The role of (lattice) QCD in Flavor Physics

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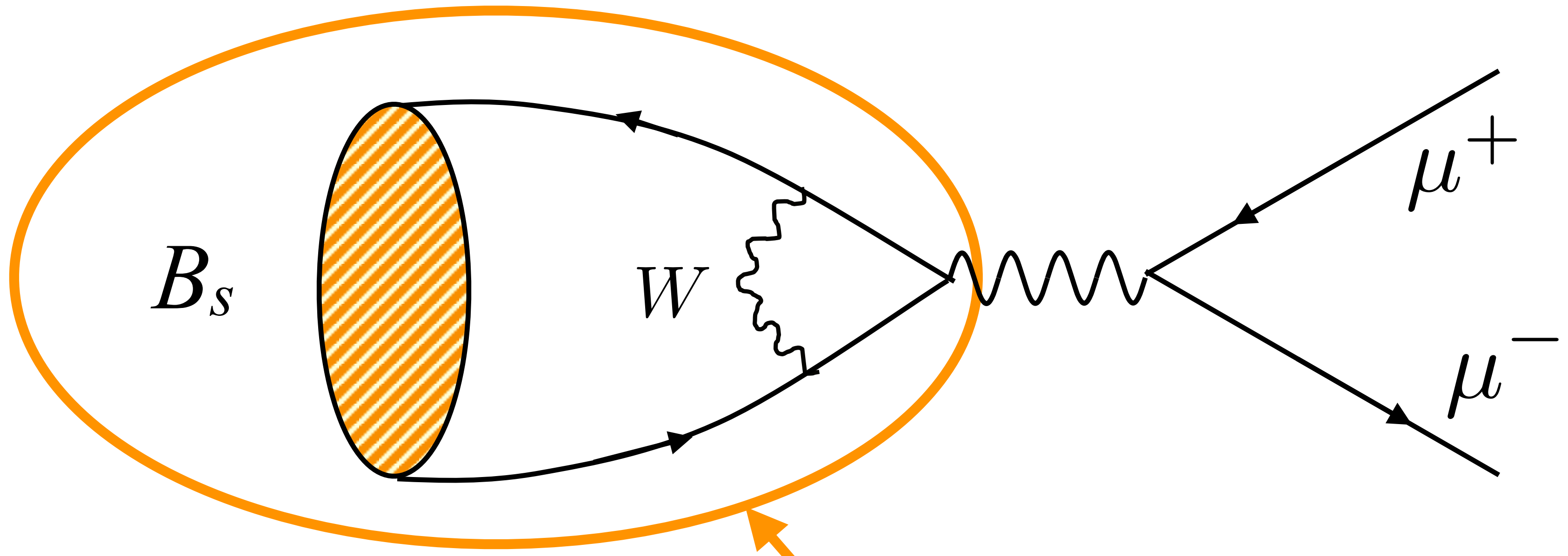
Outline

- Motivation: Open Questions
- The role of (lattice) QCD in flavor physics
  - History
  - Challenges
- Examples
  - Leptonic kaon, pion decay
  - Semileptonic kaon decay
  - First row CKM unitarity
  - Semileptonic D-meson decay
  - Leptonic B-meson decay

The role of (lattice) QCD in flavor physics

example: $B_s \to \mu^+ \mu^-$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

$\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right)$

$d\Gamma(B^0 \to D^{*-} \mu^+ \nu_\mu), \ldots$

$B(B_s \to \mu\mu), \ldots$

$\Delta m_{d(s)} \quad \cdots$

The role of (lattice) QCD in flavor physics

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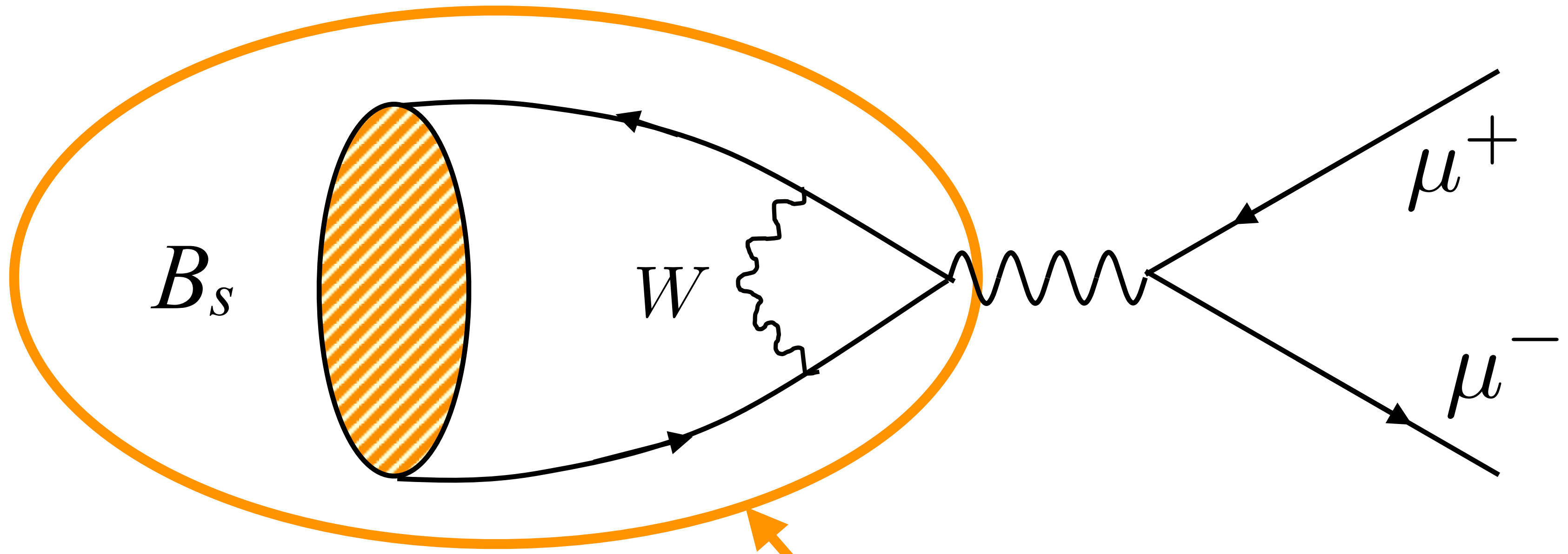

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Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

The role of (lattice) QCD in flavor physics

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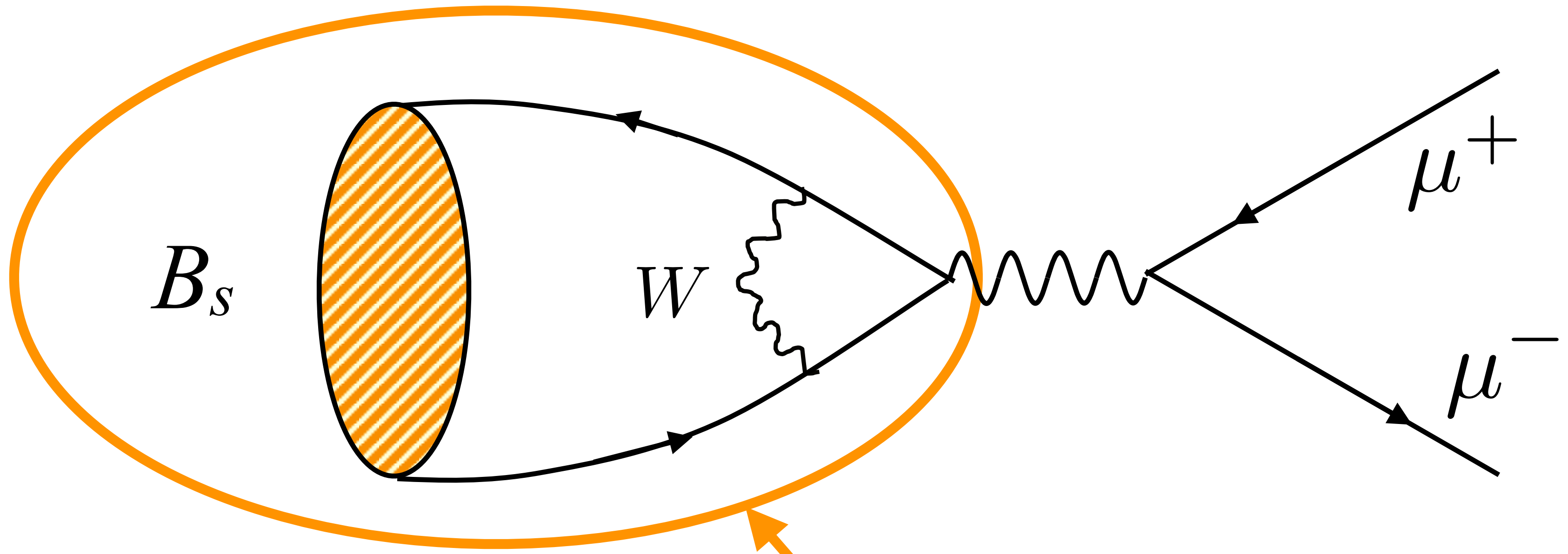

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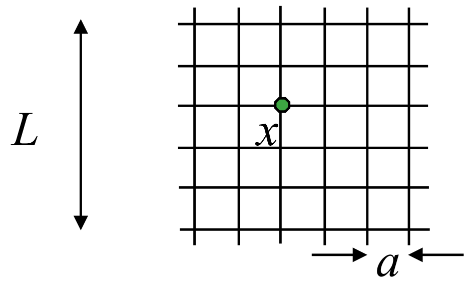
Two main purposes:

- ✦ combine experimental measurements with LQCD results to determine SM parameters.
- ✦ confront experimental measurements with SM theory using LQCD inputs.

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...



Lattice QCD Introduction

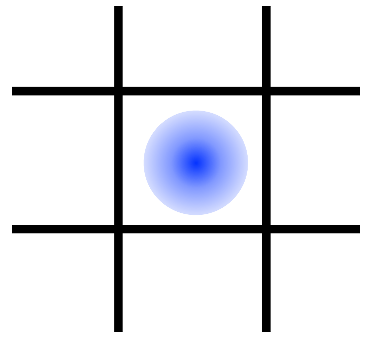
systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on Effective Field Theory (EFT) descriptions of QCD ➙ ab initio

- finite a:
  Symanzik EFT ⬅ Asymptotic Freedom & Renormalizability
- light quark masses:
  Chiral Perturbation Theory ⬅ Chiral Symmetry & Spontaneous Symmetry Breaking
- heavy quarks:
  HQET ⬅ Heavy Quark symmetry
- finite L:
  finite volume EFT ⬅ Confinement & S-matrix

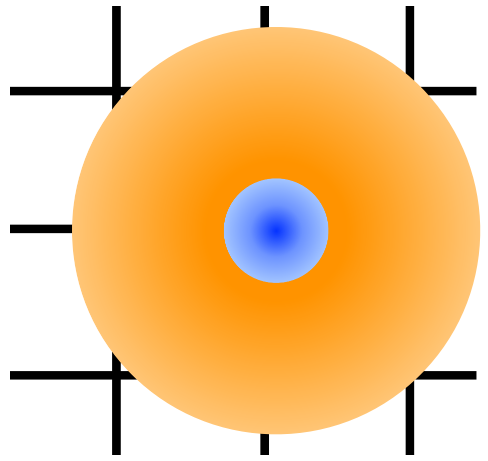
Lattice QCD Introduction: heavy quarks

b quark



$m_b \gtrsim a^{-1} \gg \Lambda$ ➠ leading discretization errors $\sim (am_b)^2$
(using same action as for light quarks)

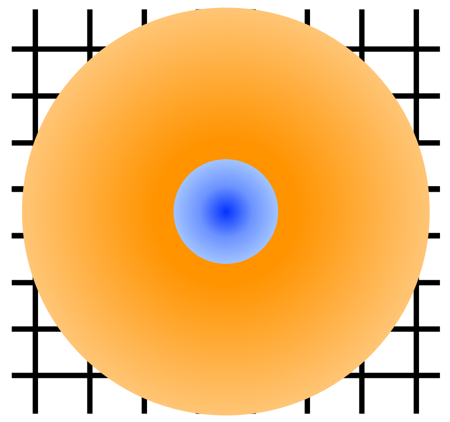
B meson



use EFT (HQET, NRQCD) ➠ Λ/m_b expansion

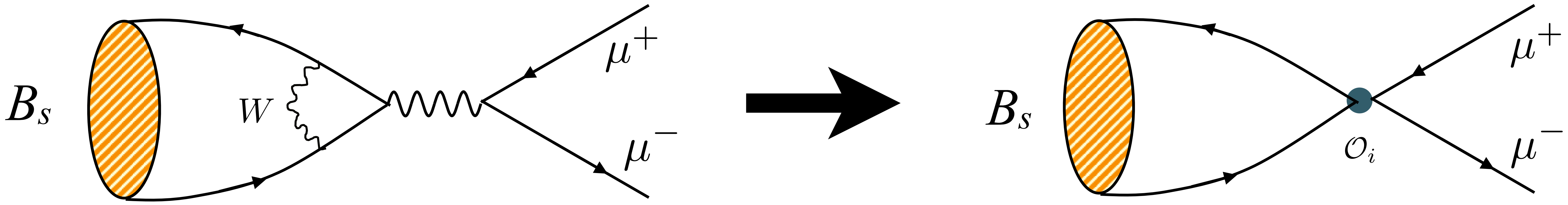
EFTs co-developed continuum/lattice

- lattice HQET, NRQCD: use EFT to construct lattice action
  complicated continuum limit
  nontrivial matching and renormalization ➠ (few-5)% errors
- relativistic heavy quarks: Fermilab (1996), also Tsukuba (2003), RHQ (2006)
  matching relativistic lattice action via HQET to continuum
  nontrivial matching and renormalization ➠ (1-3)% errors



$a^{-1} > m_b \gg \Lambda$ + highly improved light quark action
➠ same action for all quarks
➠ simple renormalization (Ward identities) ➠ < 1% errors

Rare leptonic decay $B_s \to \mu^+\mu^-$

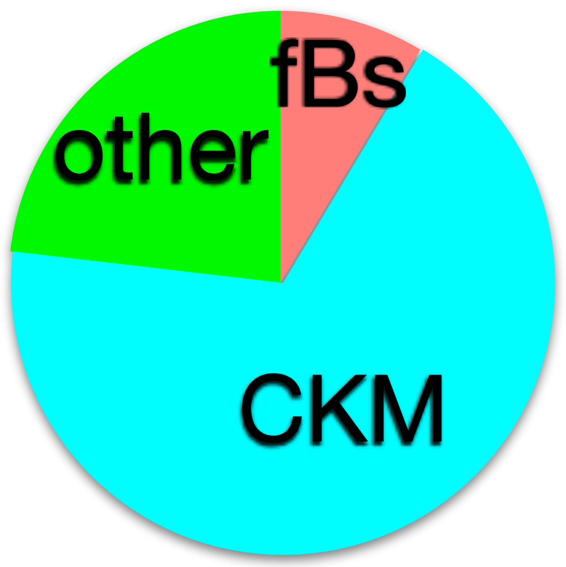


Standard Model predictions: Buras, et al [arXiv:1303.3820, JHEP 2013], Bobeth, et al [arXiv:1311.0903, PRL 2014; arXiv:2104.09521], Beneke et al [arXiv:1908.07011, JHEP 2019].

$$\overline{\mathcal{B}}(B_q \to \mu\bar{\mu}) = \frac{|\mathcal{N}_q|^2\, M_{B_q}^3\, f_{B_q}^2}{8\pi\, \Gamma_q^H}\beta_{q\mu}\left(\frac{m_\mu}{M_{B_q}}\right)^2 \left|C_{10}^{\text{eff}}\right|^2, \qquad \beta_{q\mu} \equiv \sqrt{1 - \frac{4\, m_\mu^2}{M_{B_q}^2}},$$

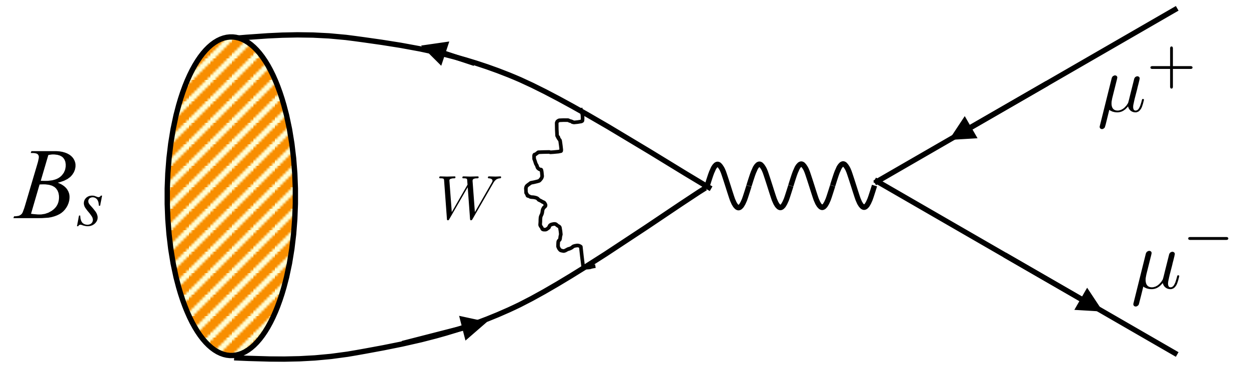
$$\overline{\mathcal{B}}(B_s \to \mu\bar{\mu})_{\text{SM}} = (3.66 \pm 0.14) \cdot 10^{-9}$$

error²

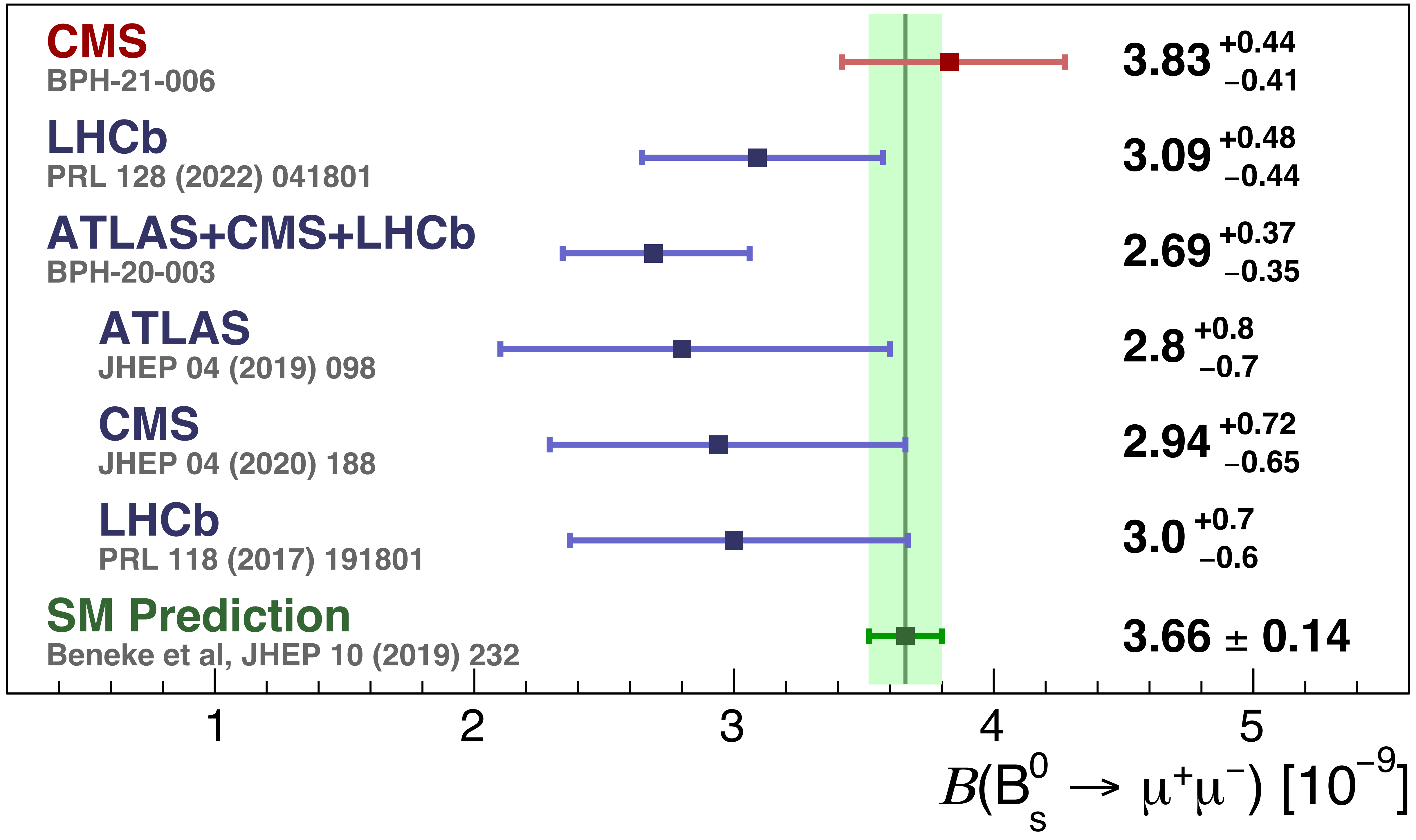


- includes structure-dependent QED corrections
- dominant uncertainty due to $|V_{cb}|$
- LQCD decay constant sub dominant source of uncertainty

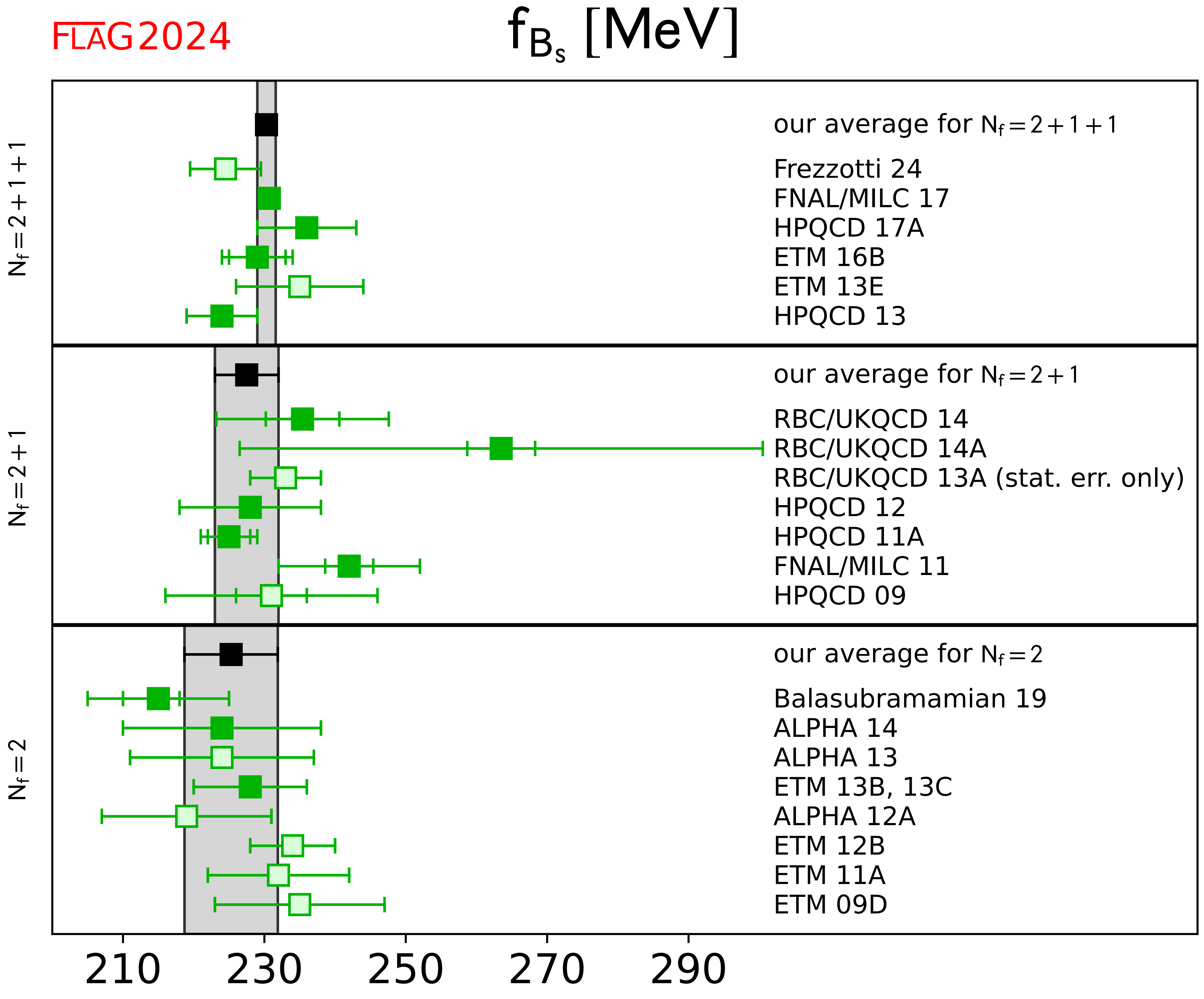
Rare leptonic decay $B_s \to \mu\mu$

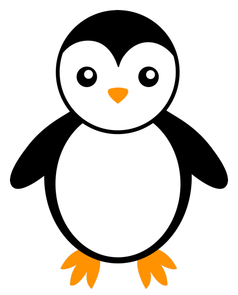


[CMS, Phys Lett B 2023]



S. Aoki et al [FLAG 2024 review, arXiv:2411.04268]





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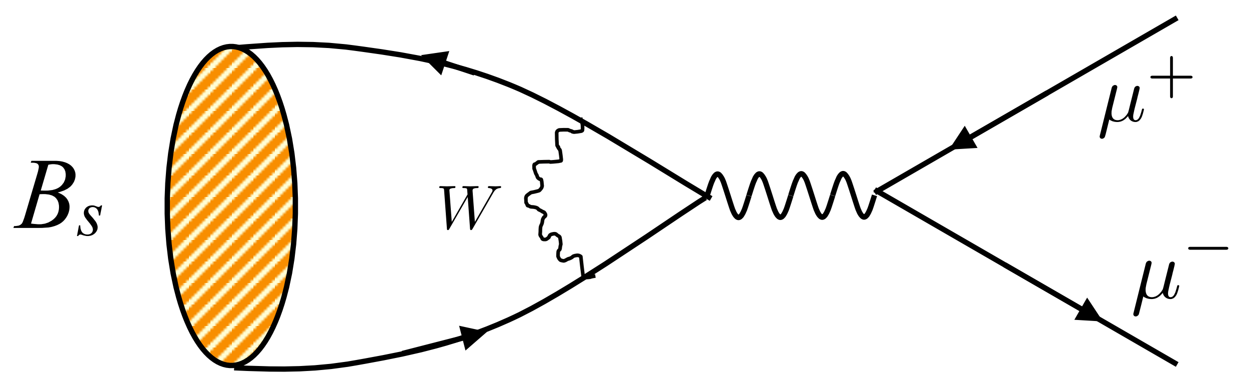

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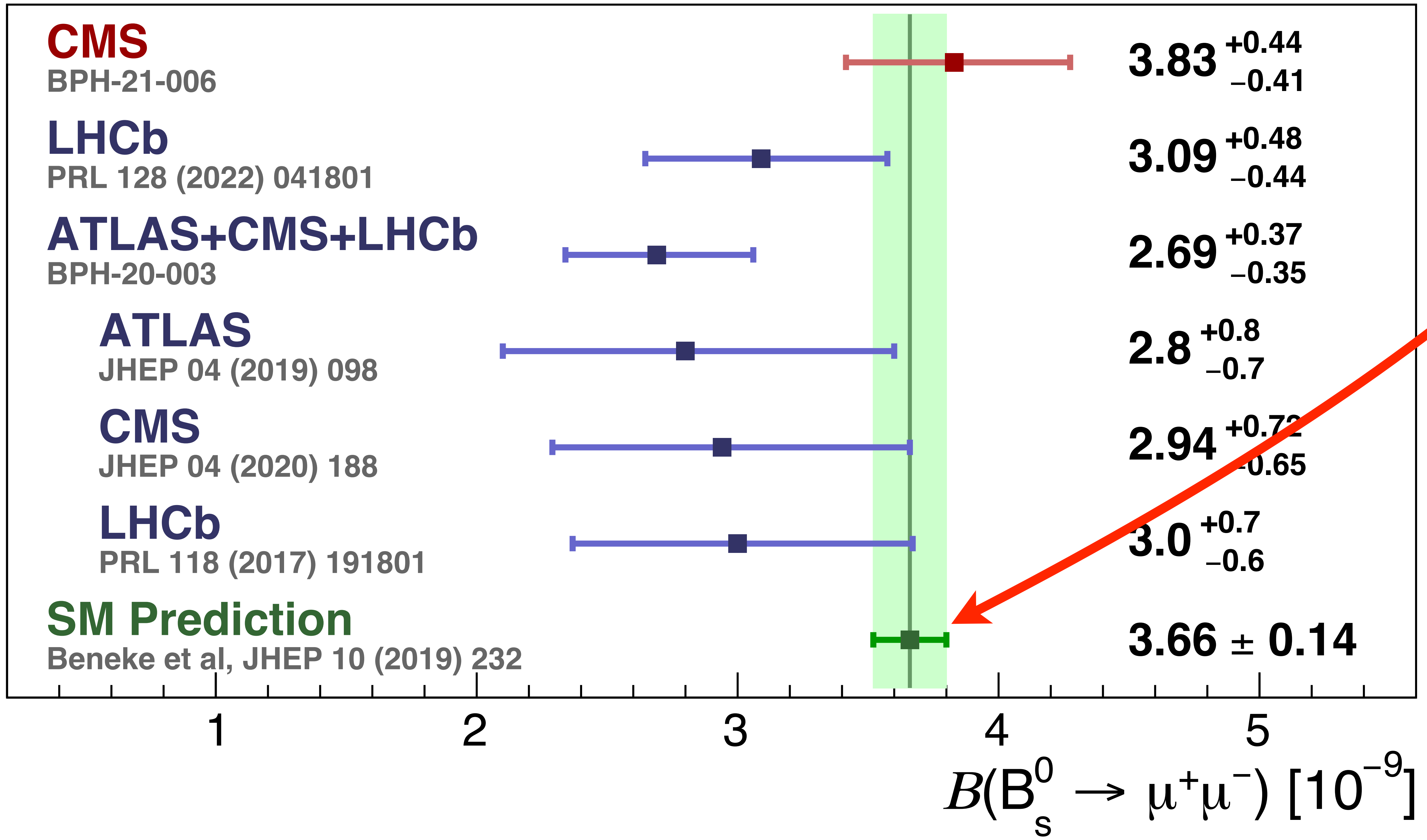
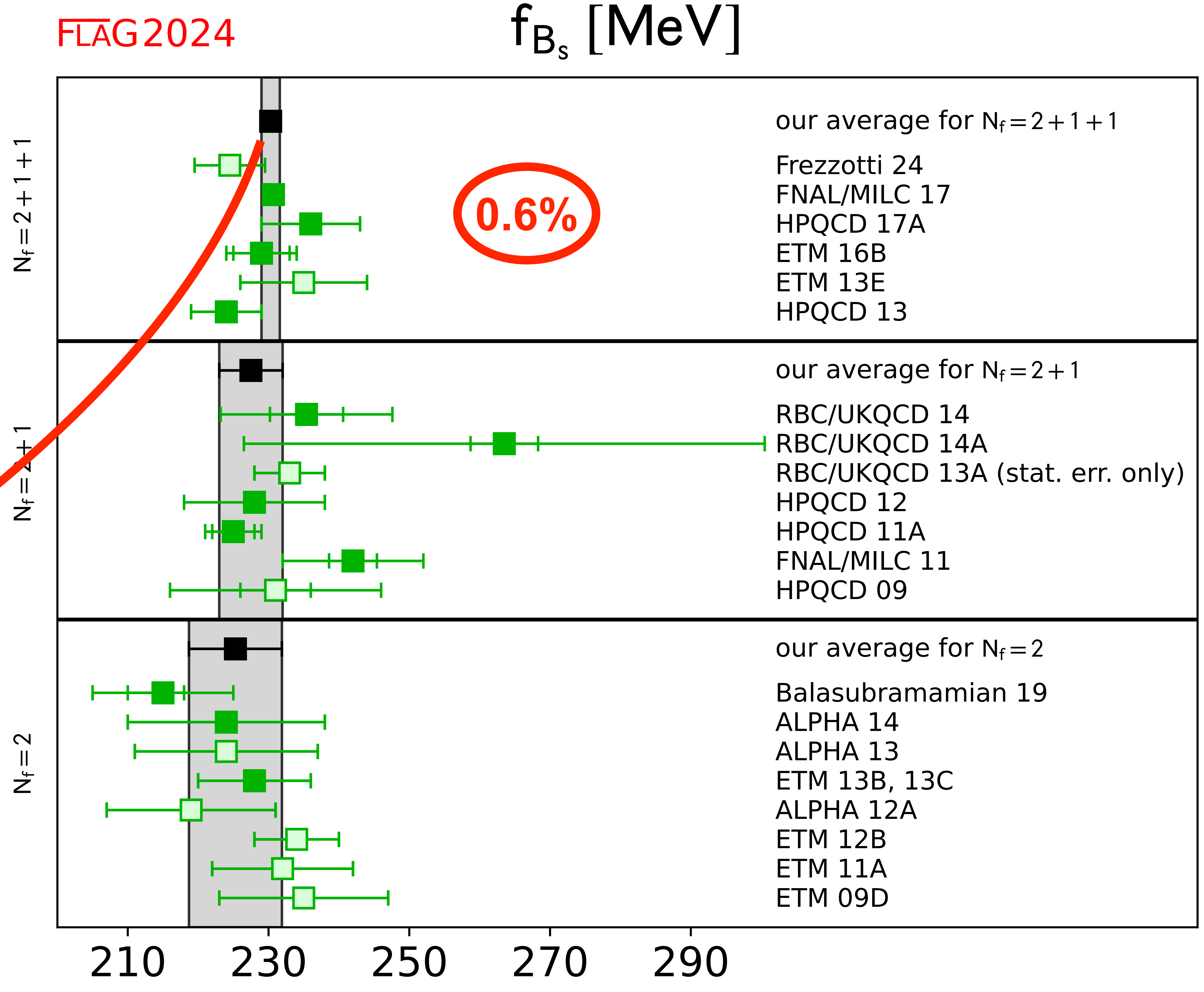
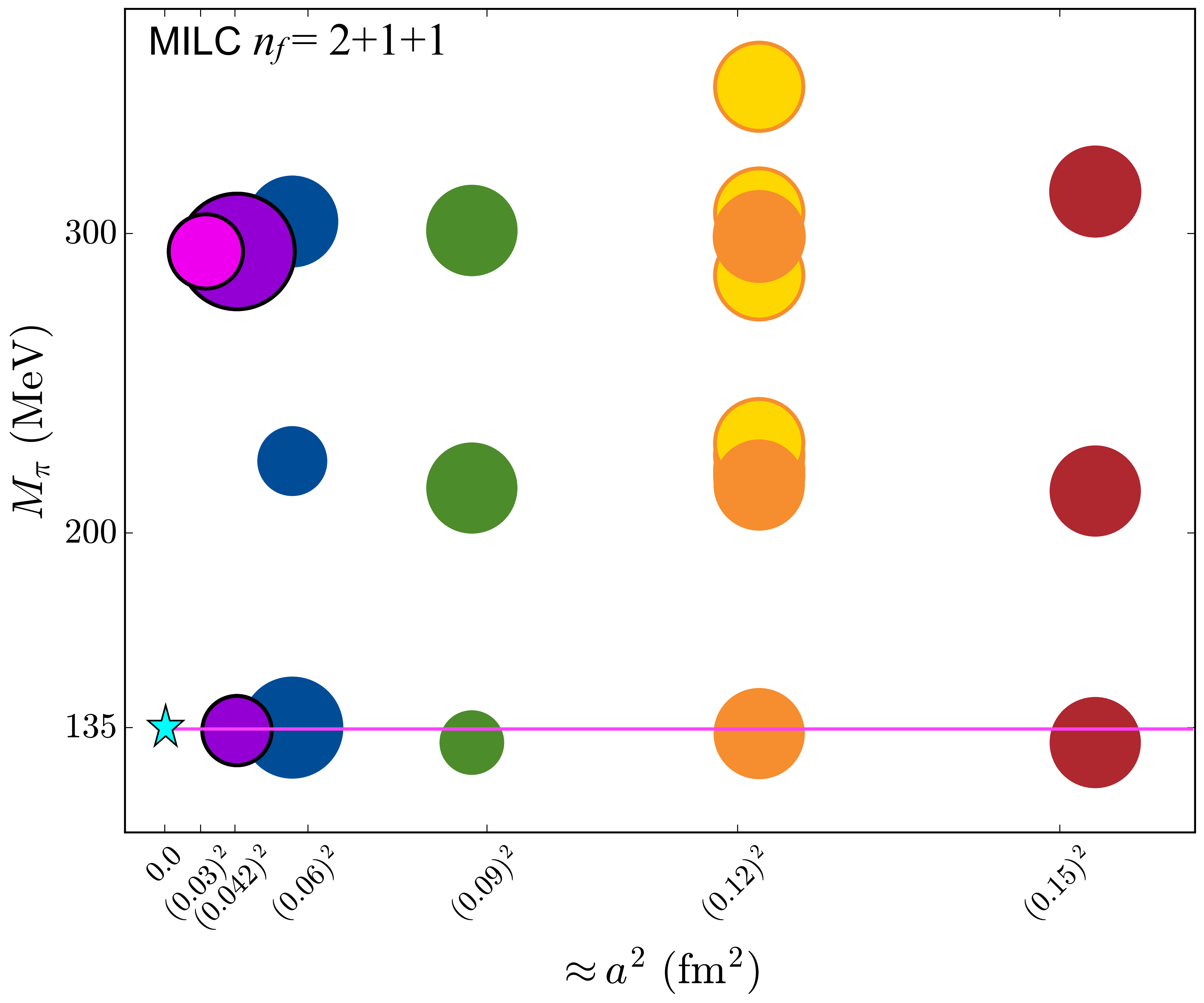
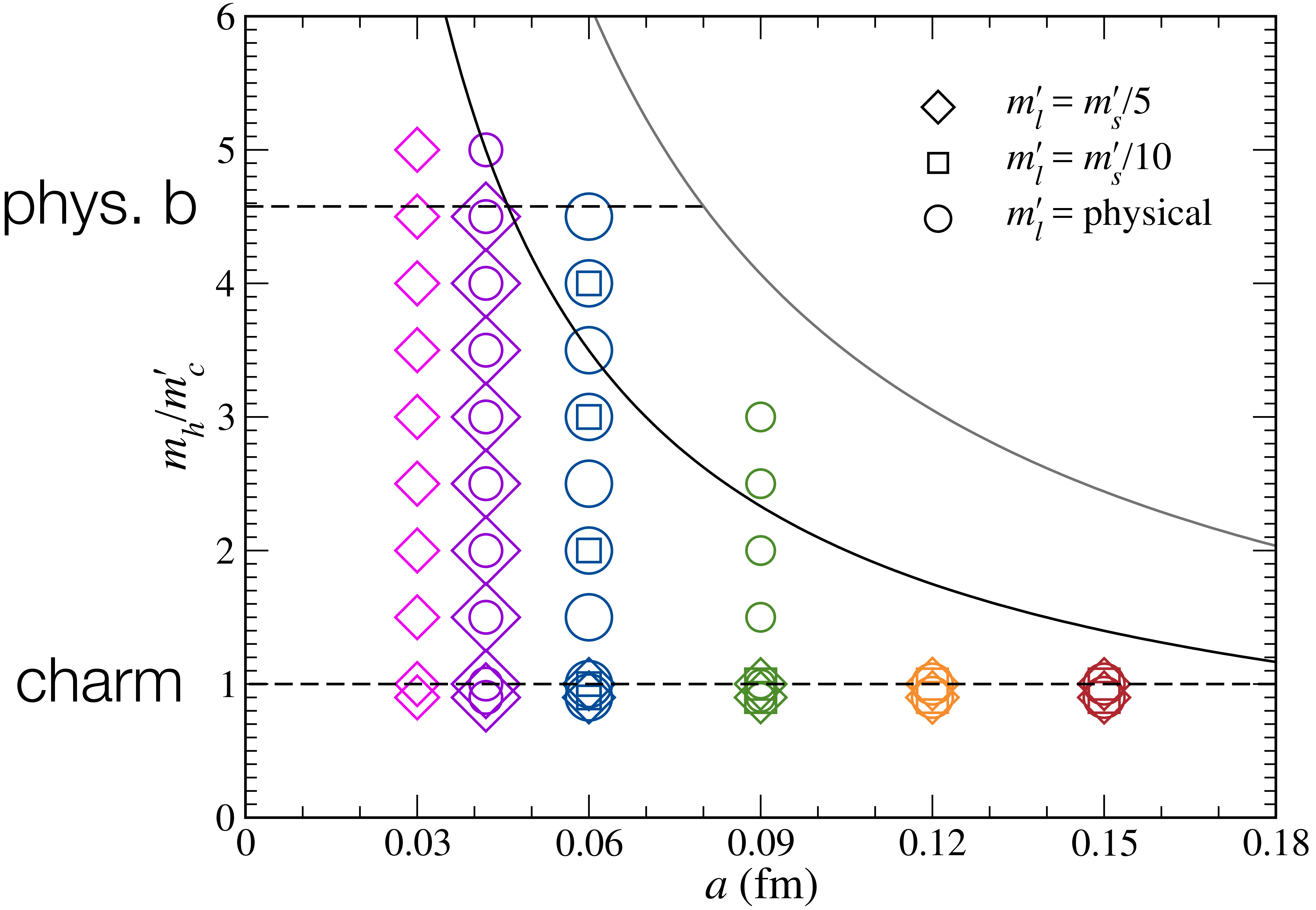




B, D meson decay constant results

A. Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]





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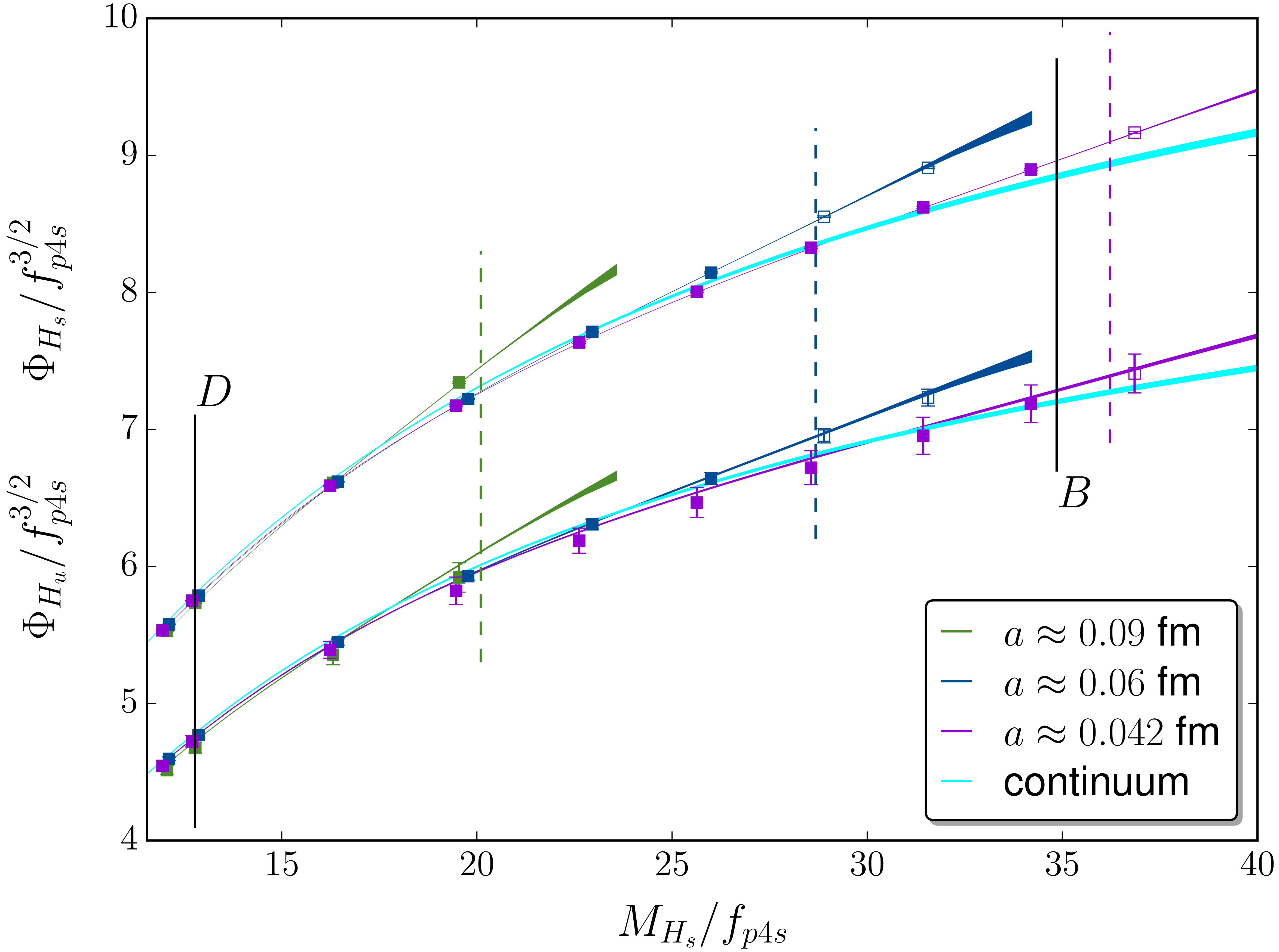

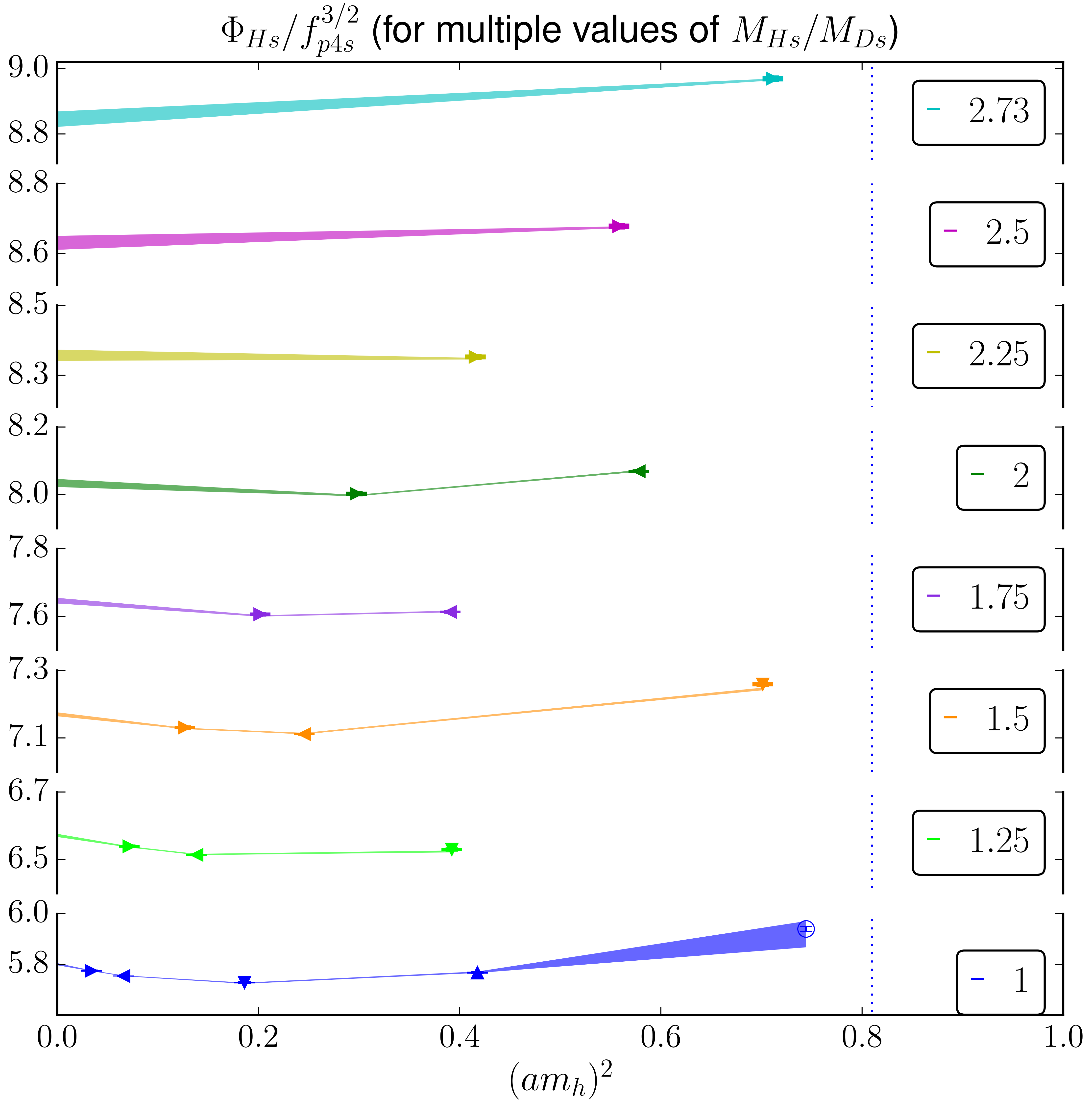

- small errors due to **physical light quark masses**
- improved quark action with small discretization errors
- no renormalization (Ward identity)

Heavy Quark discretization errors for HISQ fermions

Follana et al [HPQCD, hep-lat/0610092, 2007 PRD]
Monahan et al [arXiv:1211.6966, 2013 PRD]
Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]

Tree-level HISQ action:

$$S = \sum_x \overline{\psi}(x) \left\{ \sum_\mu \gamma_\mu \left[a\Delta_\mu - \frac{\mathbb{N}}{6} a^3 \Delta_\mu^3 \right] + am_0 \right\} \psi(x)$$

$$a\Delta_\mu \psi(x) = \tfrac{1}{2}[\psi(x+\hat{\mu}a) - \psi(x-\hat{\mu}a)],$$

Naik term:

$$m_1 \equiv E(0) \qquad \frac{m_1}{m_2} = \lim_{\boldsymbol{p}\to\boldsymbol{0}} \frac{E^2(\boldsymbol{p}) - E(0)^2}{\boldsymbol{p}^2} = 1 \quad \Rightarrow \quad am_0 = \sinh(am_1)\,\frac{1+\sqrt{1+3X}}{3}$$

$$\Rightarrow \quad \mathbb{N}(am_1) = \frac{4 - 2\sqrt{1+3X(am_1)}}{\sinh^2(am_1)} \qquad X(am_1) = \frac{2am_1}{\sinh(2am_1)}$$

Normalization of heavy-light bilinears:

$$Z_{J_{hx}} = \widetilde{\mathcal{C}h}_h^{1/2} \qquad \widetilde{\mathcal{C}h} = \cosh am_1 \left(1 - \tfrac{1}{2}\mathbb{N}\sinh^2 am_1\right)$$

remaining discretization errors:
$\sim (am_h)^4, \alpha_s(am_h)^2$