Computer Excercise 1.1.2 (mostly the same as 1.1.1)

The following program will evaluate the finite difference quotient of $f(x) = 1/(1+x^2)$ for different values of h as an approximation for the first derivative of f(x); the program will loop through different values of h multiplying the previous value of h by 0.25 on each iteration. The program will then evaluate the error of the approximation by using f(x)'s known derivative, $f'(x) = -2x/(1+x^2)^2$, and will seek out and store a smallest error value and its corresponding index.

side note: $f(x)$ is a Lorentzian function

On each iteration, desired values will be stored in an array which will then be casted as a table for display.

```
%Initialize values
n=30;
x=0.5;
h = 1;
emin = 1;

%Initialize output arrays which will then be used to display a table
%This portion is for display purposes

i_out = (1:30)';
h_out = zeros(30,1);
y_out = zeros(30,1);
error_out = zeros(30,1);

%approximation program loop for f(x)

%function and derivative are written at the bottom of the script

for i = 1:n
    h = 0.25*h;
    y = (f(x + h) - f(x))/h;
    error = abs( deriv(x) - y );

    %update the output arrays
    h_out(i) = h;
    y_out(i) = y;
    error_out(i) = error;

    %seek out minimum error and record its index
    if error < emin
        emin = error;
        imin = i;
    end
end

%cast output arrays as a table

T = table(i_out, h_out, y_out, error_out);
disp(T)
```

i_out	h_out	y_out	error_out
1	0.25	-0.64	1.11022302462516e-16
2	0.0625	-0.645697329376855	0.00569732937685485
3	0.015625	-0.64185149469624	0.00185149469624035
4	0.00390625	-0.64049064823493	0.000490648234930391
5	0.0009765625	-0.640124414425145	0.000124414425144992
6	0.000244140625	-0.640031213384646	3.1213384645512e-05
7	6.103515625e-05	-0.64000781021241	7.81021240980895e-06
8	1.52587890625e-05	-0.640001952982857	1.95298285687873e-06
9	3.814697265625e-06	-0.640000488288933	4.88288933397918e-07
10	9.5367431640625e-07	-0.640000122133642	1.22133642421751e-07
11	2.38418579101562e-07	-0.64000003086403	3.08640301094343e-08
12	5.96046447753906e-08	-0.640000008046627	8.04662703135506e-09
13	1.49011611938477e-08	-0.640000008046627	8.04662703135506e-09
14	3.72529029846191e-09	-0.640000015497208	1.54972076282789e-08
15	9.31322574615479e-10	-0.640000104904175	1.04904174791365e-07
16	2.3283064365387e-10	-0.640000343322754	3.43322753892927e-07
17	5.82076609134674e-11	-0.64000129699707	1.29699707029918e-06
18	1.45519152283669e-11	-0.639999389648438	6.10351562513323e-07
19	3.63797880709171e-12	-0.6400146484375	1.46484374999867e-05
20	9.09494701772928e-13	-0.6400146484375	1.46484374999867e-05
21	2.27373675443232e-13	-0.64013671875	0.000136718749999987
22	5.6843418860808e-14	-0.640625	0.000624999999999987
23	1.4210854715202e-14	-0.640625	0.000624999999999987
24	3.5527136788005e-15	-0.65625	0.01625
25	8.88178419700125e-16	-0.75	0.11
26	2.22044604925031e-16	-1	0.36
27	5.55111512312578e-17	0	0.64
28	1.38777878078145e-17	0	0.64
29	3.46944695195361e-18	0	0.64
30	8.67361737988404e-19	0	0.64

```
%display the minimum error and its index

fprintf('\n The minimum error occurs at i = %d has a value of %d \n', imin, emin)
```

```
 The minimum error occurs at i = 1 has a value of 1.110223e-16
```

Remarkable! Our lowest error occurs exactly on the first iteration! Note that:

```
fprintf('The derivative of f(x) at x = 0.5 is: %5.2f', deriv(0.5))
```

```
The derivative of f(x) at x = 0.5 is: -0.64
```

The first derivative of $f(x)$ at $x = 0.5$ is exactly equal to $\frac{f(x+h) - f(x)}{h}$ where $h = 0.25$ (the nonzero error arises instead of zero due to the limitations of floating point arithmetic). At first glance this appears to be related to the mean value theorem, but so called mean value point in this case would be $c = 0.5$ which is also an endpoint, so perhaps this is coincidential.

This function and its derivative are a bit chunky so I'm going to write them separately to make things neater

```
function y=f(x)
    y = 1/(1 + x^2);
end

function y=deriv(x)
    y = -(2*x)/((1 + x^2)^2);
end
```