Computer Exercise 2.2.2

The following program will use Gaussian elimination with scaled partial pivoting to solve the following $\mathbf{Ax} = \mathbf{b}$ system:

$$\begin{bmatrix} 0.4096 & 0.1234 & 0.3678 & 0.2943 \\ 0.2246 & 0.3872 & 0.4015 & 0.1129 \\ 0.3645 & 0.1920 & 0.3781 & 0.0643 \\ 0.1784 & 0.4002 & 0.2786 & 0.3927 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.4043 \\ 0.1550 \\ 0.4240 \\ 0.2557 \end{bmatrix}$$

Here, procedures *Gauss* and *Solve* have been combined into one method, *gespp*, which outputs the solution $\mathbf{x}$ along with the final states of $\mathbf{A}$ and $\mathbf{b}$. Only the solution $\mathbf{x}$ will be displayed.

```
A = [0.4096, 0.1234, 0.3678, 0.2943;
    0.2246, 0.3872, 0.4015, 0.1129;
    0.3645, 0.1920, 0.3781, 0.0643;
    0.1784, 0.4002, 0.2786, 0.3927];
b = [0.4043, 0.1550, 0.4240, 0.2557]';
[Amod, bmod, x] = gespp(A,b);
x
```

```
x = 4×1
    3.4606
    1.5610
   -2.9342
   -0.4301
```

Just to make sure that this is the right solution, we can check that $\mathbf{Ax}$ provides the same $\mathbf{b} = \begin{bmatrix} 0.4043 \\ 0.1550 \\ 0.4240 \\ 0.2557 \end{bmatrix}$

```
A*x
```

```
ans = 4×1
    0.4043
    0.1550
    0.4240
    0.2557
```

Indeed, we end up with the same $\mathbf{b}$, so this is the correct solution.

```
function [Amod, bmod, x] = gespp(A,b)
    n = length(b);
    %set index vector
    l = (1:n);
    %set scale vector
    s = zeros(length(l), 1);
```

```
    for i = 1:n
        s(i) = max(abs(A(i, :)));
    end
    %forward elimination
    for k = 1:(n-1)
        max_r = 0;
        pivot_index = l(1);
        for i = k:n
            if (abs(A(l(i), k))/s(l(i))) > max_r
                pivot_index = i;
                max_r = (abs(A(l(i), k))/s(l(i)));
            end
        end
        a = l(pivot_index);
        l(pivot_index) = l(k);
        l(k) = a;
        for i = (k+1):n
            mult = A(l(i), k)/A(l(k), k);
            for j = k:n
                A(l(i), j) = A(l(i), j) - mult*A(l(k), j);
            end
            b(l(i)) = b(l(i)) - mult*b(l(k));
        end
    end
    Amod = A;
    bmod = b;
    %back substitution
    x = zeros(n, 1);
    x(n) = b(l(n))/A(l(n), n);
    for u = (n-1):-1:1
        sum = 0;
        for v = (u+1):n
            sum = sum + (A(l(u), v)*x(v));
        end
        x(u) = (b(l(u)) - sum)/(A(l(u), u));
    end
end
```