Computer Exercise 5.4.1

This program contains an algorithm written based on formula (8) (on page 243 in the textbook**); the program also uses tests the algorithm on the indicated test function $f(x) = \frac{1}{e^{x^2}}$ in the textbook using the same test parameters: $a = 0,\ b = 1$. The objective is to match the answer indicated in the textbook: $\int_0^1 f(x)dx \approx 0.746814584$.

**Reference: Cheney, E.W. and Kincaid, D.R. Numerical Mathematics and Computing 7th edition

Formula (8) evaluates $f(x)$ on the interval $[-1, 1]$ and is given by

$$\int_{-1}^{1} f(x)dx \approx \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$$

When modified to be evaluated on the interval $[a, b]$, formula (8) becomes:

$$\int_a^b f(x)dx \approx \frac{b-a}{2}\left(\frac{5}{9} f\left(-\frac{b-a}{2}\sqrt{\frac{3}{5}} + \frac{b+a}{2}\right) + \frac{8}{9} f\left(\frac{b+a}{2}\right) + \frac{5}{9} f\left(\frac{b-a}{2}\sqrt{\frac{3}{5}} + \frac{b+a}{2}\right)\right)$$

```
%initiate function
f = @(x) exp(-(x^2));
%initiate parameters
a=0;
b=1;
%execute algorithm
sum1 = formula8(f, a, b);
%display approximation
fprintf('integral approximation = %9.9f', sum1)
```

```
integral approximation = 0.746814584
```

We see that the approximation acquired here is exactly the same as the one acquired in the textbook.

```
function sum = formula8(f, a, b)
    x1 = (-(b-a)/2)*(sqrt(3/5)) + (b+a)/2;
    x2 = (b+a)/2;
    x3 = ((b-a)/2)*(sqrt(3/5)) + (b+a)/2;
    sum = ((5/9)*f(x1) + (8/9)*f(x2) + (5/9)*f(x3))*((1/2)*(b-a));
end
```