Faraday Rotation

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I. INTRODUCTION

Throughout the mid-1800s, Faraday sought to characterize light. It then became a race for him to seek proof that light would be affected by electromagnetic forces; a symptom of EM phenomena. Faraday finally saw proof of this when he shone light through a tempered glass in the direction of a magnetic field, leading to a change in the polarization angle of the incident beam. Since proving this relationship between light and magnetic forces, the EM nature of light was shown to be empirical, and this change in polarization angle (the so-called Faraday effect, or Faraday rotation) has since been seen in semiconductors, organic materials, and even in the ionosphere, a well-known plasma, where it is a topic of interest for space physicists.[1]

Plane-polarized light propagating through a material with a non-trivial Verdet constant, in a uniform magnetic field, remains plane-polarized, but the plane of light is seen to rotate through space proportionally to the strength of the magnetic field, and proportional to the material's Verdet constant. This phenomena is referred to Faraday rotation (FR for short). Take a linearly polarized ray through a non-trivial material: this ray can be thought of as the superposition of two circularly-polarized rays. When propagating through this material, both of these rays that make up the incident beam experience different delays. Due to this delay, when they recombine upon exiting the propagation material, the output beam takes on a different polarization angle.

One can relate the angle of rotation of polarization and the magnetic field of a relevant material simply by:

$$\theta = \nu B d \tag{1}$$

where θ is the angle of rotation, ν is the Verdet constant of the material, B is the magnetic field, and d is the length of the path traveled through the propagation material[2]. Of note, particularly, is that the Verdet constant is not necessarily positive. $\nu > 0$ implies counter-clockwise rotation when a wave is propagated parallel to the magnetic field, and clockwise when anti-parallel. Likewise, $\nu < 0$ corresponds to clockwise rotation when parallel to the magnetic field, and counter-clockwise when anti-parallel.

One can also characterize this phenomena with Malus' law, which states the relationship when adding a polarizer to this process:

$$I = I_i cos^2(\phi - \theta) \tag{2}$$

with I_i incident intensity, and for a polarizer set ϕ to incident waves[2]. In this report, the aforementioned equation

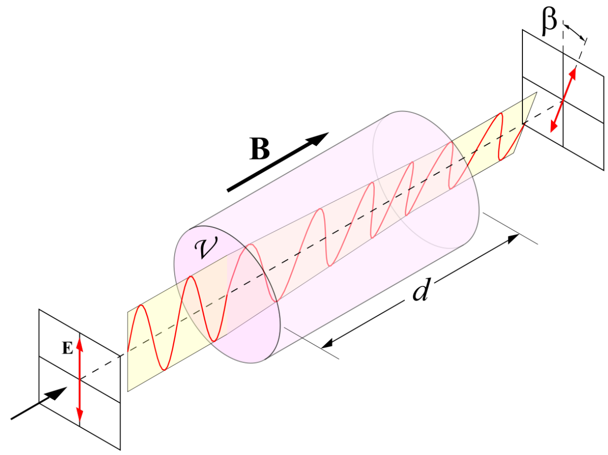

Figure 1: FR displayed through a Faraday rotator. Plane-polarized light changes its polarization angle when propagating through a non-trivial material with uniform magnetic field.

with no Faraday rotation, $\theta = 0$, is considered, to characterize the change in intensity through small Faraday rotation angles. One can then find a maximum for the change in intensity for given polarization angles:

$$\frac{\partial I}{\partial \phi} = 2I_i sin(2\phi) \Longrightarrow \max\{\frac{\partial I}{\partial \phi}\} = \frac{\partial I}{\partial \phi}\Big|_{\phi=45^o}$$

Then:

$$I(45^o) = \frac{I_i}{2} \equiv I_0, \text{ and } \frac{\partial I}{\partial \phi}\Big|_{\phi=45^o} = 2I_0$$

which when combined with the power series expansion of $I(\theta, \phi)$ with regards to theta, gives[2]:

$$I(\theta, \phi) = I_0 + \Delta I \approx I_0 + 2I_0\theta \Longrightarrow \Delta I \approx 2I_0\theta \tag{3}$$

This report will highlight methodology to characterize the relationship between the magnetic field and polarization angle for light incident and propagating through transparent material, in accordance with Faraday rotation. Malus' law will be verified by fitting relevant data from the experimental setup. The Faraday rotation directly resulting from a DC field will be characterized. Finally, the Faraday rotation resulting from an AC field will be displayed in two different procedures, so as to properly understand the magnitude and implications of this particular phenomena.

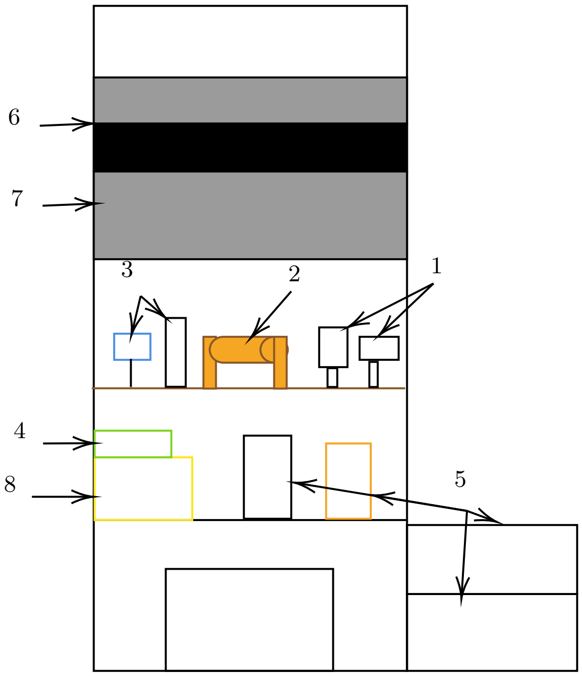

Figure 2: Experimental setup. (1) Light source and polarizer. (2) Solenoid. (3) Polarizer and photodetector. (4) Current transformer. (5) Multimeters. (6) Lock-in amplifier. (7) Crown audio amplifier. (8) DC power source.

II. EXPERIMENTAL PROCEDURE

The setup consists of a multitude of components pictured in Fig.2. There is a light source (650 nm 3mw) with a polarizer prior to the propagating material (1). A solenoid provides a uniform magnetic field, with l = 150mm, n = 1400, size = 18g, and R = 2.6Ω. This solenoid encases the propagation material (a glass rod for the original experiment, and an unknown transparent material for an addendum) (2). Another polarizer, along with a photo-detector are placed for actual measurements (3). The former of these is equipped with a micrometer and a vernier scale, and the latter can serve as a current source. The current transformer transforms current from the current detector to a voltage for the oscilloscope to read (4). It is important to note that the current transformer is wound 10 times, but due to the voltage/current of the oscilloscope, this comes out as a 1:1 ratio of current to voltage. 4 multimeters allow for measurements of voltage and current (5). The lock-in amplifier outputs a DC signal for AC signal inputs (6). It also supplies an AC signal to the solenoid. The crown audio amplifier amplifies signal from the lock-in to provide an AC signal to the solenoid (7). The DC power supply provides a DC signal for the solenoid (8).

There will be 4 specific methods for collecting data. In the first setup, the solenoid will not be used. The photodiode will be connected to the right-most multimeter for VDC measurements. The angle of maximum intensity will be recorded (the apparatus isn't quite tuned so the maximum is at 0^o) by taking the midpoint of two angle measurements. Afterwards, the left polarizer will be rotated by 5^o, and voltage measurements will be taken multiple times with this increment. This data will be graphed to relate the two variables as per Malus' law.

The second setup serves to quantify FR due to a DC field. DC current will now be measured by the Fluke 75 multimeter. The solenoid is inserted, and the polarizer is rotated to 45^o from the maximum. The change in voltage, for 5+ midpoints between 1 and 3 A is then measured. The intensities will be graphed with regards to the currents, and the Verdet constant will be determined for the glass rod with this data.

The third setup, along with the fourth setup involve determining FR due to an AC field. In the third, the polarizer is set to 45^o from the max. A current is passed through the solenoid through the lock-in. The DC offset voltage from the photo-diode is recorded. The oscilloscope is the used to measure the p-p voltage, and the FLUKE multimeter is used to measure the AC current (in rms). This measurement is repeated for 5 currents from the lock-in amplifier. The change in angle vs. the field strength is then graphed to determine the Verdet constant.

In the fourth setup, the lock-in amp is used to the voltage. The polarizer is set as in the third, and one can choose to use the multimeter, or the oscilloscope to measure the solenoid current (differences will be developed in conclusions). The voltage for 5 different currents from the lock-in will be recorded. The lock-in voltage vs. B will be plotted, and compared to prior experiments. This plot will then be converted to a $\Delta\theta$vs. B graph by tuning the slope. From this, one can once again determine the Verdet constant of the glass rod.

III. RESULTS AND ANALYSIS

A. Malus's Law Experiment

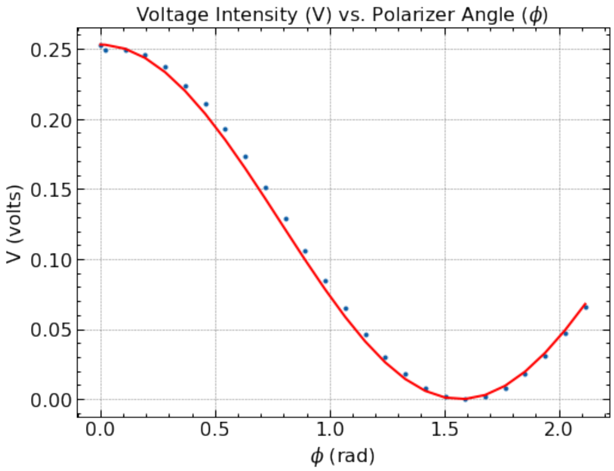

Figure 3: Malus's Law Exprimental Data and Fitted Curve

Figure 3 depicts the voltage detected at the photodiode and read on a multimeter (with the sample and solenoid away from the path of the laser beam) vs. the measured rotated angle of the polarizer. The fitted curve is of the form:

$$V(\phi) = V_0 \cos^2(\phi)$$

Indeed, Malus's Law holds up nicely with respect to our acquired data.

B. Faraday Rotation due to a DC field

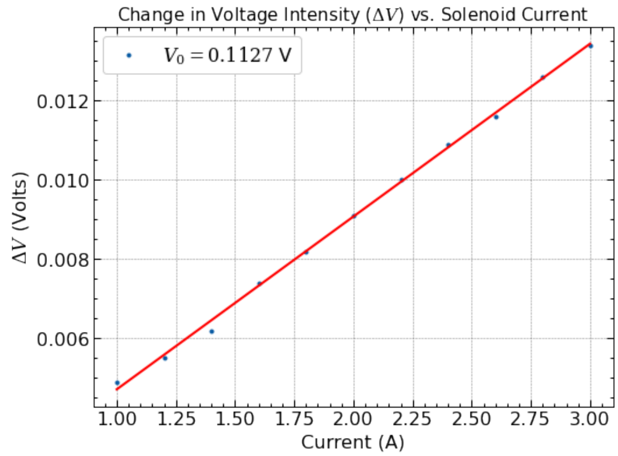

Figure 4: DC Faraday Rotation Experimental Data and Fitted Line

Figure 4 indicates the voltage change with respect to a reference voltage V_0 versus the current through the solenoid. Using equation 3 and equation 1 to arrive at the following relationship:

$$\Delta V = 2V_0 \nu l m I \tag{4}$$

where $m = 11.2 \times 10^{-3} / \frac{rad}{m \cdot T}$, $l = 0.075/m$, ν is the Verdet constant and I is the current.

Then, we use the slope of the fitted line in figure 4, call it s_1, to find the Verdet constant characterizing the Faraday rotation of the laser beam using the following equation:

$$\nu = \frac{s_1}{2V_0 l m}$$

Doing so yields $\nu = 23.023\ \frac{\mathrm{rad}}{m \cdot T}$.

C. Faraday Rotation due to a AC field

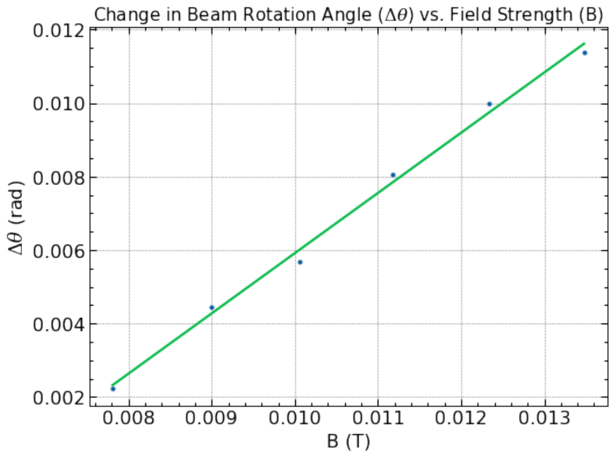

Figure 5: AC Faraday Rotation Experimental Data and Fitted Line

Displayed in figure 5 is the rotation angle of the laser beam plotted against the magnetic field strength in the solenoid. The magnetic field strength is related to the current in the solenoid (where I is the current and $m = 11.2 \times 10^{-3}\ \frac{rad}{m \cdot T}$) by the equation:

$$B = mI$$

We had two options for acquiring our solenoid current values: through the output of the transformer to the oscilloscope or by using the value detected by the multimeter. The current used for this equation was acquired by reading the voltage signal off of the oscilloscope; the transformer is calibrated such that the peak value of the voltage signal is also the magnitude of the current running through the solenoid.

This is perhaps the more accurate method versus reading off the multimeter since the multimeter provides an rms value and would have to be multiplied by a factor of $\sqrt{2}$;however, the multimeter would be the more precise approach since it provides more digits for measurement, but this approach would be more prone to error whereas the value provided by the output of the transformer provides a more direct measurement. In order to evaluate $\Delta\theta$, we assumed without the magnetic field, $\theta = 0$ so that $\Delta\theta = \theta - 0$, and used equations 3 and 4 where we used $V_0 = 5.182 \times 10^{-3}\ V$ (our DC offset) for ΔV. After evaluation, we end up with $\nu = 21.85\ \frac{\text{rad}}{m \cdot T}$ as our Verdet constant.

D. AC Field w/ Lock-in Amp

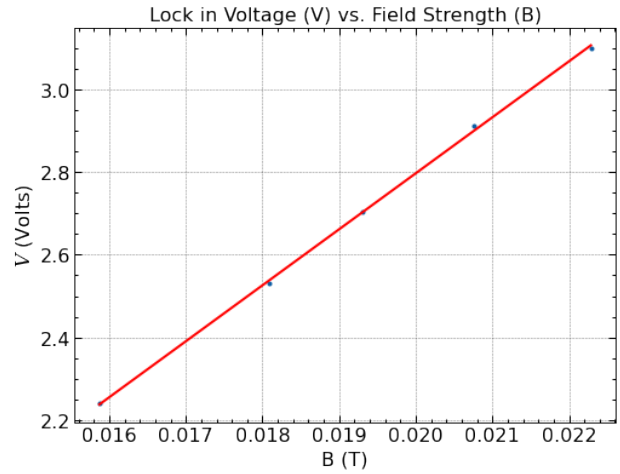

Figure 6: AC Faraday Rotaion with Lock in Amp: Voltage and Field Strength Data with Fitted Line

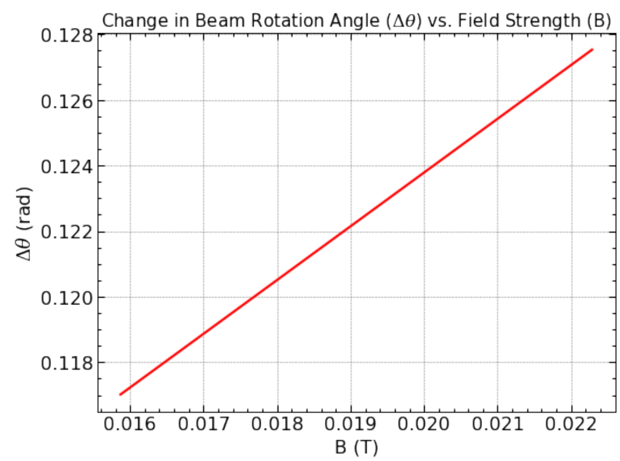

Figure 7: AC Faraday Rotaion with Lock in Amp: approximated Angular change and Field Strength Data with Fitted Line

Figure 6 depicts the voltage from the lock in amplifier plotted against the current detected from the oscilloscope; we chose to measure the current from the oscilloscope since, as said in part C, it is the more accurate approach since it provides a more direct measurement. We did not have a reference voltage V_0 to allow us to plot ΔV, but the linear relationship between the lock in voltage and the magnetic field strength is similar in behavior to the previous linear plots which confirms the linear relationship between the change in voltage and the output voltage of the photodiode. Figure 7 depicts the rotation angle of the laser beam versus the magnetic field such that the slope of the plot is the slope of figure 6 scaled to match the slope of figure 5. Of course, since the slope is the same as in figure 5, we end up with $\nu = 21.85\ \frac{\text{rad}}{m \cdot T}$

as our verdet constant.

Overall, it turns out that the DC method is the most precise when noting the expected value of the verdet constant for a sf-59 glass rod is 28.5 $\frac{rad}{m \cdot T}$. For both AC methods,using the transformer instead of the multimeter provided us with more accurate results; the multimeter would provide more precise results. Of course, there is a trade off in selection when considering which approach to use for reading the solenoid current values. In deciding to read our current values via the output of the transformer, we made a sacrifice in precision, but this was a more desirable choice since one of our goals was to verify the validity of the Faraday rotation equation indicated by equation 1. Moreover, if we were to choose using the multimeter for our current values, we would end up sacrificing accuracy since we would have to assume to factor the acquired values by $\sqrt{2}$ which may not be exactly the case.

IV. CONCLUSIONS

Through the multitude of experiments, the implications of FR can be quantified. Experiment 1 provided for evidence of the relationship seen in Malus' law. The data taken resembles that predicted by eq. 2 functionally. Through experiment 2, one can obtain a Verdet constant of 23.023 $\frac{rad}{m \cdot T}$, which poses a great approximation to the actual verdet constant for the given glass rod of 28.5 $\frac{rad}{m \cdot T}$. Experiments 3 and 4 also provided for a Verdet constant somewhat close to this aforementioned value, especially considering the crude methodology for collecting data. There are a multitude of errors that can confound these experiments. One of the most glaring, is the lack of a "zero", i.e, a calibration point in experiment 4. Besides that, the light source appeared to not fully hit the photodetector. This would likely result in lower Verdet constants all-around, since current readings would be lower. In addition, having unclean, or improperly cut materials can reduce the Verdet constant unwillingly.

V. AUTHOR CONTRIBUTIONS

All members took measurements. Nuno M. wrote the "introduction", and "experimental methods" sections of the report. Lendel D. wrote the "results and analysis" section. Bryant N. wrote the conclusion and perused resources.

[1] F. B. Daniels and S. J. Bauer, Journal of the Franklin Institute **267**, 187 (1959).
[2] Pre-Laboratory for Physical Measurements Laboratory (PHYS 4373).