EXERCISE 1

Find out what the largest/most prominent nuclear-physics experiments in your region are (city, state, nearby state). Among the questions posed in this lecture, which ones are the physics target of the experiments your region?

EXERCISE 2


Isospin symmetry, that is a SU(2) symmetry in the space of up and down quark flavors, is a good approximate symmetry of strong interactions. Consider two-nucleon (*NN*) systems in nature under this symmetry. Classify positive-parity *NN* systems according to their spin, isospin, and angular momentum. Recall that fermionic wavefunctions must be fully antisymmetric under exchange of the fermions. Each nucleon has spin 1/2 and isospin 1/2. Proton (p) and neutron (n) have $I_3 = 1/2$ and $I_3 = -1/2$, respectively, where I_3 is the third component of isospin. Write down the states explicitly according to their spin and flavor content. Which one of these channels exhibit a bound state in nature?

EXERCISE 3

Consider the QCD Lagrangian in a previous slide and ignore all flavors of quarks but up (u), down (d), and strange (s). This problem guides you to verify and make sense of various symmetries of the interactions. In the following: $q \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$.

a) Set the mass of quarks to zero (that is called the chiral limit). Show that the Lagrangian is invariant under a global $U(1)_V$ transformation of the quark fields:

$$q \to q' = e^{i\alpha} q$$

What is the corresponding Noether current? What does the conserved charge represent?

b) Show that the Lagrangian is invariant under independent rotations of left- and right-handed quarks:

$$q_L \to q_L' = e^{i\alpha_L^a T^a} q_L, \;\; q_R \to q_R' = e^{i\alpha_R^a T^a} q_R$$

where $q_L = \dfrac{1-\gamma^5}{2} q, \;\; q_R = \dfrac{1+\gamma_5}{2} q$. This is called chiral symmetry. T^a are generators of SU(3).

c) Show that for massive quarks, chiral symmetry breaks. This is an explicit chiral symmetry breaking. The symmetry is broken even if quarks were massless through a spontaneous symmetry-breaking mechanism, see the next slide.

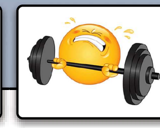
EXERCISE 4


By expanding the leading-order chiral perturbation theory Lagrangian for pseudo-scalar mesons, obtain the Lagrangian for the pion fields. Keep only terms up to two pions. This form should allow you to obtain a relation between the mass of the pions and the mass of u and d quarks. Notice that in the chiral limit, pions (and all pseudo-scalar mesons) couple derivatively to all orders, meaning that they do not interact at zero momentum. This feature continues even when these mesons couple to baryons and other fields in the EFT. Argue why this is an expected feature.

EXERCISE 5

There could be in principle two other spin-isospin structure for the two-nucleon contact potential at leading order. What are they? Argue why only the two terms written for the contact potential in the previous slide suffice to describe *NN* interactions at this order.

EXERCISE 6

In chiral perturbation theory, the leading-order interactions among nucleons and pions can be read from the Lagrangian:

$$\mathcal{L}_{\pi N} = -\frac{g_A}{\sqrt{2} f_\pi} N^\dagger \tau^a \sigma \cdot \nabla \pi^a N$$

a) Consider the Feynman diagram for the exchange of one pion among two nucleons. In the static limit, derive the (instantaneous) potential in this process. Note that in this limit, the pion propagator is:

$$S_\pi(\mathbf{q}) = \frac{-i\delta_{a,b}}{\mathbf{q}^2 + m_\pi^2}$$

Obtain both the momentum-space potential $V_{\mathrm{OPE}}(\mathbf{q})$, where **q** is the momentum exchange among nucleons, and the position-space $V_{\mathrm{OPE}}(\mathbf{r})$, where **r** is nucleons' relative distance.

b) From the position-space potential, identify the "central" and "tensor" components.

c) How does the strength of the two-pion potential compares between the isosinglet (*I*=0) and isotriplet (*I*=1) two-nucleon channels?

EXERCISE 7

Consider the expansion of two-nucleon scattering amplitude within the KSW power-counting scheme. This exercise will guide you to fill some of the missing details.

a) [Bonus] In order to arrive at an amplitude that is p^{-1} at leading order, it was important for the loop function I_0 to scale as p, where p is the magnitude of the relative (spatial) momentum of the nucleons on shell. Evaluate this loop using dimensional regularization and notice that it is not UV divergent in 3+1 dimensions but it is in 2+1 dimensions. Subtract the divergence at 2+1 dimensions from the result. This is called power-divergence subtraction scheme and is what is used in the expansion of amplitudes. Here, you can consider the nonrelativistic nucleon propagators in the loops:

$$S_N(p) = \frac{i}{p^0 - \frac{\mathbf{p}^2}{2M_N} + i\epsilon}.$$

b) How should the C_2 LEC scale as a function of p such that the next-to-leading order amplitude $\mathcal{M}^{(NLO)}$ scales as p^0 ?

c) Derive the expression for leading and next-to-leading amplitude in the pionless EFT in terms of the leading and next-to-leading order couplings (LECs) C_0 and C_2.