Characterizing the Hall Effect

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I. INTRODUCTION

Since its discovery in 1879, the Hall Effect has been used in various applications such as implementation in sensors for many everyday electronics as well as damping of attitude motion and measurement of magnet strength in satellites[1].

The Hall Effect arises due to the behavior of charge carriers in conductors. According to Band Theory in solids, there exists an energy gap between the valence band and the conduction band that dictates the ability of charge carriers to move freely within the material. If the energy gap is too large, almost all of the electrons within the material will be stuck in the valence band and unable to conduct electricity[2].

In semiconductors, increasing the temperature will thermally excite some charge carriers to the conduction band and increase the material's conductivity. Moreover, the addition of impurities to the material serves to increase or decrease the amount of charge carriers, depending on whether the element substituted has more or less valence electrons than the original material, respectively. An element with more valence electrons adds more charge carriers to the conduction band known as "donors" (n-type material) while an element with fewer valence electrons takes away charge carriers from the conduction band, creating a "hole" (p-type material)[3].

Applying an electric field E to a semiconductor causes the electron carriers with mass m^* and charge q to behave according to:

$$m^* \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t} + m^* \frac{\overrightarrow{v}}{\tau} = q\overrightarrow{E} \tag{1}$$

$$\overrightarrow{v_d} = \frac{q\overrightarrow{E}\tau}{m^*} \tag{2}$$

where v_d is the terminal velocity (drift velocity) and τ is the relaxation time in the steady state.

Given that there are n carriers/unit volume of charge q moving at the drift velocity, there is a resulting current density:

$$\overrightarrow{J} = nq\overrightarrow{v_d} = \left(\frac{nq^2\tau}{m^*}\right)\overrightarrow{E} = \sigma\overrightarrow{E} \tag{3}$$

σ is the electrical conductivity.

The Hall Effect is observed once a magnetic field B is applied perpendicularly to the current and the charge carriers are affected by the resulting Lorentz force:

$$\overrightarrow{F_m} = q\overrightarrow{v_d} \times \overrightarrow{B} = \frac{\overrightarrow{J} \times \overrightarrow{B}}{n} \tag{4}$$

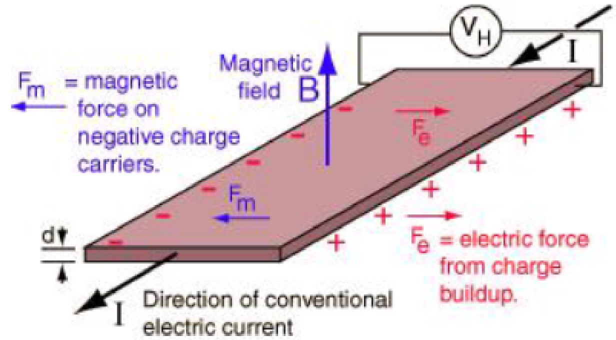

FIG. 1: Hall Effect for Negative Carrier[3]

This force creates a separation of the charges on opposite edges of the sample (Figure 1), establishing the Hall Voltage (V_H) and the Hall Electric Field (E_H). For a sample with width w and thickness d:

$$\overrightarrow{E_H} = \frac{JB\widehat{y}}{nq} \tag{5}$$

$$\left|V_H\right| = \frac{IB}{n|q|d} = R_H\frac{IB}{d} \tag{6}$$

R_H is the Hall Constant and the mobility of the charge carriers is given by:

$$\mu = R_H\sigma = \frac{q\tau}{m^*} \tag{7}$$

In this report, the results of an experiment to highlight the hall effect in Germanium semiconductors will be outlined. The experimental procedure for this experiment will be shown for 3 types of Germanium semiconductors (p-type, n-type, undoped). Hall voltage, and voltage of the Germanium sample will be plotted with regards to several variables of interest (particularly, current, temperature of the sample, and magnetic field applied), and physicals constants will be derived and compared to theoretical results. Through these relationships, conclusions about the consequences of the Hall effect will be drawn.

II. EXPERIMENTAL PROCEDURE

Figure 2 depicts the primary components of the apparatus used for our experiment. First, the power supply sends an AC current to the Console, which is then converted to DC and sent to the Germanium sample. Then, the power supply provides a DC current to the

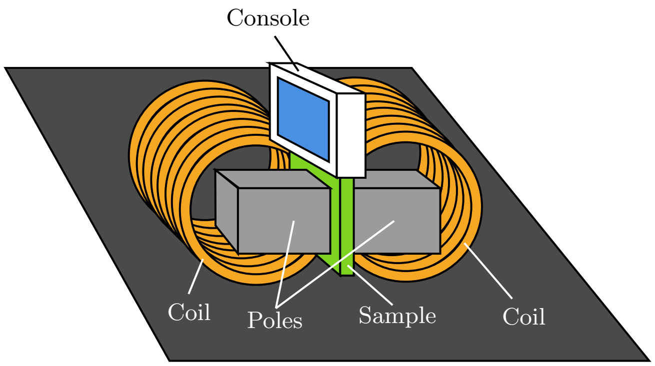

FIG. 2: Primary Components of Apparatus

coils and establishes a magnetic field. The poles connected to the coils then reorient that magnetic field to point perpendicularly to the current flowing through the Germanium sample. As mentioned before, this produces the Hall Voltage which can be measured and read from the Console as well as the sample's voltage and current, magnetic field values, and the temperature.

As mentioned before, the Hall voltage and voltage of the Germanium sample will be plotted with regards to several variables of interest in order to derive and compare physical constants to theoretical results[3].

III. RESULTS AND ANALYSIS

For all three samples, as seen in the bottom halves of figure 3, figure 4, and figure 5, we observe that the relationship between sample voltage and current is primarily ohmic. This is expected behavior since these measurements were taken at a constant sample temperature and since Ge has a nonempty conduction band population at room temperature.

In p-type materials, we would expect that hole concentration exceeds electron concentration. Assuming a positive constant magnetic field is applied to the sample, for a known positive direction of the current, hole carriers will be deflected via the Hall effect such that a positive Hall voltage would be measured; for a known negative direction of the current, hole carriers will be deflected via the Hall effect such that a negative Hall voltage would be measured.

According to equation (6), this means that if we start from a negative current and measure the Hall voltage each time we increase the current, we should observe a positive slope for the linear relationship indicated by (6) between the Hall voltage and current. Indeed, this is what we observe in the top half of figure 3.

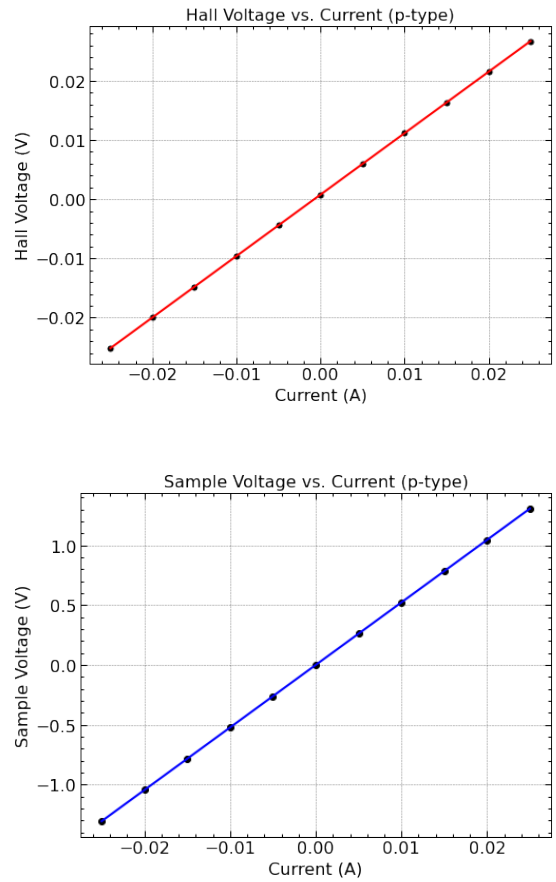

FIG. 3: P-type Current Trial

In n-type materials, we would expect that electron concentration exceeds hole concentrations. Again, assuming a positive constant magnetic field is applied to the sample, for a known positive direction of the current, electrons will be deflected via the Hall effect such that a negative Hall voltage would be measured; for a known negative direction of the current, electrons will be deflected via the Hall effect such that a positive Hall voltage would be measured.

According to equation (6), this means that if we start from a negative current and measure the Hall voltage each time we increase the current, we should observe a negative slope for the linear relationship indicated by (6) between the Hall voltage and current. Indeed, this is what we observe in the top half of figure 4.

In the top half of figure 5, we see that the undoped relationship between Hall voltage is a flat line, so an increase in current causes neither an increase or decrease in Hall Voltage. This behavior can be attributed to the fact that in an undoped semiconductor sample, the concentrations of electrons and holes are approximately equal.

Consequently, increasing the electron current is coupled with an equivalent increase in hole current. However, we do observe a constant negative Hall voltage for the undoped sample with respect to current. This is because in Ge, electron mobility is higher than hole mobility (insert reference here). Thus, electrons are more influenced by the applied magnetic field to the undoped sample[4].

In figures 6 (p-type) and 7 (n-type), the behaviors of

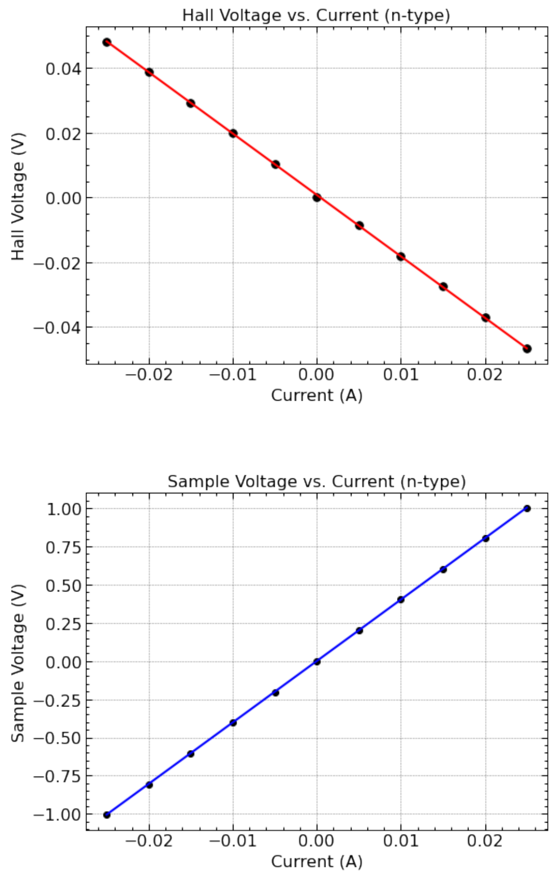

FIG. 4: N-Type Current Trial

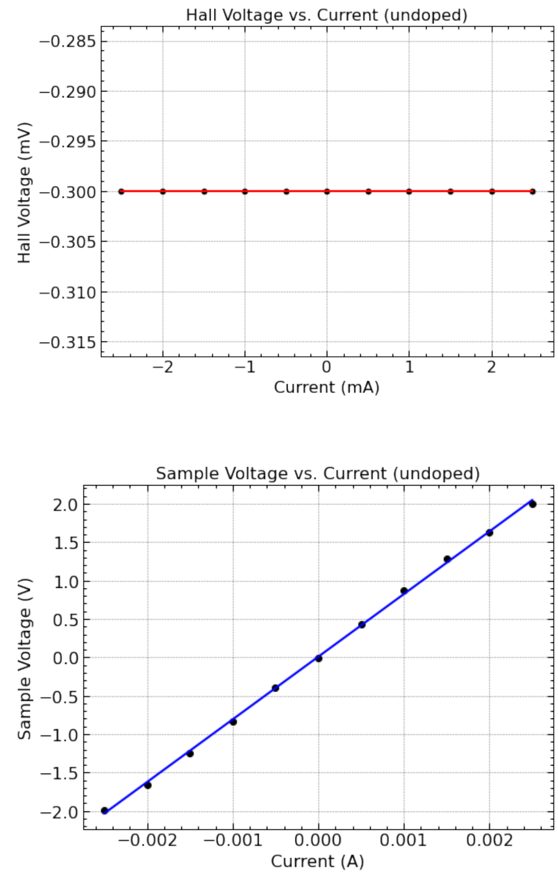

FIG. 5: Undoped Current Trial

the Hall voltage (top portion), sample voltage (middle portion), and conductivity (bottom portion) are plotted with respect to temperature. The middle portions and bottom portions demonstrate the same behavior in both figures. For both doped samples, an increase in temperature causes a corresponding increase in atomic vibrations present in the samples. Just as in ordinary conductors, this causes more resistance and hence, a decrease in conductivity.

Simultaneously, as temperature increases, more and more electrons from Ge atoms enter the conduction band. Up until a certain threshold temperature (about 80°C for both samples), these "freed" electrons did not have much of an effect on the sample voltage; however, past that point, enough electrons have entered the conduction band to where current flow can noticeably overcome the increasing resistance of the sample with respect to temperature. Thus, conductivity increases and sample voltage decreases past this threshold temperature.

In the top portions of figures 6 and 7, we can see that the samples depict opposing behavior (both look like ladles facing opposite directions).

For the p type sample, hole concentration is dominant and up until about 60°C, the Hall voltage remains approximately constant since freed electrons due to increasing temperature from the semiconductor lattice have not made a significant enough contribution. Past 60°C, we can see a significant decrease in hall voltage with respect to further increases in temperature. This is due to the fact that the freed electrons from the Ge atoms balance out the holes in the current.

Starting past 100°C, we can see that the plot stops decreasing at a negative Hall voltage and past about 120°C, begins to increase again. Theoretically, we would expect the Hall voltage around this region to level off at a constant negative voltage as depicted in the top half of figure 8. This unexpected behavior is likely due to the sample reaching a high enough temperature to where the electrons from the dopant atoms begin to enter the conduction band. As such, the current becomes more populated by electrons; the mobility of the electrons would hence, decrease and would thus become less influenced by the applied magnetic field on the sample which would correspond to more of an influence by the hole current and

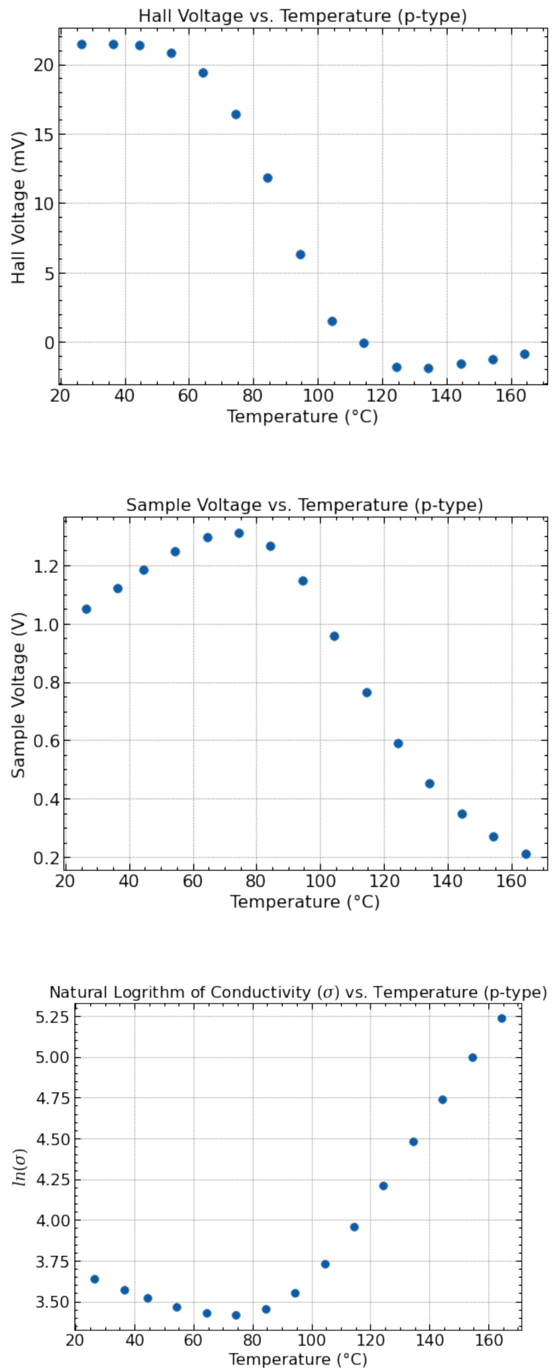

FIG. 6: P-type Temperature Trial

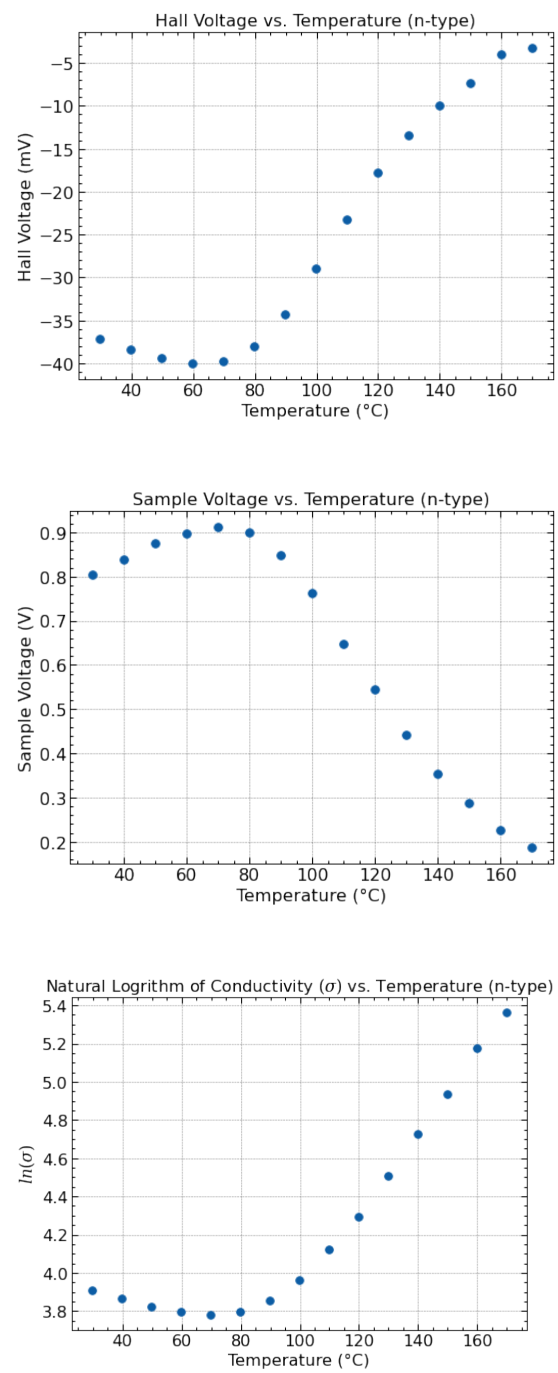

FIG. 7: N-type Temperature Trial

thus a more positive Hall voltage.

For the n type sample, electron concentration is dominant due to donor electrons present from the dopant atoms. Up until about 60°C, electrons seem to have an increasing influence on the Hall voltage likely due to the freed electrons from the semiconductor lattice. Past 60°C, we are still having more and more Ge electrons enter the conduction band. However, maintaining the sample current fixed with an increasing supply of electrons must be coupled with a decrease in electron mobility in the sample current.

As such, as more electrons enter the current, the current becomes less influenced by the applied magnetic field. We see this trend continue up until about 160°C, to where the plot appears to level off — likely to the steady Hall voltage depicted in the top half figure 8.

Compared to the doped samples, undoped Ge displays

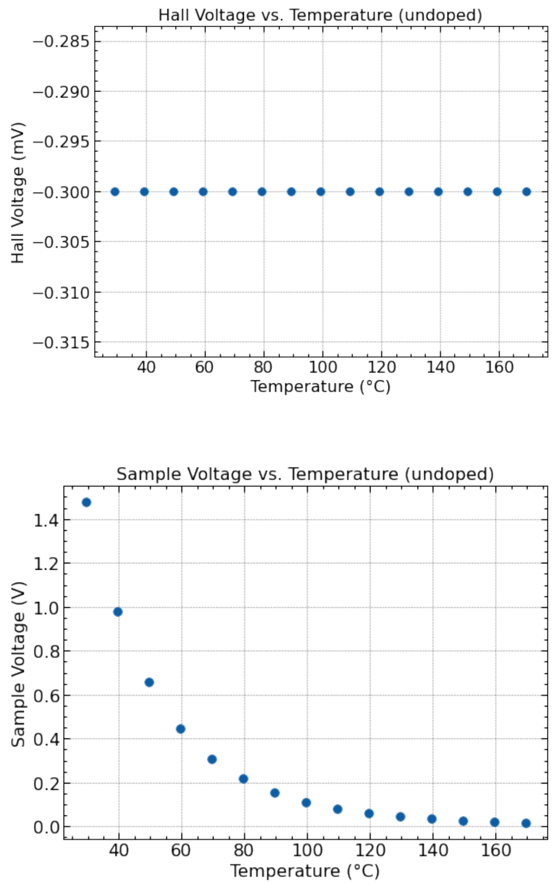

FIG. 8: Undoped Temperature Trial

significantly different qualitative behavior. Ultimately, this difference can be attributed to the extrinsic carrier concentrations from the dopants in the doped samples which results in either a hole or electron dominance. For undoped Ge, similar to V_H vs. current, for V_H vs. temperature, the concentration of holes and electrons are approximately equal in the sample which explains the flat line, but again, electron mobility dominates (insert corresponding reference here) which is why we see a net negative hall voltage.

Unlike in the current trial, as temperature increases, more intrinsic electrons enter the conduction band. However, the increase in electrons involved in the current causes electron mobility to decrease to maintain the same current value.[1]

In figure 9, the conductivity for the undoped sample is plotted differently compared to the doped samples; instead it is plotted against T^{-1} instead of T. The reason why this is the case here is because conductivity in pure semiconductors is related to temperature by the following equation[5]

$$\sigma = \sigma_0 e^{\frac{-E_g}{kT}} \tag{8}$$

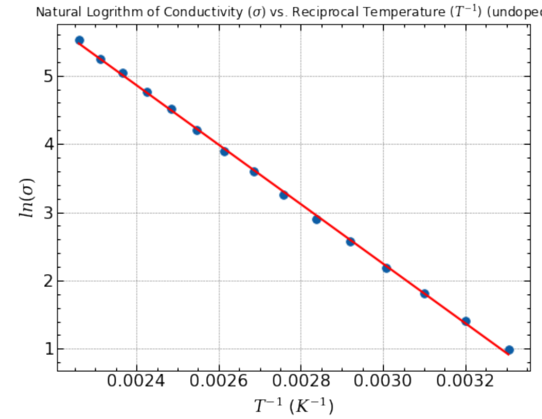

FIG. 9: Conductivity Analysis of Undoped Sample

where k is the Boltzmann constant, T is temperature, σ_0 is some reference conductivity, and E_g is the gap energy for charge carriers to enter the conduction band.
From equation (8), we can take the natural logarithm of both sides to yield

$$ln(\sigma) = \frac{-E_g}{kT} + ln(\sigma_0). \tag{9}$$

Equation (9) indicates that we can find the band gap energy of a pure semiconductor by utilizing the linear relationship between σ and $1/T$. Let s_1 denote the slope of the plot in figure 9. Then, by equation 9, we can calculate the gap energy using

$$E_g = -ks_1 \tag{10}$$

Using equation 10 from the slope based on our data, we get $E_g = 0.75\ eV$.

Figures 10 and 11 depict Hall voltages and sample voltages vs. the magnetic field applied to the sample at a constant applied current. From equation (6), we see that the Hall voltage is also linearly related to the applied magnetic field. Indeed, this is observed from the positive (for p type) and negative slopes (for n type) present in the top halves of figures 10 and 11.

In the bottom halves of figures 10 and 11, we can observe a nonlinear relationship between the sample voltage and magnetic field. This is probably due to the fact that as current travels through the sample, the path the current takes is deflected by the magnetic field and so charge carriers in the current have to travel a longer path than it would otherwise travel without the magnetic field.

The net effect of this deflection is that for a constant current, the sample voltage increases which means that the resistance of the sample to current flow also increases. Moreover, the magnetic deflection causes the charge carriers to accelerate transversely to their original path which might result in some electromagnetic radiation and hence, a loss of energy. When considering these

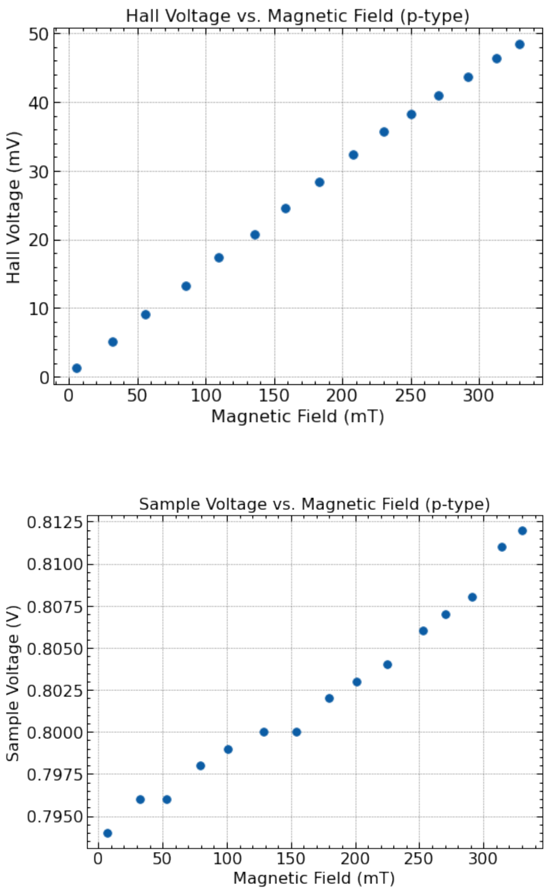

FIG. 10: P-Type Magnetic Field Trial

possibilities, the nonlinear behavior depicted for sample voltage vs. magnetic field is hardly surprising.

A. Acquired Data

	σ $(\Omega \cdot m)^{-1}$	μ $[m^2(V \cdot s)^{-1}]$	n (m^{-3})
P-Type	38.249	0.312	7.64 $\times 10^{20}$
N-type	49.780	0.767	4.05 $\times 10^{20}$
Undoped	2.45	2.41	3.17 $\times 10^{18}$

TABLE I: Acquired Data for conductivity (σ), mobility (μ), and charge carrier density (n) for all three samples

In order to evaluate the conductivity of the samples, we used the sample voltage vs. current plots (bottom half figures 3, 4, and 5) and evaluated the slope via linear regression which gave us the resistance R. Using R, we can use the provided dimensions of the conductor sample and the resistance formula (in terms of conductivity).

$$R = \frac{L}{\sigma w d} \tag{11}$$

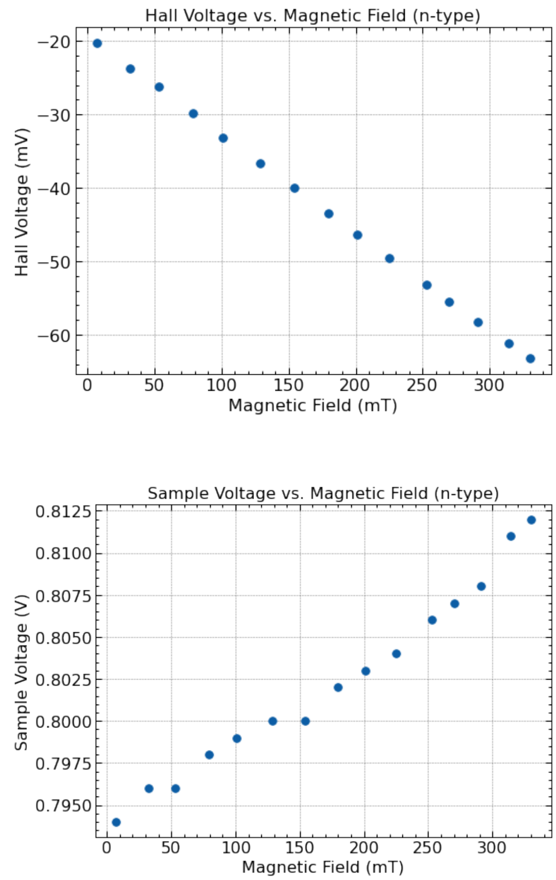

FIG. 11: N-Type Magnetic Field Trial

where L is length, w is width, and d is depth. The provided values are $L = 20mm$, $w = 10mm$, $d = 1mm$. Then, rearranging 11 gives

$$\sigma = \frac{L}{Rwd} \tag{12}$$

For example, for the p type sample, we got $R = 52.289\ \Omega$. Using equation 12, we get

$$\sigma = \frac{0.02m}{52.289\ \Omega \cdot 0.0100m \cdot 0.001m} = 38.249\ \frac{1}{\Omega \cdot m}$$

For acquiring the mobilities and carrier densities, we had to take different approaches for evaluating these quantities for doped and undoped samples. For the doped samples, the process was relatively straight forward. In order to evaluate the mobilities and carrier densities, we used the Hall voltage vs. current plots (top half of figures 3 and 4). We also used linear regression to evaluate the slope, denote it by s_2, which gave us, according to equation (6)

$$s_2 = \frac{R_H B}{d} I$$

which gives

$$R_H = \frac{s_2 d}{B}$$

Now, R_H is related to the density by

$$R_H = \frac{1}{nq}$$

which leads to the following expression for n:

$$n = \frac{B}{s_2 dq} \tag{13}$$

Moreover, we can use R_H our conductivity data to find the mobility via equation (7).

For instance, for the p type sample, we applied a constant magnetic field of about $B = 127\ mT$ to the sample and we acquired a slope of $s_2 = 1.038$ which yields

$$n = \frac{0.127}{1.038 \cdot 0.001 \cdot (1.602 \times 10^{-19})} = 7.64 \times 10^{20} m^{-3}$$

$$\mu = \frac{s_2 d\sigma}{B} = \frac{1.038 \cdot 0.001 \cdot 38.249}{0.127} = 0.312\ \frac{m^2}{V \cdot s}$$

For the undoped sample, we used the following equation for the desnsity[5]

$$n = 2\left(\frac{kT}{2\pi\hbar^2}\right)^{3/2} (m_n m_h)^{3/4} e^{-E_g/kT} \tag{14}$$

where m_n and m_h are the effective masses of the electrons and holes respectively, $\hbar$ is a form of planck's constant, T is the temperature ($T = 300K$ for our purposes), and E_g is the gap energy we calculated earlier. We used the following mass ratios (where m_0 is the electron rest mass, $m_0 = 9.109 \times 10^{-31} kg$) to evaluate the effective masses[6]: $m_n/m_0 = 0.56$ and $m_h/m_0 = 0.29$.

We also used the following equation[5] by making the simplifying assumption that $\mu = \mu_n = \mu_h$

$$\sigma = nq(\mu_h + \mu_n)$$

$$\Rightarrow \sigma = 2nq\mu$$

which yields

$$\mu = \frac{\sigma}{2nq} \tag{15}$$

IV. CONCLUSIONS

Charge carriers can be distinguished by the plots of Hall voltage vs. current for each type of Germanium semiconductor. The positive slope in the p-type semiconductor Hall voltage vs. current plot indicates holes as the primary charge carriers in the sample, the negative slope in the n-type semiconductor indicates electron as the primary charge carriers, and the constant slope in the undoped sample points toward neither being predominant in the sample. Plots for each set of variables were strikingly similar to theoretical values. For example, gap energy from experimental data was 0.75 eV (undoped). With a real value of 0.67 eV[3], this leads to a 12% error between values. Considering the somewhat crude methods of measurement, this appears to be rather satisfactory, and symptomatic of proper data collection. Regardless, values for charge density/mobility appear to be relatively off.

Unsurprisingly, our mobility value was quite off when compared to that found in established literature[4] where $\mu_n = 0.39 \frac{m^2}{V \cdot s}$ and $\mu_h = 0.19 \frac{m^2}{V \cdot s}$. It was a leap to assume that these two mobility values were equal in the first place; however, the assumption was made since finding these quantities would be pretty difficult to do unempirically.

Differences in functional behavior of Hall voltage vs. temperature can be explained by the lack of predominant behavior in undoped samples vs. the asymmetry in doped samples. The equality of concentration of electrons vs. holes for the undoped sample leads to a lack in change of the Hall voltage with temperature, while dominance in either electrons or holes for doped samples leads to an increase in Hall voltage for the n-type sample, and a decrease in Hall voltage for the p-type sample. In both cases, this can be attributed to the increasing amount of freed electrons caused by the increase in temperature.

It appears that sources of error were few and far between, with data being near theoretical values. There was some slight "error" in our Hall voltage vs. temperature trial for the p-type semiconductor (an upwards trend where there shouldn't be), but this might be explained by the temperature becoming so high that dopant atoms start to release conduction band Electrons. One source of error could be the fluctuating temperature when heating up each sample. It is very difficult to gather data points for an exact temperature, so most data points were taken within a range (although less than 1 Celsius for each point). There is slight inaccuracies in some data points that can be explained by frictional error, but trends for plots appear to be as expected theoretically. Finally, an error that confounded results at first was improper wiring for the apparatus; reversing functional behaviors by changing the sign of the slope. This was fixed by changing the sign of representative plots (a simple multiplication by -1).

V. AUTHOR CONTRIBUTIONS

Data was taken by all members. Bryant N. wrote the introduction, and experimental methods, and made figures of interest for the aforementioned sections. Lendel Deguia wrote the "Results and analysis" section, in addition to creating most of the salient graphs for the report. Nuno M. added references, wrote the conclusion, and proofed the report. All members looked for references.

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