LGT4HEP TRAINEESHIP

EFFECTIVE FIELD THEORIES OF HADRONIC AND NUCLEAR PHYSICS

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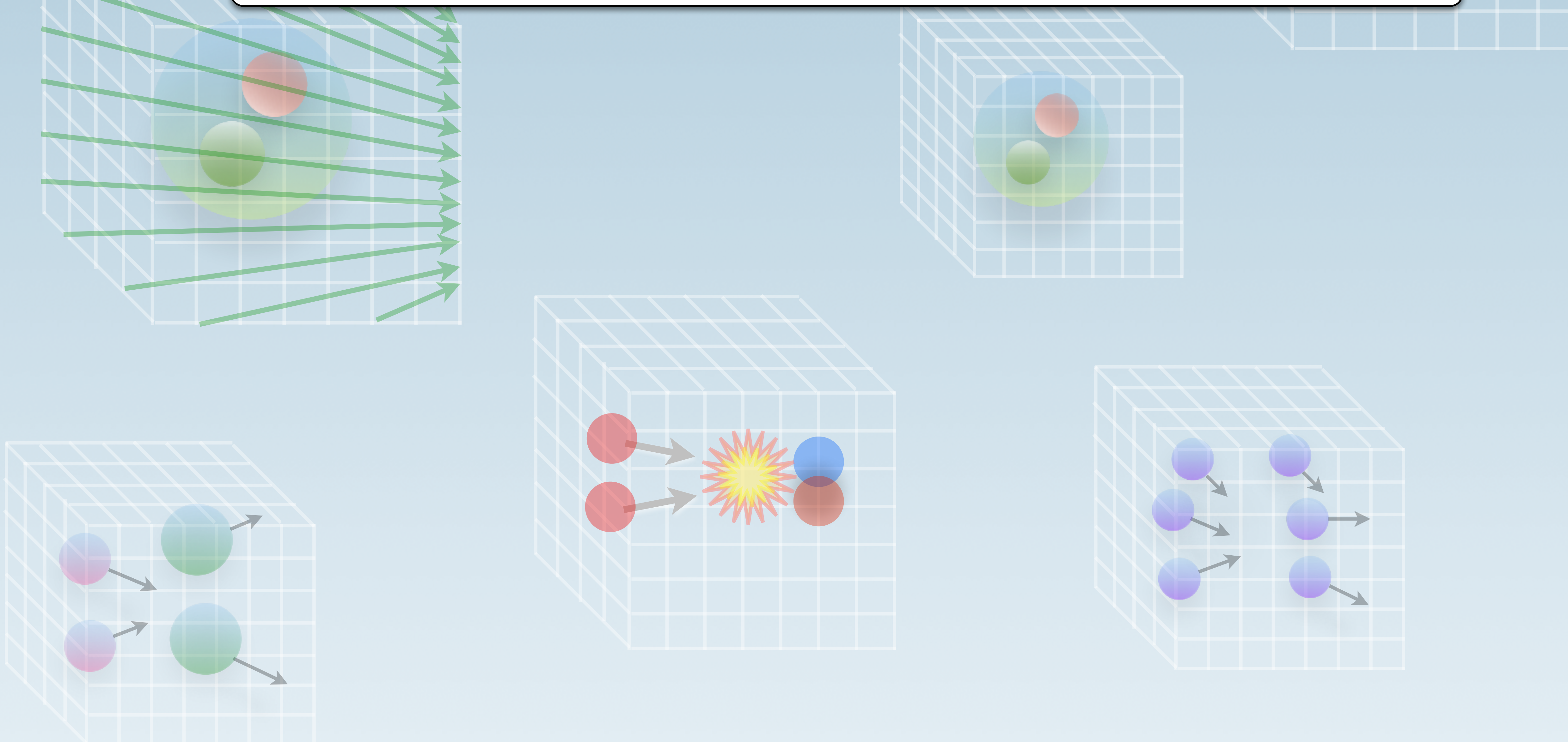
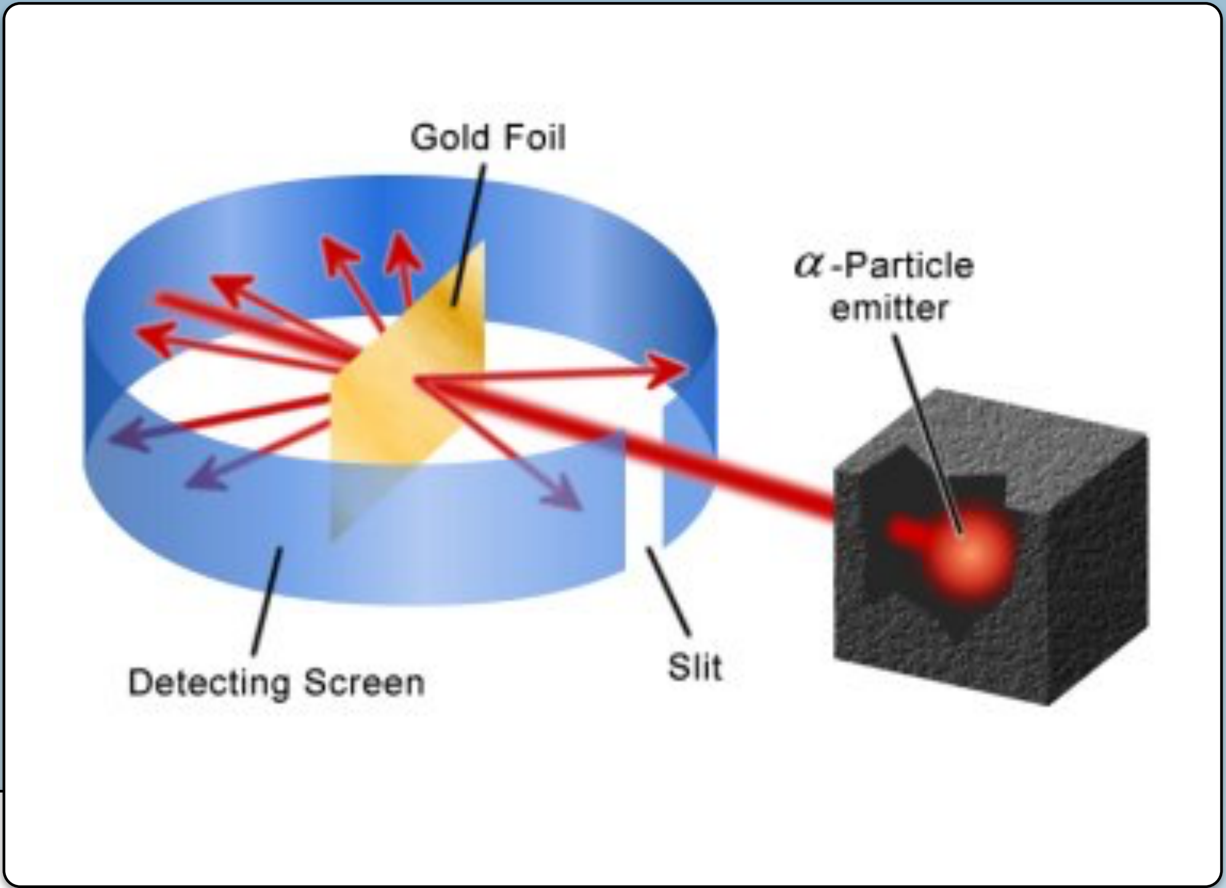


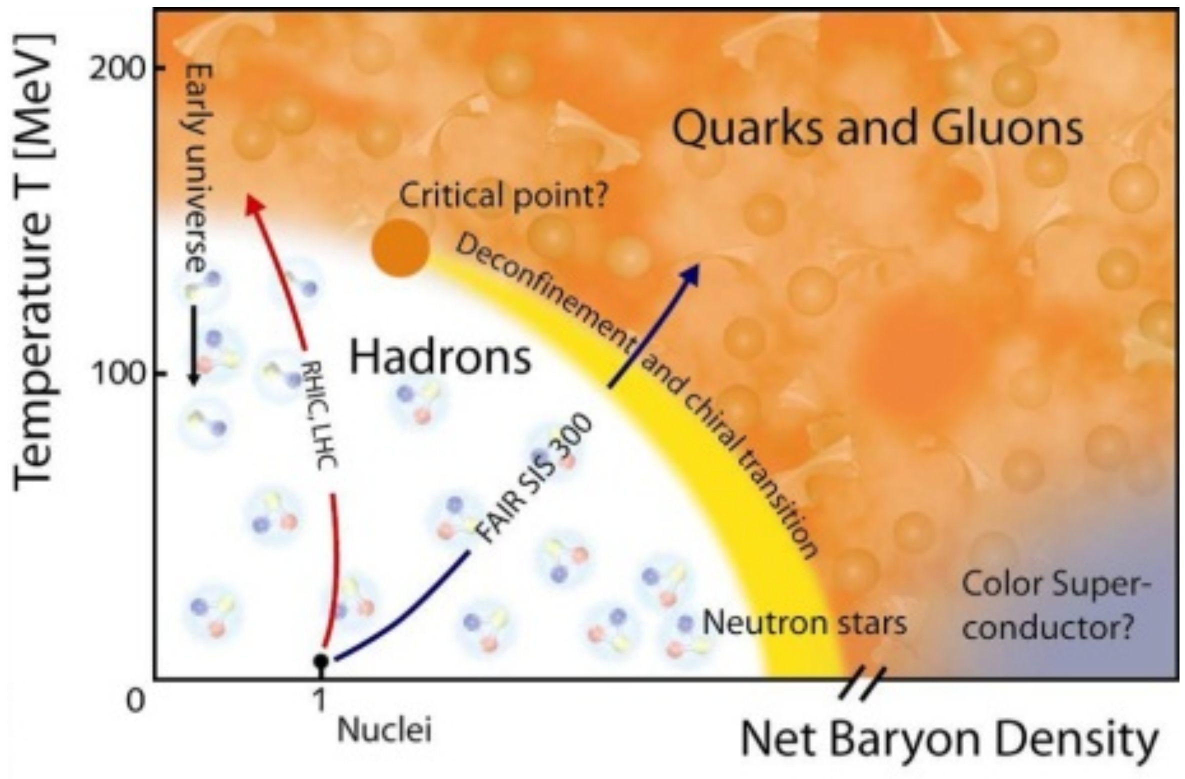
WHAT ARE THE BIG QUESTIONS IN HADRONIC AND NUCLEAR PHYSICS TODAY?



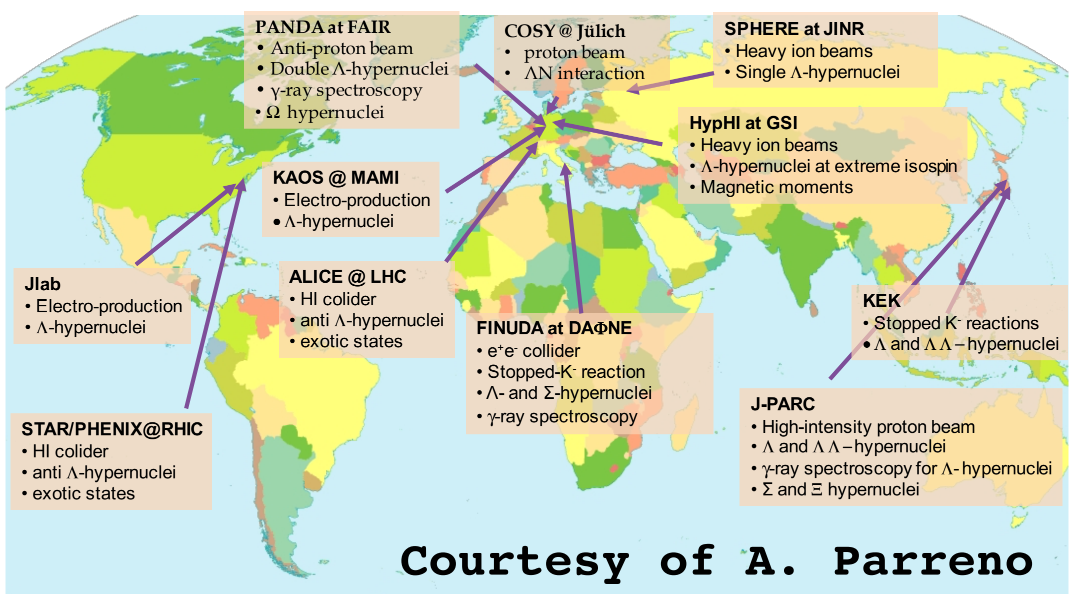
Discovery of a nucleus by Geiger, Marsden, and Rutherford in 1911 marks the start of nuclear physics…

…yet major questions in nuclear physics remain unanswered. questions like:

- What is the nature of dense matter in universe?
- What constitutes the interior of a neutron star?
- What are the phases of strongly interacting matter?
- Can rare exotic isotopes made in laboratories give some clues?

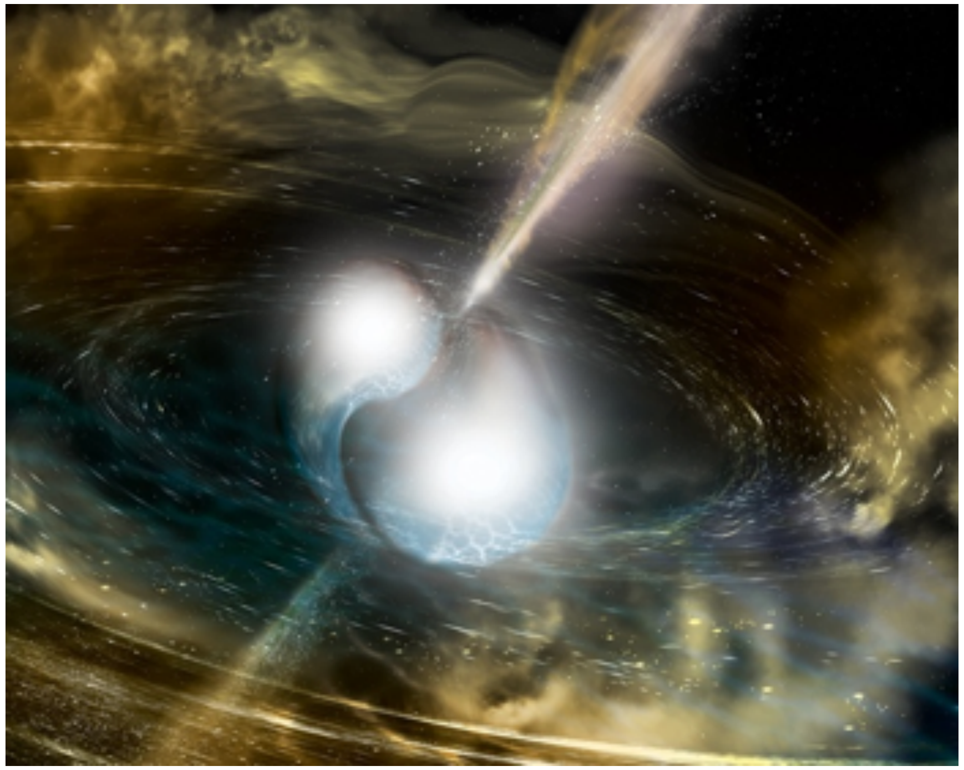


Source: The Facility for Antiproton and Ion Research (FAIR),GSI, Darmstadt, Germany.

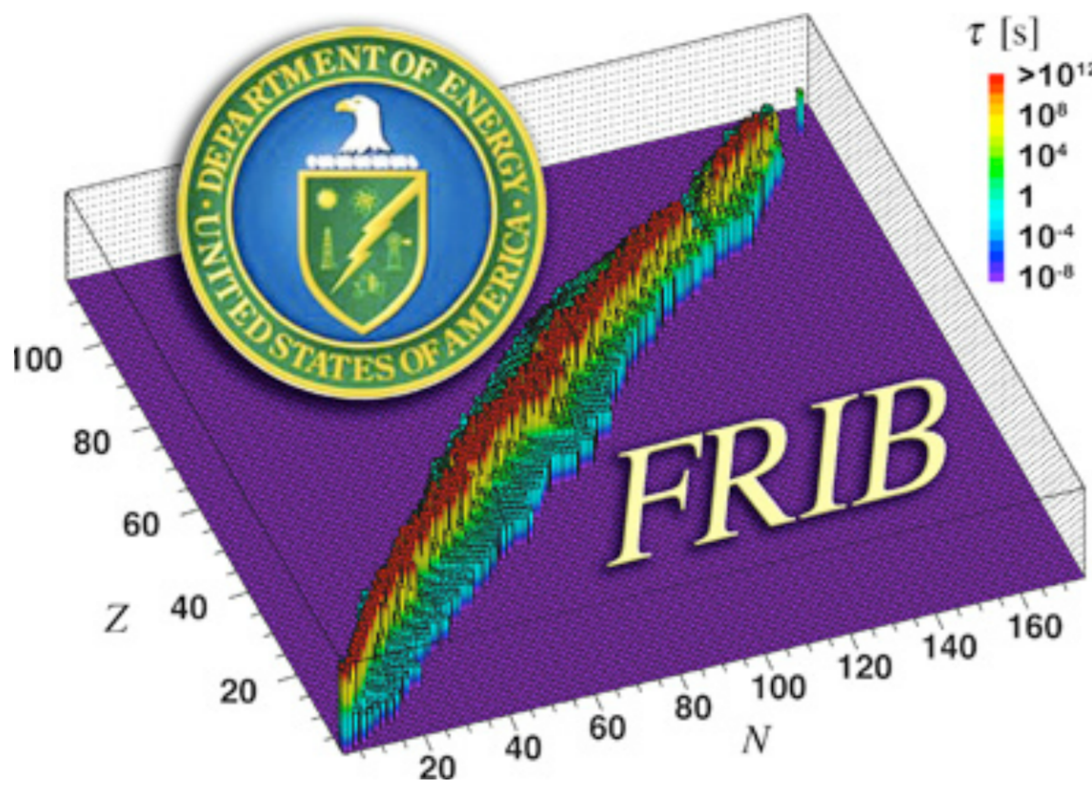


updated from J. Pochodzalla, Int. Journal Modern Physics E, Vol 16, no. 3 (2007) 925-936

- What processes occur in core-collapse supernovae and neutron-star mergers?
- Where do the heavy elements on earth come from?
- How far can we go in making neutron-rich isotopes on earth and can we understand their properties better?

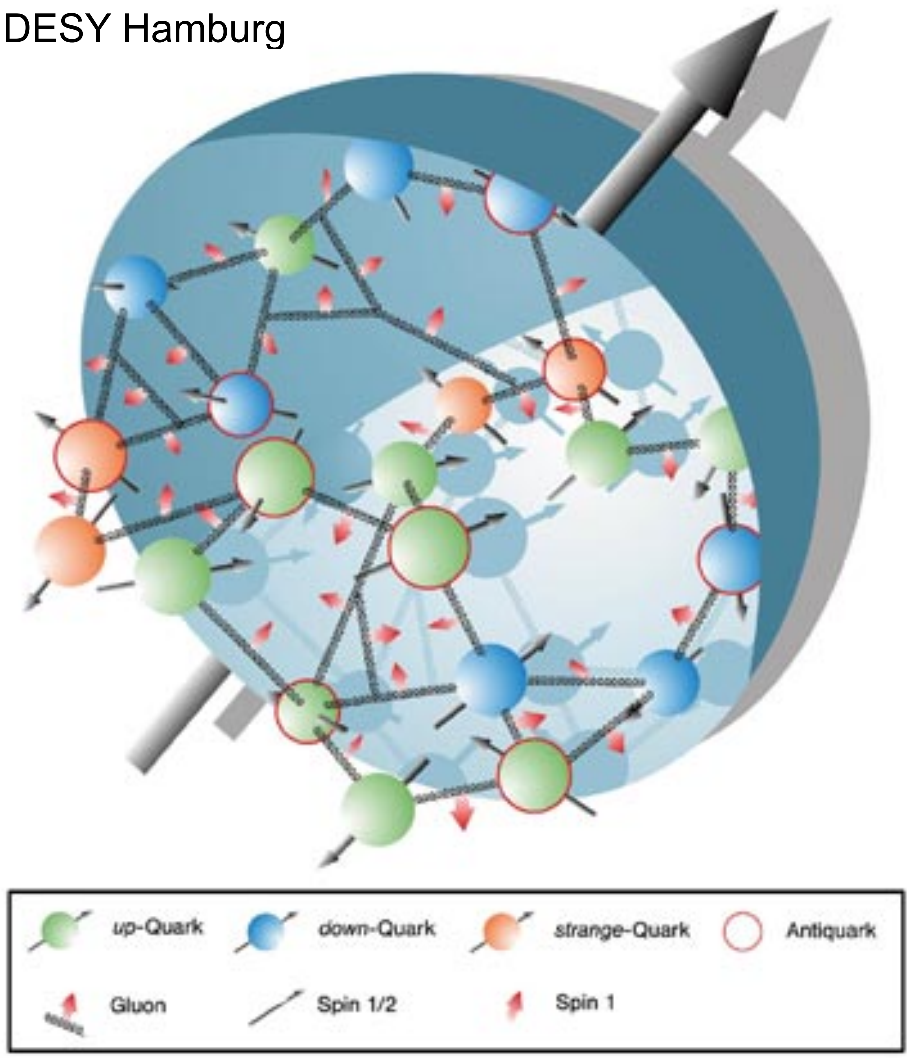


National Science Foundation/LIGO/Sonoma State University/A. Simonnet



- What role the partonic (quark and gluon) degrees of freedom play in the structure of nucleons?
- Does the nature of nucleon change when bound in a heavy nucleus?
- How important is the role of gluons in nucleon and nuclei properties?

DESY Hamburg



- Are fundamental symmetries of Standard Model violated?
- Are new interactions in play in nature?
- Nuclei serve as a laboratory to make discovery in this area, but do we know well how they interact with external probes?

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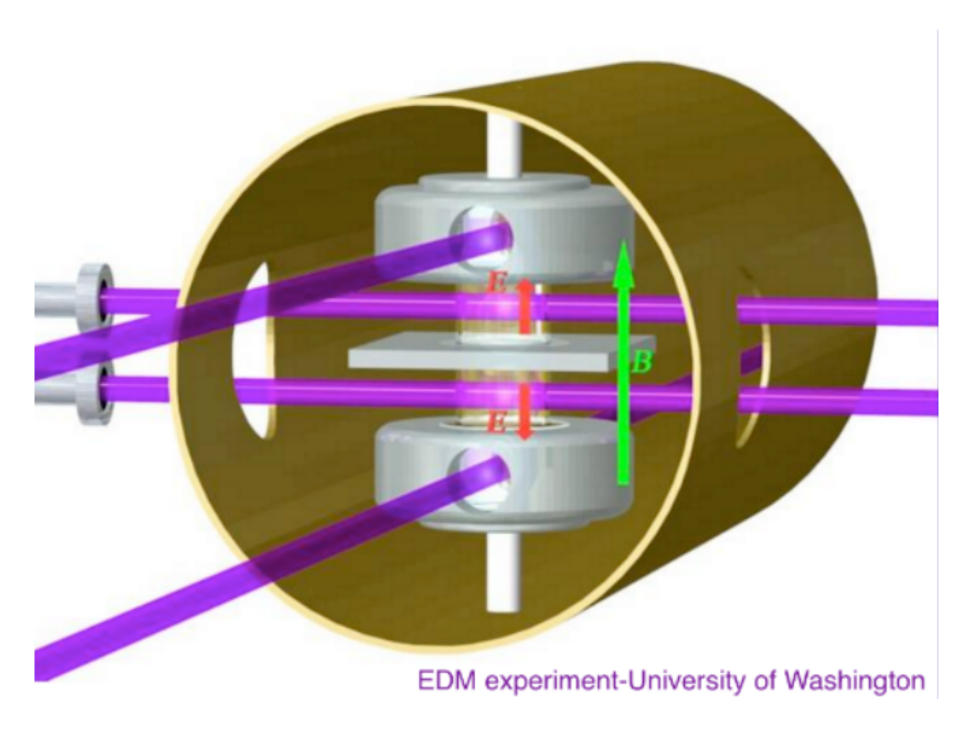
Example 1) Electric dipole moment of nucleon, nucleus, and atoms and time reversal invariance



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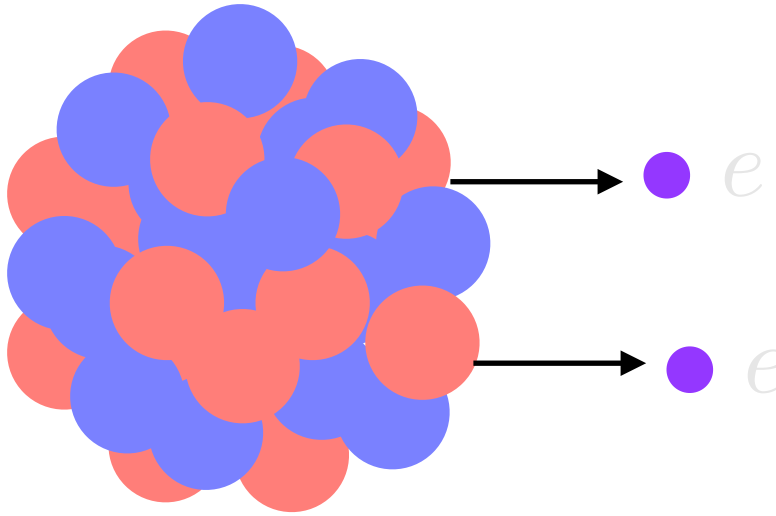
Example 2) Neutrinoless double-β decay and lepton-number violation



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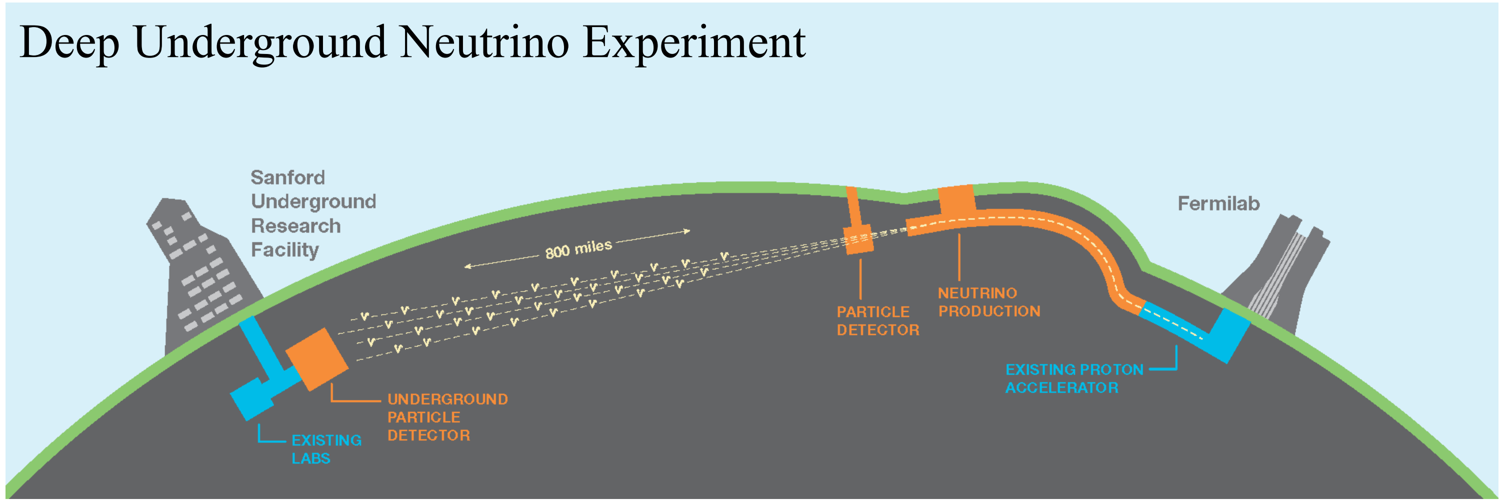
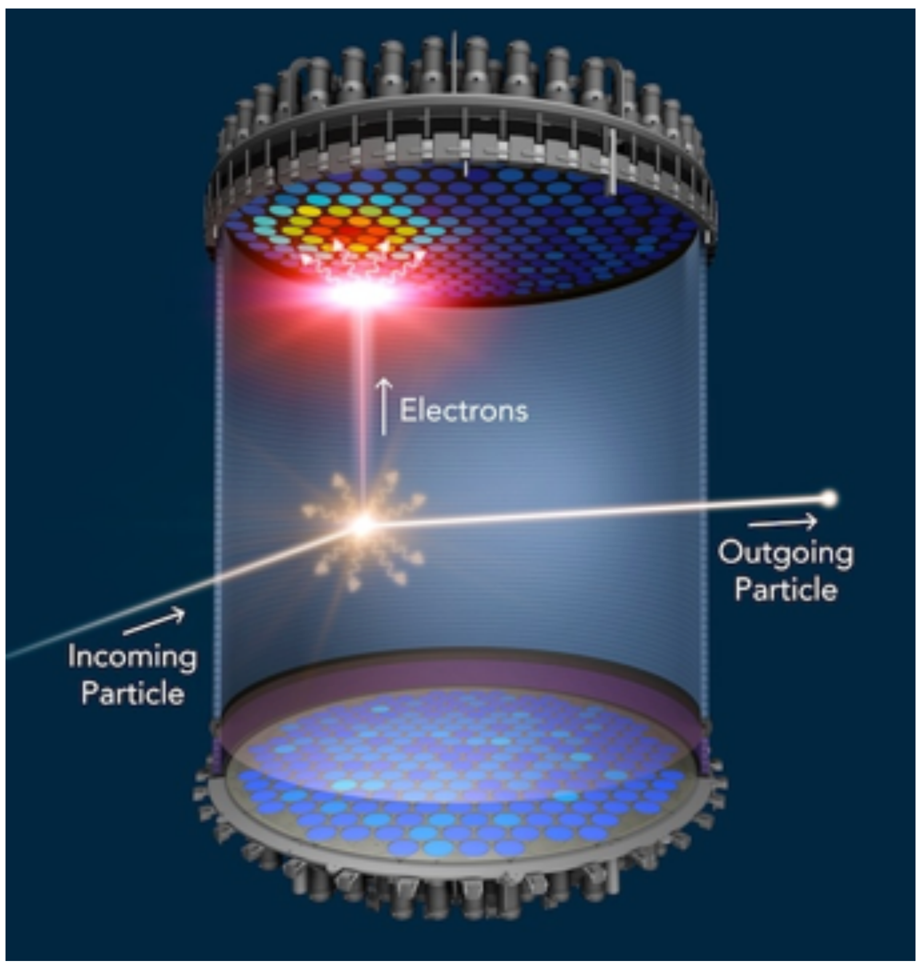
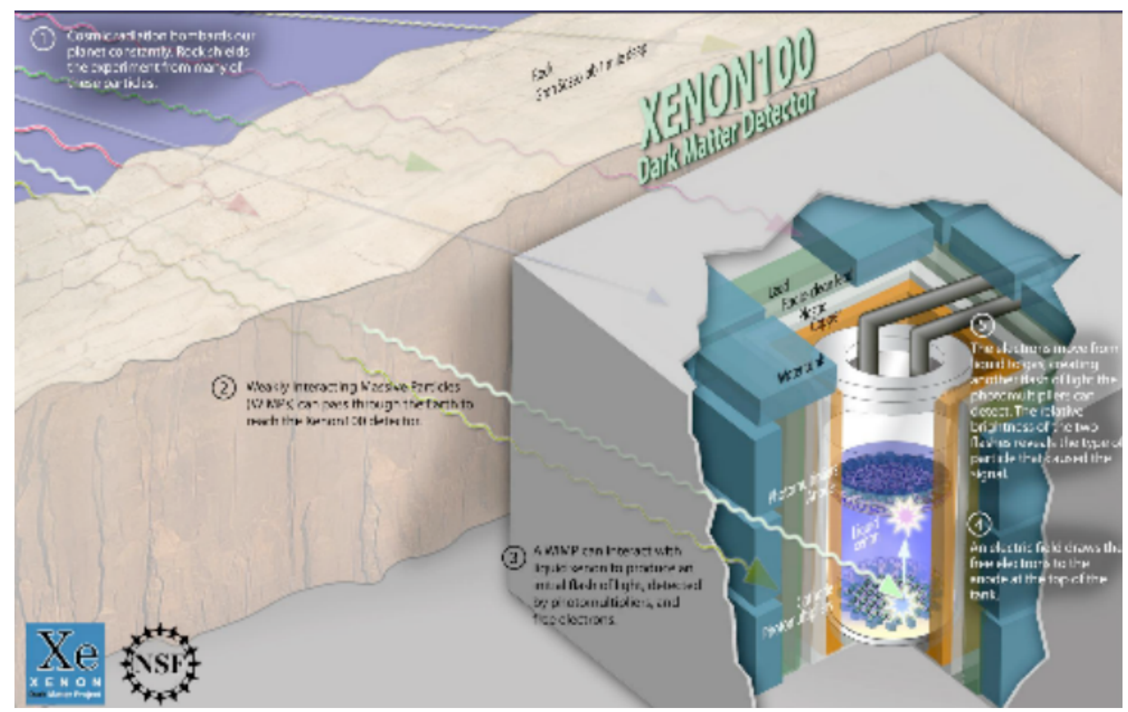
- Nuclei serve as a laboratory to make discovery in this area, but do we know well how they interact with external probes?

Example 3) Neutrino oscillations and CP violation

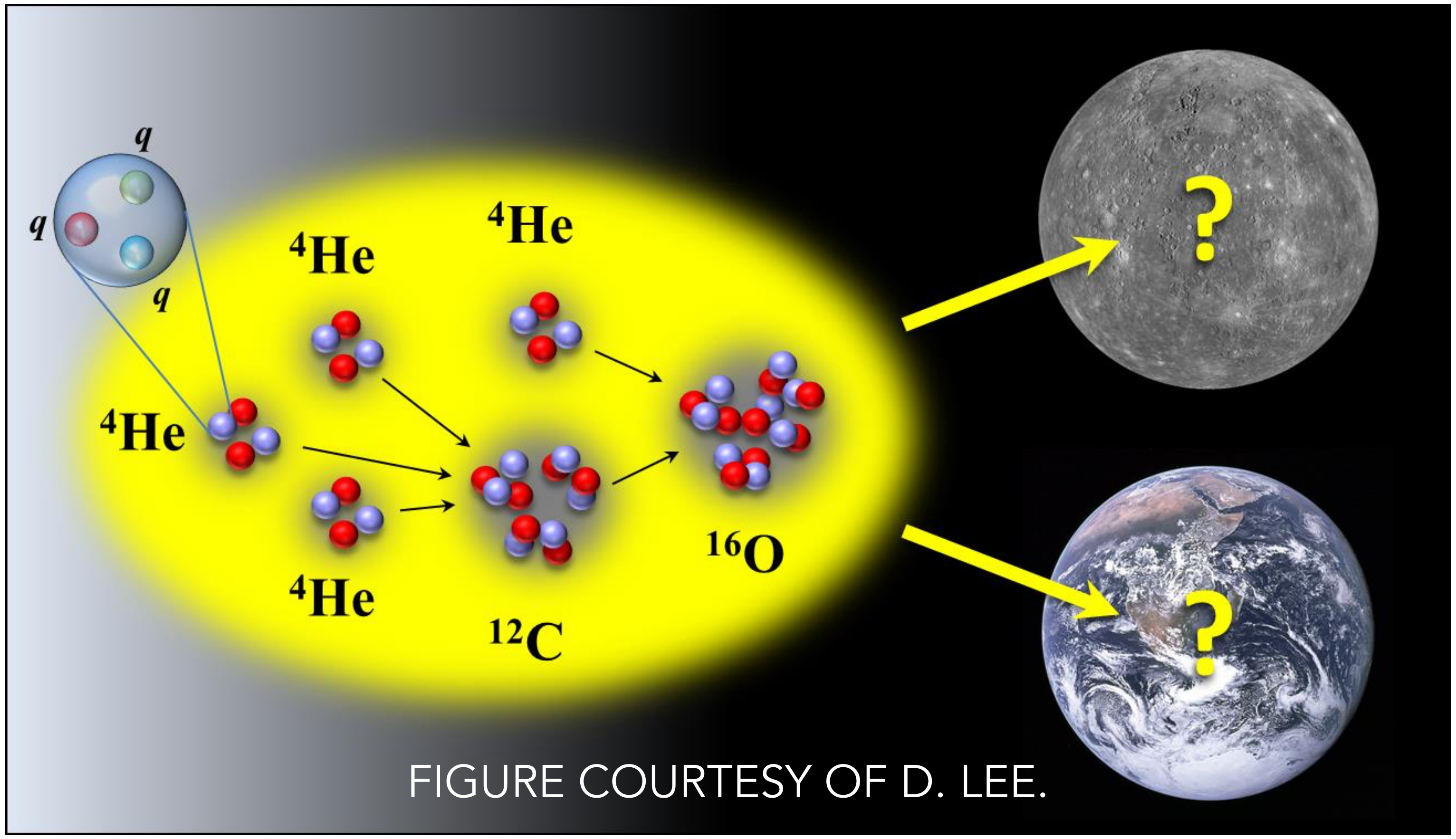


- Can a dark-matter candidate be discovered through its interactions with matter?
- How well can we predict potential dark matter-nucleus interactions?





- Would have we existed if the input parameters of the Standard Model had been set differently in nature?
- What is the fate of nuclear physics if quark were lighter or heavier?



EXERCISE 1

Find out what the largest/most prominent nuclear-physics experiments in your region are (city, state, nearby state). Among the questions posed in this lecture, which ones are the physics target of the experiments your region?

HOW TO APPROACH THESE PROBLEMS?

Traditionally:

i) Build nuclear potentials using experimental data in few-nucleon systems

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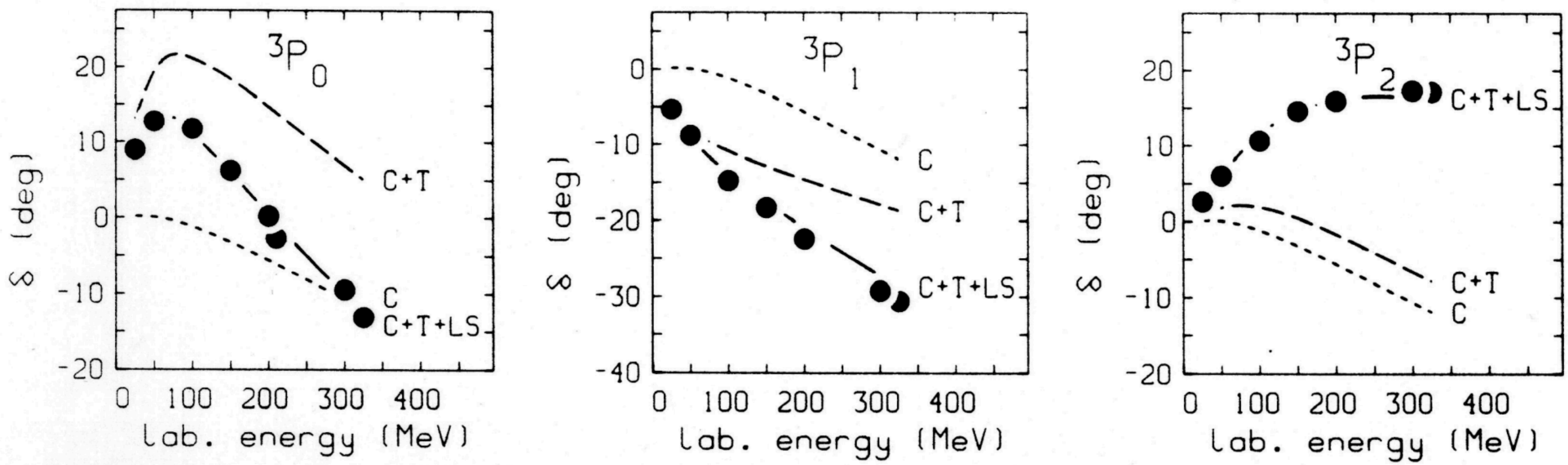

http://www.int.washington.edu/PROGRAMS/talent13

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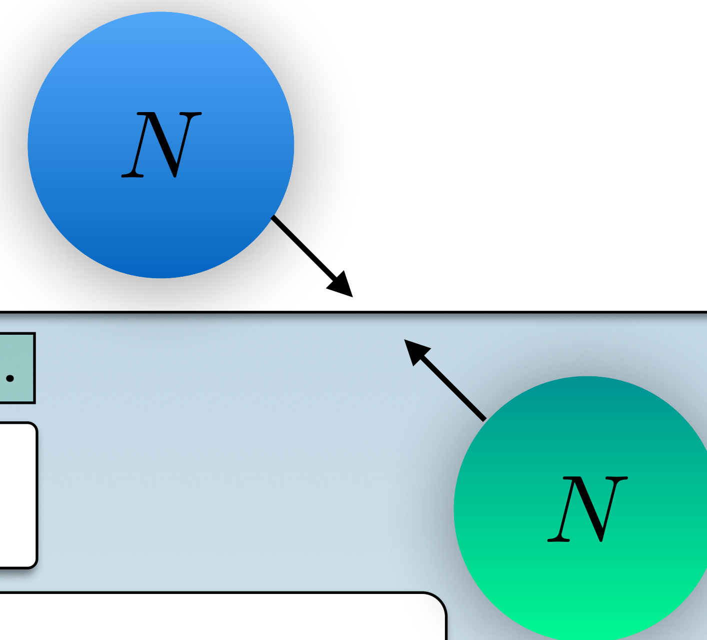

Wiringa, Stoks, Schiavilla, Phys.Rev.C51:38-51 (1995).

Example: *NN* potential $V_{\mathrm{AV18}} = V_\pi + V_{\mathrm{sd}} + V_{\mathrm{EM}}$

For example the one-pion exchange potential is:

$$V_\pi(\mathbf{r}) \propto f^2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left[(3\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2})+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right]\frac{e^{-m_\pi r}}{r}$$

14 OPERATOR STRUCTURES…

$$\{1,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, S_{12}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, (\mathbf{L}\cdot\mathbf{S})^2\}\otimes\{1,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\}$$

…and 4 more.

Traditionally:

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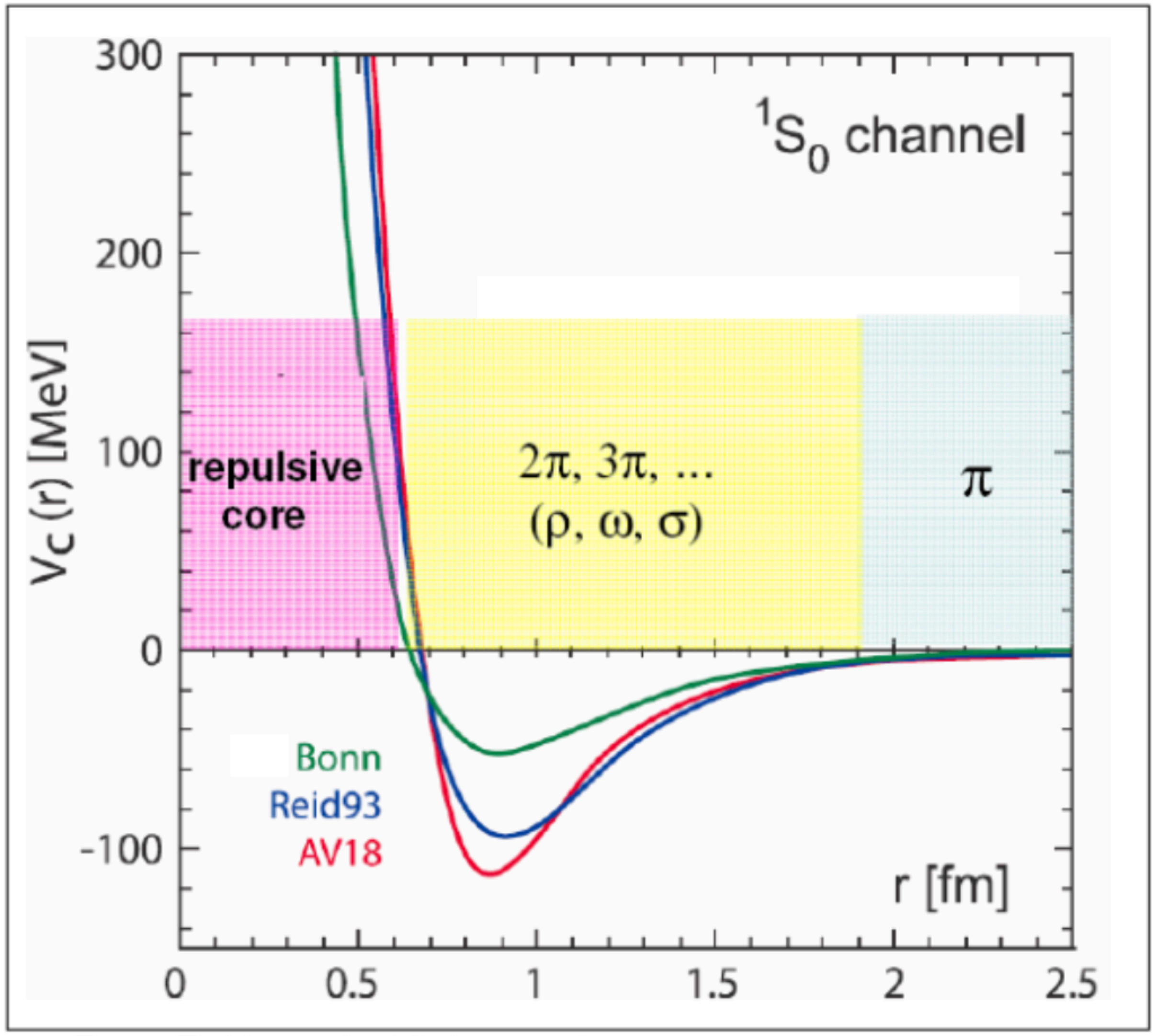
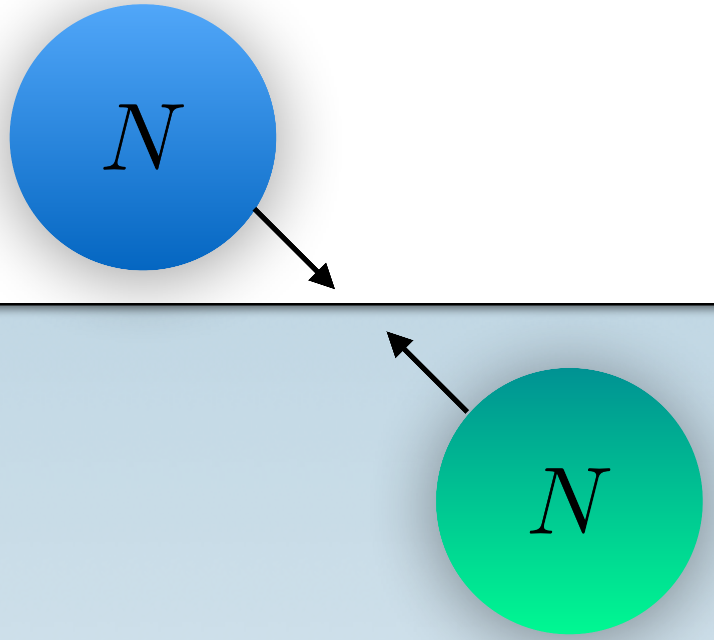


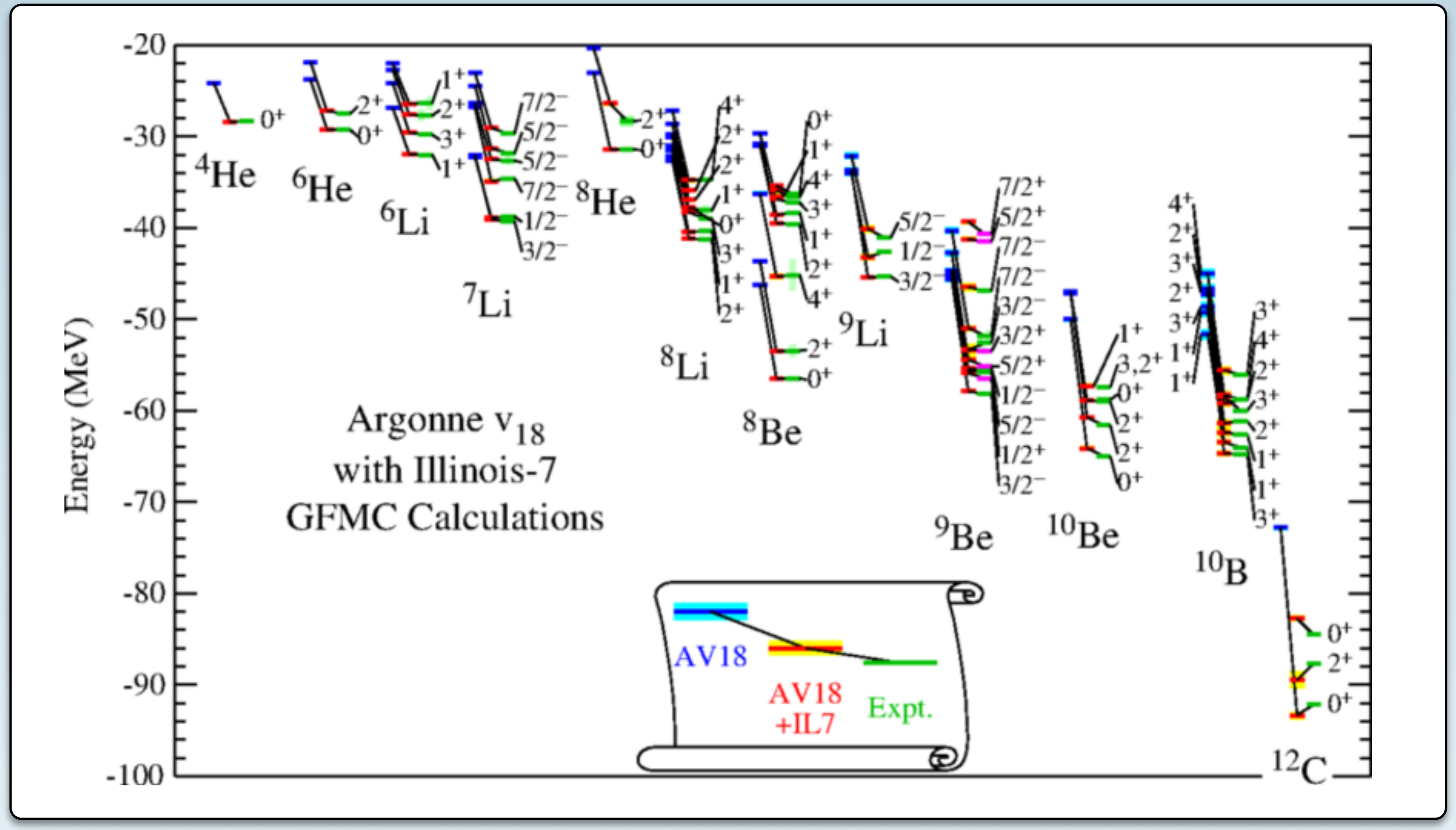


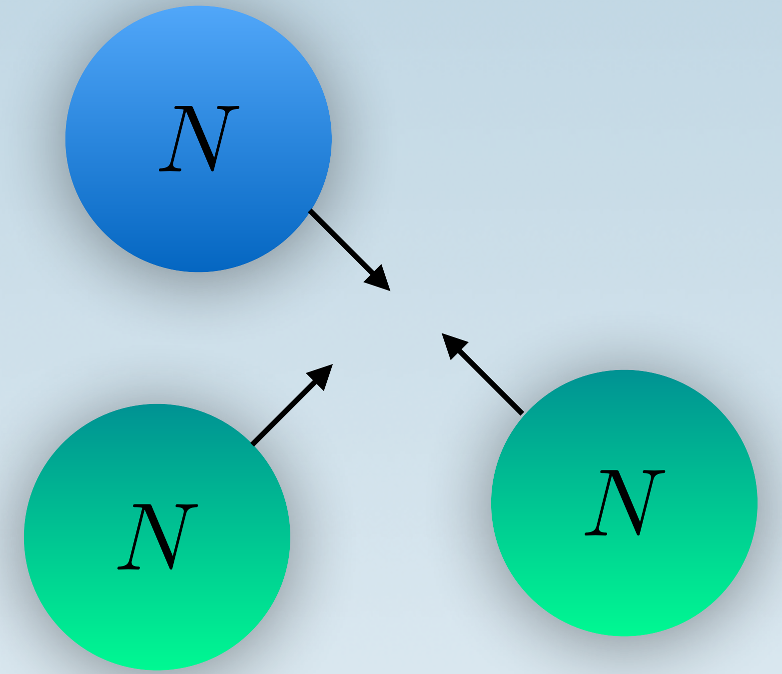
T. Hatsuda, Oslo 2008.

Traditionally:

i) Build nuclear potentials using experimental data in few-nucleon systems

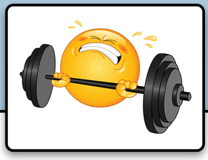
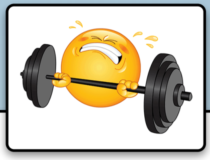
ii) solve the many-body Schrodinger equation for larger systems





Carlson et al., Review of Modern Physics (2014).

EXERCISE 2

Isospin symmetry, that is a SU(2) symmetry in the space of up and down quark flavors, is a good approximate symmetry of strong interactions. Consider two-nucleon (*NN*) systems in nature under this symmetry. Classify positive-parity *NN* systems according to their spin, isospin, and angular momentum. Recall that fermionic wavefunctions must be fully antisymmetric under exchange of the fermions. Each nucleon has spin 1/2 and isospin 1/2. Proton (*p*) and neutron (*n*) have $I_3 = 1/2$ and $I_3 = -1/2$, respectively, where I_3 is the third component of isospin. Write down the states explicitly according to their spin and flavor content. Which one of these channels exhibit a bound state in nature?

SO WHAT IS MISSING?

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- No uncertainty can be assigned. Not clear where the model fails. Cannot improve systematically.
- No connection to underlying theory of strong interactions. Not fully respecting chiral symmetry, etc.
- Unrelated to other strong interaction channels (nucleon-pion, etc.)
- Unable, by its nature, to answer some questions: phases of dense matter, fine tunings in nuclear physics, etc.

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QCD is the fundamental theory, so ideally we should start from there…

QUANTUM CHROMODYNAMICS (QCD)

QCD is a SU(3) Yang-Mills theory augmented with several flavors of massive quarks:

Quark kinetic and mass term

Quark/gluon interactions

$$\mathcal{L}_{QCD} = \sum_{f=1}^{N_f} \left[\bar{q}_f (i\gamma^\mu \partial_\mu - m_f) q_f - g A^i_\mu \bar{q}_f \gamma^\mu T^i q_f \right]$$

$$-\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \frac{g}{2} f_{ijk} F^i_{\mu\nu} A^{i\mu} A^{j\nu} - \frac{g^2}{4} f_{ijk} f_{klm} A^j_\mu A^k_\nu A^{l\mu} A^{m\nu}$$

Gluons kinetic and interaction terms

QUANTUM CHROMODYNAMICS (QCD)

Observe that:

i) There are only $1 + N_f$ input parameters plus QED coupling. Fix them by a few quantities and all aspects of nuclear physics is predicted (in principle)!

ii) QCD is asymptotically free such that: $\alpha_s(\mu') = \dfrac{1}{2\underbrace{b_0}_{\text{Positive constant for } N_f \leq 16} \log \frac{\mu'}{\Lambda_{QCD}}}$

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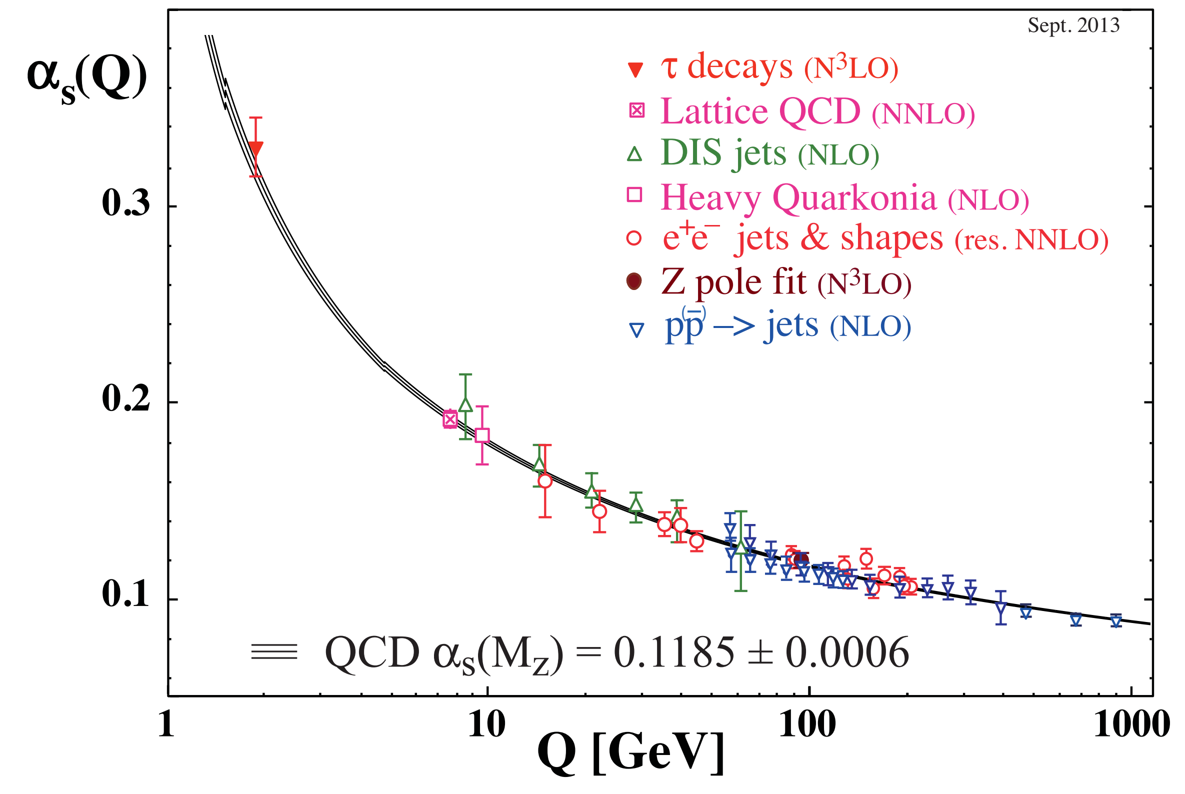

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Positive constant for $N_f \leq 16$

WHAT CAN WE DO AT LOW ENERGIES?

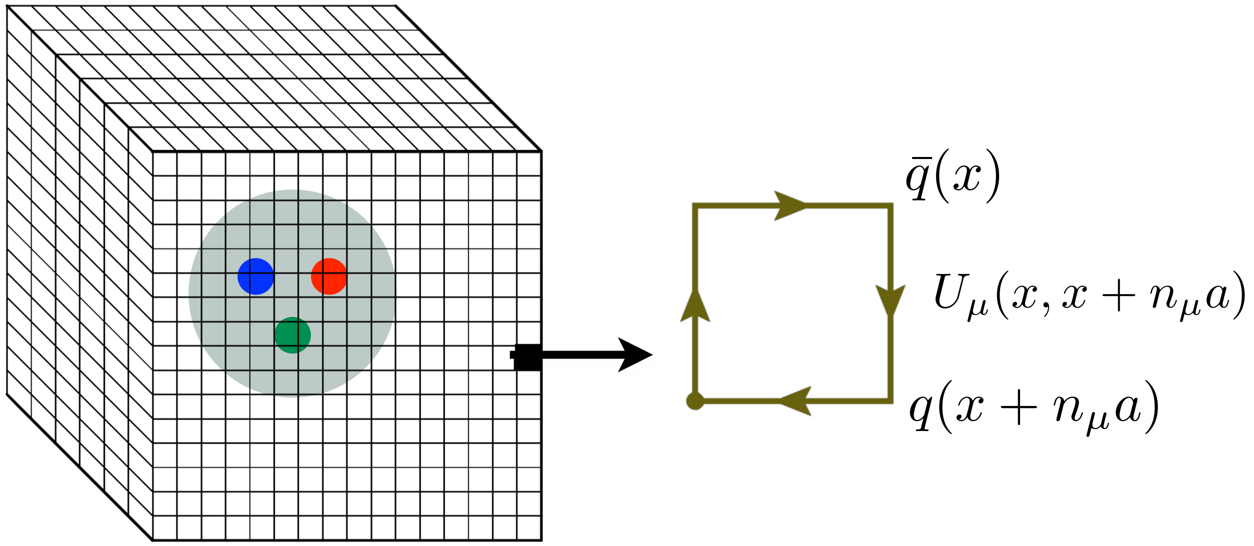
Solve it nonperturbatively:
Lattice QCD

$$\mathcal{L}_{QCD}[q, \bar{q}, A; m_q, \alpha_s]$$

$$\int d^4x \to a^4 \sum_{\mathbf{n}}$$

$$\mathcal{L}_{LQCD}[q, \bar{q}, U[A]; m_q a, \beta]$$

Extrapolate to infinite volume, zero lattice spacing



WHAT CAN WE DO AT LOW ENERGIES?

Or

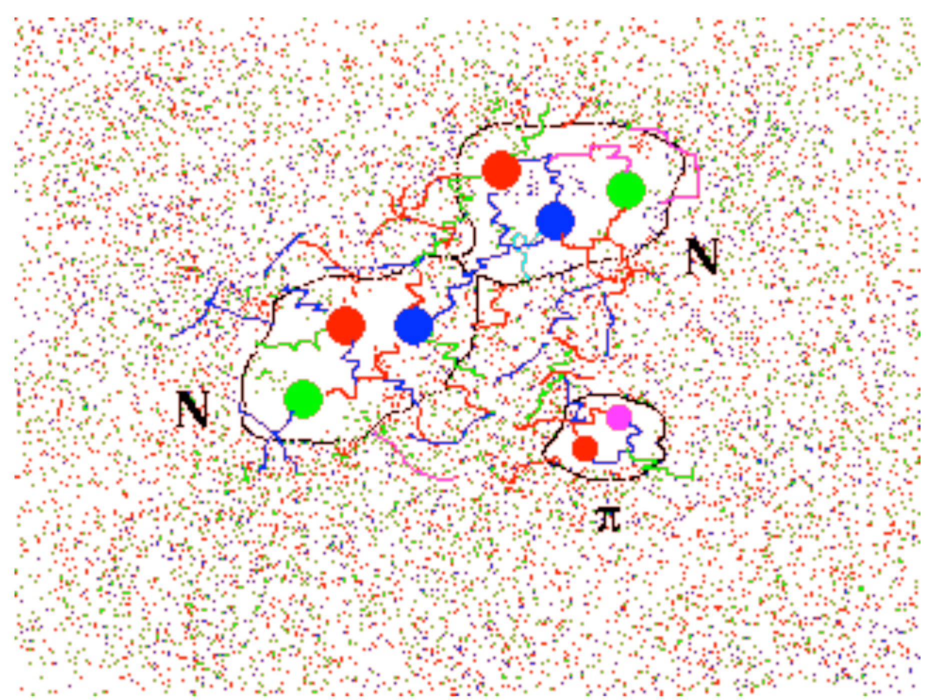
Write down effective interactions consistent with it: Effective field theories

$$\mathcal{L}_{QCD}[q, \bar{q}, A; m_q, \alpha_s]$$

$$p, m << \Lambda$$

$$\mathcal{L}_{EFT}[\pi, N, ...; m_\pi, m_N, ..., C_i]$$

Low-energy constants



WHAT CAN WE DO AT LOW ENERGIES?

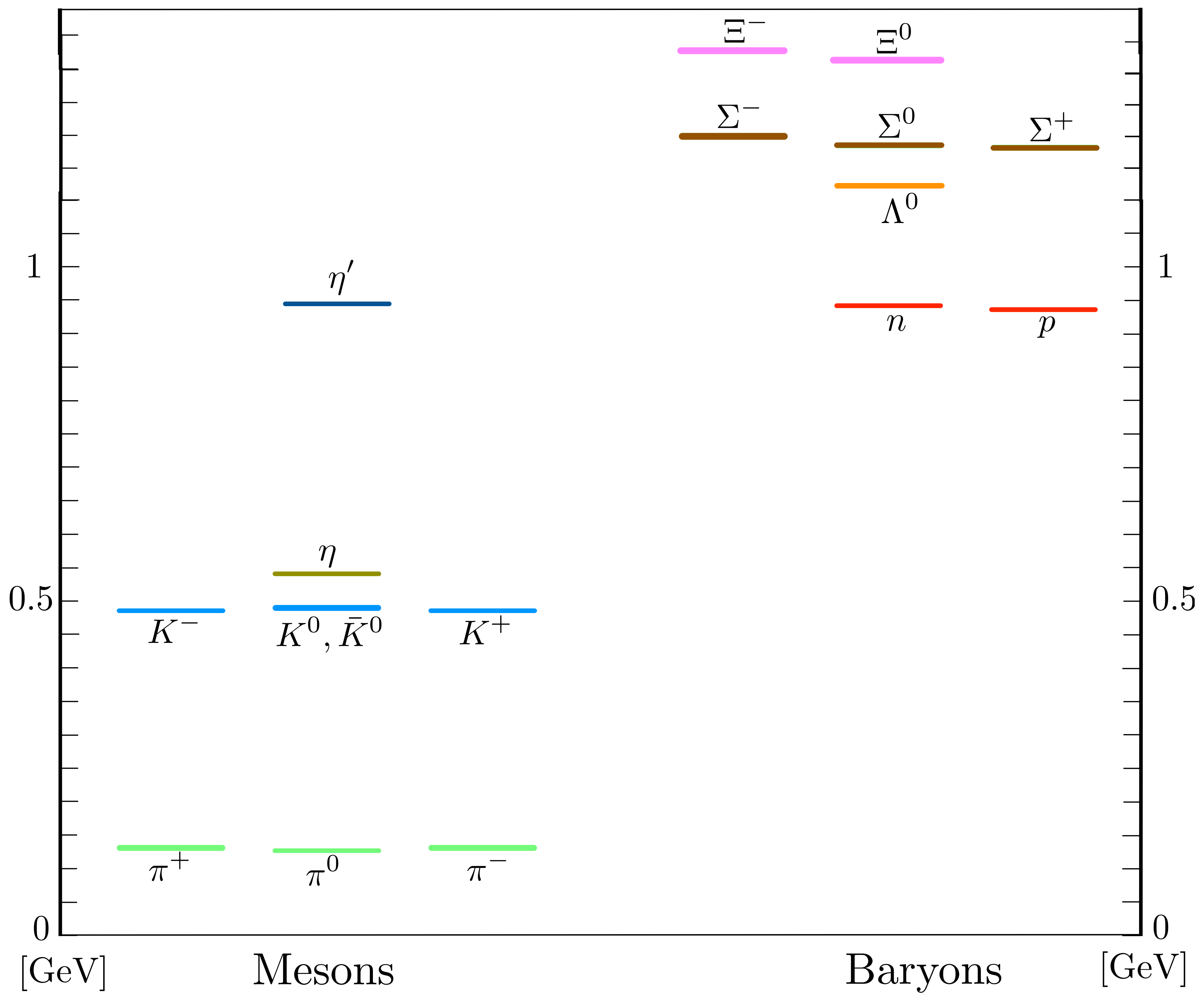
Both?

A BRIEF GUIDE TO HADRONIC AND NUCLEAR EFTs…

EFT PROCEDURE:

- Inspect the low-energy spectrum of hadrons with regard to symmetries of QCD and possible symmetry breakings
- Write the most general interactions consistent with symmetries using only low-energy degrees of freedom
- Rank the interactions according to a momentum power counting
- Keep as many terms as needed for your precision goal
- Fix the coefficients of low-energy interactions by experiment (or lattice QCD)!
- Use the constrained EFT to make prediction for all other observables

- Inspect the low-energy spectrum of hadrons with regard to symmetries of QCD and possible symmetry breakings

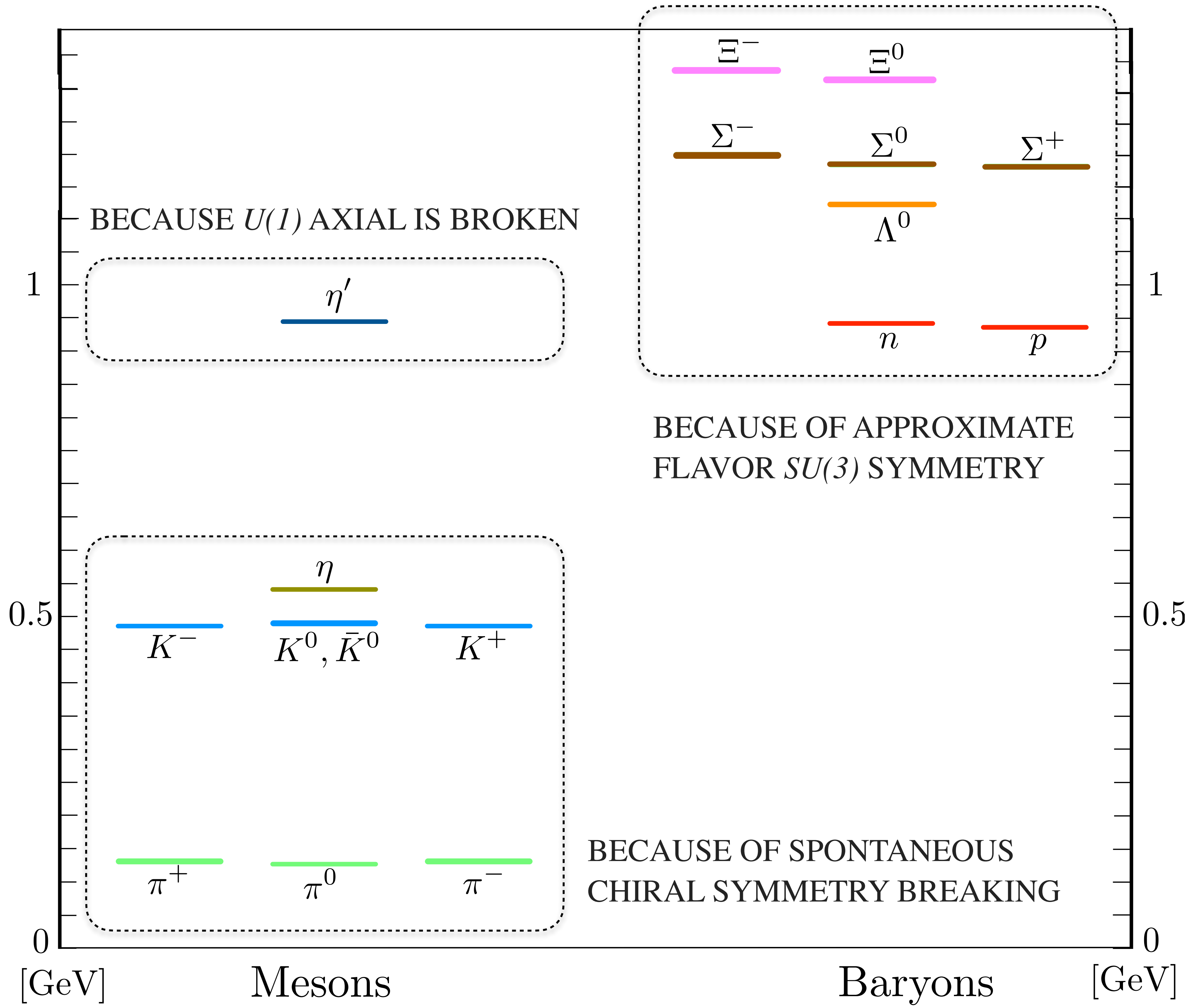


- With 3 massless quarks, Lagrangian is invariant under:

$$SU(3)_L \otimes SU(3)_R \otimes U(1)_V \otimes U(1)_A$$

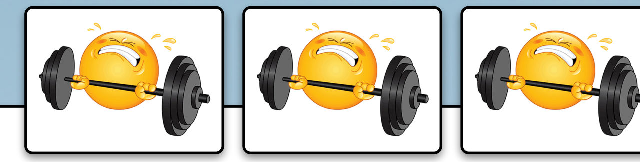
where L and R refer to chirality of quarks.

- $U(1)_V$ gives the conservation of baryon number.
- $U(1)_A$ is broken by a quantum anomaly.
- $SU(3)_L \otimes SU(3)_R$ is broken down to $SU(3)_V$ with a nonzero quark condensate in vacuum: 8 Goldstone bosons are created.
- With small masses for the quarks, chiral symmetry is approximate. There are 8 pseudo-goldstone bosons with lighter mass compared with other hadrons.
- Such spontaneous chiral symmetry breaking also breaks parity in the spectrum. e.g. N(1535) is the parity doublet of nucleon and is much heavier!



BECAUSE U(1) AXIAL IS BROKEN
η'
BECAUSE OF APPROXIMATE FLAVOR SU(3) SYMMETRY
Ξ^-
Ξ^0
Σ^-
Σ^0
Σ^+
Λ^0
n
p
η
K^-
$K^0, \bar{K}^0$
K^+
π^+
π^0
π^-
BECAUSE OF SPONTANEOUS CHIRAL SYMMETRY BREAKING
1
0.5
0
[GeV]
Mesons
Baryons

EXERCISE 3



Consider the QCD Lagrangian in a previous slide and ignore all flavors of quarks but up (u), down (d), and strange (s). This problem guides you to verify and make sense of various symmetries of the interactions. In the following: $q \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$.

a) Set the mass of quarks to zero (that is called the chiral limit). Show that the Lagrangian is invariant under a global $U(1)_V$ transformation of the quark fields:

$$q \to q' = e^{i\alpha} q$$

What is the corresponding Noether current? What does the conserved charge represent?

b) Show that the Lagrangian is invariant under independent rotations of left- and right-handed quarks:

$$q_L \to q'_L = e^{i\alpha^a_L T^a} q_L, \;\; q_R \to q'_R = e^{i\alpha^a_R T^a} q_R$$

where $q_L = \dfrac{1-\gamma^5}{2} q, \;\; q_R = \dfrac{1+\gamma_5}{2} q$. This is called chiral symmetry. T^a are generators of SU(3).

c) Show that for massive quarks, chiral symmetry breaks. This is an explicit chiral symmetry breaking. The symmetry is broken even if quarks were massless through a spontaneous symmetry-breaking mechanism, see the next slide.

- Write the most general interactions consistent with symmetries using only low-energy degrees of freedom
- Rank the interactions according to a momentum power counting
- Keep as many terms as needed for your precision goal

Example: Meson and baryon chiral perturbation theory

See e.g. the review by Scherer, arXiv:hep-ph/0210398.

Let's investigate more the chiral symmetry breaking. In the chiral limit (massless quarks), while the Lagrangian is chirally invariant, the vacuum state is not:

$$\boxed{\langle 0 \,|\bar{q}_{R,j} q_{L,i}|\, 0\rangle = -\Lambda^3 \delta_{ij},} \quad i,j = u,d,s$$

which is clear from:

$$\langle 0 \,|\bar{q}_{R,j} q_{L,i}|\, 0\rangle \to L_{ii'} R^{\dagger}_{j'j} \,\langle 0 \,|\bar{q}_{R,j'} q_{L,i'}\,|\, 0\rangle = -\Lambda^3 (LR^{\dagger})_{ij}.$$

$SU(3)_V$ vector subgroup associated with:

$$L = R$$

remains unbroken with 8 conserved isovector vector currents:

$$J^{\mu,a} = \bar{q}\gamma^{\mu} T^a q$$

$SU(3)_A$ axial subgroup associated with:

$$L \neq R$$

is broken with 8 Goldstone bosons corresponding the the 8 generators of the broken symmetry:

$$J^{\mu 5,a} = \bar{q}\gamma^{\mu}\gamma^5 T^a q$$

$*\,T^a$ with $a = 1, \ldots, 8$ are the 8 generators of $SU(3)\cdot$

Example: Meson chiral perturbation theory

See e.g. the review by Scherer, arXiv:hep-ph/0210398.

Leading-order interactions that are invariant under chiral symmetry:

$$\Sigma \equiv e^{2i\boldsymbol{\pi}(x)/f} \quad \text{with} \quad \boldsymbol{\pi} \equiv \pi^a T^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

$$\propto \langle 0 \,|\bar{q}_{R,j} q_{L,i}|\, 0\rangle$$

$$\mathcal{L}^{(2)}_{pGB} = \frac{f_\pi^2}{8}\,\mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger + 2B(M\Sigma^\dagger + M^\dagger \Sigma)\right]$$

Fixed by weak decay of pion:

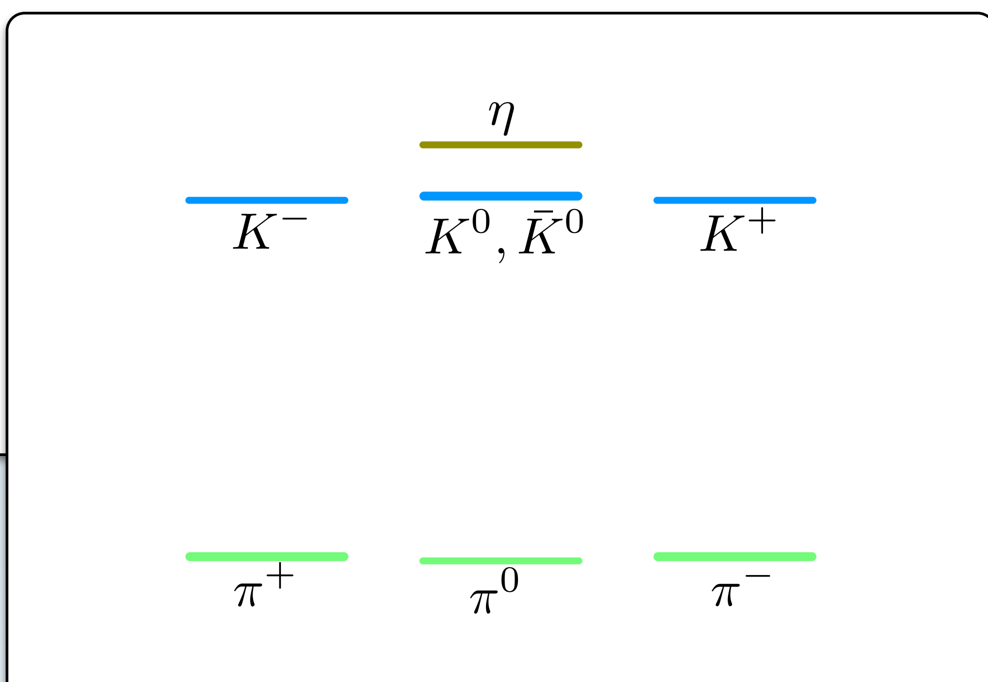
$$f_\pi = 130.41 \pm 20 \ \text{MeV}$$

Related to the condensate, e.g. obtained by lattice QCD

Quark mass matrix:

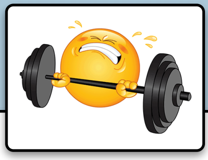
$$M \equiv \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$m_u = 2.15 \pm 0.15 \ \text{MeV}, \ m_d = 4.70 \pm 0.20 \ \text{MeV}, \ m_s = 93.5 \pm 2.5 \ \text{MeV}$$



* Masses from PDG at a renormalization scale of 2 GeV.

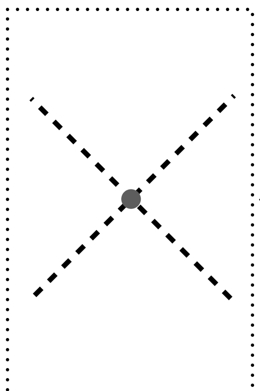
EXERCISE 4

By expanding the leading-order chiral perturbation theory Lagrangian for pseudo-scalar mesons, obtain the Lagrangian for the pion fields. Keep only terms up to two pions. This form should allow you to obtain a relation between the mass of the pions and the mass of u and d quarks. Notice that in the chiral limit, pions (and all pseudo-scalar mesons) couple derivatively to all orders, meaning that they do not interact at zero momentum. This feature continues even when these mesons couple to baryons and other fields in the EFT. Argue why this is an expected feature.

- Use the constrained EFT to make prediction for all other observables.

Example: two-pion scattering



$$\sim \frac{p^2}{f_\pi^2}$$

Leading order only depends on one LEC, pion decay constant.

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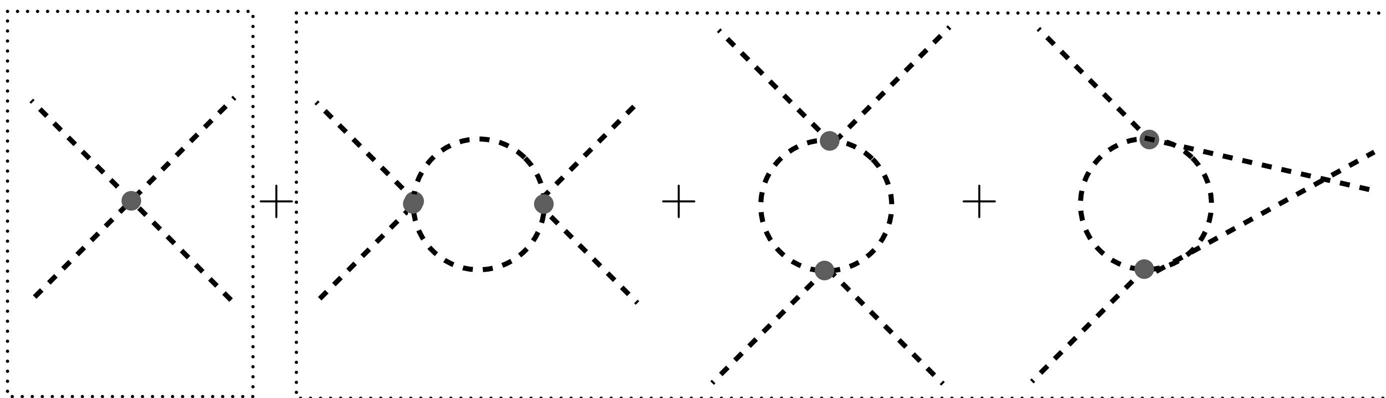

$\sim \frac{p^2}{f_\pi^2}$

$\sim \frac{p^4}{f_\pi^4} \log(\frac{p^2}{\mu^2})$

Leading order only depends on one LEC, pion decay constant.

Leading loop corrections with the LO LEC. UV divergent hence scale deponent.

- Use the constrained EFT to make prediction for all other observables.

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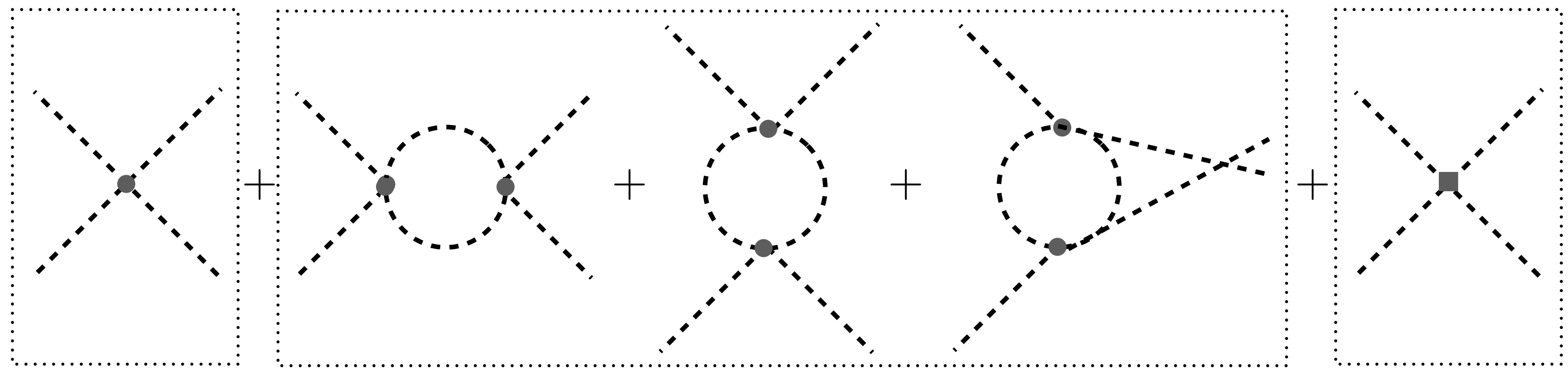


$$\sim \frac{p^2}{f_\pi^2} \qquad \sim \frac{p^4}{f_\pi^4}\log(\frac{p^2}{\mu^2}) \qquad \sim \frac{p^4}{f_\pi^4}$$

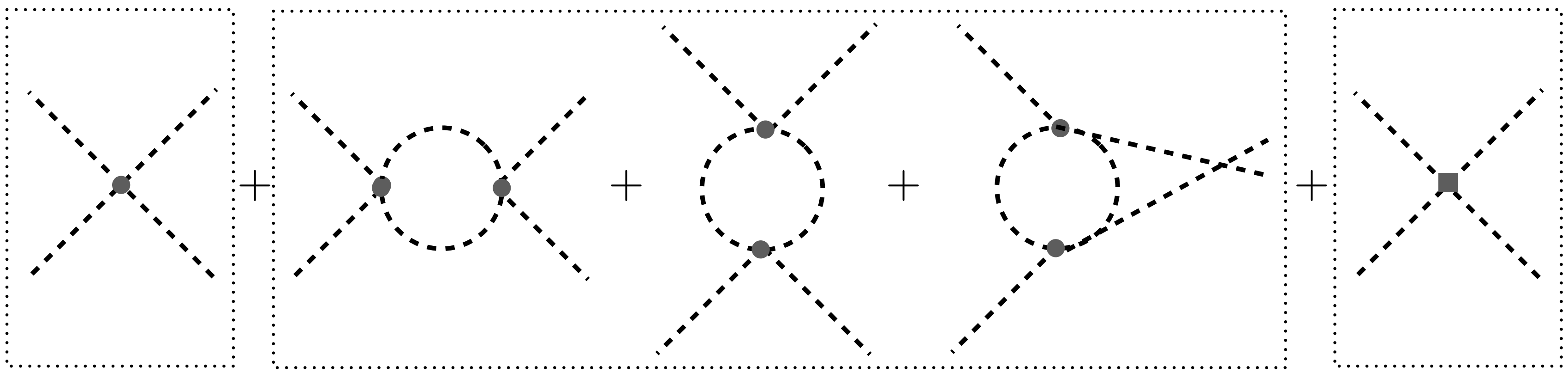
Leading order only depends on one LEC, pion decay constant.

Leading loop corrections with the LO LEC. UV divergent hence scale deponent.

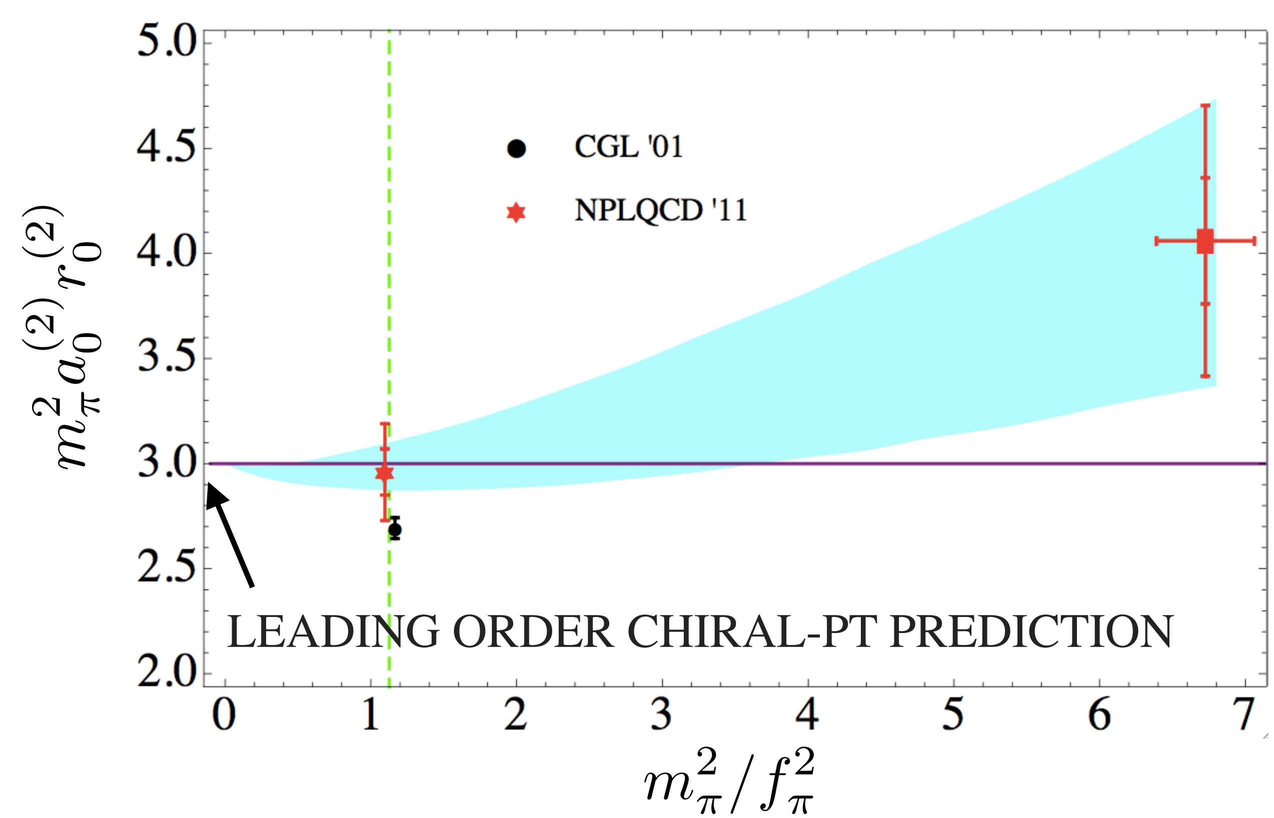
Next to leading order depends on 8 LECs (Gasser-Leutwyler coeff.). Their scale dependence cancels that of the second box.

- Use the constrained EFT to make prediction for all other observables.

Example: two-pion scattering



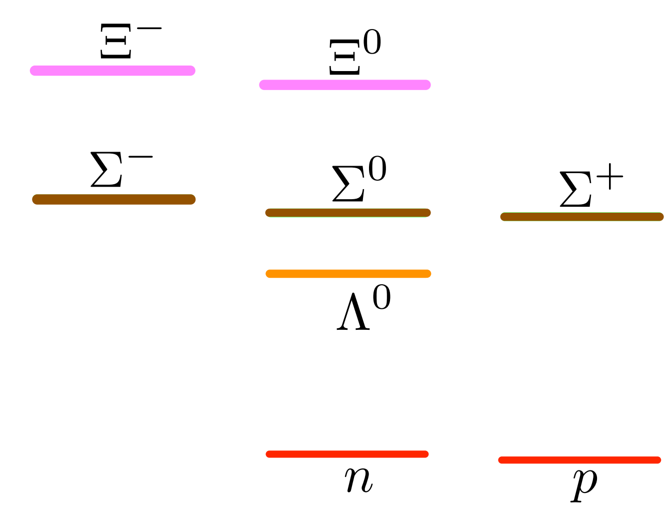
$$\sim \frac{p^2}{f_\pi^2} \qquad \sim \frac{p^4}{f_\pi^4}\log\left(\frac{p^2}{\mu^2}\right) \qquad \sim \frac{p^4}{f_\pi^4}$$



Beane, et al. (NPLQCD collaboration), 2012.

Example: Baryon chiral perturbation theory

See e.g. the review by Scherer, arXiv:hep-ph/0210398.



$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

$$\mathcal{L}^{(1)}_{B,pGB} = \mathrm{Tr}\left[\overline{B}(i\gamma^\mu D_\mu - M_B)B\right] - D\,\mathrm{Tr}\left[\bar{B}\gamma^\mu\gamma^5\{\mathcal{A}_\mu, B\}\right] - F\,\mathrm{Tr}\left[\bar{B}\gamma^\mu\gamma^5[\mathcal{A}_\mu, B]\right]$$

Fixed by semi-leptonic weak decays of baryons

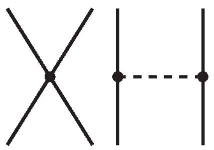
Now let's go back and form an EFT version of nuclear forces…

Two common approaches in ranking interactions (power counting):

- Weinberg power-counting that builds chiral potentials
- Kaplan-Savage-Wise power counting used that builds scattering amplitudes

IN WEINBERG SCHEME:

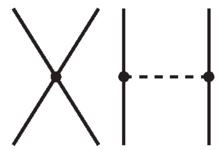
See the review by Machleidta and Entem, arXiv:1105.2919 [nucl-th].

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$			

Contact interactions plus one pion exchange

IN WEINBERG SCHEME:

See the review by Machleidta and Entem, arXiv:1105.2919 [nucl-th].

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$			

Contact interactions plus one pion exchange

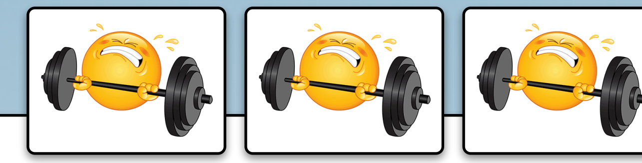
$$V^{(0)}_{\text{cont.}} = C_S + C_T\, \sigma_1 \cdot \sigma_2$$

$$V^{(0)}_{1\text{PE}}(\mathbf{q}) = -\frac{g_A^2}{2f_\pi^2}(\tau_1 \cdot \tau_2)\frac{(\sigma_1 \cdot \mathbf{q})(\sigma_2 \cdot \mathbf{q})}{\mathbf{q}^2 + m_\pi^2}$$

EXERCISE 5

There could be in principle two other spin-isospin structure for the two-nucleon contact potential at leading order. What are they? Argue why only the two terms written for the contact potential in the previous slide suffice to describe *NN* interactions at this order.

EXERCISE 6



In chiral perturbation theory, the leading-order interactions among nucleons and pions can be read from the Lagrangian:

$$\mathcal{L}_{\pi N} = -\frac{g_A}{\sqrt{2} f_\pi} N^\dagger \tau^a \sigma \cdot \nabla \pi^a N$$

a) Consider the Feynman diagram for the exchange of one pion among two nucleons. In the static limit, derive the (instantaneous) potential in this process. Note that in this limit, the pion propagator is:

$$S_\pi(\mathbf{q}) = \frac{-i\delta_{a,b}}{\mathbf{q}^2 + m_\pi^2}$$

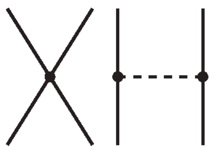
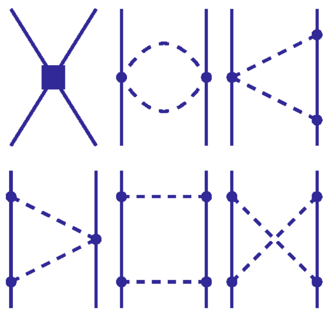
Obtain both the momentum-space potential $V_{\mathrm{OPE}}(\mathbf{q})$, where **q** is the momentum exchange among nucleons, and the position-space $V_{\mathrm{OPE}}(\mathbf{r})$, where **r** is nucleons' relative distance.

b) From the position-space potential, identify the "central" and "tensor" components.

c) How does the strength of the two-pion potential compares between the isosinglet (I=0) and isotriplet (I=1) two-nucleon channels?

IN WEINBERG SCHEME:

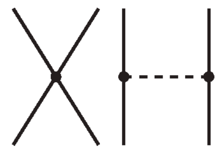
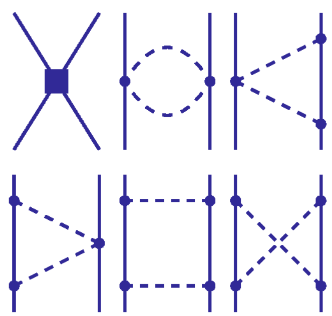
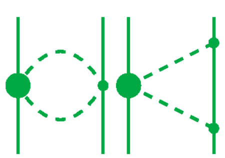
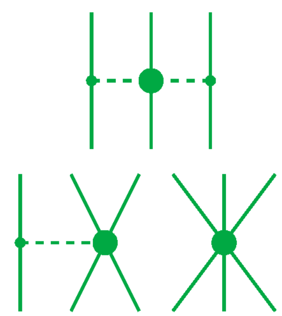
See the review by Machleidta and Entem, arXiv:1105.2919 [nucl-th].

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			

More contact interactions including derivative couplings. More pion exchanges.

IN WEINBERG SCHEME:

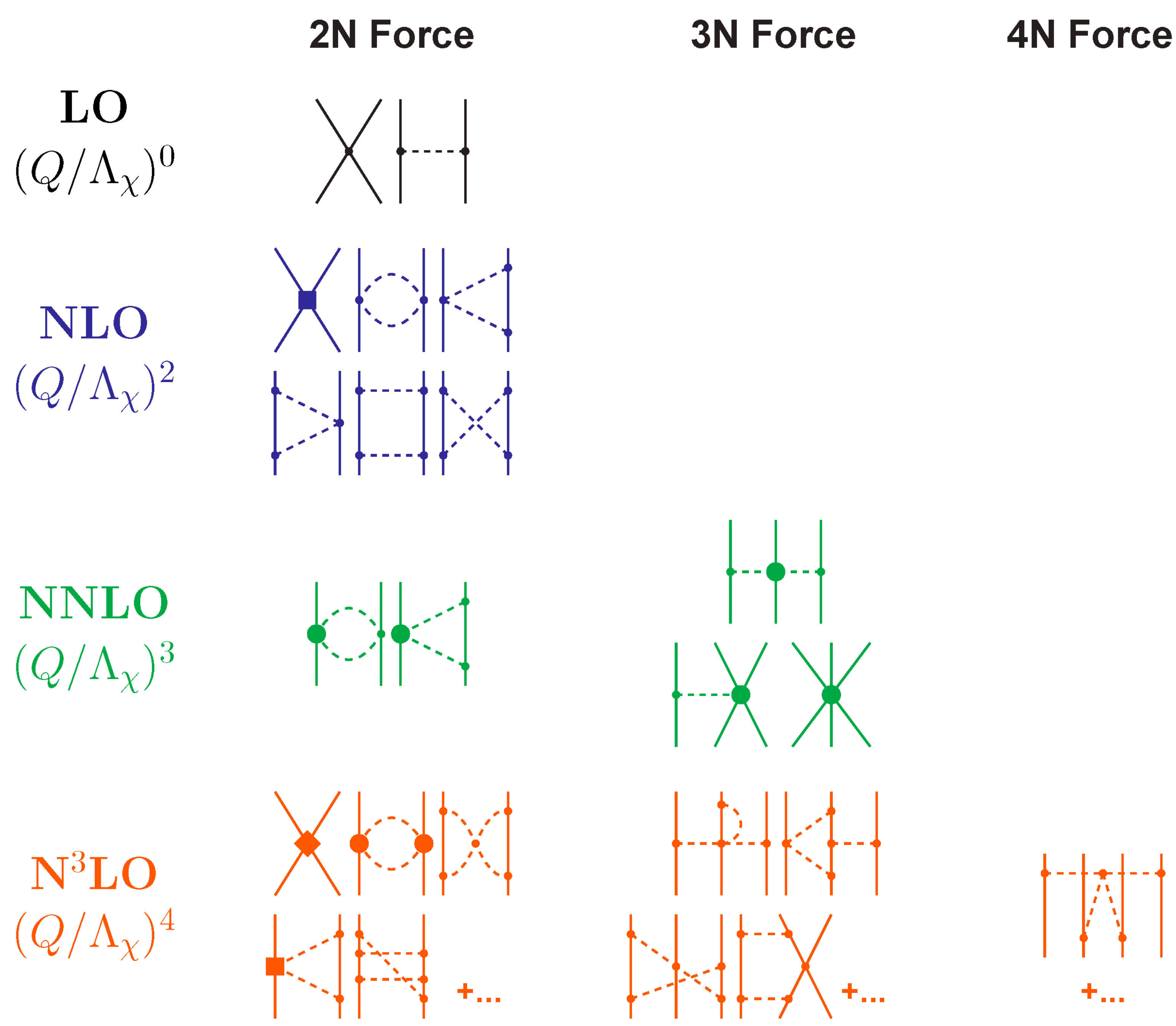
See the review by Machleidta and Entem, arXiv:1105.2919 [nucl-th].

	2N Force	3N Force	4N Force
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			
NNLO $(Q/\Lambda_\chi)^3$			

The first order at which three-nucleon couplings enter.

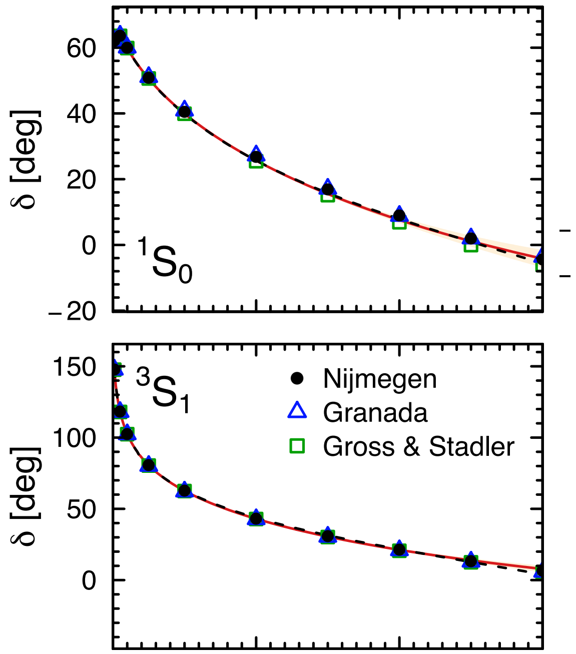
IN WEINBERG SCHEME:

See the review by Machleidta and Entem, arXiv:1105.2919 [nucl-th].



The first order at which four-nucleon couplings enter.

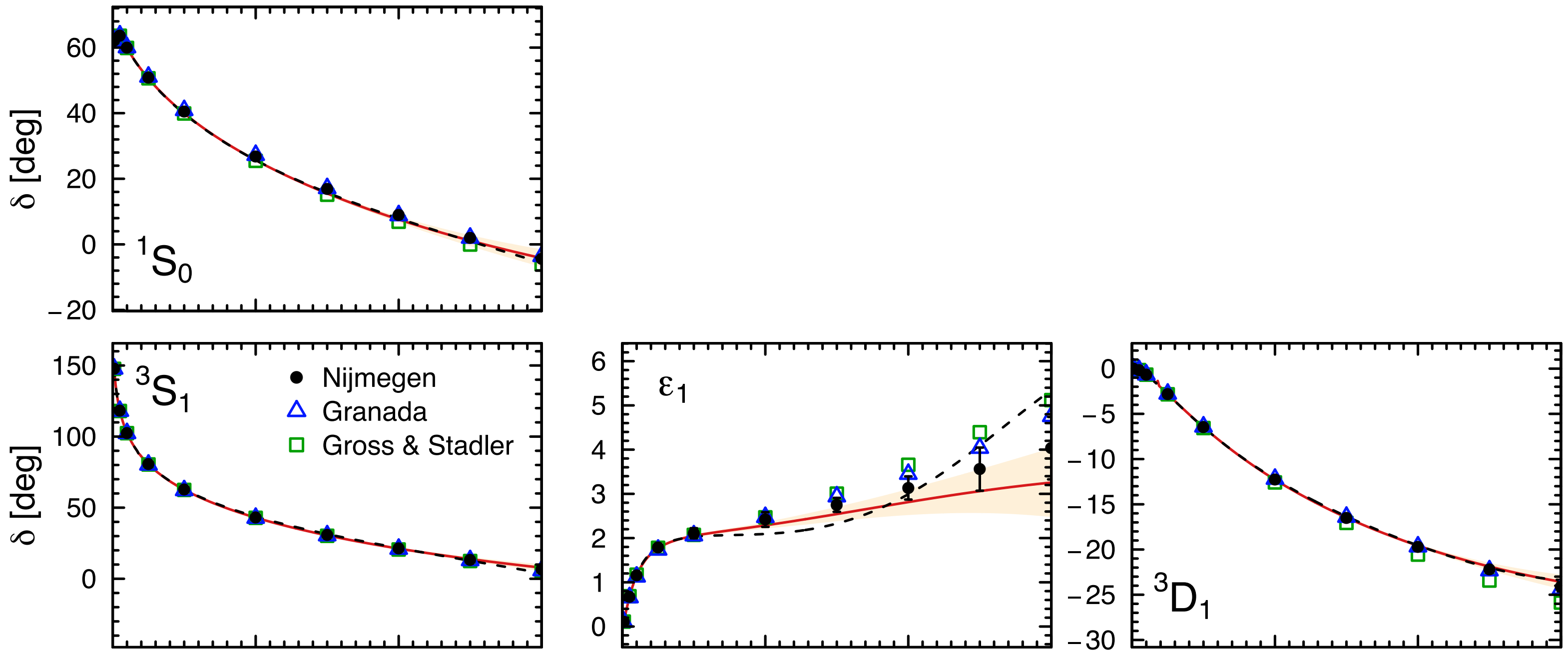
Leading to impressive results…such as phase shifts and mixing angles in the *NN* scattering channels once a few LECs are fixed:



Lowest-order partial waves

From a review by: Epelbaum, Krebs, and Reinert, arXiv:1911.11875 [nucl-th].

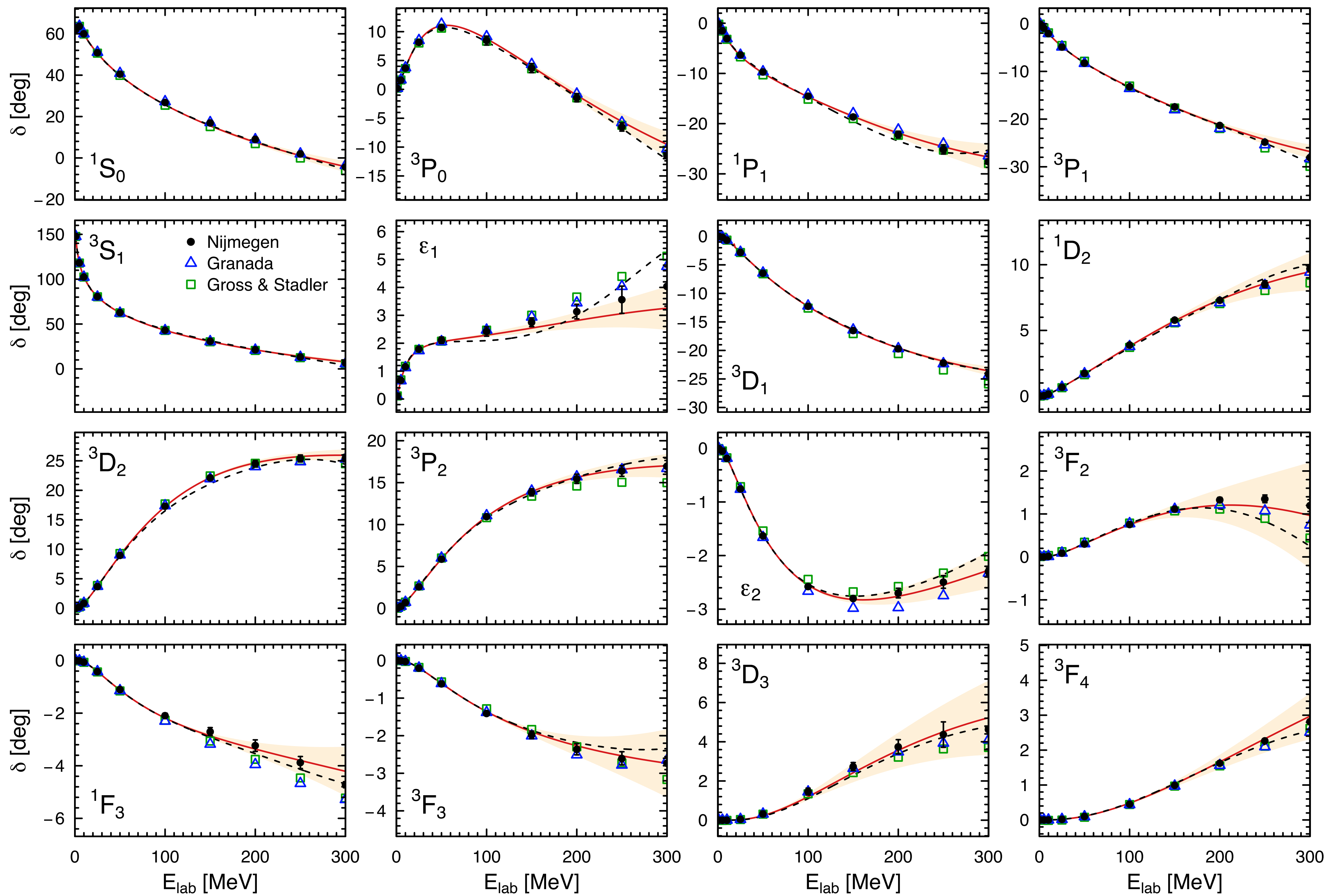
Leading to impressive results…such as phase shifts and mixing angles in the *NN* scattering channels once a few LECs are fixed:



And D wave and S-D mixing parameter in the isosinglet channel.

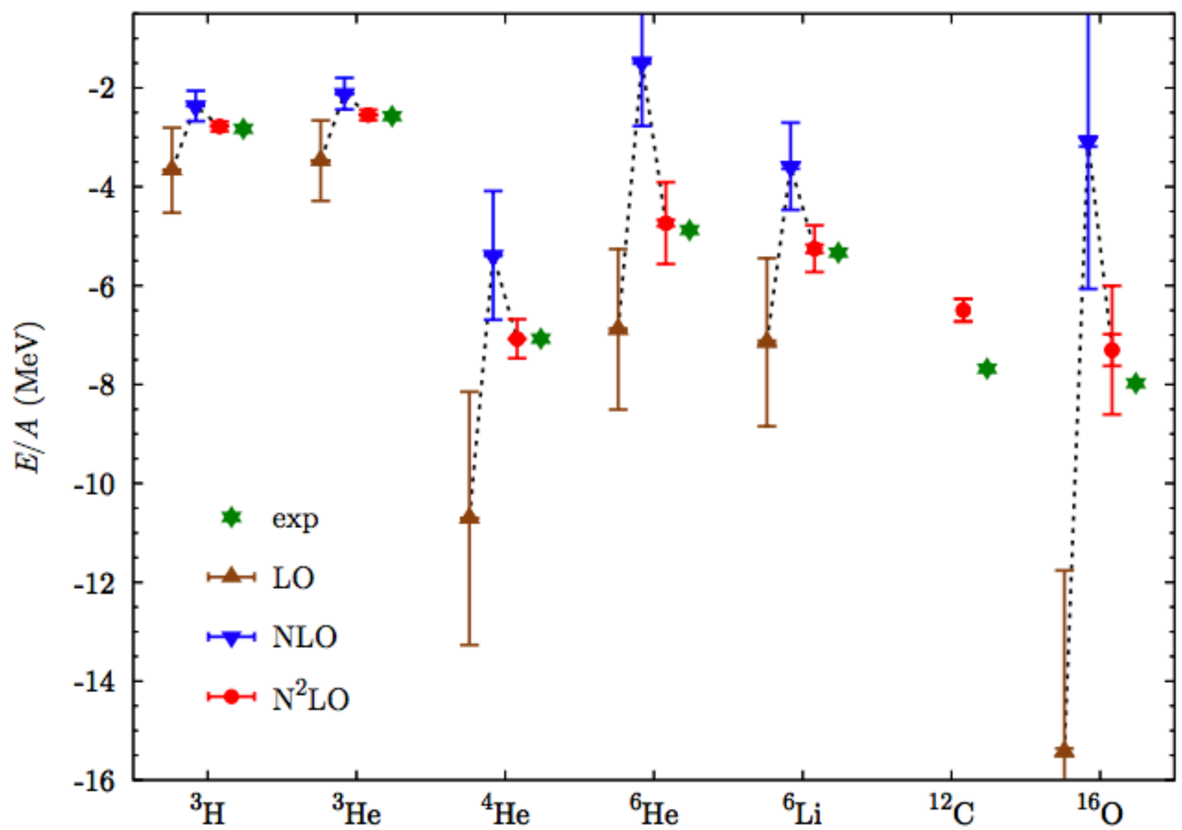
From a review by: Epelbaum, Krebs, and Reinert, arXiv:1911.11875 [nucl-th].

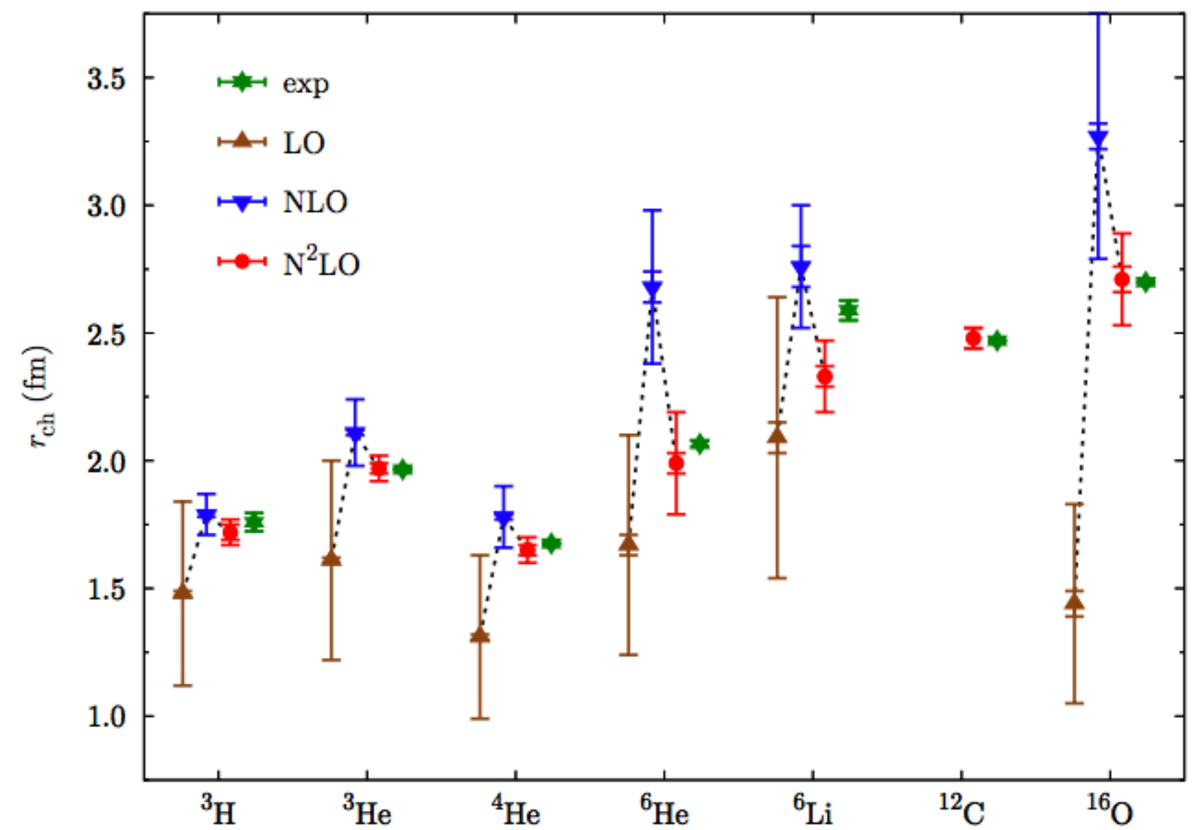
Leading to impressive results…such as phase shifts and mixing angles in the *NN* scattering channels once a few LECs are fixed:



From a review by: Epelbaum, Krebs, and Reinert, arXiv:1911.11875 [nucl-th].

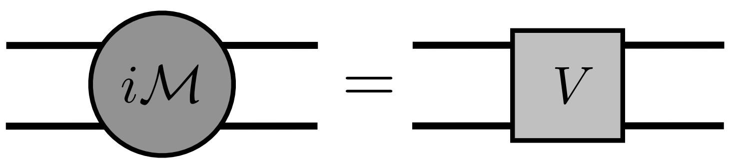
…as well as spectrum and charge radii of light to medium mass nuclei.





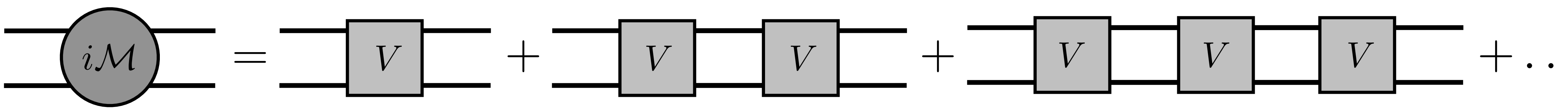
Lonardoni et al., arXiv:1709.09143[nucl-th].

However while the potential is systematically counted, the amplitude is not:



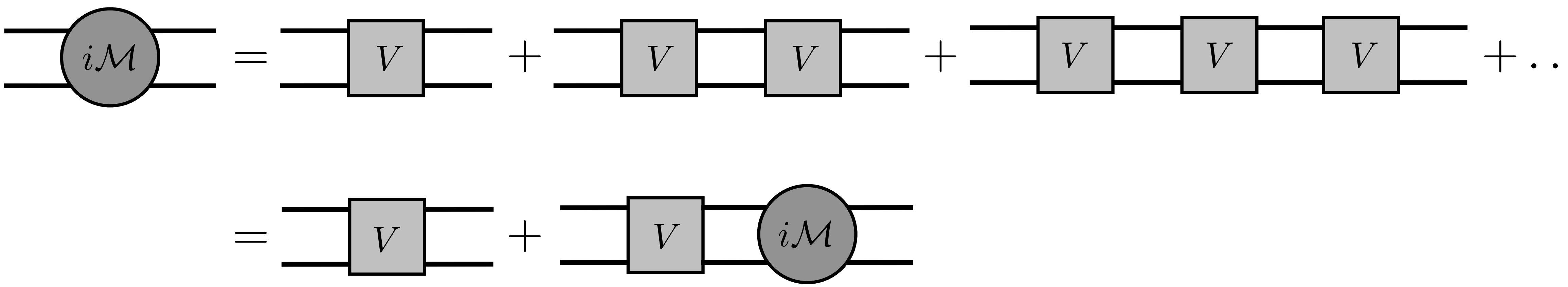
At leading order, amplitude is just the potential.

However while the potential is systematically counted, the amplitude is not:



But according to Lipmann-Schwinger equation, potential must be iterated to all orders to give the amplitude.

However while the potential is systematically counted, the amplitude is not:



So calculations need to deal with a residual cutoff dependence at any order in the EFT.

Let us another look at how to organize interactions according to their importance. In the Weinberg power counting, the pion exchange as well as contact interactions come at zeroth order. However, this “naive dimensional analysis” is not necessarily the best way to go.

Consider the simplified form of contact Lagrangian at leading and next-to-leading orders (ignoring all spin-isospin structures):

$$\mathcal{L}_{NN}^{(0)} \propto \boxed{C_0} N^\dagger N^\dagger N N$$

Could be the contact LECs of Weinberg scheme or that of KSW scheme.

$$\mathcal{L}_{NN}^{(2)} \propto \boxed{C_2} \nabla\nabla N^\dagger N^\dagger N N$$

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$$\mathcal{L}_{NN}^{(2)} \propto \boxed{C_2} \nabla\nabla N^\dagger N^\dagger N N$$

Then the mass dimensions of the LECs are:

$$[\mathcal{L}] = \mathrm{Mass}^4$$

$$[\nabla] = \mathrm{Mass}^1$$

$$[N] = \mathrm{Mass}^{3/2}$$

$$\Rightarrow$$

$$\boxed{[C_0]} = \mathrm{Mass}^{-2}$$

$$\boxed{[C_2]} = \mathrm{Mass}^{-4}$$

Now consider two expansions of the S-wave amplitude in terms of an effective range expansion:

Natural interactions

$$\mathcal{M} = \frac{4\pi}{M_N}\frac{1}{\boxed{p\cot\delta} - ip} = \begin{cases} -\dfrac{4\pi a}{M_N}\left[1 - iap + (\dfrac{ar}{2} - a^2)p^2 + \cdots\right] & \text{, for } \boxed{a \text{ small}}\text{, so } a \sim \dfrac{1}{\Lambda} \\ -\dfrac{4\pi}{M_N}\dfrac{1}{1/a + ip}\left[1 + \dfrac{r/2}{(1/a + ip)}p^2 + \cdots\right] & \text{, for } \boxed{a \text{ large}}\text{, so } a \sim \dfrac{1}{p} \end{cases}$$

Unnatural interactions

$$\boxed{p\cot\delta = -\frac{1}{a} + \frac{1}{2}rp^2 + \cdots}$$

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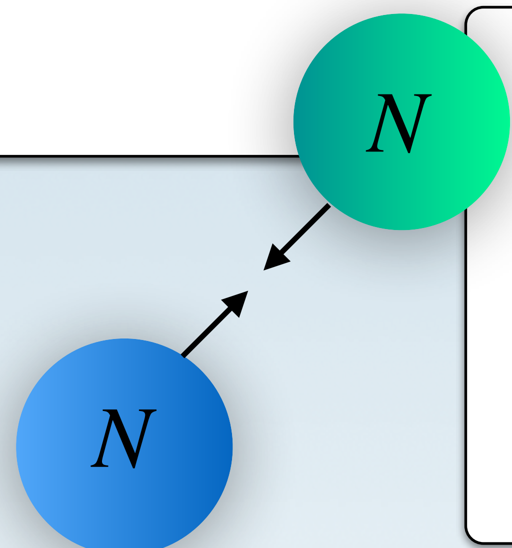
Unnatural interactions

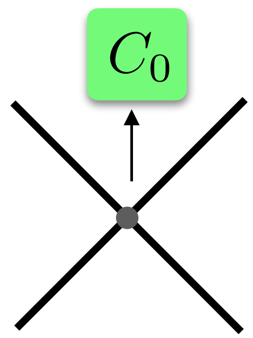


Unnaturalness is what we see in NN systems in nature (compare with atomic systems near Feshbach resonance)!

$$a^{({}^1S_0)} \approx -23\ [\mathtt{fm}] \gg\text{-}1/m_\pi$$

$$a^{({}^3S_1)} \approx \quad 5\ [\mathtt{fm}] \gg 1/m_\pi$$

But the amplitude at leading order is:

$\mathcal{M} =$ 

$+ \ldots \quad \Rightarrow \quad \begin{cases} C_0 \propto \dfrac{1}{\Lambda M_N} & \text{, for } a \text{ small, so } a \sim \dfrac{1}{\Lambda} \\ C_0 \propto \dfrac{1}{p M_N} & \text{, for } a \text{ large, so } a \sim \dfrac{1}{p} \end{cases}$

Natural interactions

Unnatural interactions

So in the natural case the leading contact LEC has its naive dimensional scaling, and can be included perturbatively, but for the unnatural case, the leading contact LEC is promoted to a lower order. It is therefore large and its contribution should be included to all orders.

Alternative: In the KSW power counting, the leading contact interactions are summed to all orders, and pion exchange comes at next-to-leading order.

Kaplan, Savage, and Wise (1998).

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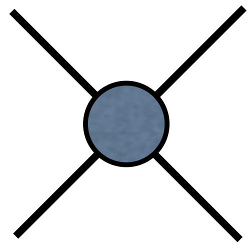
$\mathcal{M}^{(LO)}$:

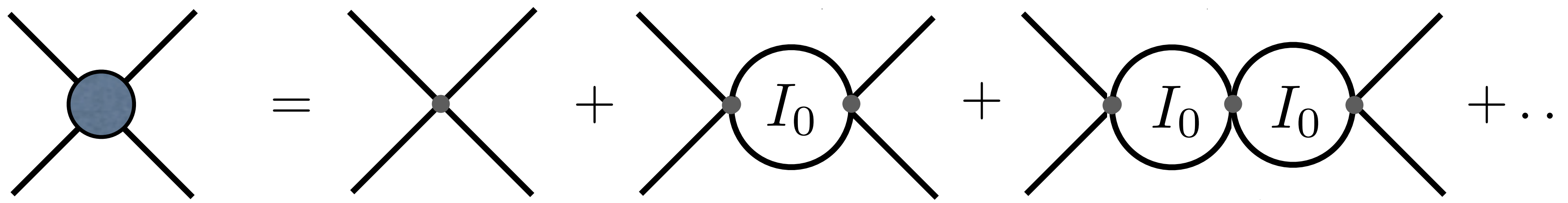


$\sim \frac{1}{p}$

Unlike the Weinberg scheme, the LO amplitude here is assumed to go as inverse of small momentum.

Alternative: In the KSW power counting, the leading contact interactions are summed to all orders, and pion exchange comes at next-to-leading order.

$\mathcal{M}^{(LO)}:$



$$\sim \frac{1}{p} \qquad \sim \frac{1}{p} \qquad \sim \frac{1}{p}\, p\, \frac{1}{p} \qquad \sim \frac{1}{p}\, p\, \frac{1}{p}\, p\, \frac{1}{p}$$

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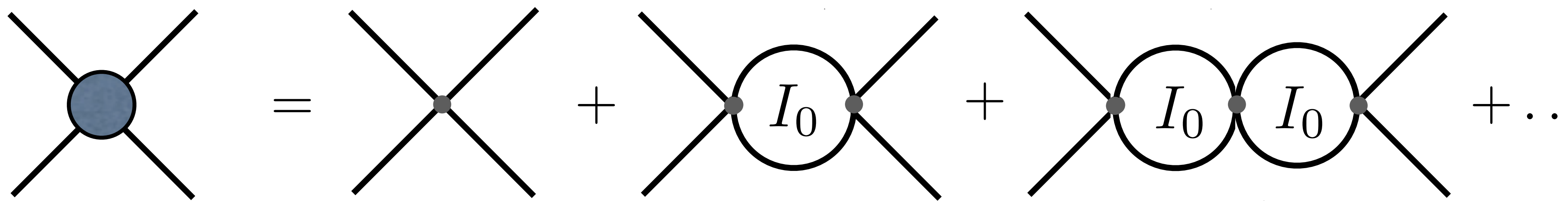
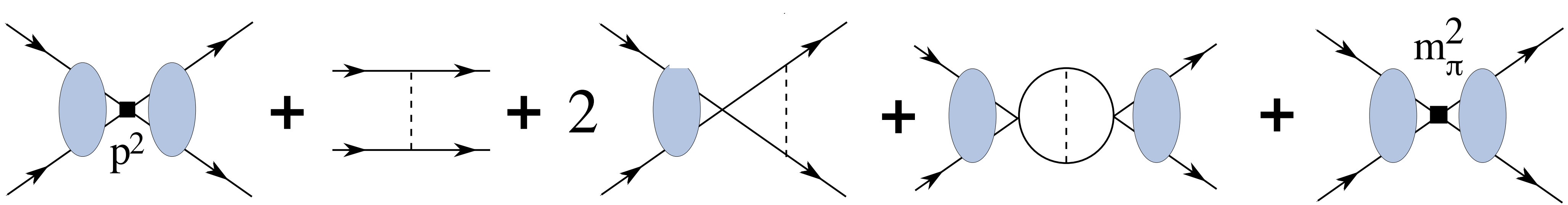
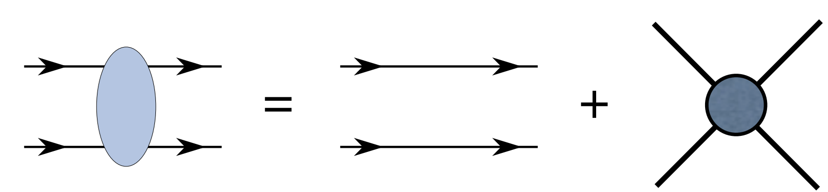


$$\sim \frac{1}{p} \qquad \sim \frac{1}{p} \qquad \sim \frac{1}{p}\, p\, \frac{1}{p} \qquad \sim \frac{1}{p}\, p\, \frac{1}{p}\, p\, \frac{1}{p}$$

$\mathcal{M}^{(NLO)}$:



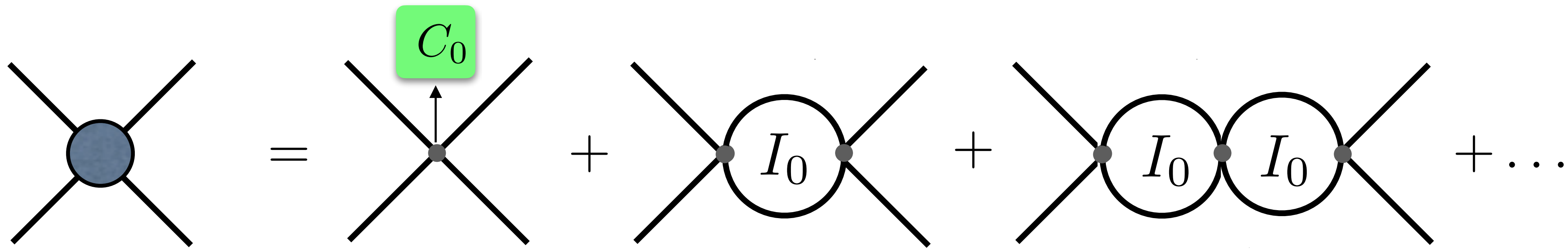
$\sim p^0$



Pionless EFT is a version of this expansion that is valid for $p \ll m_\pi$:

Kaplan, Savage, and Wise (1998), van Kolck (1999).

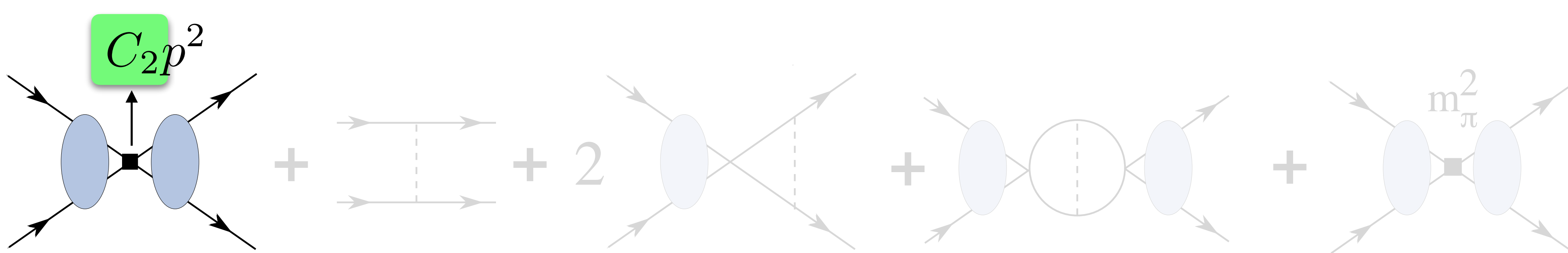
$\mathcal{M}^{(LO)}$:



$$\sim \frac{1}{p} \qquad \sim \frac{1}{p} \qquad \sim \frac{1}{p}\, p\, \frac{1}{p} \qquad \sim \frac{1}{p}\, p\, \frac{1}{p}\, p\, \frac{1}{p}$$

$$\mathcal{M}_{\not\pi}^{(LO)} = \frac{-C_0}{\left[1 + \frac{C_0 M}{4\pi}(\mu + ip)\right]}$$

$\mathcal{M}^{(NLO)}$:



$\sim p^0$

$$\mathcal{M}_{\not\pi}^{(NLO)} = \frac{-C_2 p^2}{\left[1 + \frac{C_0 M}{4\pi}(\mu + ip)\right]^2}$$

EXERCISE 7

Consider the expansion of two-nucleon scattering amplitude within the KSW power-counting scheme. This exercise will guide you to fill some of the missing details.

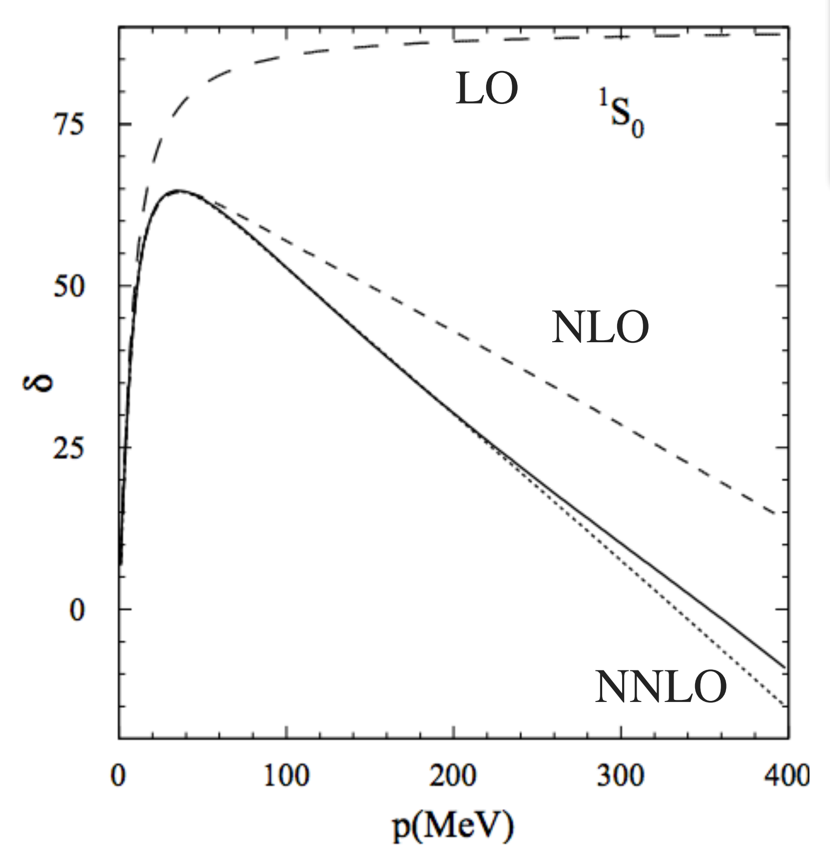
a) [Bonus] In order to arrive at an amplitude that is p^{-1} at leading order, it was important for the loop function I_0 to scale as p, where p is the magnitude of the relative (spatial) momentum of the nucleons on shell. Evaluate this loop using dimensional regularization and notice that it is not UV divergent in 3+1 dimensions but it is in 2+1 dimensions. Subtract the divergence at 2+1 dimensions from the result. This is called power-divergence subtraction scheme and is what is used in the expansion of amplitudes. Here, you can consider the nonrelativistic nucleon propagators in the loops:

$$S_N(p) = \frac{i}{p^0 - \frac{\mathbf{p}^2}{2M_N} + i\epsilon}.$$

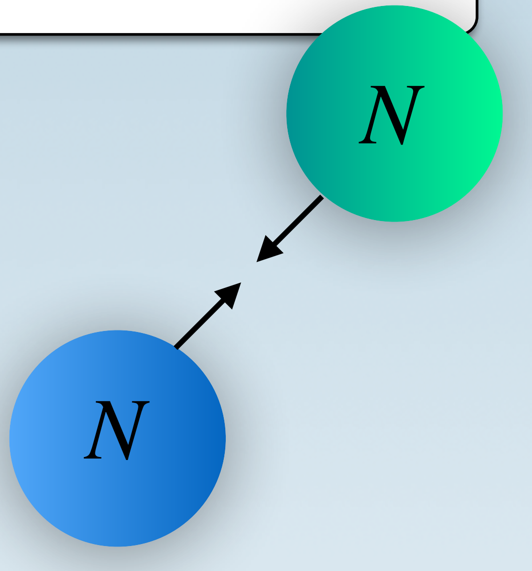
b) How should the C_2 LEC scale as a function of p such that the next-to-leading order amplitude $\mathcal{M}^{(NLO)}$ scales as p^0 ?

c) Derive the expression for leading and next-to-leading amplitude in the pionless EFT in terms of the leading and next-to-leading order couplings (LECs) C_0 and C_2.

In the KSW power counting, therefore no cutoff (or renormalization-scale) dependence remains order by order in the expansion in small momenta, which is the desired feature of an EFT.



It has been worked out to higher orders and is shown to work well for the $^1S_0$ NN channel.



Mehen and Stewart (1999).

KEEP IN MIND THAT…

- Both Weinberg and KSW power counting have convergence issues, in one channel or another. Other power countings exist with better convergence in both channels but are more involved.

  See e.g., Beane, Bedaque, Savage, and van Kolck (2002).

- Whether a given power counting is applicable to heavier nuclei must be established carefully. This puts questions on how predictive EFT is in larger systems.

- One still needs experiment or lattice QCD to constrain EFT coefficients, for example for 3n/4n forces, hyperon-nucleon and hyperon-hyperon forces, matrix elements of various currents, etc.

So given these limitations, why not calculating everything with lattice QCD? We come back to this question later…