Dendel Deguia

4.1.5

X	1	-2	0
Y	3	-3	-7

Table 5.1

$$P(x) = 3 + 2(x-1) + 4(x-1)(x+2) \quad (5.2)$$
$$q(x) = 4x^2 + 6x - 7 \quad (5.3)$$

$$P(1) = 3 + 2\cancel{(1-1)} + 4\cancel{(1-1)}(1+2) = 3 + 0 + 0 = 3$$
$$P(-2) = 3 + 2(-2-1) + 4(-2-1)\cancel{(-2+2)} = 3 - 6 + 0 = -3$$
$$P(0) = 3 + 2(0-1) + 4(0-1)(0+2) = 3 - 2 - 8 = -7$$

$\Rightarrow$ P(x) interpolates table 5.1

$$q(1) = 4(1)^2 + 6(1) - 7 = 4 + 6 - 7 = 3$$
$$q(-2) = 4(-2)^2 + 6(-2) - 7 = 16 - 12 - 7 = -3$$
$$q(0) = 4(0)^2 + 6(0) - 7 = 0 + 0 - 7 = -7$$

$\Rightarrow$ q(x) interpolates table 5.1

By the existence and uniqueness theorem, this means that $p(x) = q(x)$; this becomes evident after multiplying out equation (5.2) and combining terms:

$$P(x) = 3 + 2x - 2 + 4(x^2 + x - 2) = 1 + 2x + 4x^2 + 4x - 8$$
$$\Rightarrow P(x) = 4x^2 + 6x - 7$$

$\therefore$ $P(x) = q(x)$, so the existence and uniqueness theorem is not violated.

4.1.7 b) From the given table:

$$(x_0, x_1, x_2, x_3) = (-1, 1, 3, 4);\ (f[x_0], f[x_1], f[x_2], f[x_3]) = (2, -4, 46, 99.5)$$

and $f[x_2, x_3] = 53.5$

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{-4-2}{1-(-1)} = -3$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{46-(-4)}{3-1} = 25$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{25-(-3)}{3-(-1)} = 7$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{53.5-25}{4-1} = 9.5$$

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} = \frac{9.5-7}{4-(-1)} = \frac{1}{2}$$

x	f[ ]	f[,]	f[, ,]	f[, , ,]
-1	$\boxed{2}$			
		$\boxed{-3}$		
1	-4		$\boxed{7}$	
		25		$\boxed{0.5}$
3	46		9.5	
		53.5		
4	99.5			

$$\Rightarrow \boxed{P_3(x) = 2 - 3(x+1) + 7(x+1)(x-1) + \frac{1}{2}(x+1)(x-1)(x-3)}$$

4.1.11

Year	1950	1960	1970	1980	1990
Population (mil)	150.7	179.3	203.3	226.5	249.6

Using divided difference yields:

1950	$\boxed{150.7}$				
		$\boxed{2.86}$			
1960	179.3		$\boxed{-0.0023}$		
		2.4		$\boxed{0.0006\overline{333}}$	
1970	203.3		-0.004		$\boxed{-1.291\overline{66}\times10^{-5}}$
		2.32		$0.0001\overline{166}$	
1980	226.5		-0.0005		
		2.31			
1990	249.6				

$$\Rightarrow P_4(X) = 150.7 + 2.86(X-1950) - \frac{23}{1000}(X-1950)(X-1960)$$
$$+ \frac{19}{30000}(X-1950)(X-1960)(X-1970)$$
$$- \frac{31}{2400000}(X-1950)(X-1960)(X-1970)(X-1980)$$

Now, using $P_4(X)$ to estimate populations in 1920 and 2000:

$$P_4(1920) = -47.2 \text{ million}$$
$$P_4(2000) = 270.2 \text{ million}$$

The estimate for $X=2000$ seems acceptable since there is an increasing trend in population w.r.t. year. However, the estimate for $X=1920$ is nonsensical which indicates that the predictive (and retrospective) capabilities of an interpolating polynomial is limited within the data it interpolates; beyond that data, ludicrous predictions are inevitable.

4.1.15 We can construct Newton's interpolation polynomial for the table. By the existence and uniqueness theorem, there is an interpolating polynomial of at most degree 5 that interpolates the table. Whatever polynomial is constructed is the only one possible for the provided data.

-2	$\boxed{1}$	$\boxed{3}$	$\boxed{2}$	$\boxed{-1}$	$\boxed{0}$	$\boxed{0}$
-1	4	7	-1	-1	0	
0	11	5	-4	-1		
1	16	-3	-7			
2	13	-17				
3	-4					

Using divided differences, we see that the final two columns are zeros which indicates that the interpolating polynomial is degree 3

$$\Rightarrow \boxed{P(x) = 1 + 3(x+2) + 2(x+2)(x+1) - (x+2)(x+1)x}$$

Indeed, $P(x)$ is a 3rd degree polynomial.