Londel Degnia

2.2.3

$$A=\begin{bmatrix}1 & 0 & 3 & 0\\ 0 & 1 & 3 & -1\\ 3 & -3 & 0 & 6\\ 0 & 2 & 4 & -6\end{bmatrix} \Rightarrow \vec{S}=(3,3,6,6)$$
$$\vec{l}=(1,2,3,4)$$

$K=1$ $\max\left\{\frac{|a_{11}|}{S_1}, \frac{|a_{21}|}{S_2}, \frac{|a_{31}|}{S_3}, \frac{|a_{41}|}{S_4}\right\}$

$=\max\left\{\frac{1}{3}, 0, \frac{1}{2}, 0\right\} \Rightarrow j=3 \Rightarrow l_K \Leftrightarrow l_j$

$\Rightarrow \vec{l}=(3,2,1,4)$

$$\Rightarrow \begin{bmatrix}0 & 1 & 3 & -2\\ 0 & 1 & 3 & -1\\ 3 & -3 & 0 & 6\\ 0 & 2 & 4 & -6\end{bmatrix} \Rightarrow K=2 \quad \max\left\{\frac{|a_{22}|}{S_2}, \frac{|a_{12}|}{S_1}, \frac{|a_{42}|}{S_4}\right\}$$

$= \max\left\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\} \Rightarrow j=2$

$$\Rightarrow \vec{l}=(3,2,1,4) \Rightarrow \begin{bmatrix}0 & 0 & 0 & -1\\ 0 & 1 & 3 & -1\\ 3 & -3 & 0 & 6\\ 0 & 0 & -2 & -4\end{bmatrix}$$

2.2.4

For

i) naive Gaussian elimination, the next pivot element will be $\boxed{-0.0145}$

ii) For unscaled partial pivoting, the next pivot element will be $\max_{2\le i\le 4}|a_{i2}| = \boxed{102.7513}$

iii) For scaled partial pivoting, the next pivot element will be the one corresponding to $\max\left\{\frac{|a_{i2}|}{S_i} \,\middle|\, 2\le i\le 4\right\}$ where

$\vec{S}=[987.6543, 833.3333, 102.7513, 9876.5432] \Rightarrow \max\left\{\frac{0.0145}{833.333}, \frac{102.7513}{102.7513}, \frac{1.3131}{9876.5432}\right\}$

$\Rightarrow \boxed{102.7513}$

2.2.13

(a) The ~~system~~ system yields the following matrix equation:

$$\begin{bmatrix} 3 & 4 & 3 \\ 1 & 5 & -1 \\ 6 & 3 & 7 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \\ 15 \end{bmatrix} \Rightarrow \vec{S} = (4, 5, 7)$$

$$\vec{l} = (1, 2, 3) \leftarrow \text{initial index vector}$$

$$K=1 \Rightarrow \max\left\{\frac{3}{4}, \frac{1}{5}, \frac{6}{7}\right\} \Rightarrow j=3 \Rightarrow \boxed{\vec{l} = (3, 2, 1)}$$

switched l_K with l_j

$$\Rightarrow \begin{bmatrix} 0 & 2.500 & -0.5000 \\ 0 & 4.500 & -2.167 \\ 6.000 & 3.000 & 7.000 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 2.500 \\ 4.500 \\ 15.00 \end{bmatrix}$$

$$K=2 \Rightarrow \max\left\{\frac{|a_{l_2 2}|}{S_{l_2}}, \frac{|a_{l_3 2}|}{S_{l_3}}\right\} = \max\left\{\underbrace{\frac{|a_{22}|}{S_2}}_{j=2}, \underbrace{\frac{|a_{12}|}{S_1}}_{j=3}\right\}$$

$$= \max\left\{\frac{4.500}{5}, \frac{2.500}{4}\right\} \Rightarrow j=2 \Rightarrow l_K \leftrightarrow l_j \Rightarrow \text{no change to } \vec{l}$$

$$\Rightarrow \boxed{\vec{l} = (3, 2, 1)}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0.7039 \\ 0 & 4.500 & -2.167 \\ 6.000 & 3.000 & 7.000 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.500 \\ 15.00 \end{bmatrix}$$

$$\Rightarrow \quad 0.7039 X_3 = 0 \quad \Rightarrow X_3 = 0$$

$$4.500 X_2 - 2.167 X_3 = 4.500 \quad X_2 = \frac{4.500}{4.500} = 1$$

$$6.000 X_1 + 3.000 X_2 + 7.000 X_3 = 15.00 \quad \Rightarrow X_1 = \frac{12.00}{6.000} = 2$$

$$\therefore \boxed{\begin{aligned} X_1 &= 2 \\ X_2 &= 1 \\ X_3 &= 0 \end{aligned}}$$

8.4.4 By definition of a subordinate matrix norm:

$$\|I\| = \sup\{\|I\vec{x}\| : \vec{x} \in \mathbb{R}^n \wedge \|\vec{x}\| = 1\}$$
$$= \sup\{\|\vec{x}\| : \vec{x} \in \mathbb{R}^n \wedge \|\vec{x}\| = 1\}$$
$$= \sup\{1\} \Rightarrow \boxed{\|I\| = 1; \text{(b)}}$$

8.4.7 By the Jacobi and Gauss-Seidel convergence theorem, a sufficient condition for the convergence of the Jacobi method on $A\vec{x} = \vec{b}$ is that $\boxed{A \text{ is diagonally dominant; (b)}}$

8.4.8 By the Jacobi and Gauss-Seidel convergence theorem, a sufficient condition for the convergence of the Gauss-Seidel method on $A\vec{x} = \vec{b}$ is that $\boxed{A \text{ is diagonally dominant; (a)}}$

8.4.11 Here, we use the $\underline{l_1 \text{ norm}}$ for the matrix norm: $\|A\|_1 = \max\limits_{1 \le j \le n}\left(\sum\limits_{i=1}^{n} |a_{ij}|\right)$

(a)
$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -3/4 & -1/2 & -1/4 \\ -1/2 & -1 & -1/2 \\ -1/4 & -1/2 & -3/4 \end{bmatrix} \Rightarrow \begin{array}{l} \|A\|_1 = \max\{3, 4\} = 4 \\ \|A^{-1}\|_1 = \max\{\frac{3}{2}, 2\} = 2 \end{array}$$

$$\Rightarrow K(A) = \|A\|_1 \|A^{-1}\|_1 \Rightarrow \boxed{K(A) = 8}$$

(c)
$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{array}{l} \|A\|_1 = \max\{1, 2, 3\} = 3 \\ \|A^{-1}\|_1 = \max\{\frac{1}{3}, \frac{1}{2}, 1\} = 1 \end{array}$$

$$\Rightarrow K(A) = \|A\|_1 \|A^{-1}\|_1 \Rightarrow \boxed{K(A) = 3}$$