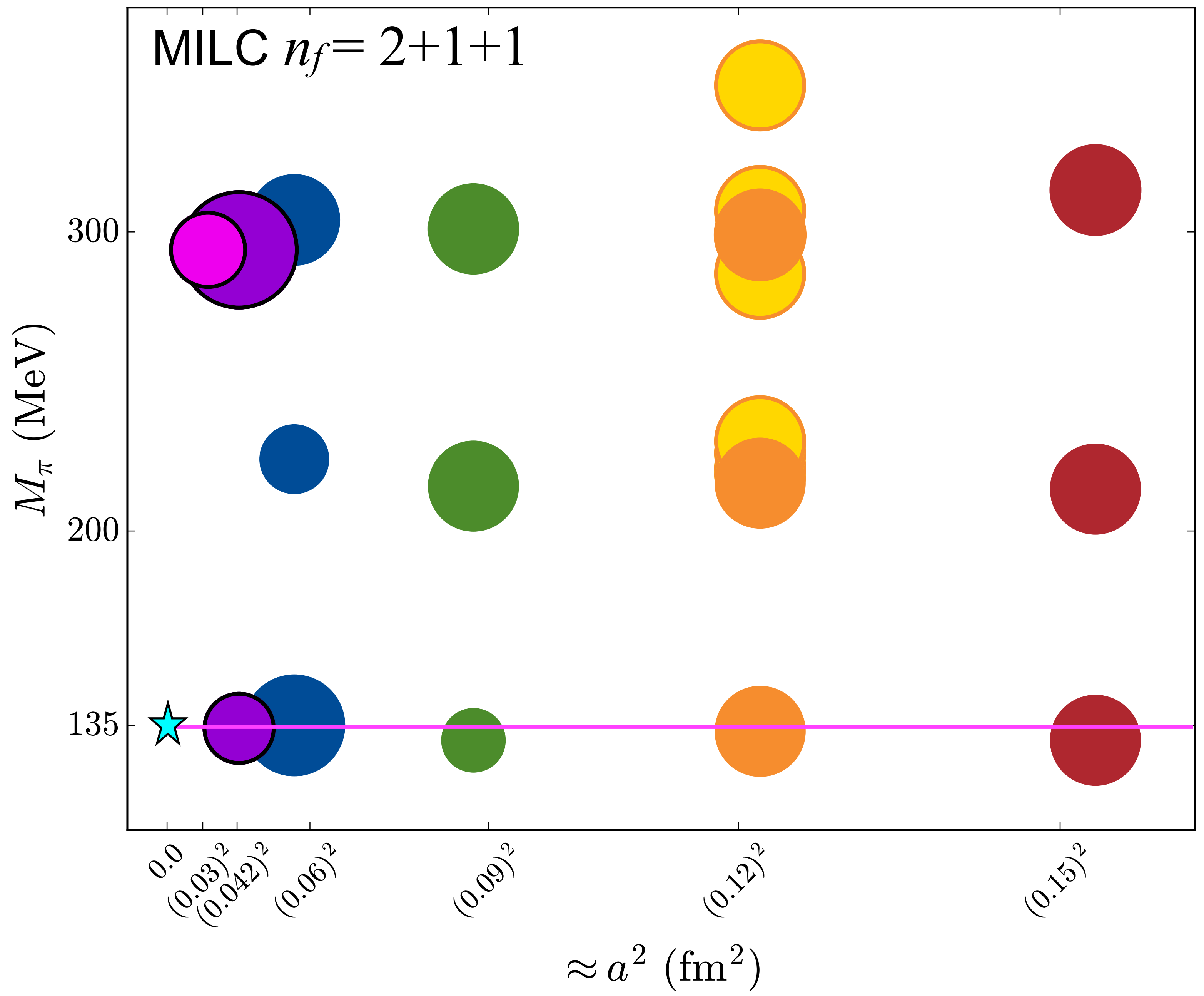
- finite a: Symanzik EFT
- light quark masses: Chiral Perturbation Theory
- heavy quarks: HQET
- finite L: finite volume EFT

In practice:
stability and control over systematic errors depends on the lattice action(s) employed, underlying simulation parameters (available computational resources), analysis choices, …

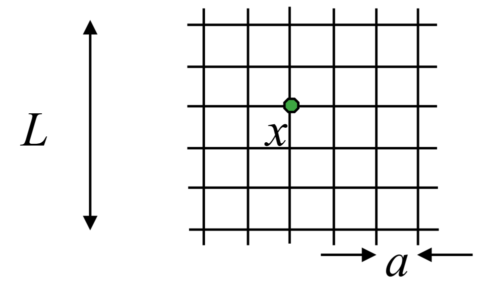


Lattice QCD Introduction: quark discretizations

Fermion doubling problem ⇔ chiral symmetry

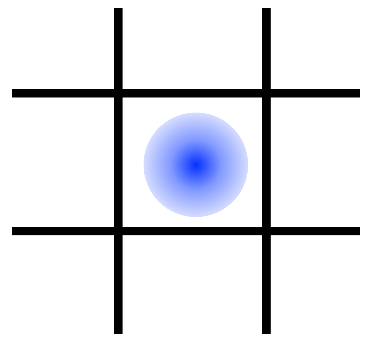
- Staggered quarks (a.k.a Kogut-Susskind)
  reduce the number of doublers (staggering) but keep some (a.k.a tastes),
  remnant chiral symmetry ensures multiplicative mass renormalization, etc…
  improved actions with discretization starting at ~ $O(\alpha_s a^2, \alpha_s^2 a^2)$ (Astad, HISQ)
  dominant discretization effects due to taste-breaking effects (can be corrected in ChPT)
  various smearing schemes to reduce taste-breaking effects
  computationally inexpensive
- (improved) Wilson quarks
  no doublers, but chiral symmetry broken explicitly
  requires improvement to remove $O(a)$ effects (NP improved, twisted mass, …)
  moderate computational cost
- Domain wall quarks (live in 5 dimensions)
  no doublers, chiral symmetry breaking exponentially suppressed
  small $O(a^2)$ discretization effects
  high computational cost
- …

- new ideas: workshop on novel fermion actions
  https://indico.mitp.uni-mainz.de/event/314/



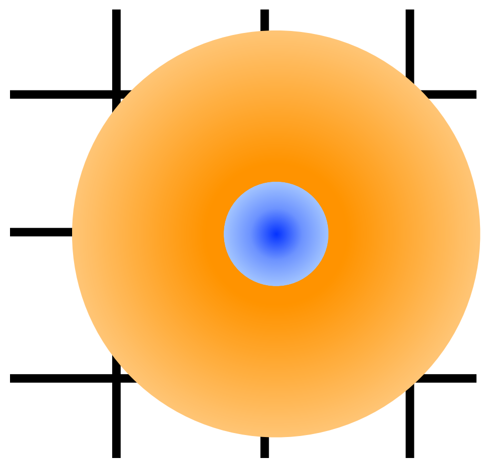
Lattice QCD Introduction: heavy quarks

b quark



$m_b \gtrsim a^{-1} \gg \Lambda$ ➠ leading discretization errors $\sim (am_b)^2$
(using same action as for light quarks)

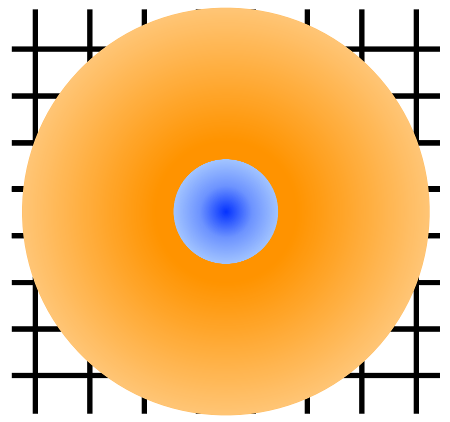
B meson



use EFT (HQET, NRQCD) ➠ Λ/m_b expansion

EFTs co-developed continuum/lattice

- lattice HQET, NRQCD: use EFT to construct lattice action
  complicated continuum limit
  nontrivial matching and renormalization ➠ (few-5)% errors
- relativistic heavy quarks: Fermilab (1996), also Tsukuba (2003), RHQ (2006)
  matching relativistic lattice action via HQET to continuum
  nontrivial matching and renormalization ➠ (1-3)% errors



$a^{-1} > m_b \gg \Lambda$ + highly improved light quark action
➠ same action for all quarks
➠ simple renormalization (Ward identities) ➠ $<$ 1% errors

The CKM Matrix

Precise Lattice QCD results **with complete systematic error budgets** now exist for all these processes ➠ precise determinations of the corresponding CKM elements

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \Lambda_b \to p\,\ell\nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D^{(*)}\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & \Lambda_b \to \Lambda_c\ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ B \to \pi\ell\ell & B \to K\ell\ell & \\ B^0 \leftrightarrow \bar{B}^0 & B_s \leftrightarrow \bar{B}_s & \end{pmatrix}$$

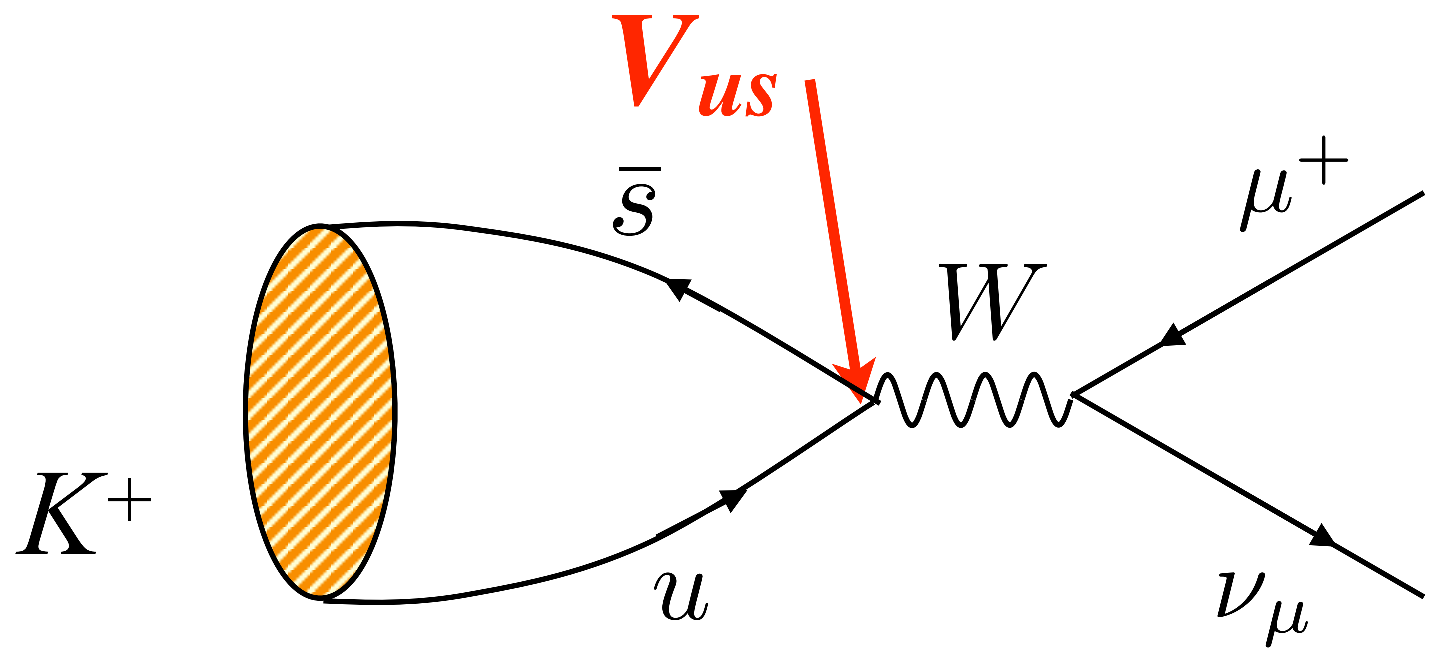
$(\bar{\rho}, \bar{\eta})$

$K^0 \leftrightarrow \bar{K}^0$

ϵ'

Leptonic decays of K, π, D, B mesons

example: $K^+ \to \mu^+ \nu_\mu$



$$\Gamma^{(0)}(K^+ \to \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} m_\ell^2 M_{K^+} \left(1 - \frac{m_\ell^2}{M_{K^+}^2}\right)^2 f_{K^+}^2 |V_{us}|^2$$

$$f_{K^+} p^\mu = \langle 0 | \bar{u} \gamma^\mu \gamma^5 s | K^+(p) \rangle$$

experimentally measured:

$$\Gamma(K^+ \to \ell^+ \nu_\ell [\gamma]) = \Gamma^{(0)}(K^+ \to \ell^+ \nu_\ell) \times (1 + \delta_{\mathrm{EM}}^\ell (K))$$

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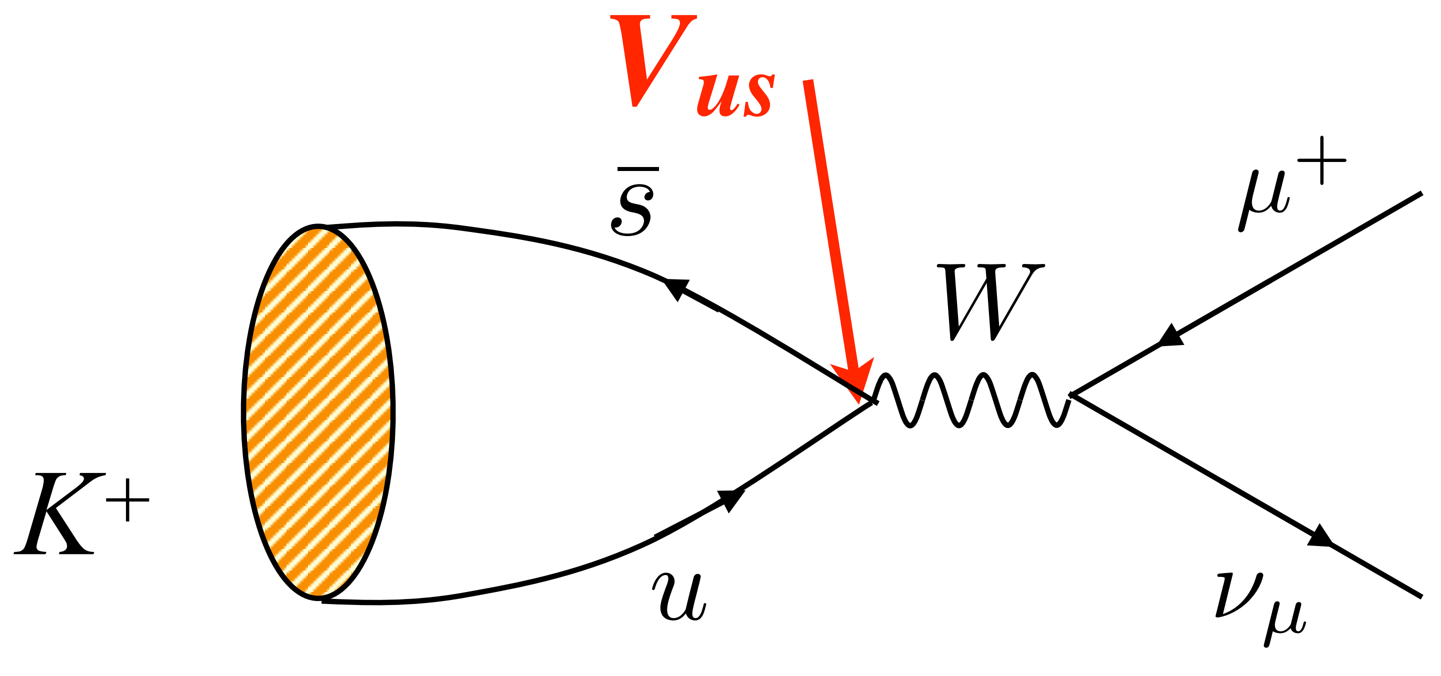


$$\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right) = (\text{known}) \times (1 + \delta^\ell_{\text{EM}}) \times |V_{us}|^2 \times f^2_{K^+}$$

experimental average (PDG 2024):

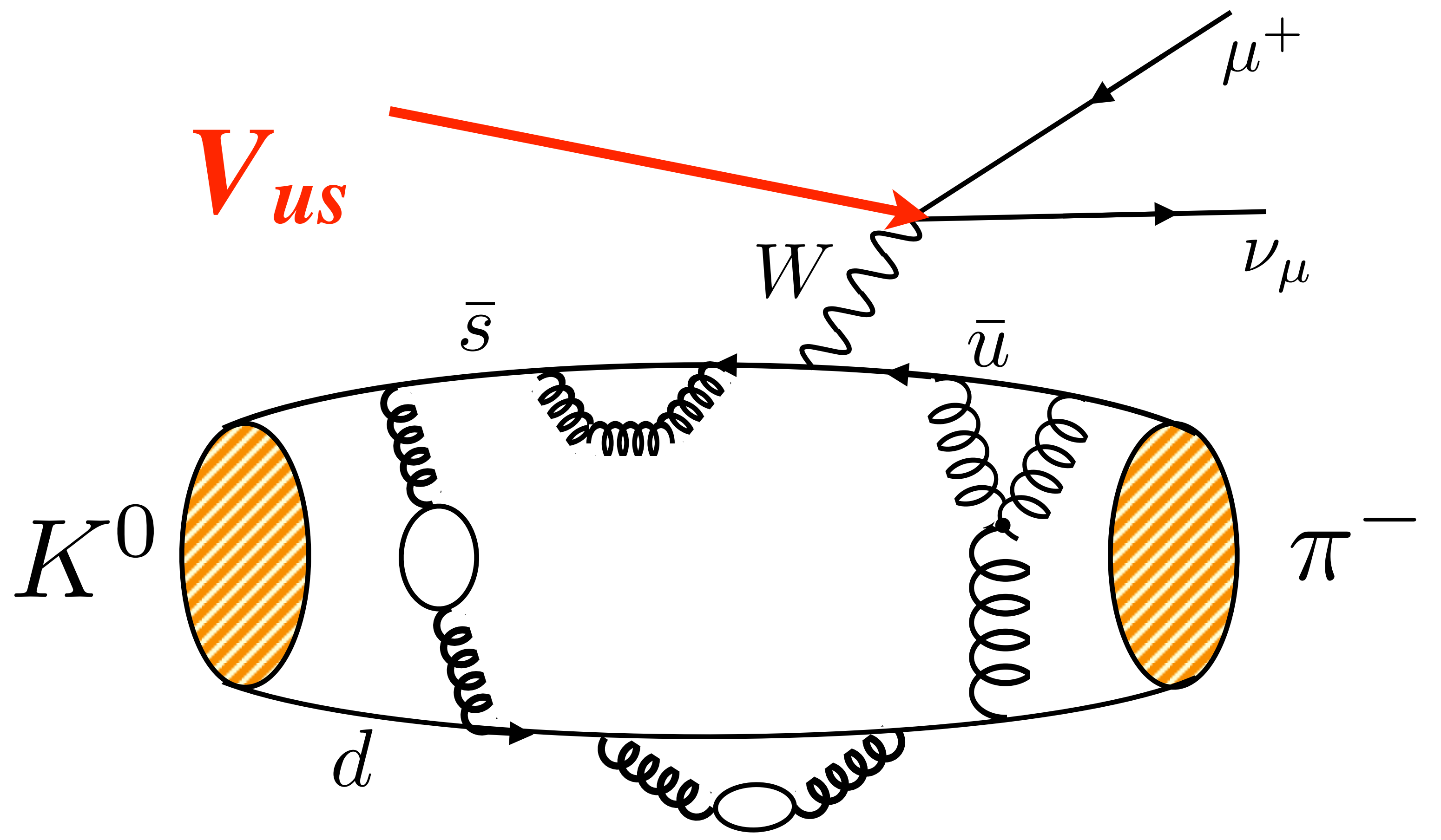
$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} = 1.3367(28)$$

$$\frac{|V_{us}| f_{K^+}}{|V_{ud}| f_{\pi^+}} = 0.27599 \pm 0.00033 \pm 0.00024$$

Needed for SM prediction of exp measured decay rate.
Can be estimated phenomenologically for K, π mesons.
[Cirigliano et al, arXiv:1107.6001, RMP 2012]

Semileptonic kaon decay

example: $K^0 \to \pi^- \ell^+ \nu_\ell$



$$\Gamma_{K\ell 3} = (\text{known}) \times \begin{pmatrix} \text{phase} \\ \text{space} \end{pmatrix} \times (1 + \delta_{\rm EM}^{K\ell} + \delta_{\rm SU(2)}^{K\pi}) \times |V_{us}|^2 \times |f_+^{K^0\pi^-}(0)|^2$$

Needed to include charged kaon decay in the experimental average.

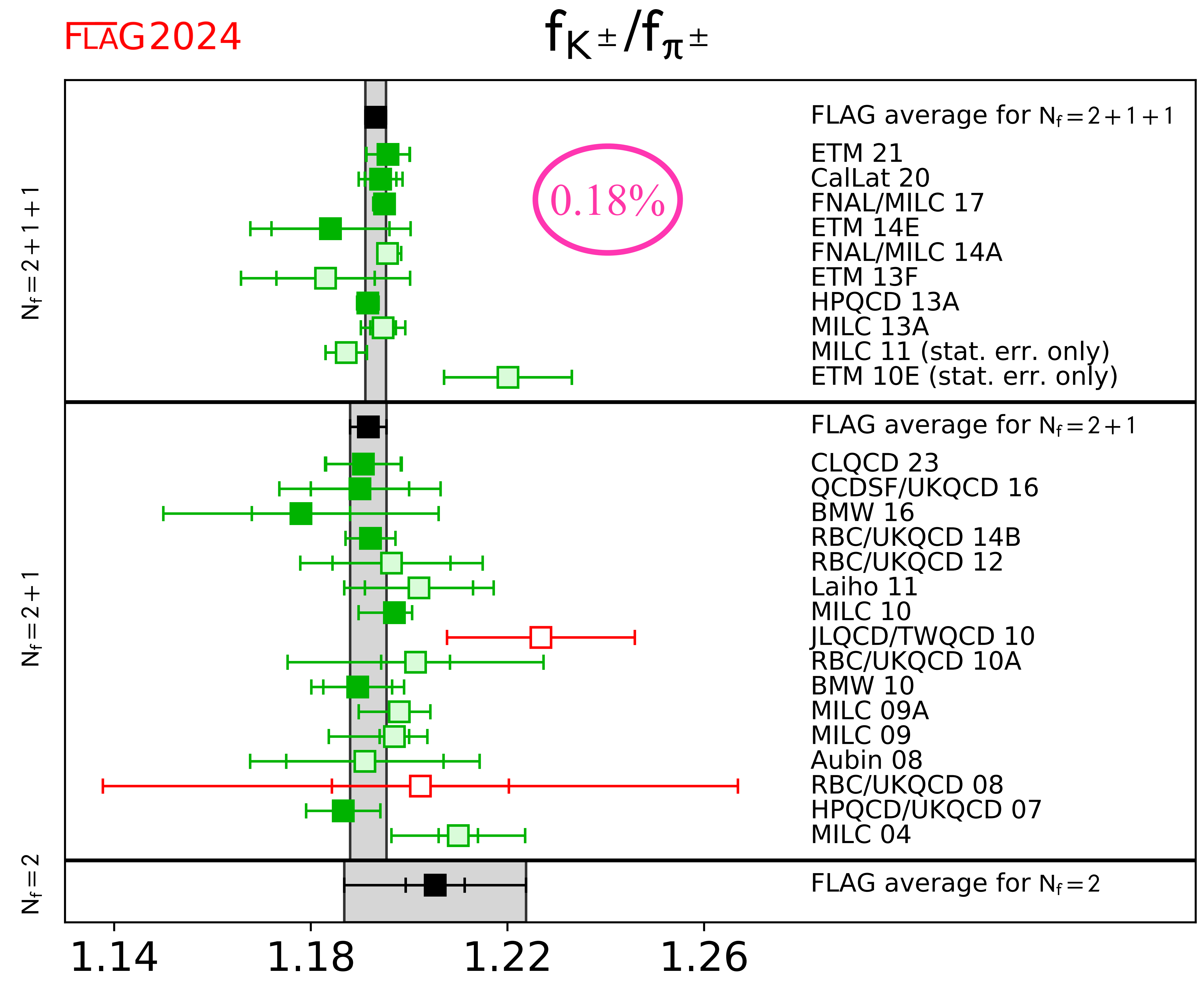
Mode dependent radiative correction

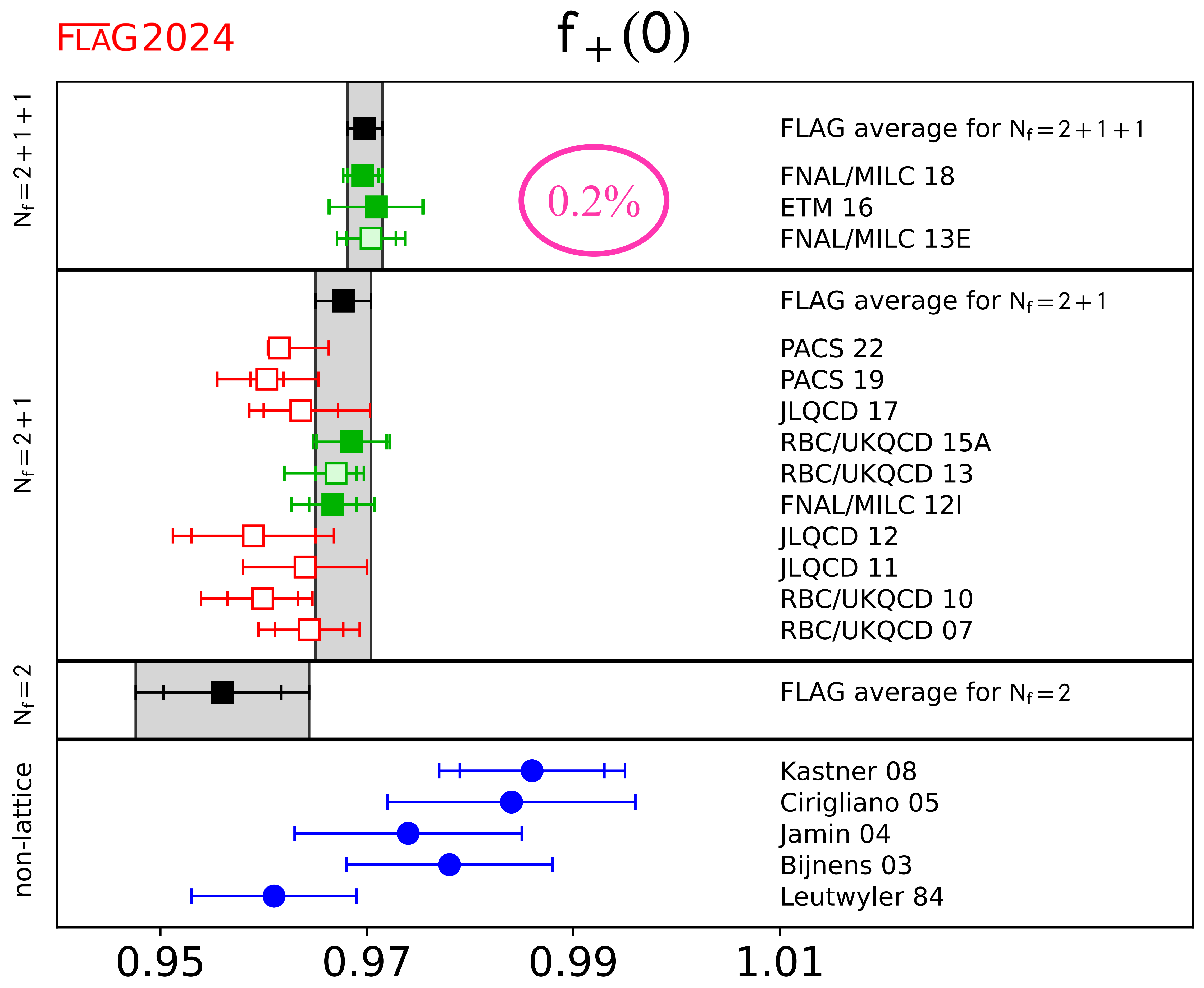
experimental average (PDG 2024)

$$f_+(0)|V_{us}| = 0.21656(35)$$

K/π decay constants and $K_{\ell 3}$ form factor results

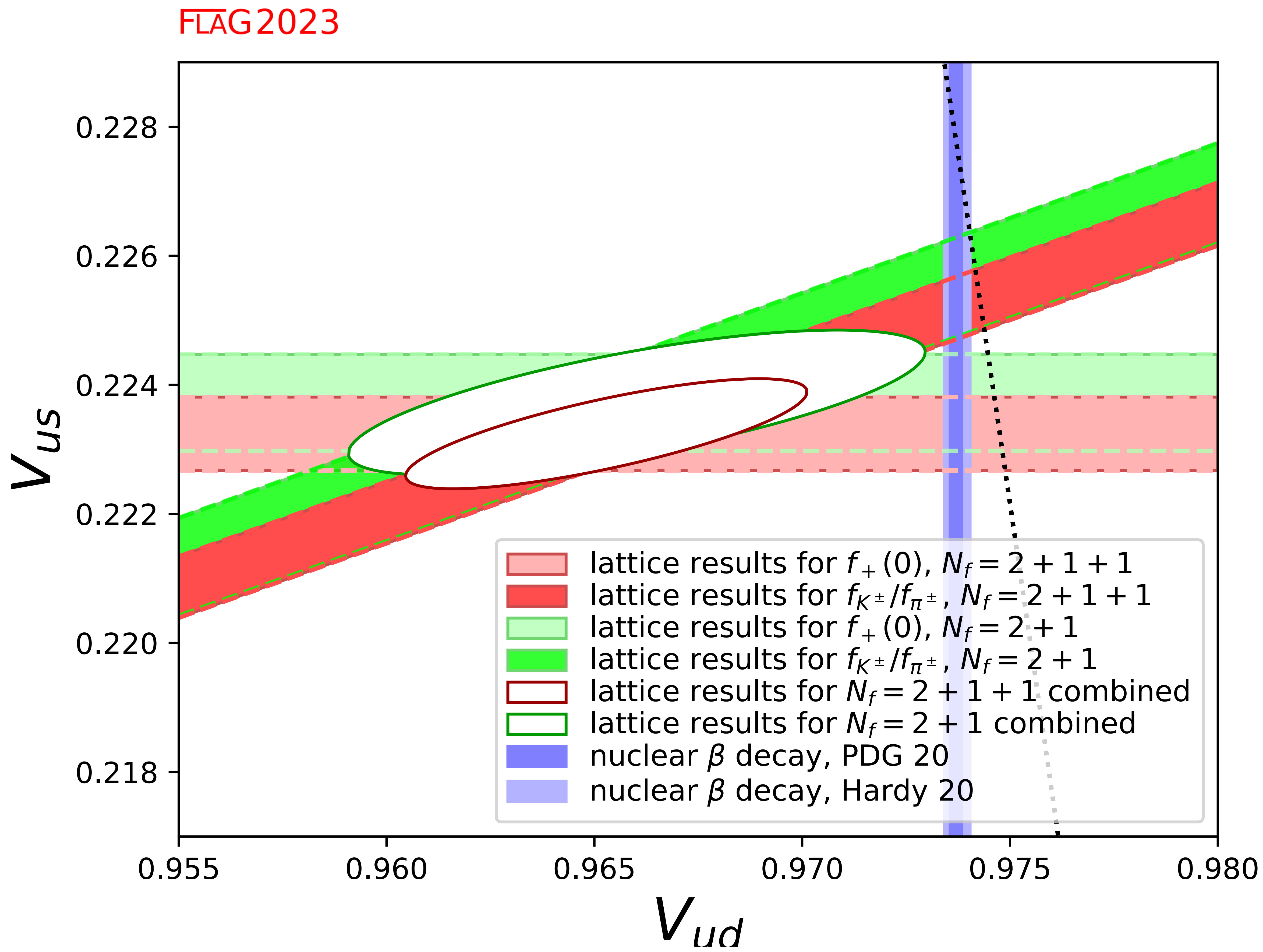
Y. Aoki et al [FLAG review, arXiv:2411.04268]





small errors due to **physical light quark masses,**
improved light-quark actions NPR or no renormalization

First row CKM unitarity



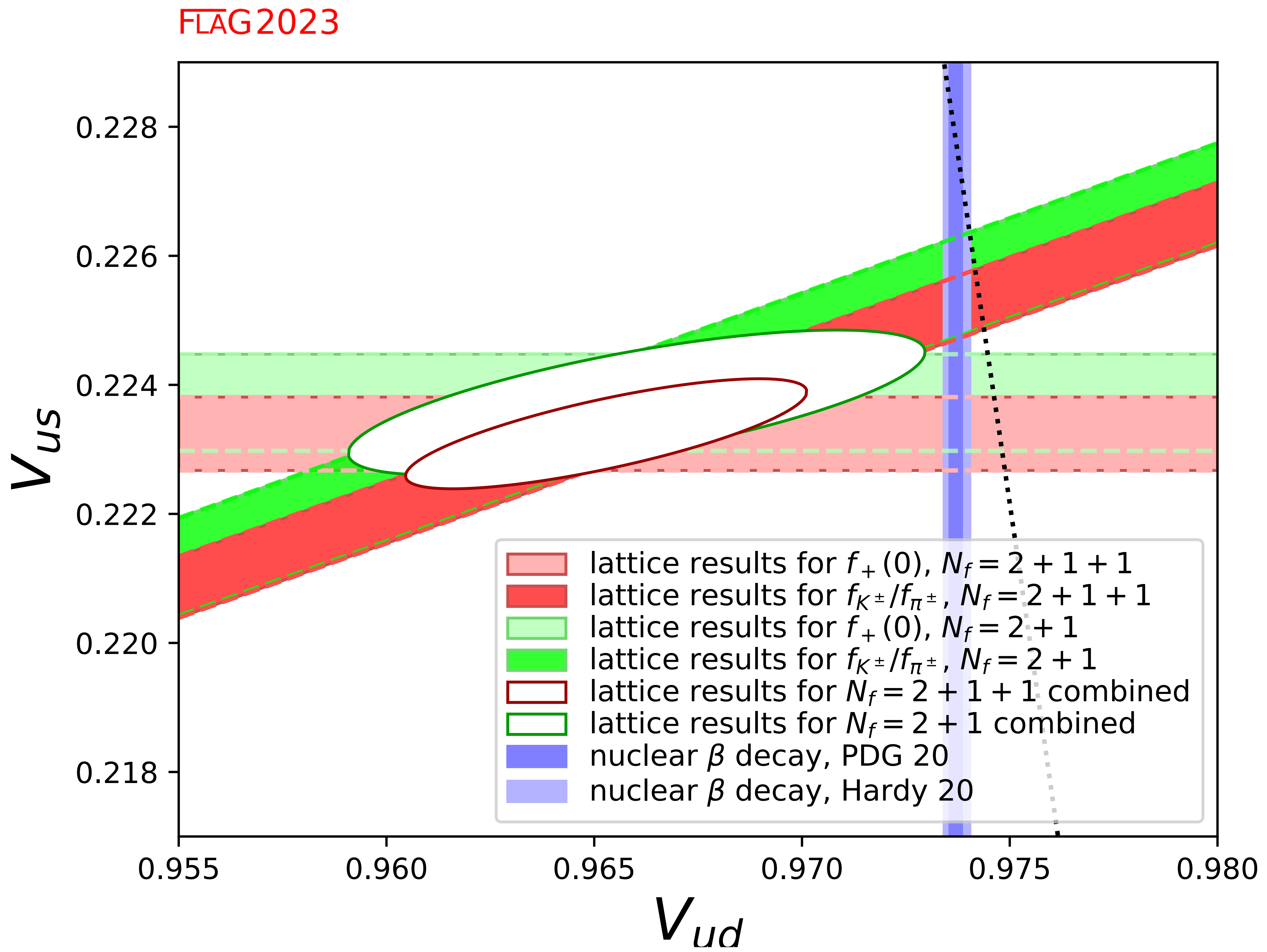
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983(6)(4)$$

$$V_{ub} \sim 0$$

$$V_{ud} = 0.97367(11)_{\rm exp}(13)_{\Delta_R^V}(27)_{\rm NS}\ (32)_{\rm total}$$

from nuclear β-decay [Cirigliano et al, arXiv:2208.11707]

First row CKM unitarity



$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983(6)(4)$$

$$V_{ub} \sim 0$$

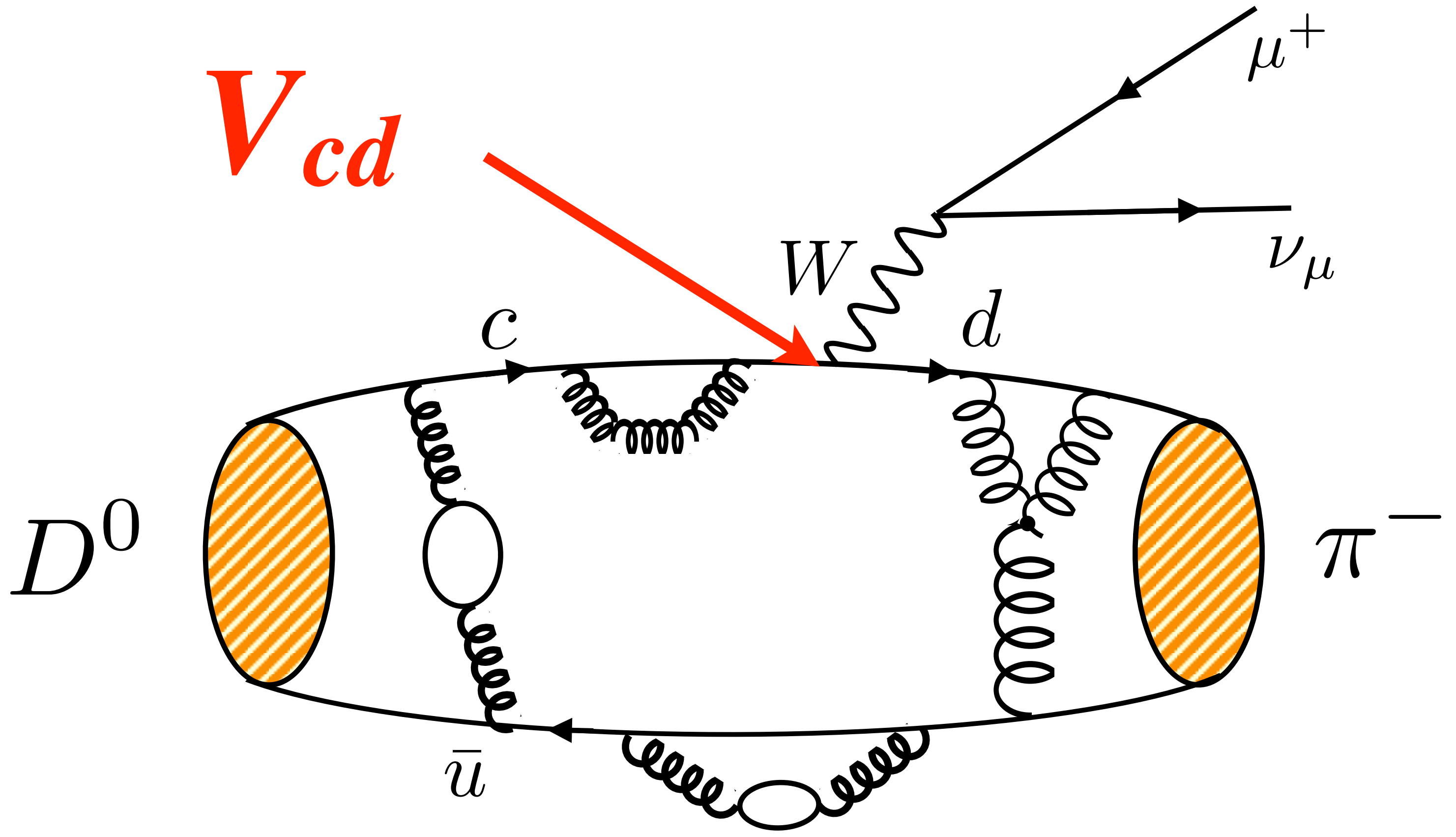
2σ tension

$$V_{ud} = 0.97367(11)_{\rm exp}(13)_{\Delta_R^V}(27)_{\rm NS}\ (32)_{\rm total}$$

from nuclear β-decay [Cirigliano et al, arXiv:2208.11707]

Semileptonic D, D_s meson decay

example: $D^0 \to \pi^- \mu^+ \nu_\mu$

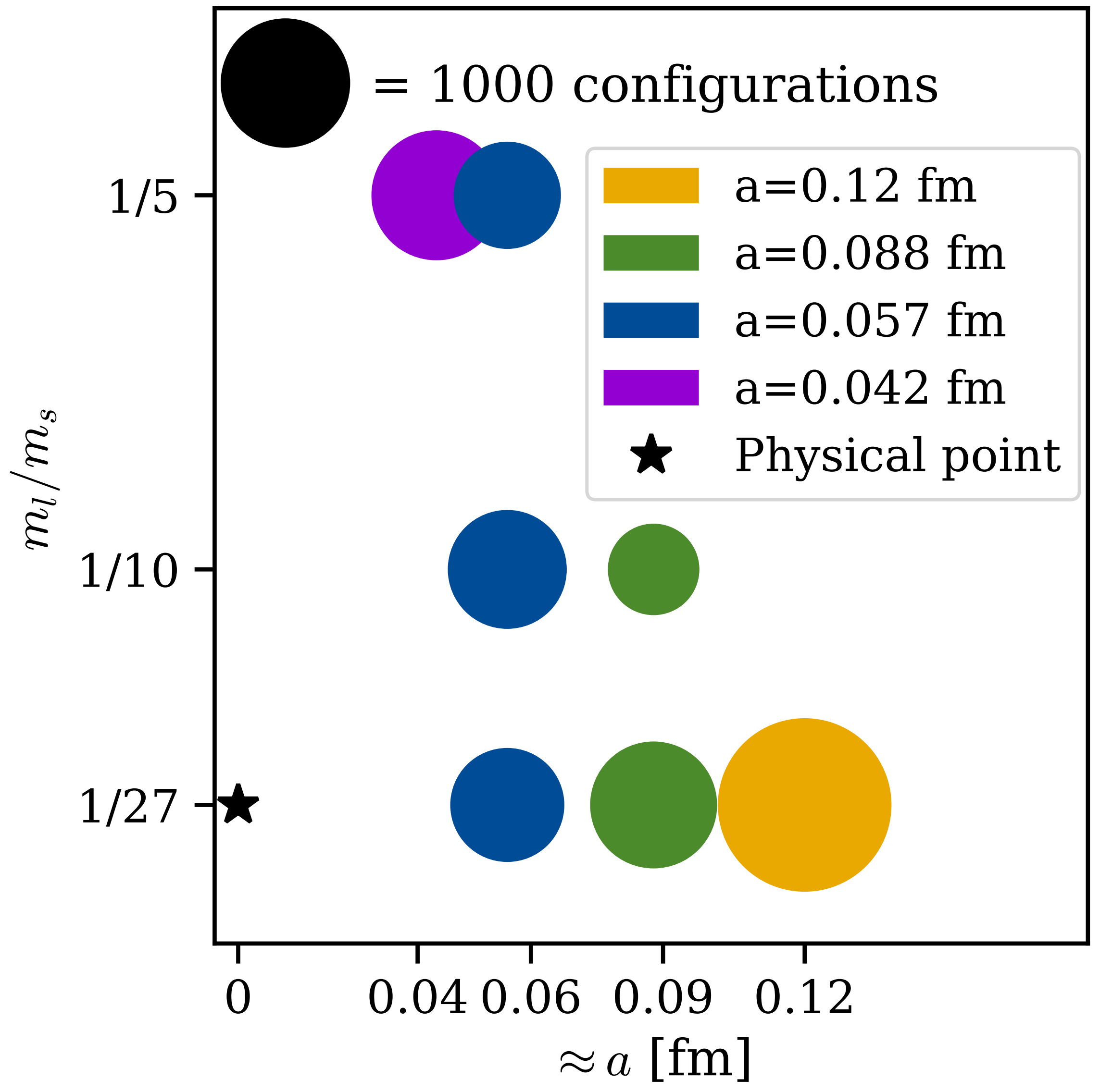


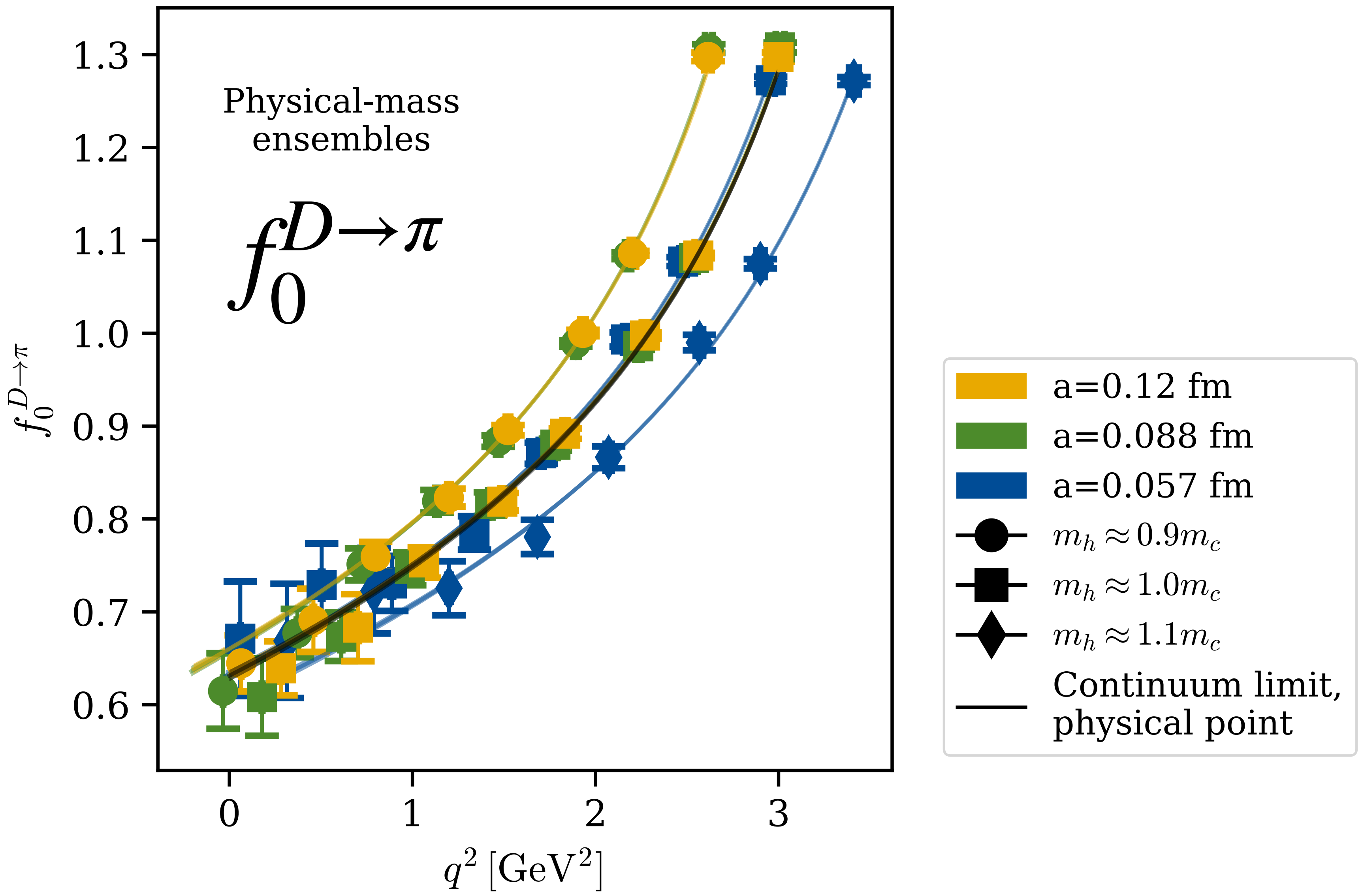
$$\frac{d\Gamma(D^0 \to \pi^- \mu^+ \nu_\mu (\gamma))}{dq^2} = (\text{known}) \times S_{\text{EW}}(1 + \delta_{\text{EM}}) \times |V_{cd}|^2 \times f_+(q^2)^2$$

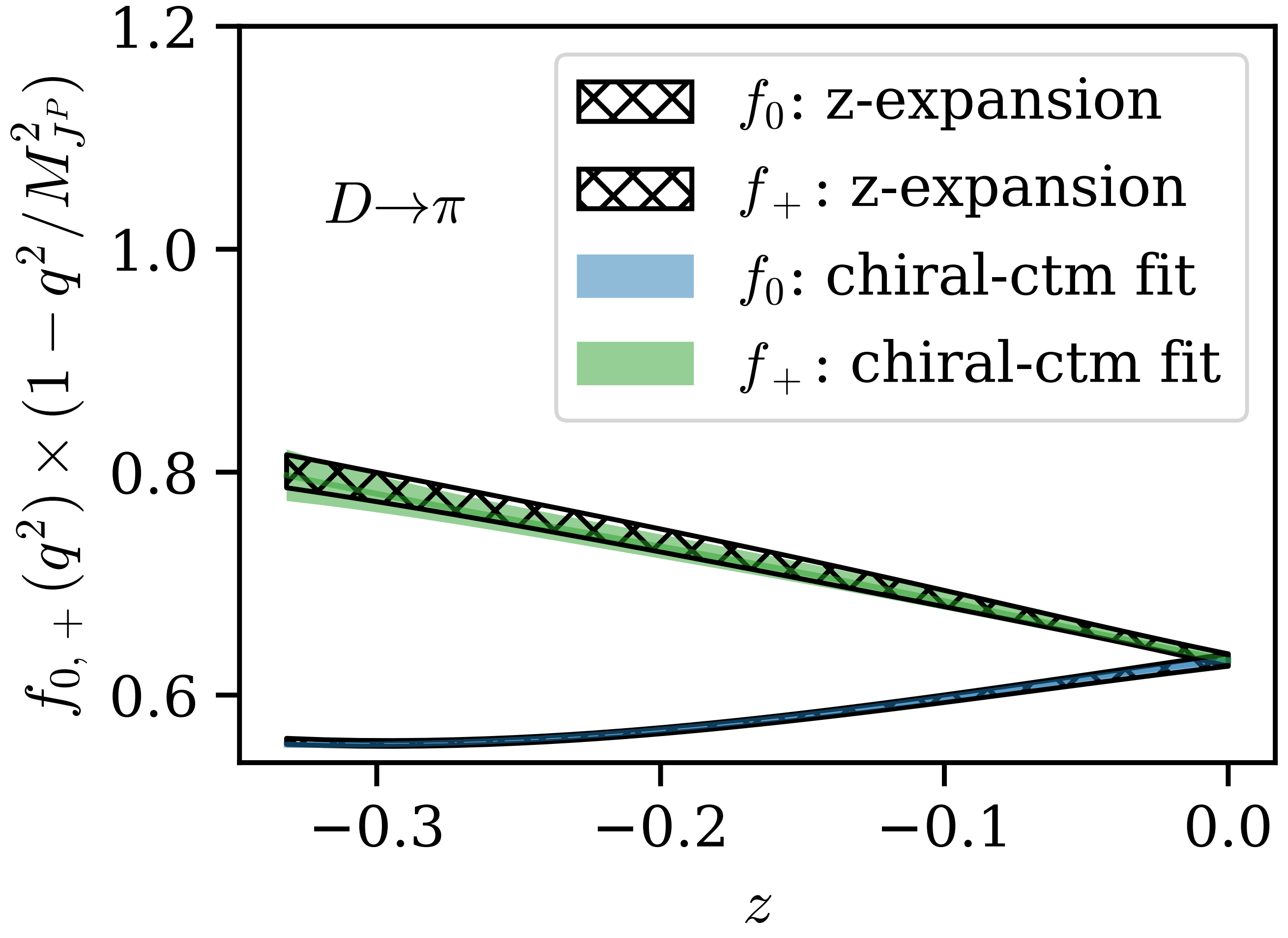
- calculate the form factors over entire q^2 range + model-independent parametrization of shape (z-expansion).
- account for EW+EM corrections in experimental rate
  - EW: [Sirlin, Nuc. Phys. 1982] ~ 1.8%
  - EM: Structure dependent: has not been calculated!
    - ➠ use guidance from $K_{\ell 3}$ ~ 1% take correction as uncertainty, inflated by $\times 2$
  - Long distance: [Kinoshita, PRL 1959] ~ 2.4% ➠ removed with PHOTOS

Semileptonic D meson decay form factors

FNAL/MILC [arXiv:2212.12648]





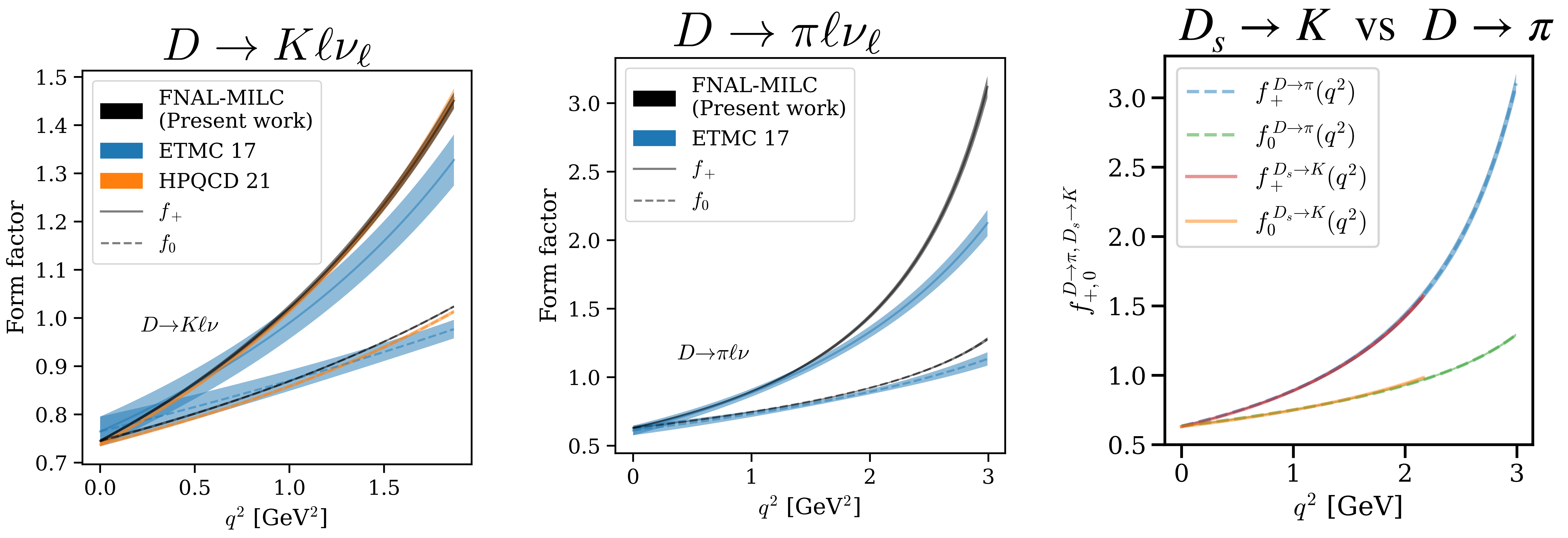


- HISQ action for all quarks.
- First step in program: D form factors on a subset of available MILC $n_f = 2+1+1$ ensemble set.
- Combined chiral-continuum interpolation & extrapolation.
- z-expansion using BCL parameterization to parameterize form factors (interpolation only).
- Blind analysis until systematic error budget was finalized.
- FNAL/MILC in progress: analysis of semileptonic $B_{(s)}$ form factors [A. Chauhan]

Semileptonic D meson decay form factors

ETM [arXiv:1706.03017, PRD 2017; arXiv:1706.03657, EPJC 2017]
HPQCD [arXiv:2104.09883,2207.12468]

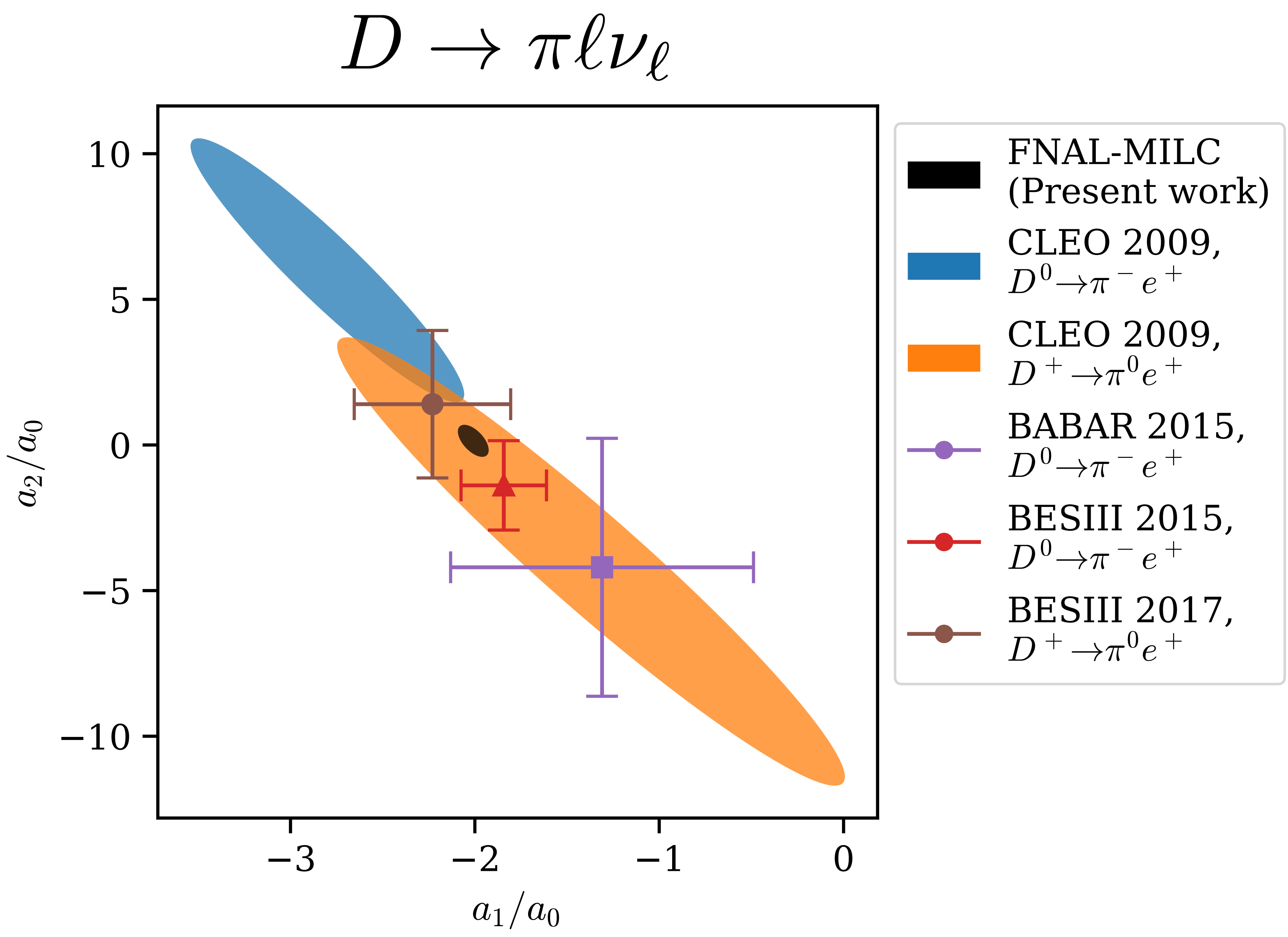
FNAL/MILC [arXiv:2212.12648]

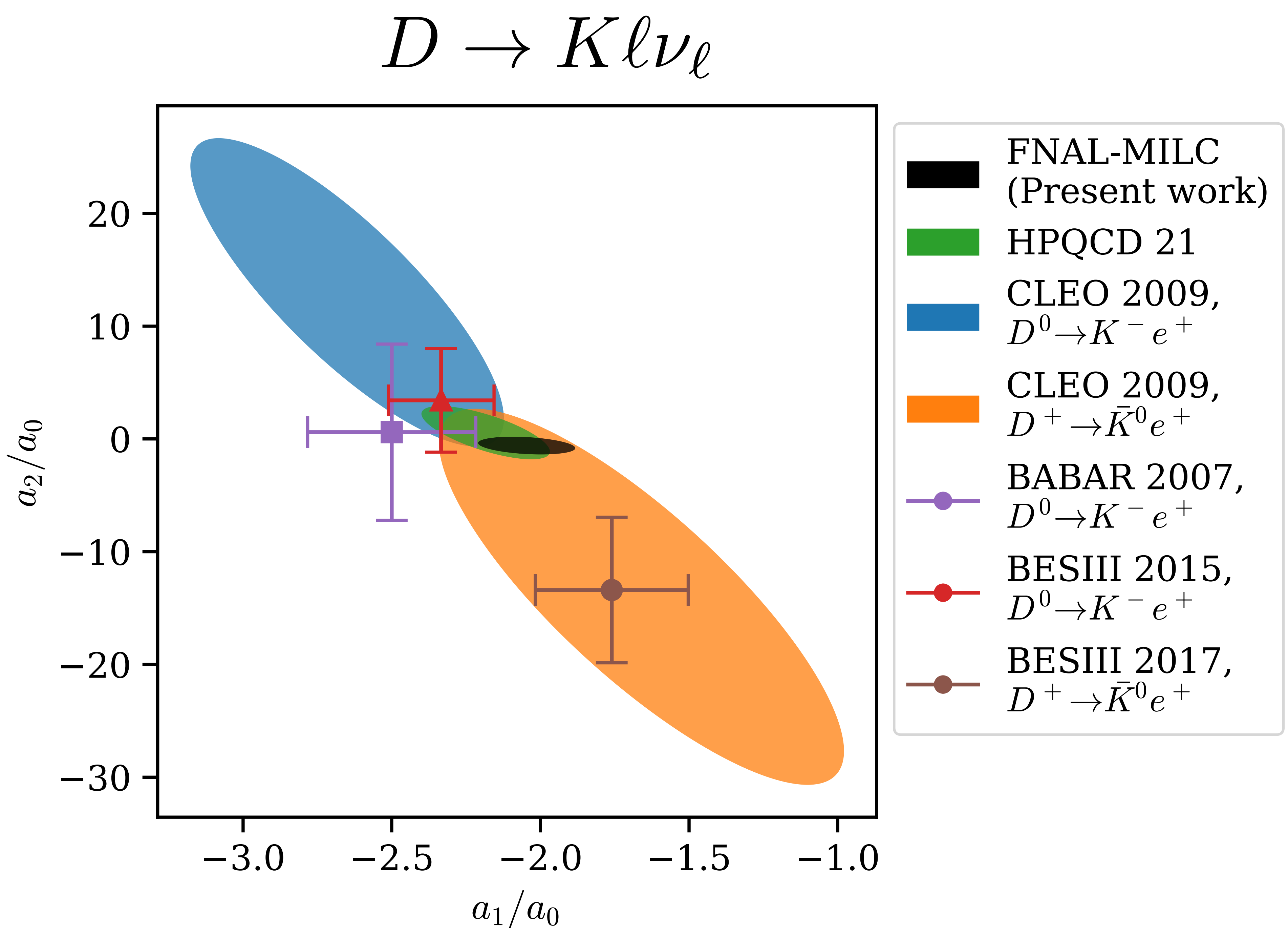


☆ Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$.

shape comparison

FNAL/MILC [arXiv:2212.12648]



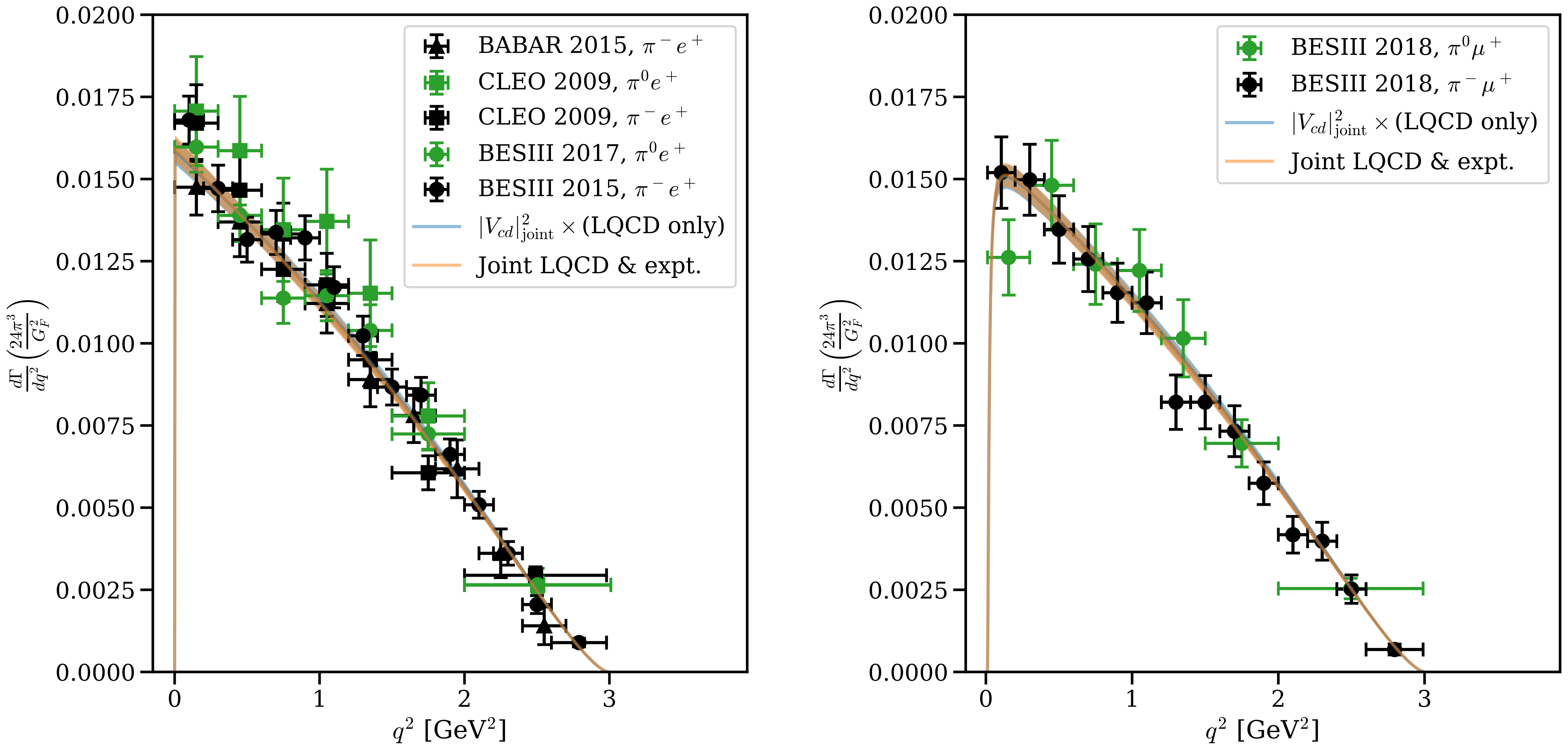


- Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$.

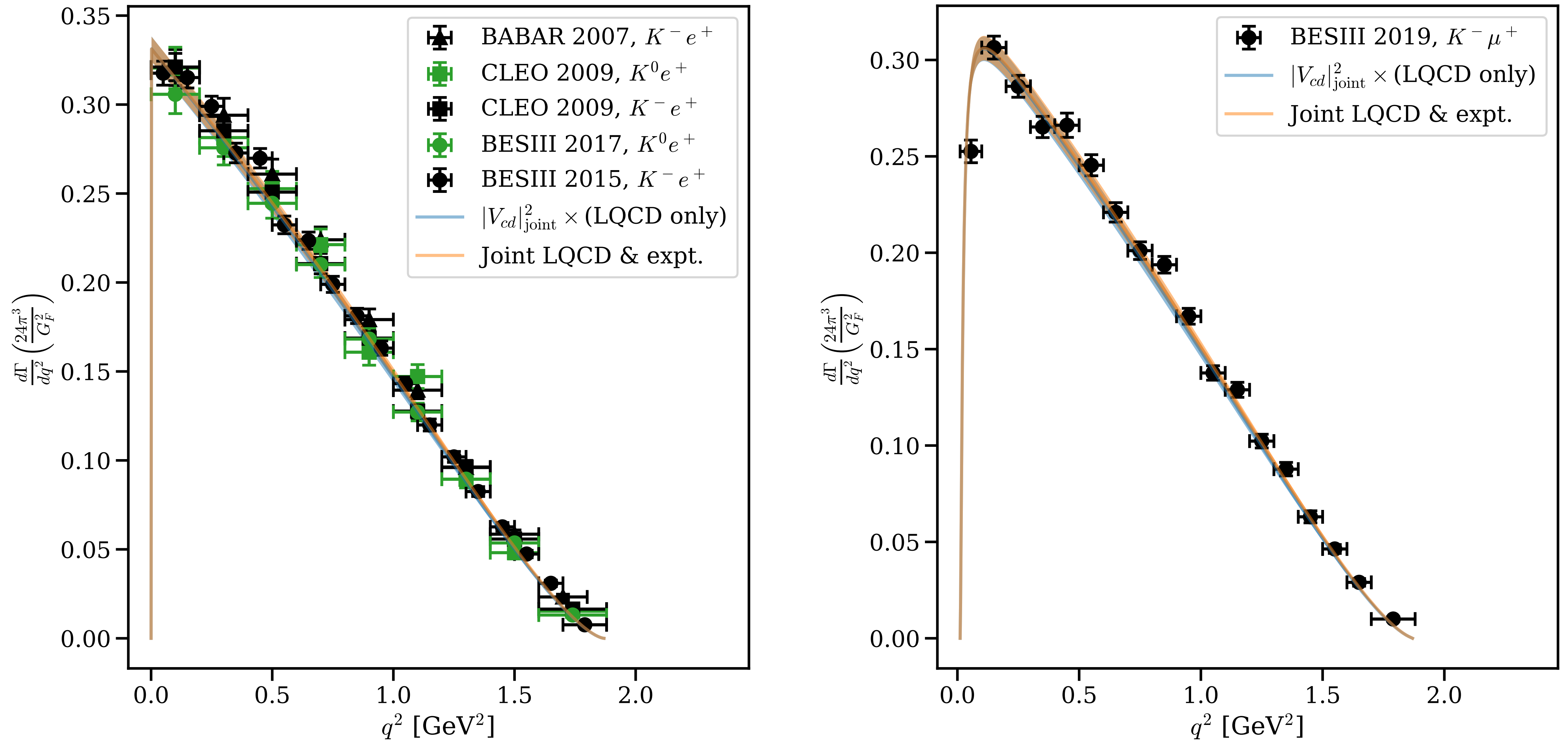
joint fits to exp. $d\Gamma/dq^2$ + LQCD form factors

FNAL/MILC [arXiv:2212.12648]

$D \to \pi \ell \nu_\ell$



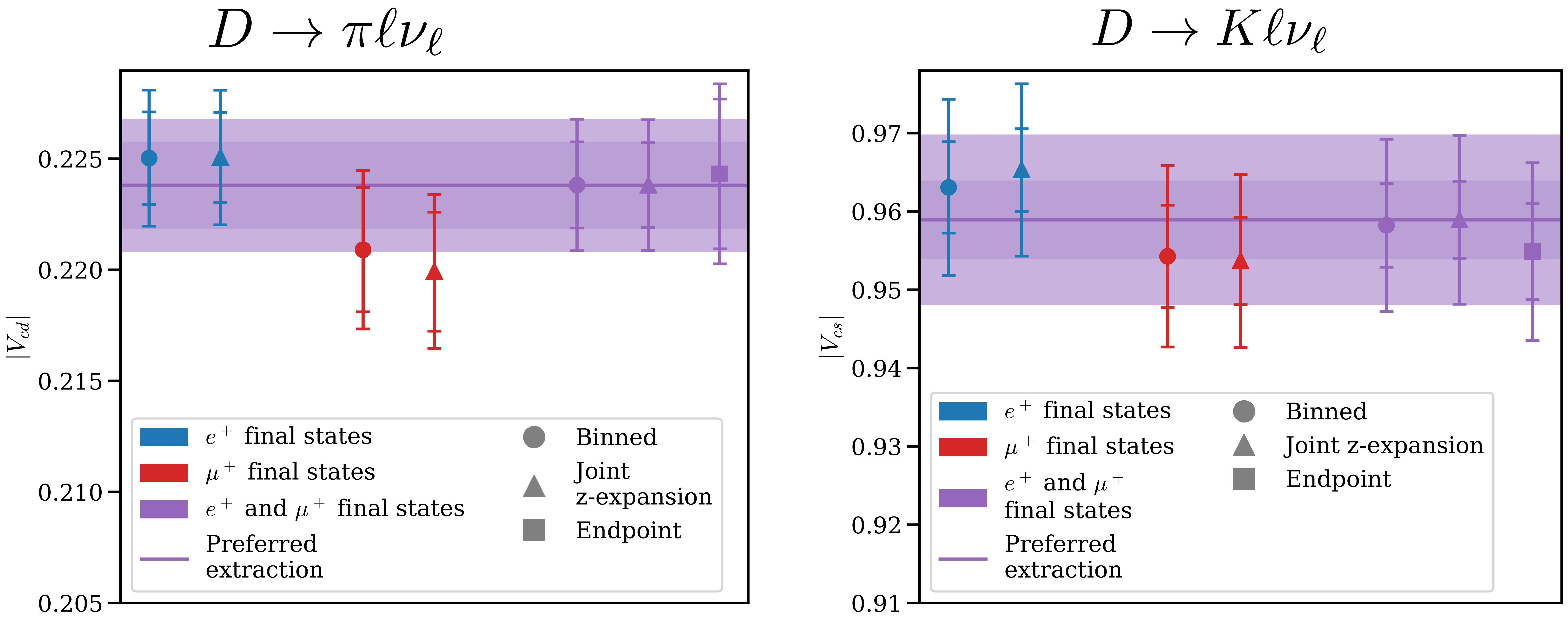
$D \to K \ell \nu_\ell$



- Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$.
- can also extract CKM elements from exp. average of $|V_{cq}| f_+(0)$
- similar analysis with Λ_c decay form factors [Meinel, arXiv:1611.09696, 2017 PRL].
- also: D-meson tensor form factors [ETM, arXiv:1803.04807, 2018 PRD]

$|V_{cd}|$ and $|V_{cs}|$ determinations

FNAL/MILC [arXiv:2212.12648]



- Compare shape of LQCD form factor with experiment and fit LQCD form factors + experimental diff. rates to determine $|V_{cd}|$ or $|V_{cs}|$ or perform binned analysis.
- can also extract CKM elements from exp. average of $|V_{cq}|f_+(0)$
- similar analysis with Λ_c decay form factors [Meinel, arXiv:1611.09696, 2017 PRL].
- also: D-meson tensor form factors [ETM, arXiv:1803.04807, 2018 PRD]

$|V_{cd}|$ and $|V_{cs}|$ determinations

For illustration: experimental averages [HFLAV 2019, arXiv:1909.12524, EPJC2021]:

$$[S_{\rm EW}(1+\delta_{\rm EM})]^{1/2}|V_{cs}|f_+^{DK}(0) = 0.7180\,(33)_{\rm exp} \quad [S_{\rm EW}(1+\delta_{\rm EM})]^{1/2}|V_{cd}|f_+^{D\pi}(0) = 0.1426\,(18)_{\rm exp}$$

From joint exp + LQCD fits:

HPQCD [arXiv:2104.09883]

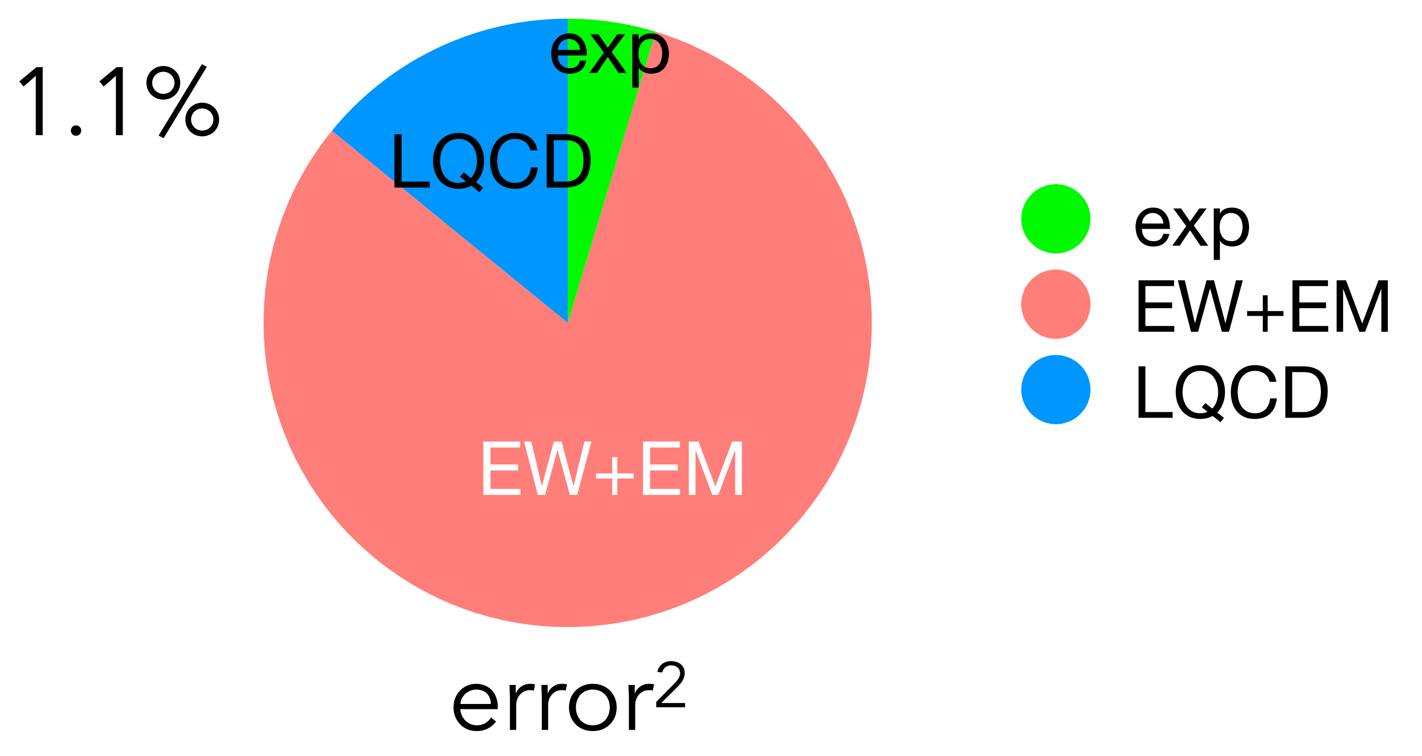
$$|V_{cs}| = 0.9663\,(39)_{\rm exp}(53)_{\rm LQCD}(19)_{\rm EW}(40)_{\rm EM}$$

ETM [arXiv:1706.03657, EPJC 2017] $|V_{cd}| = 0.2341(74)_{\rm exp+LQCD}$

FNAL/MILC [arXiv:2212.12648]

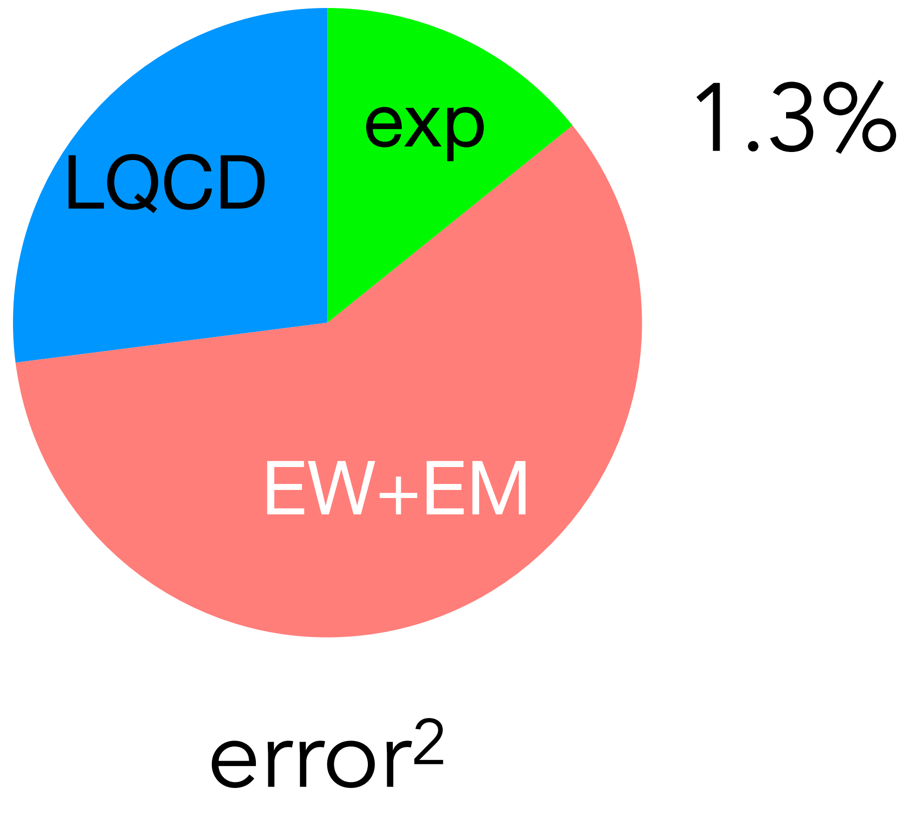
$$|V_{cs}| = 0.9589(23)_{\rm exp}(40)_{\rm LQCD}(15)_{\rm EW}(05)_{\rm SIB}[95]_{\rm QED}$$

$$|V_{cd}| = 0.2238(11)_{\rm exp}(15)_{\rm LQCD}(04)_{\rm EW}(02)_{\rm SIB}[22]_{\rm QED}$$



2nd row CKM unitarity test:

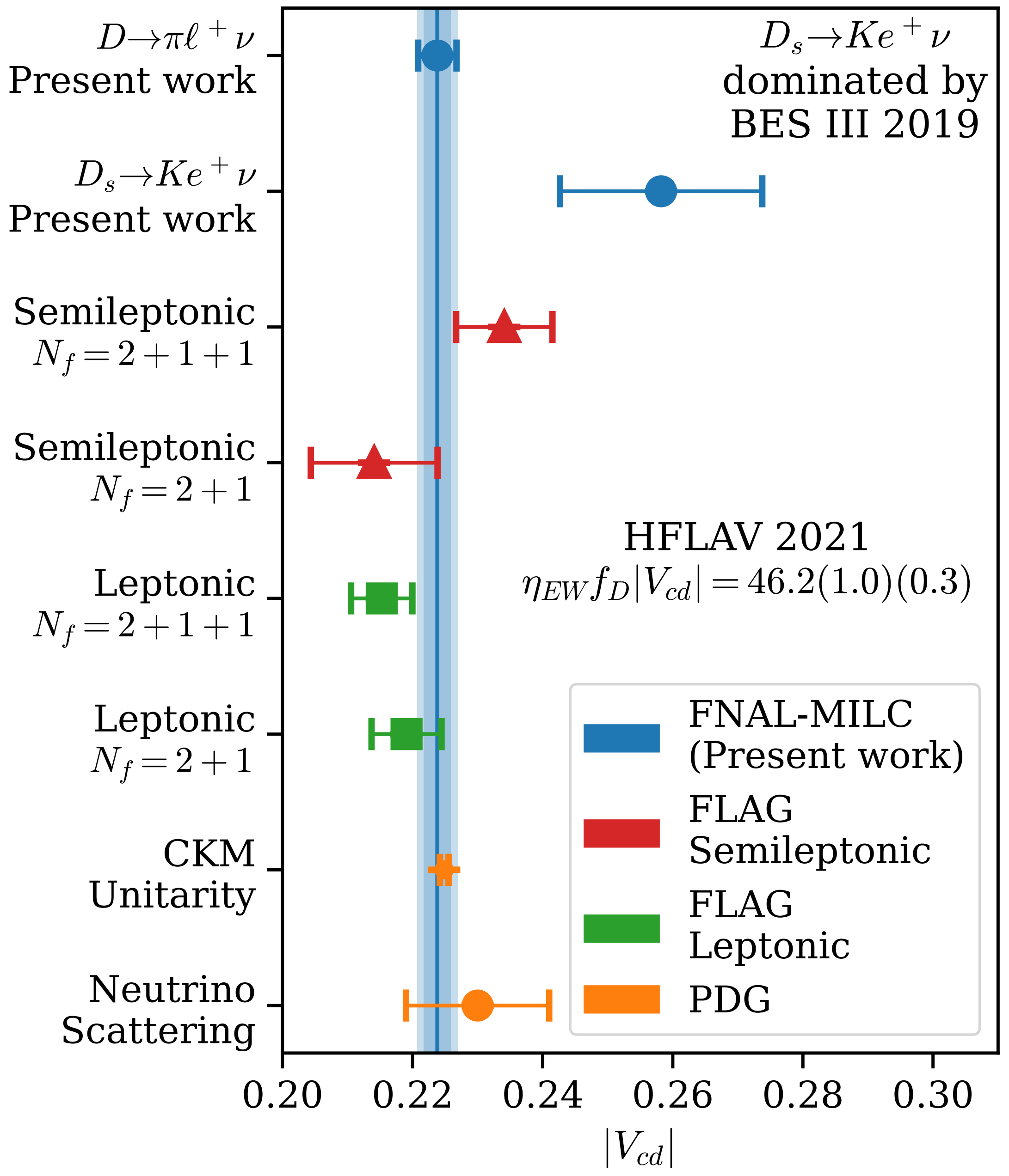
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.029(22)$$



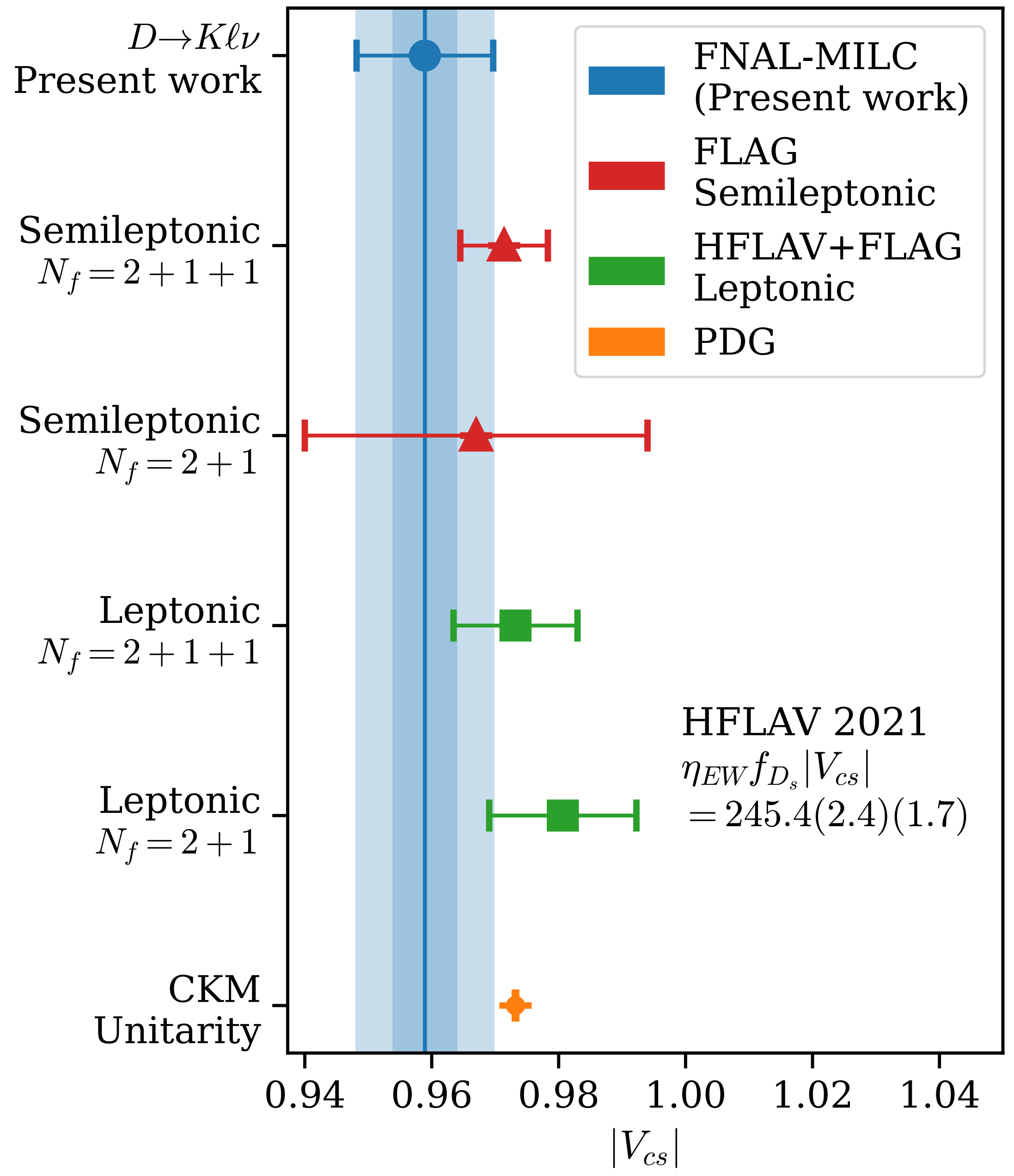
Comparisons

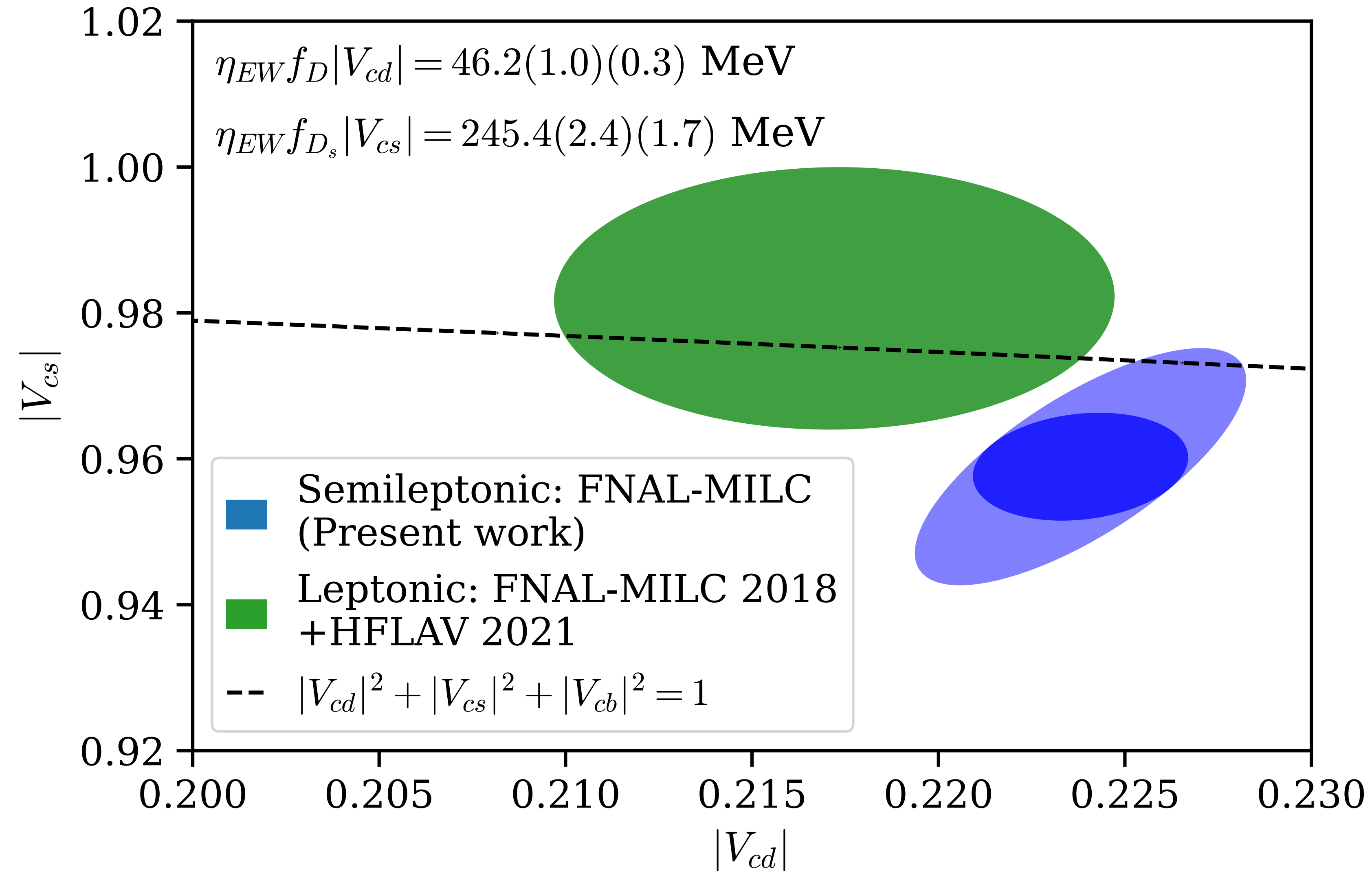
FNAL/MILC [arXiv:2212.12648]

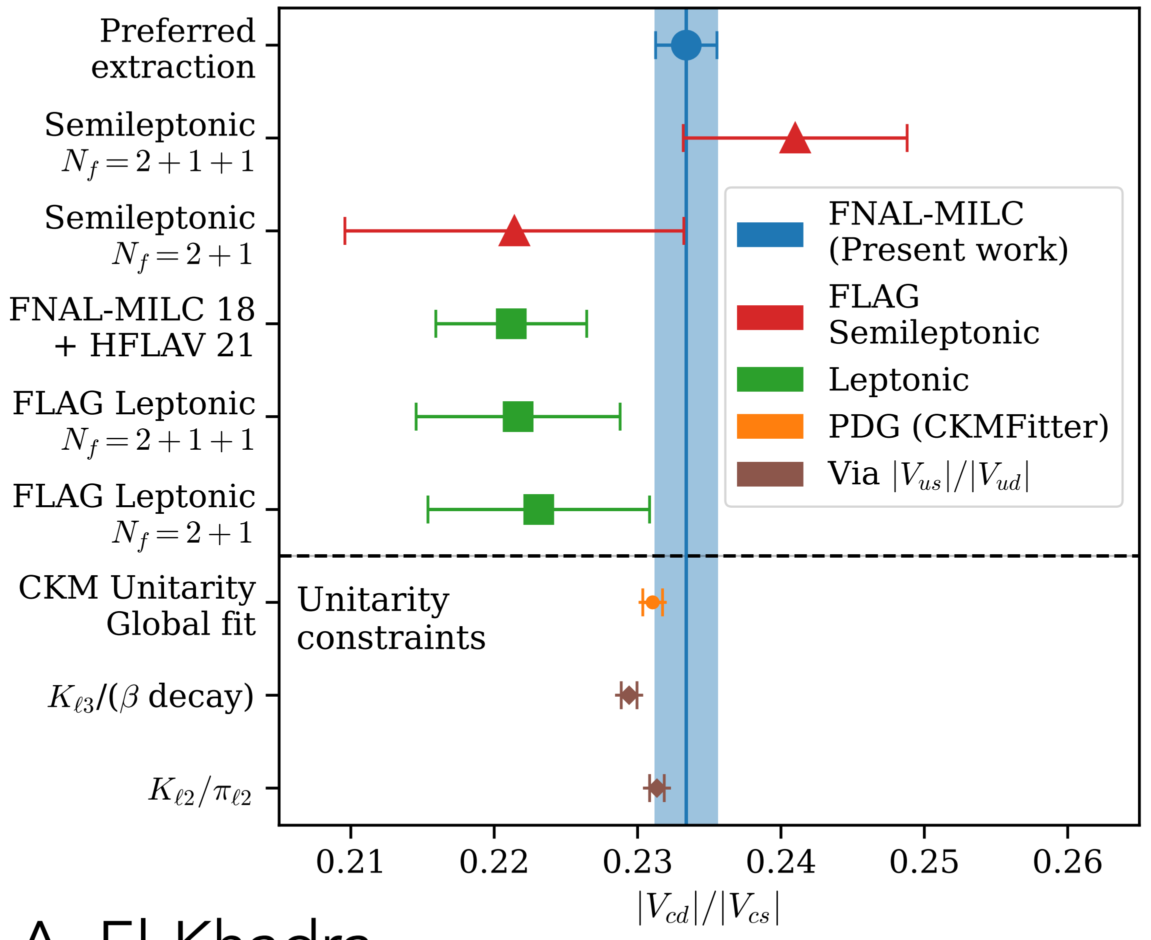
$|V_{cd}|$



$|V_{cs}|$

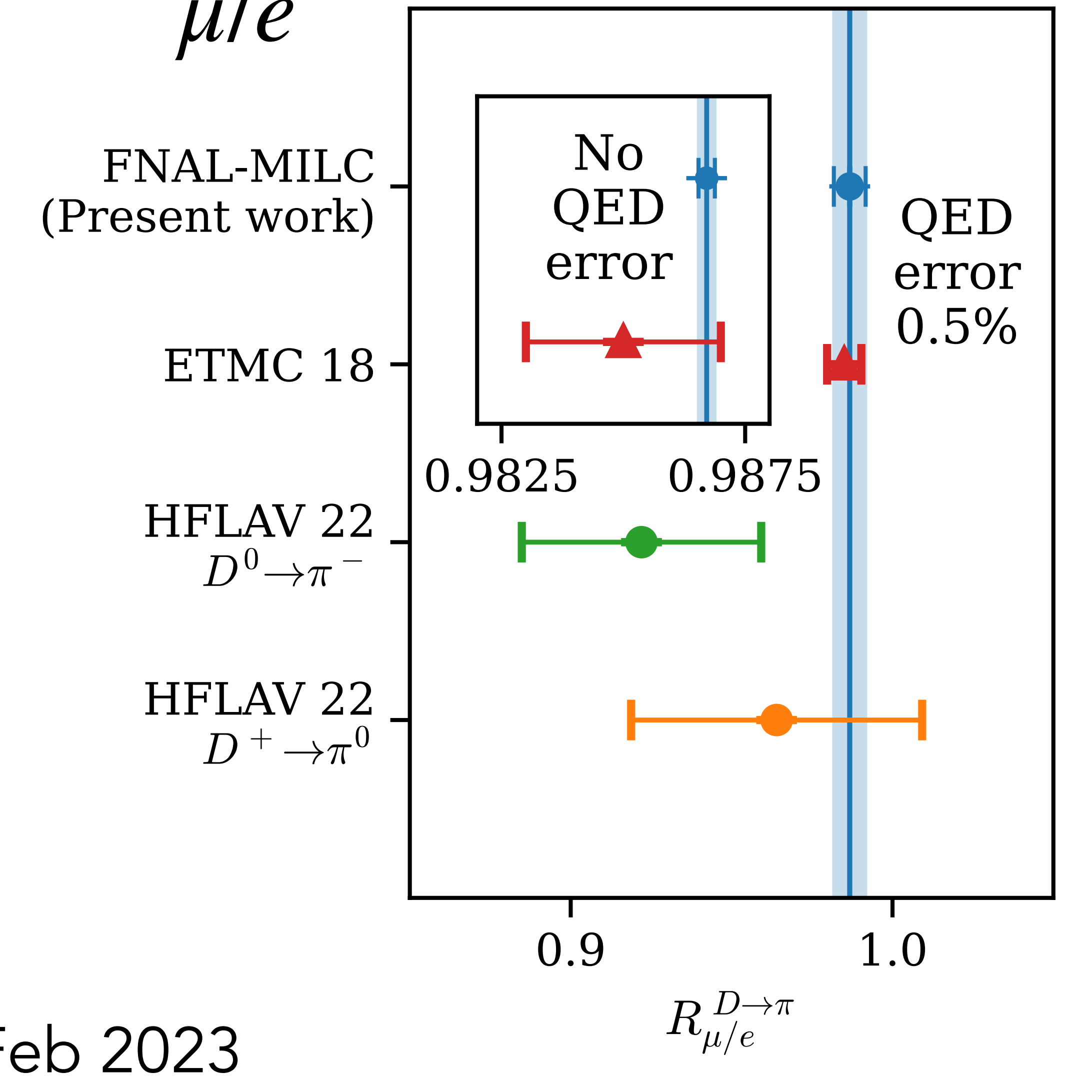




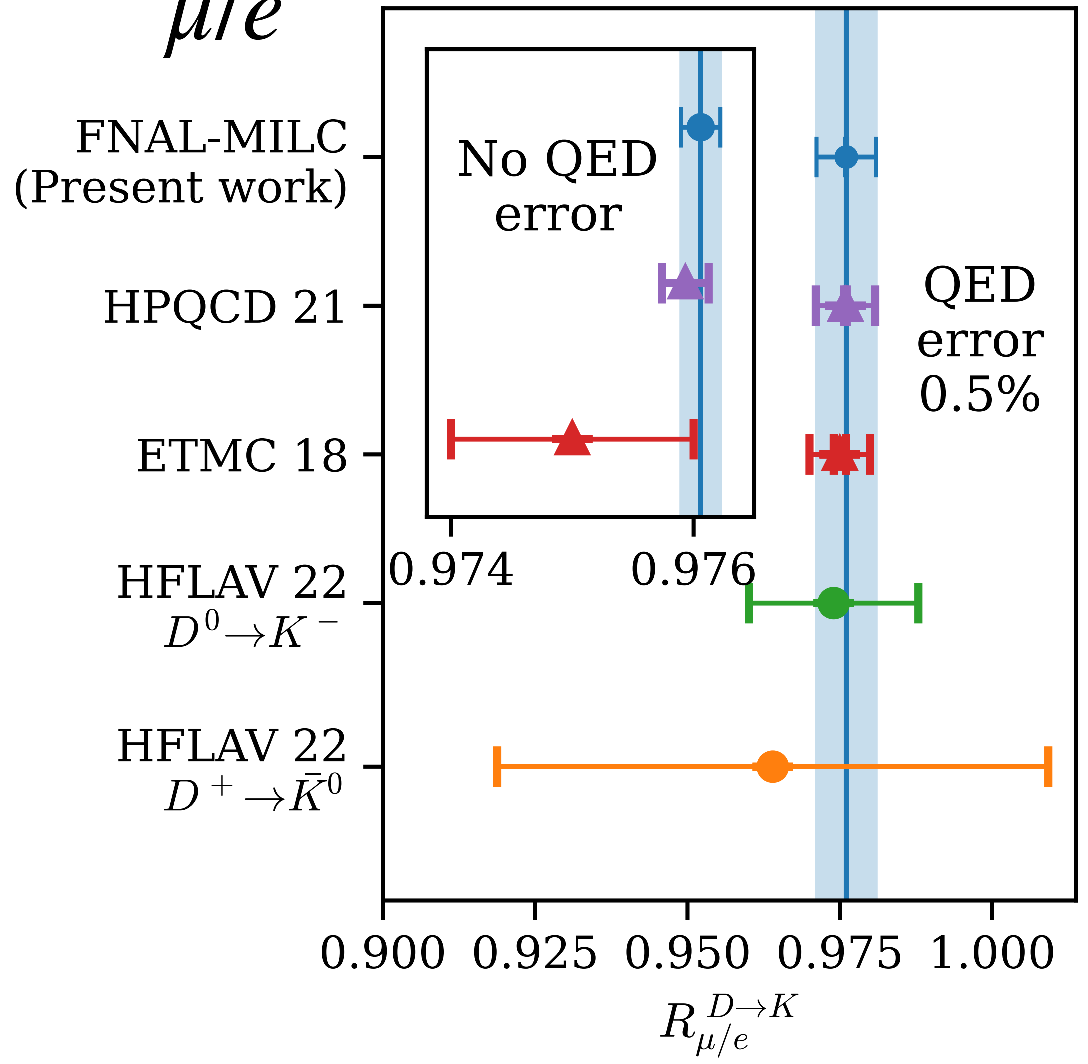


$|V_{cd}/V_{cs}|$

$R_{\mu/e}^{D\to\pi}$



$R_{\mu/e}^{D\to K}$

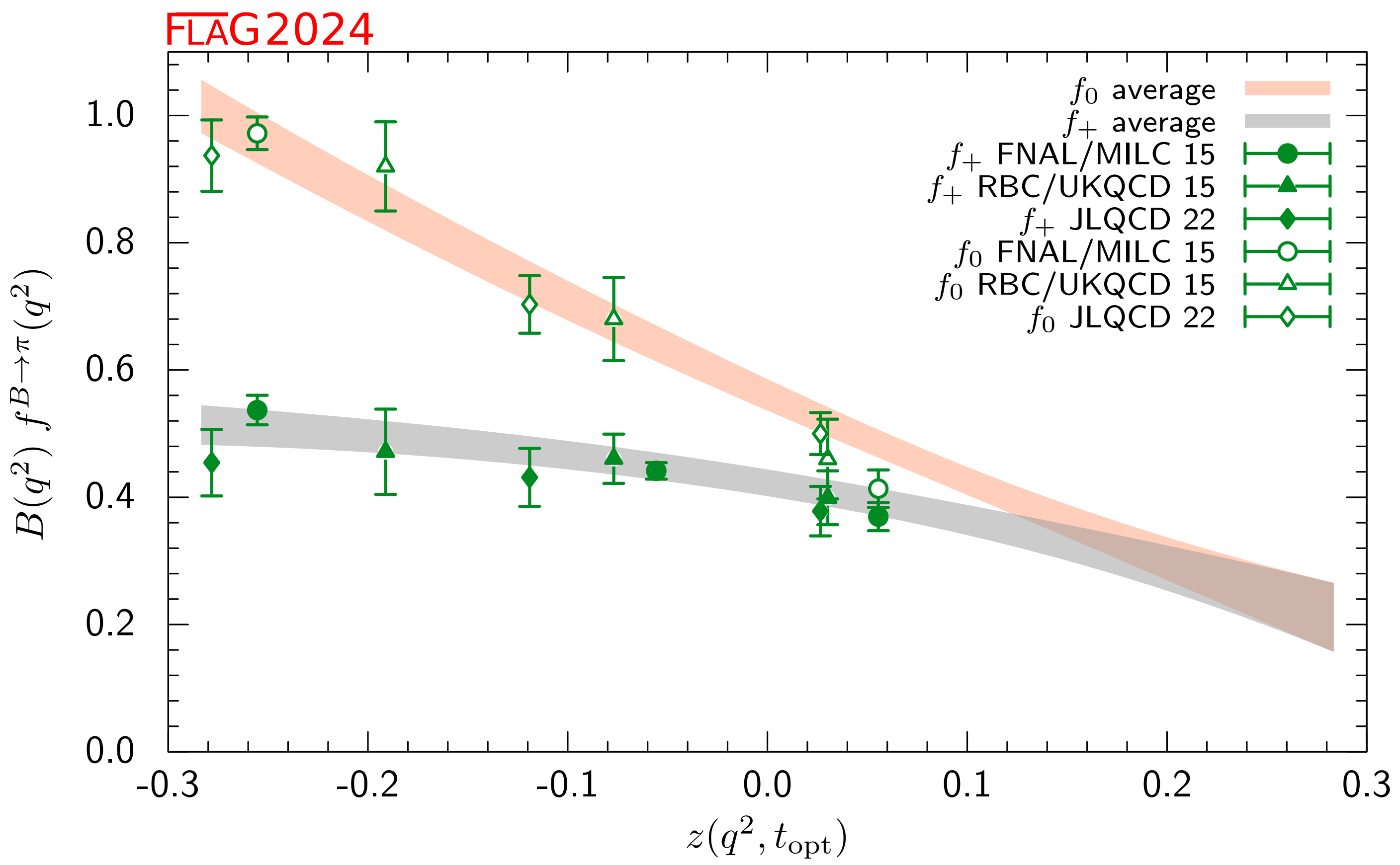


B meson semi-leptonic decays

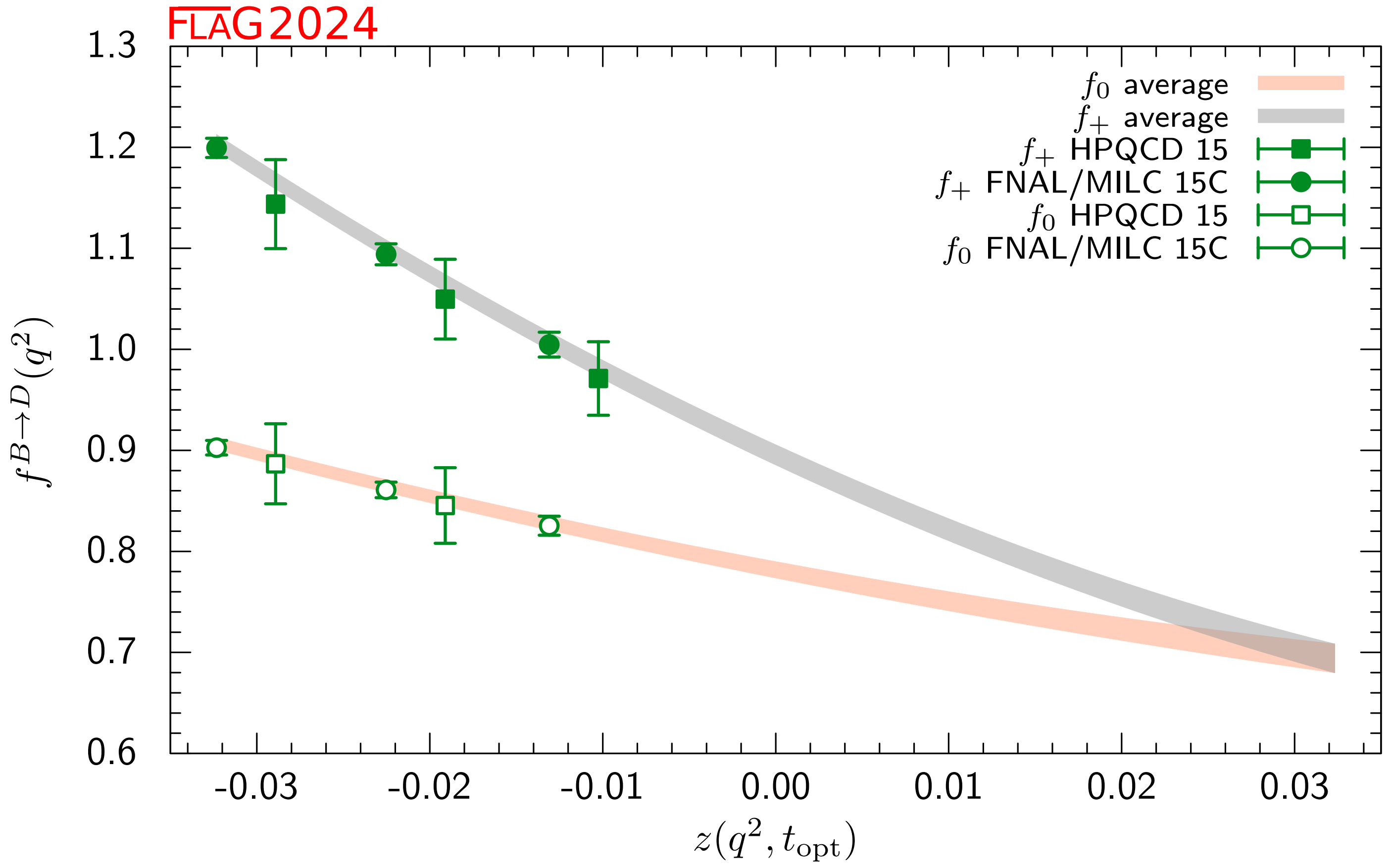
Y. Aoki et al [FLAG 2024 review, arXiv:2411.04268]

Lattice QCD

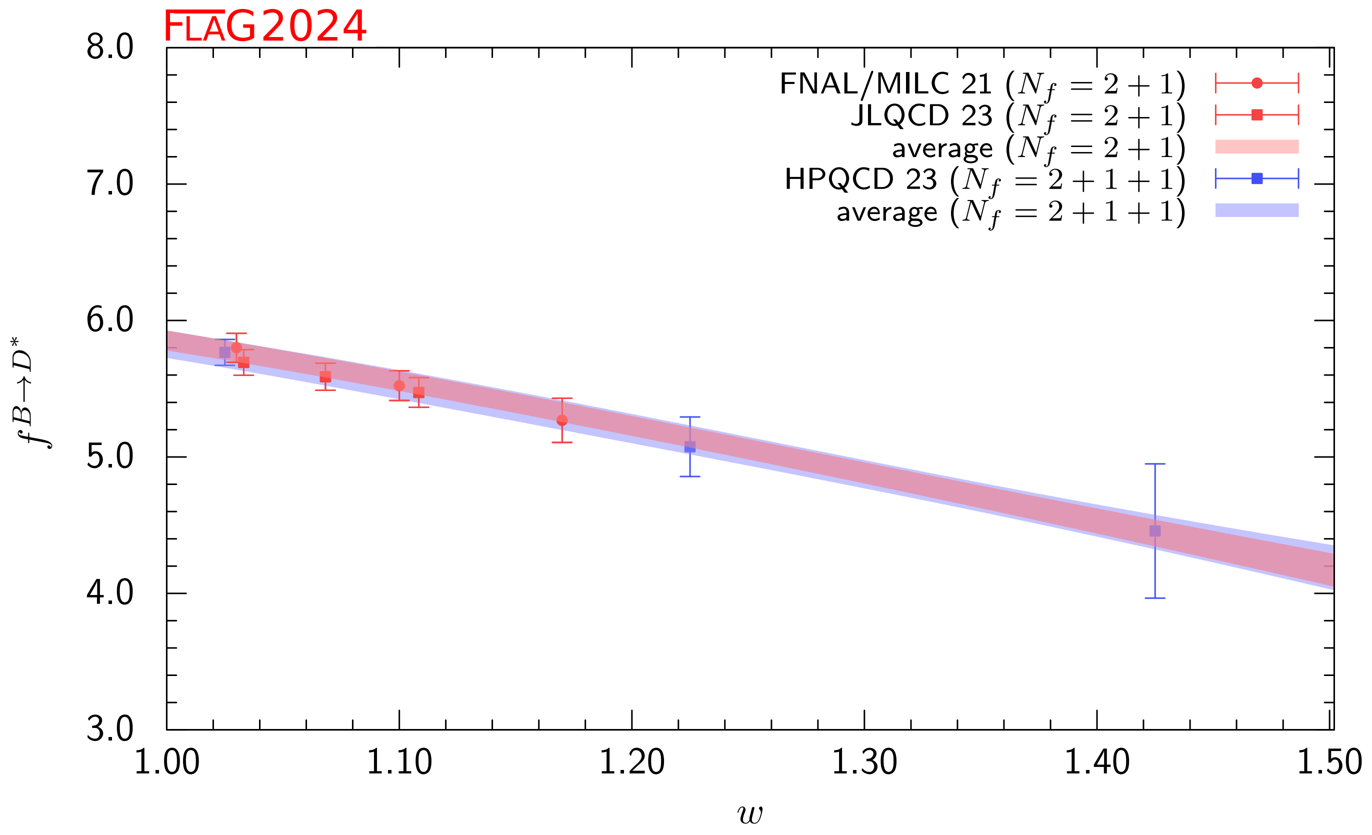
$B \to \pi \ell \nu$



$B \to D \ell \nu$



$B \to D^* \ell \nu$

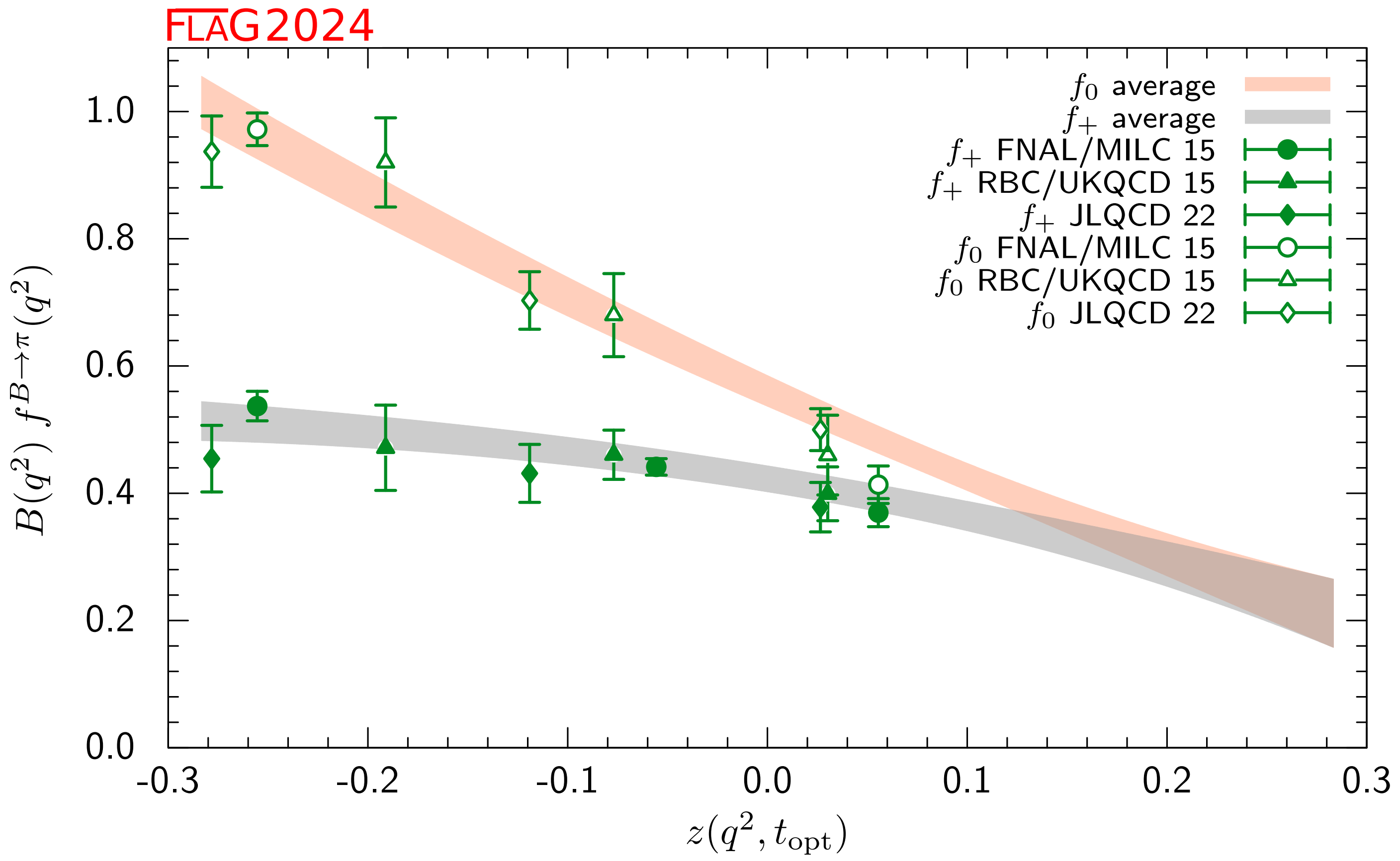


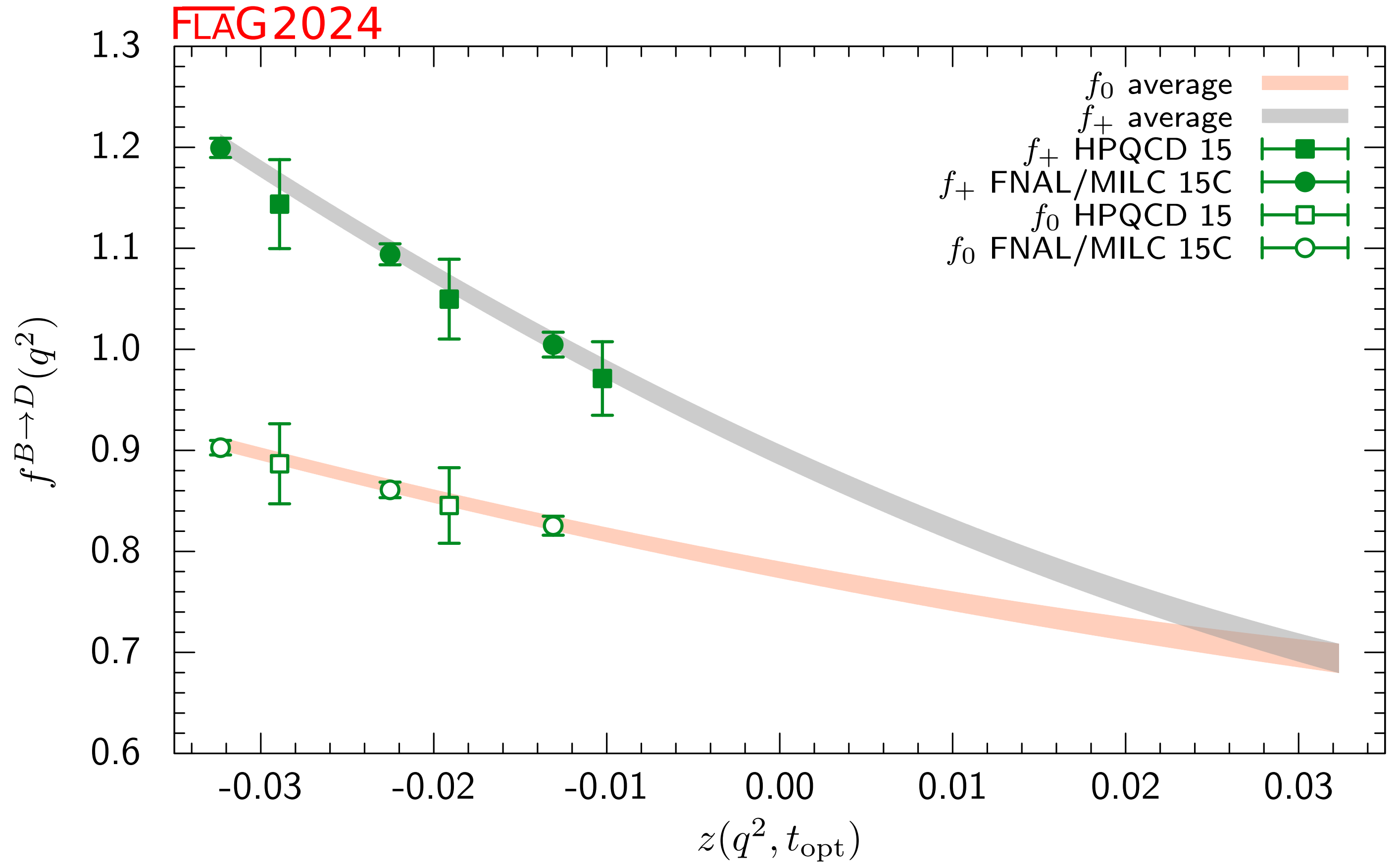
B meson semi-leptonic decays

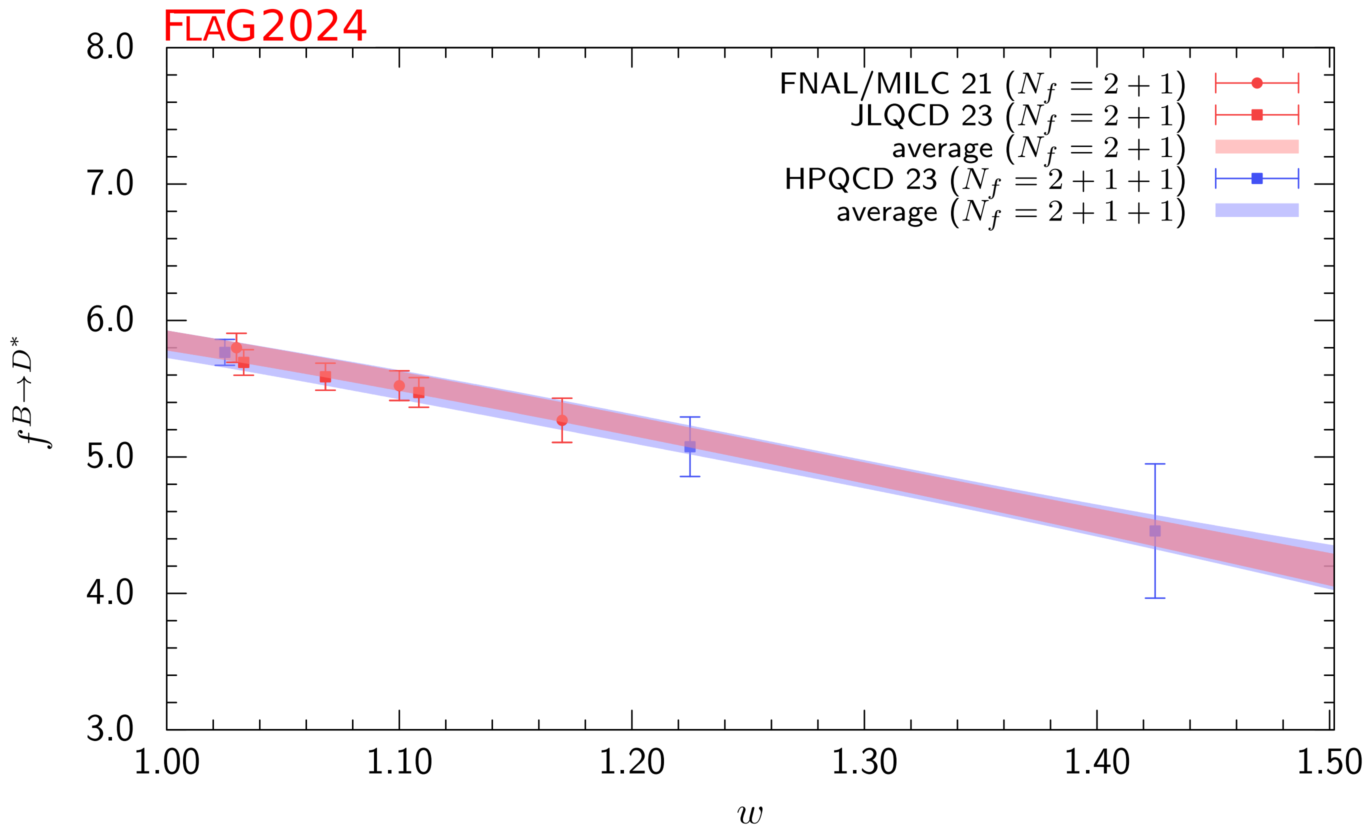
Y. Aoki et al [FLAG 2024 review, arXiv:2411.04268]

$B \to \pi \ell \nu$ | $B \to D \ell \nu$ | $B \to D^* \ell \nu$

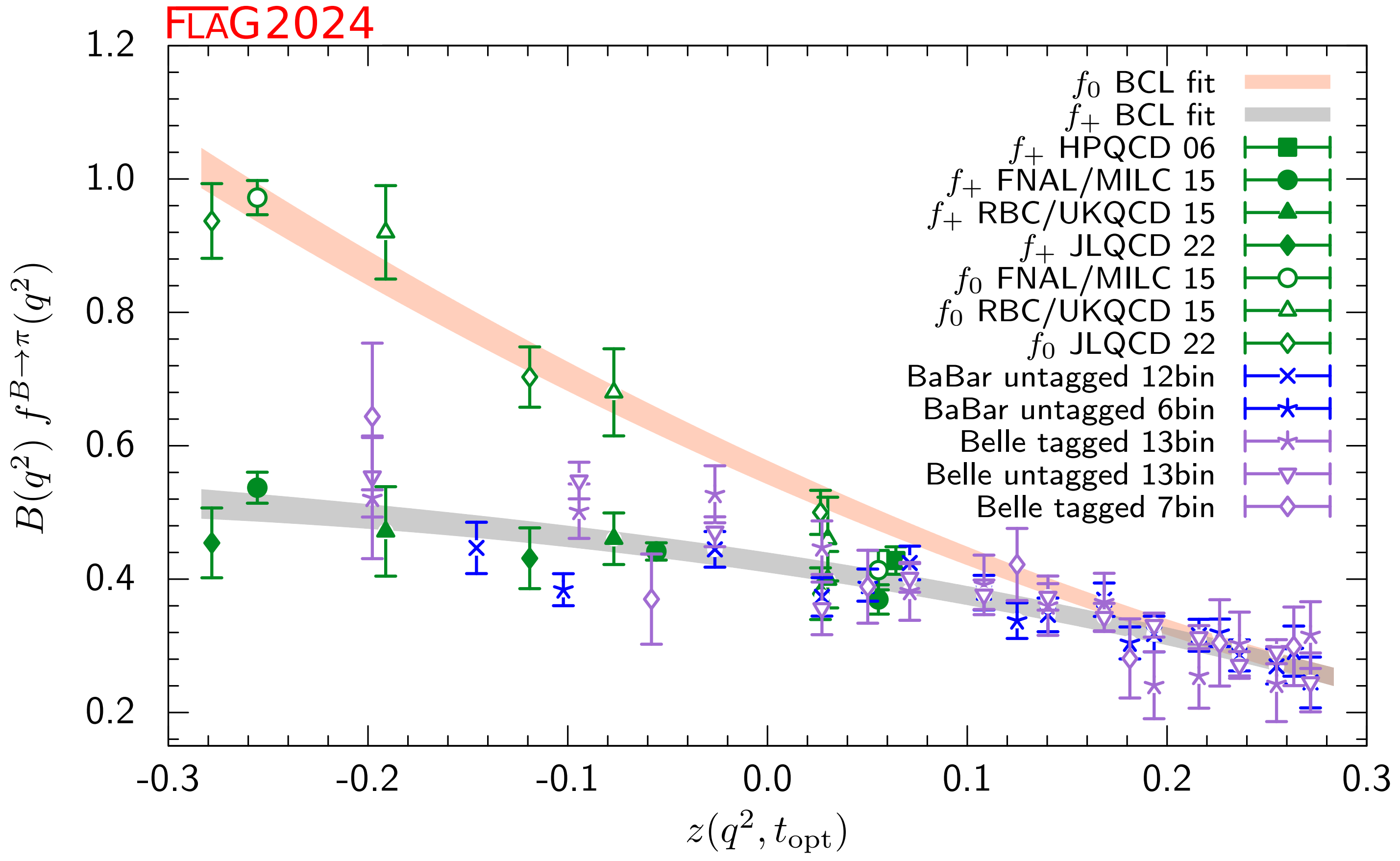
Lattice QCD

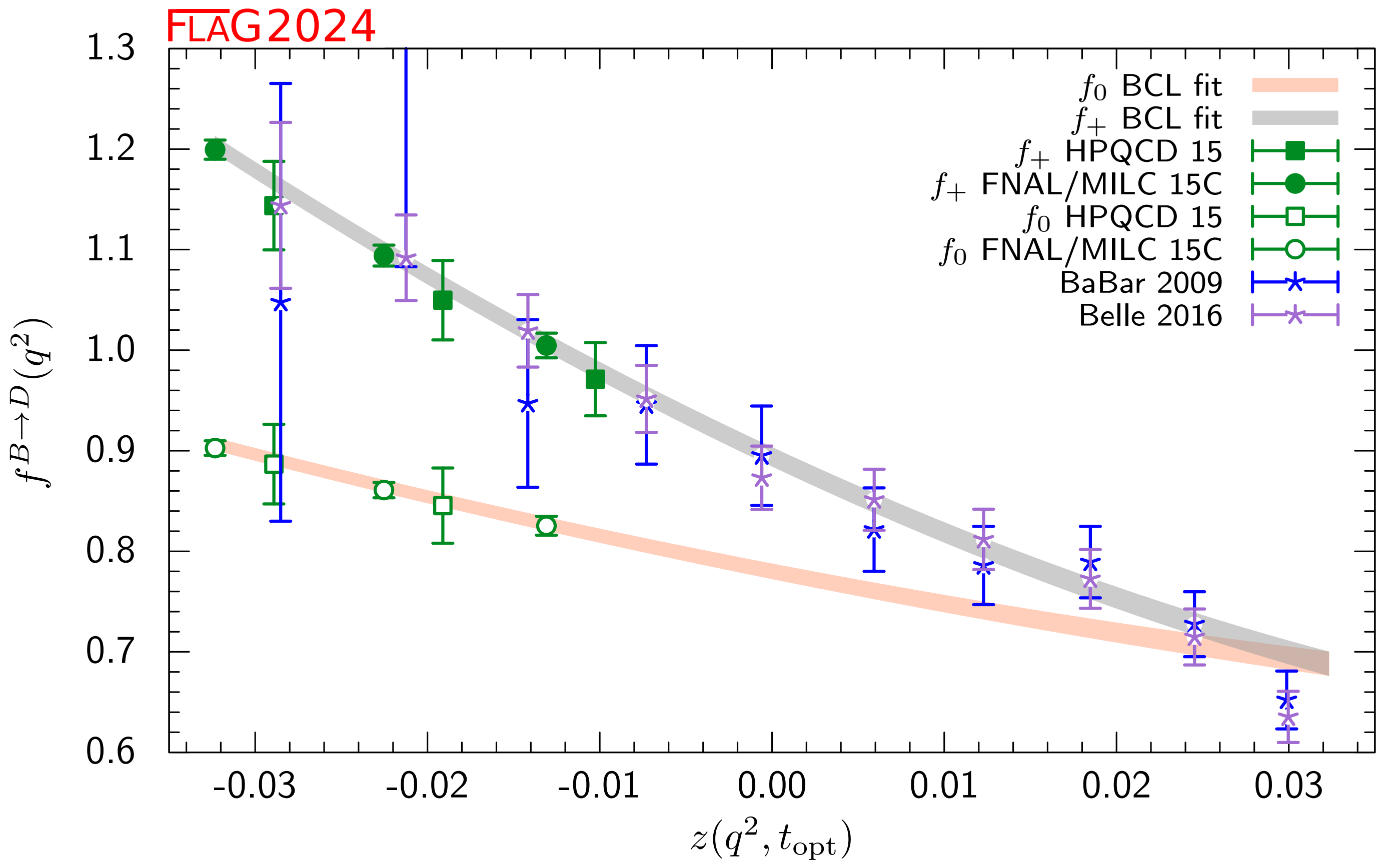


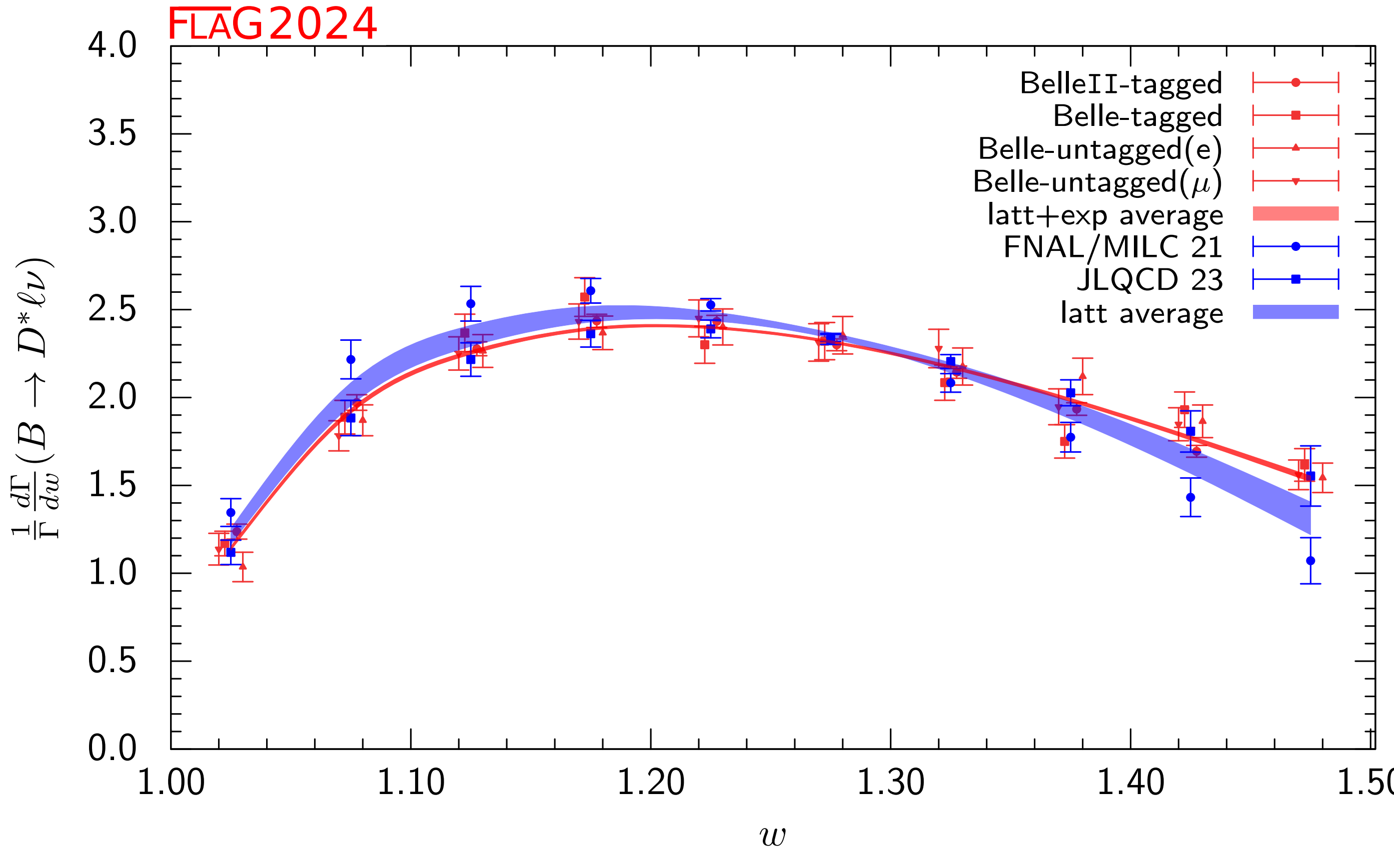




Lattice QCD + Exp.





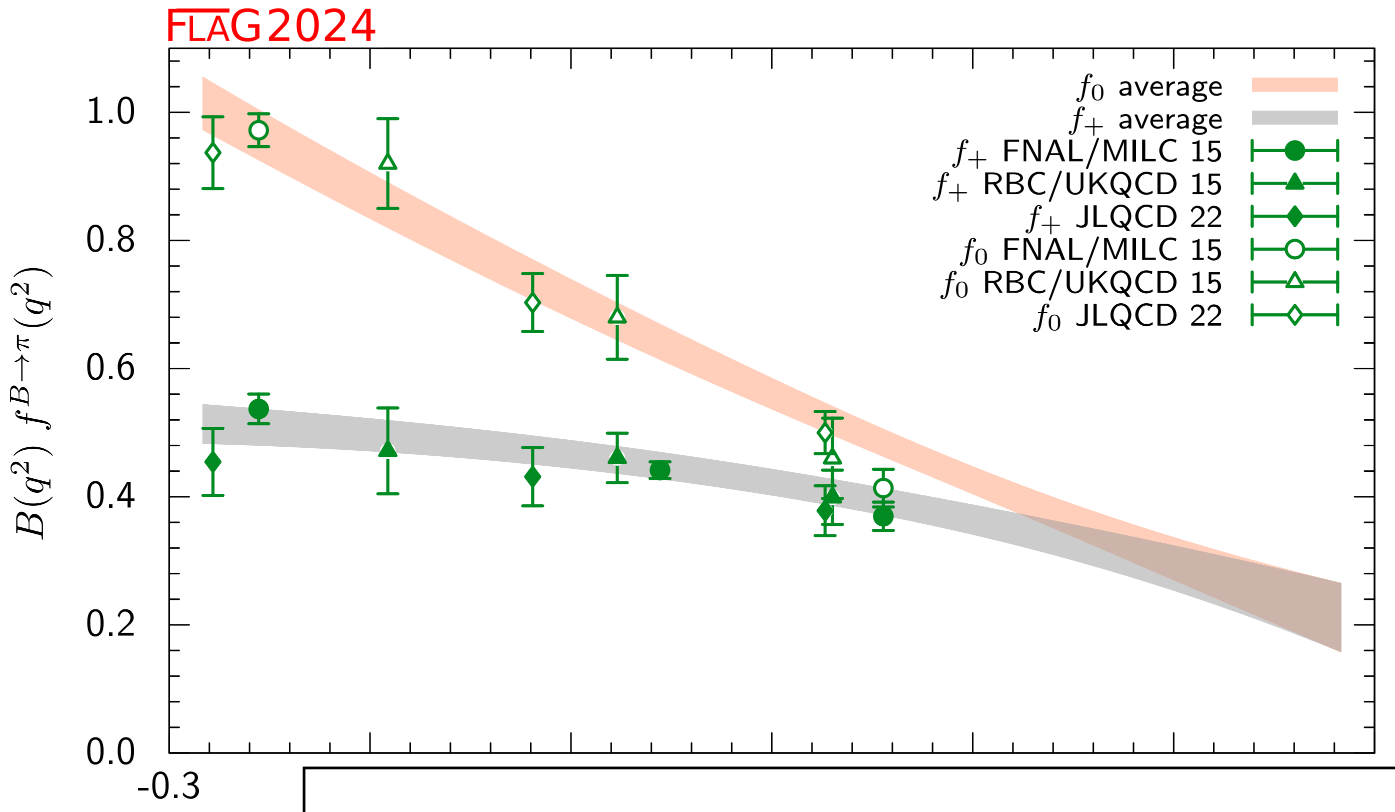


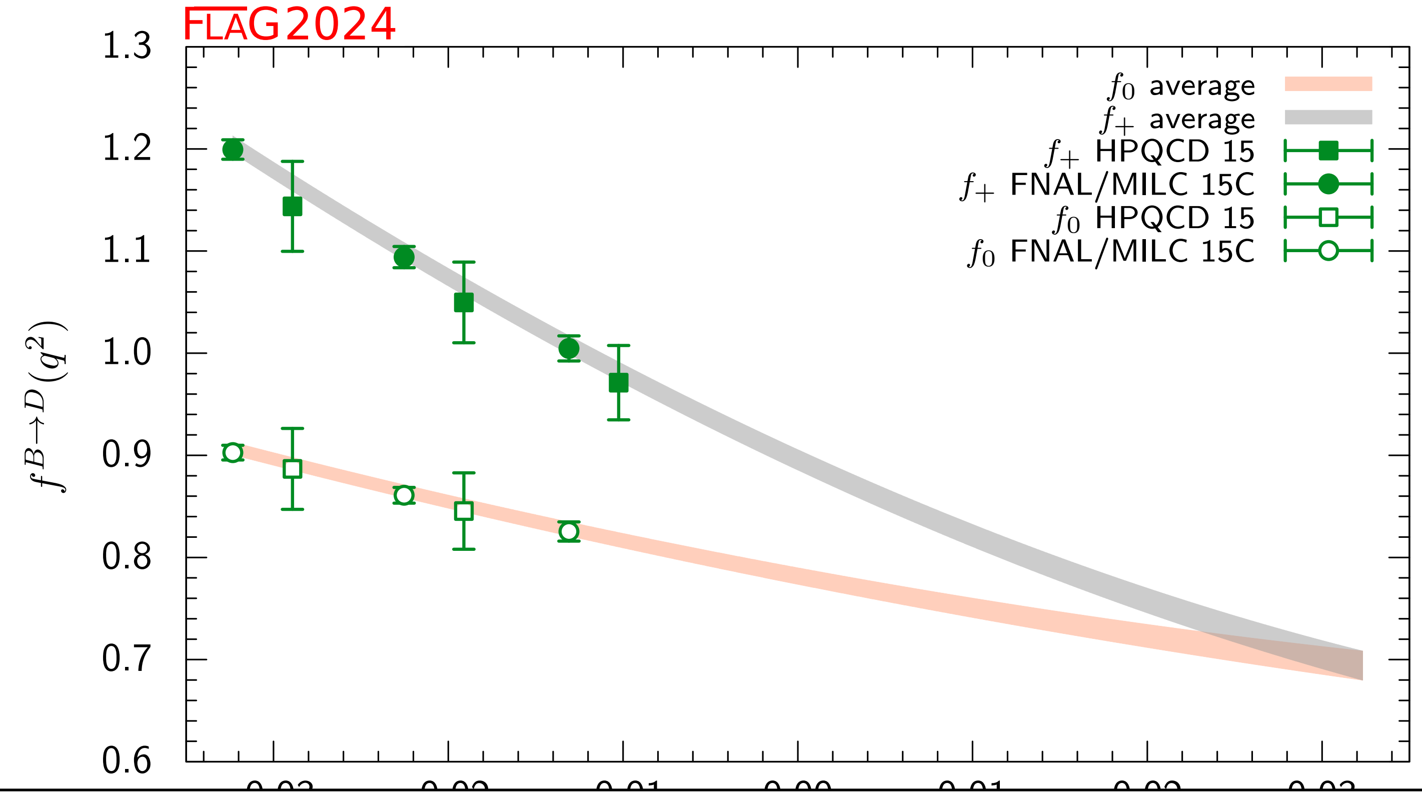
B meson semi-leptonic decays

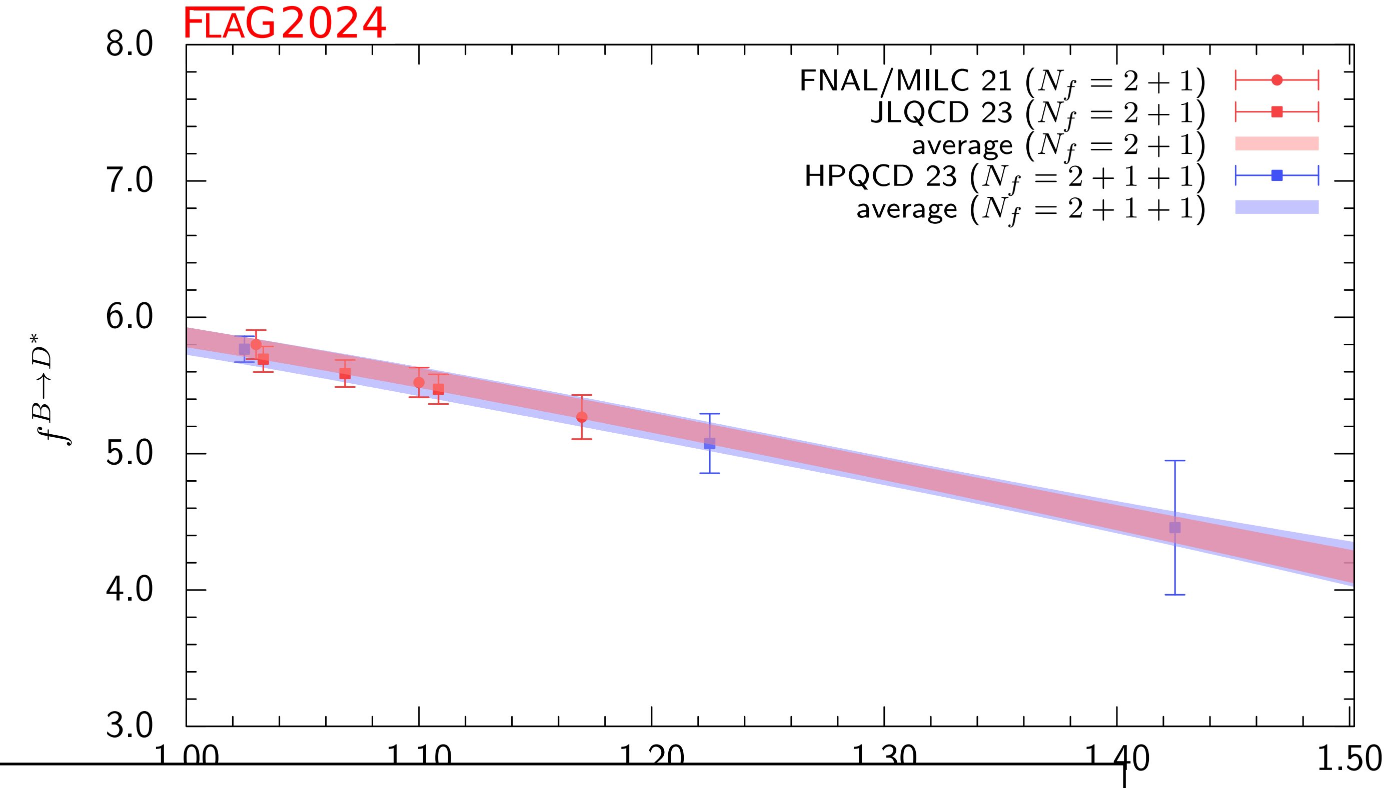
Y. Aoki et al [FLAG 2024 review, arXiv:2411.04268]

$B \to \pi \ell \nu$ | $B \to D \ell \nu$ | $B \to D^* \ell \nu$

Lattice QCD



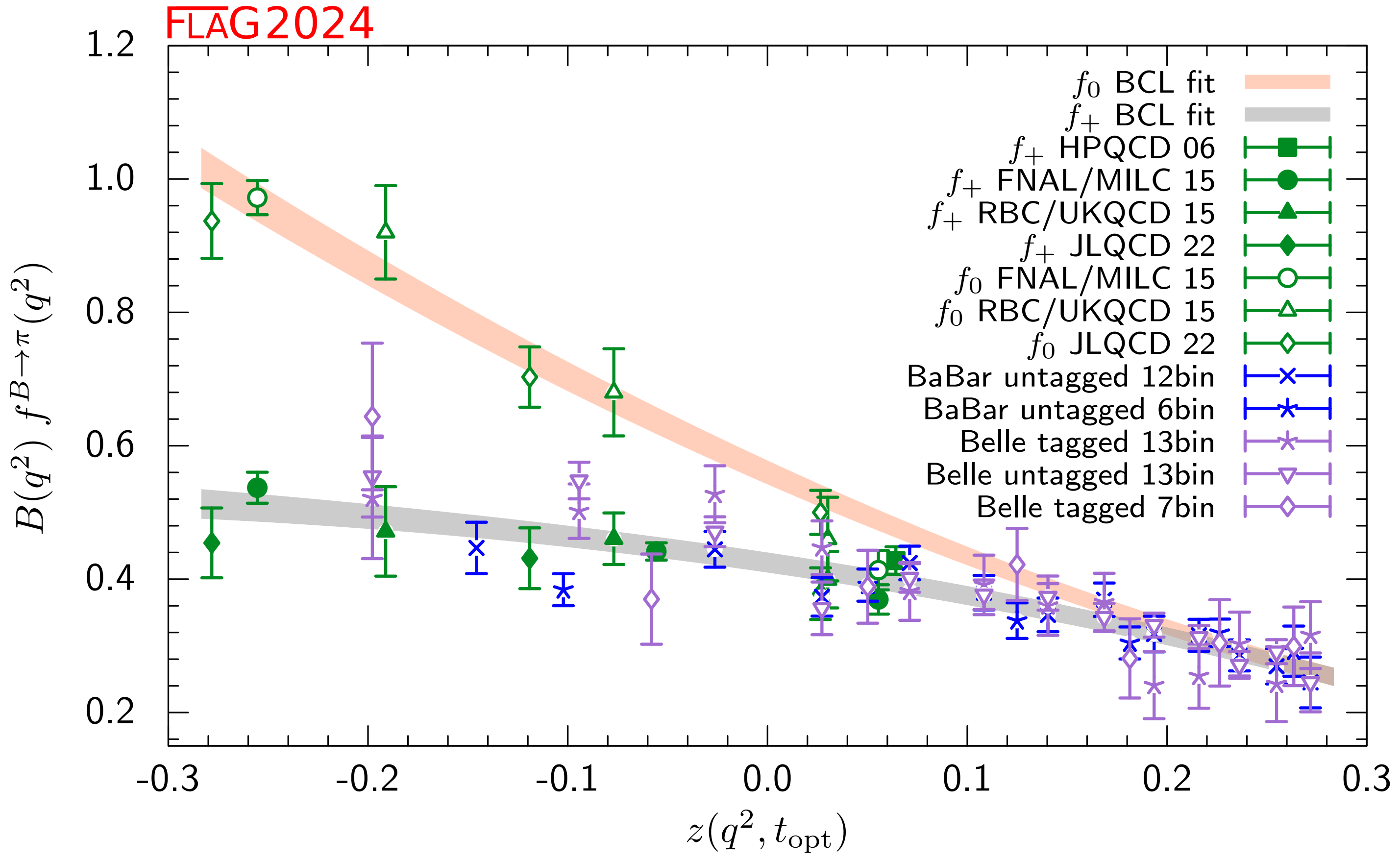


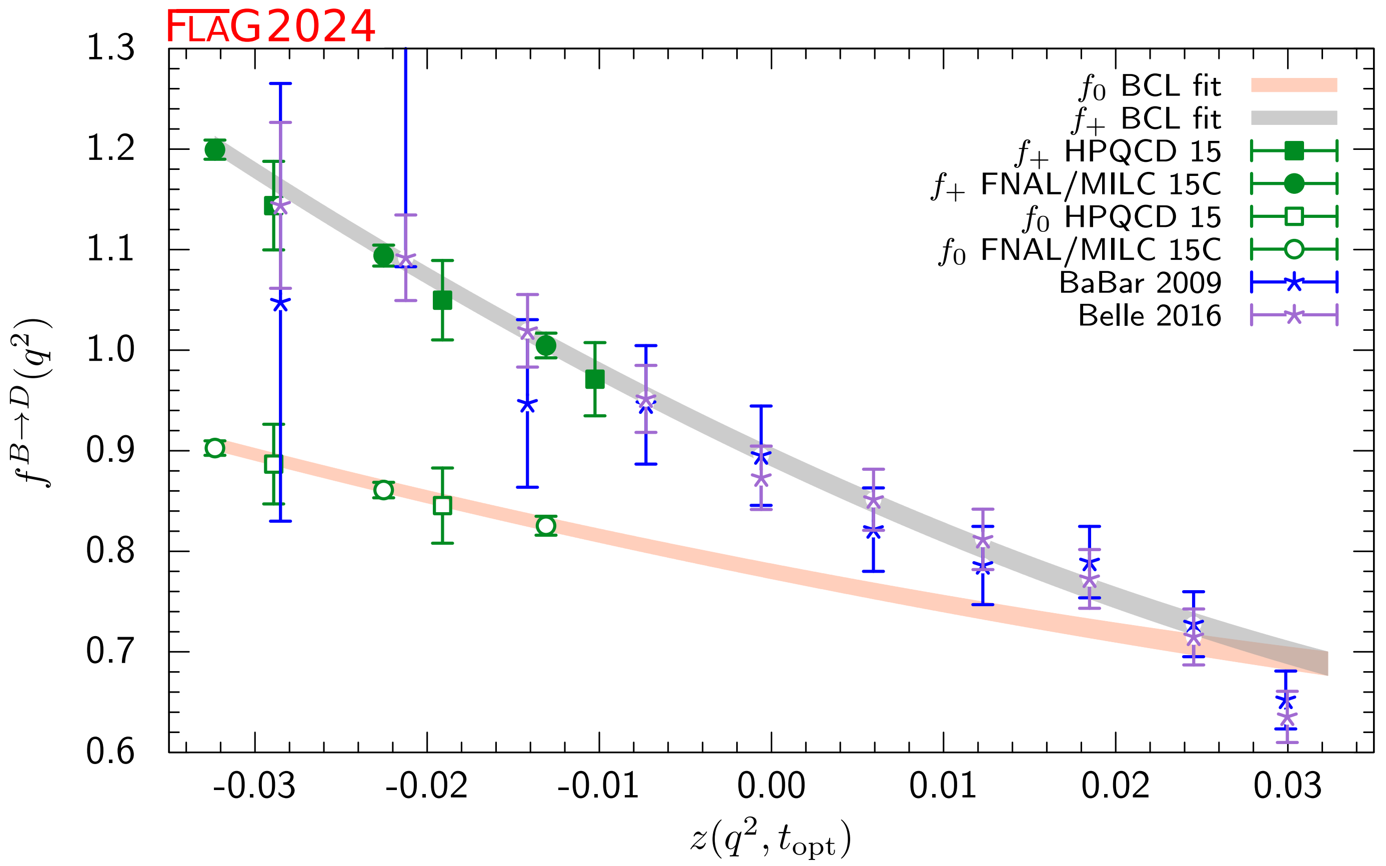


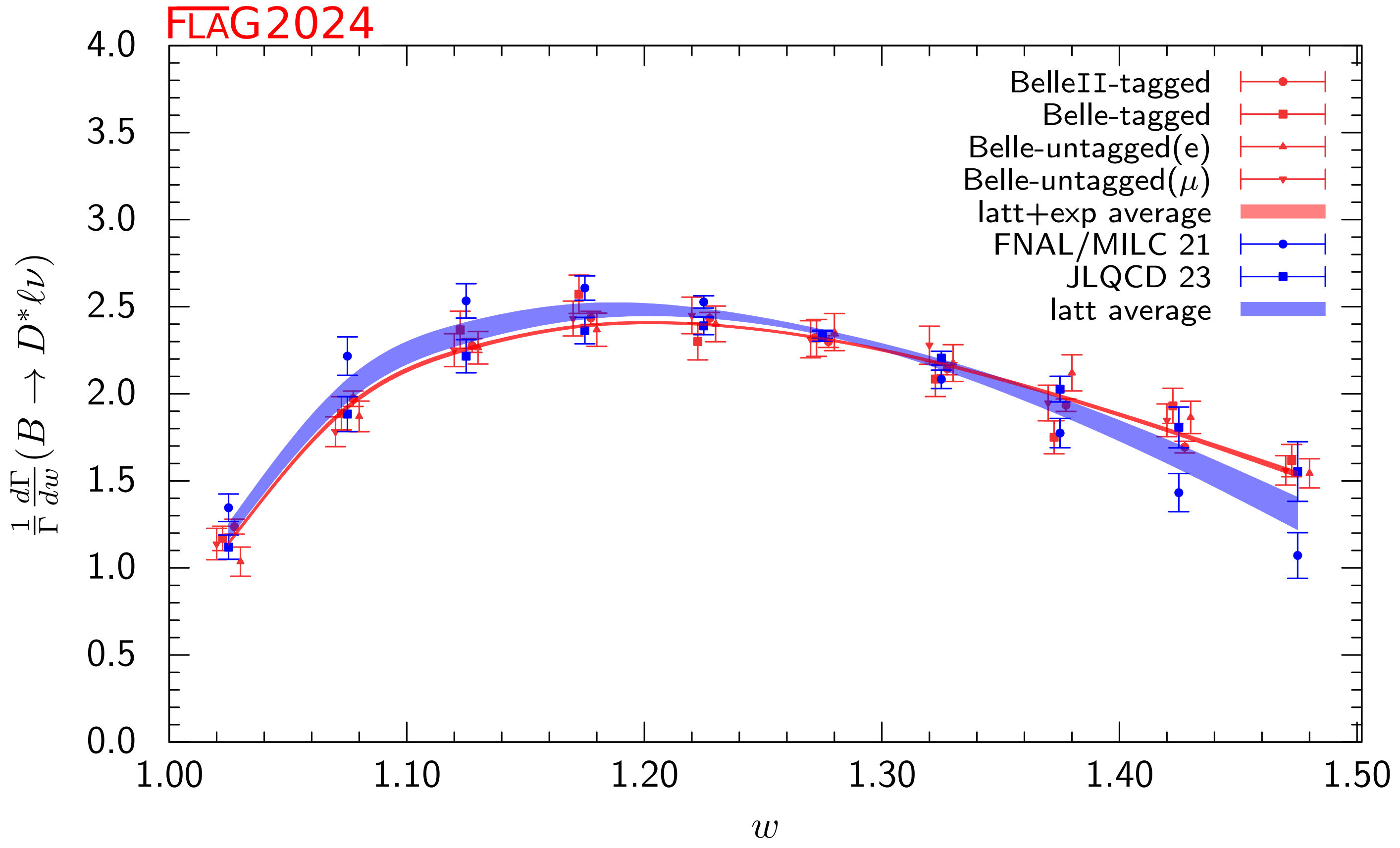
⇓

Form factors for many other B, D, K semileptonic decay channels

Lattice QCD + Exp.

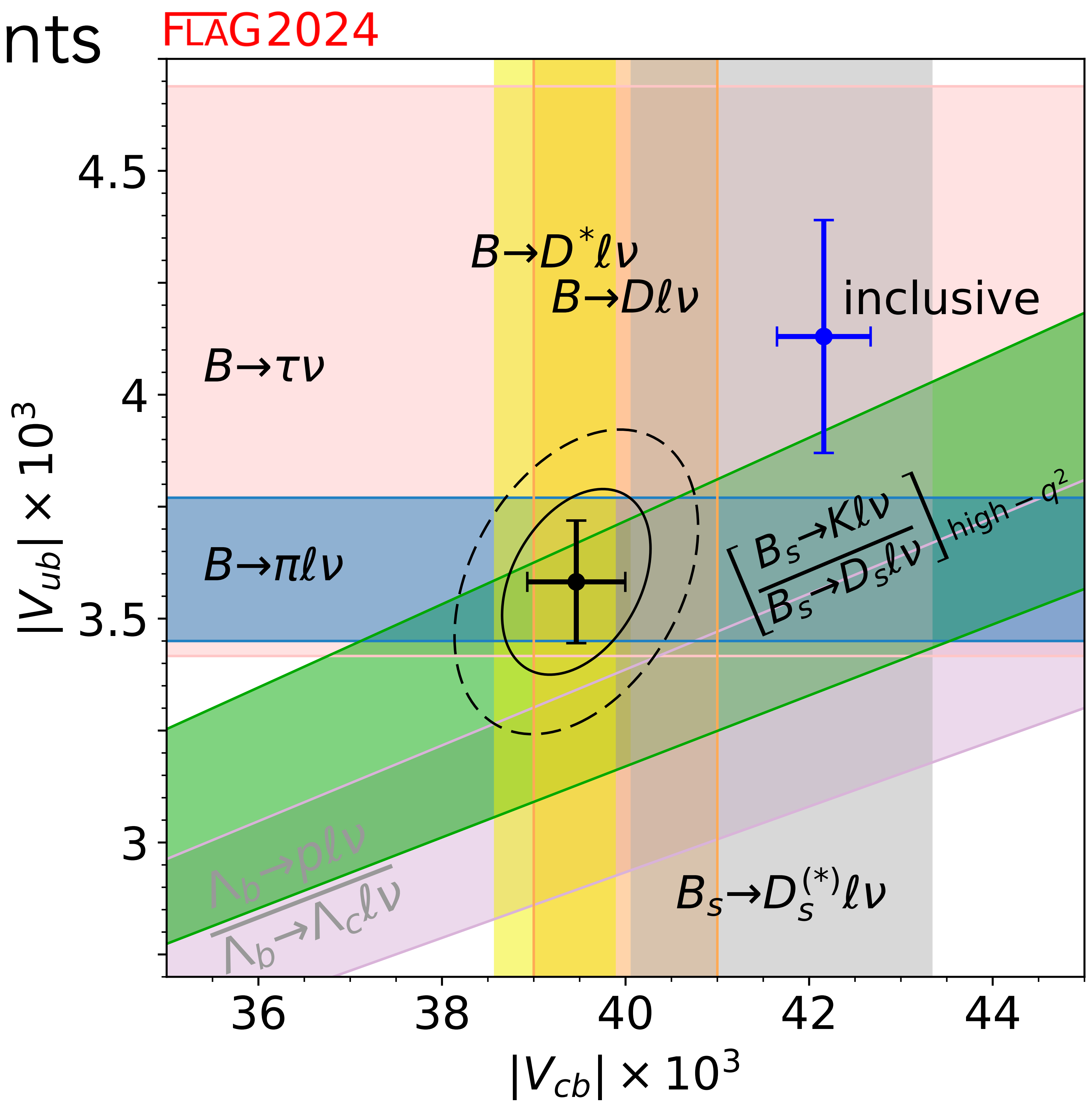






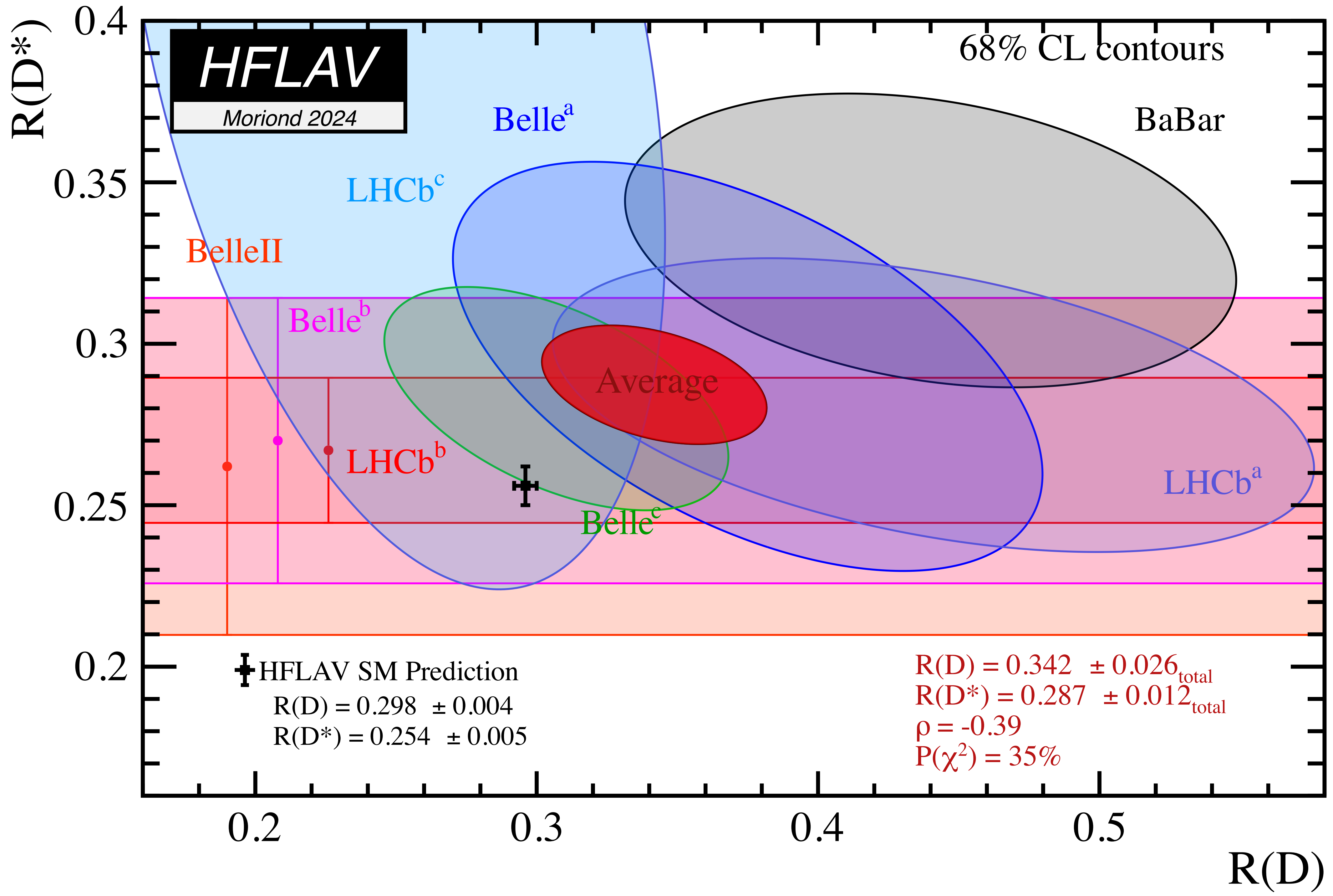
B meson semi-leptonic decays

CKM determinations from exclusive vs inclusive measurements



$\sim (2-3)\,\sigma$ tension between exclusive and inclusive $|V_{xb}|$

Lepton-flavor universality test: $\tau/(e+\mu)$



$\sim 3.3\,\sigma$ tension between experiment and SM prediction