HW 9

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5.1.2 $\int_1^2 \frac{1}{x}\,dx \Rightarrow f(x)=\frac{1}{x}$; $P=\{1, \frac{3}{2}, 2\}$

$\Rightarrow f(P)=\{1, \frac{2}{3}, \frac{1}{2}\} \Rightarrow \int_1^2 \frac{1}{x}\,dx \approx \frac{1}{2}\left[\left(\frac{3}{2}-1\right)\left(1+\frac{2}{3}\right)+\dots\right.$

$\left.\dots+\left(2-\frac{3}{2}\right)\left(\frac{1}{2}+\frac{2}{3}\right)\right] = \frac{1}{2}\left[\frac{5}{6}+\frac{7}{12}\right] \Rightarrow \boxed{\int_1^2 \frac{1}{x}\,dx \approx \frac{17}{24}}$

5.1.9 $\int_0^1 \frac{1}{1+x^2}\,dx \Rightarrow f(x)=\frac{1}{1+x^2}$; $P=\{0, \frac{1}{2}, 1\}$

$\Rightarrow f(P)=\{1, \frac{4}{5}, \frac{1}{2}\} \Rightarrow \int_0^1 \frac{1}{1+x^2}\,dx \approx \frac{1}{2}\left[\left(\frac{1}{2}-0\right)\left(1+\frac{4}{5}\right)+\dots\right.$

$\left.\dots+\left(1-\frac{1}{2}\right)\left(\frac{1}{2}+\frac{4}{5}\right)\right] = \frac{1}{2}\left[\frac{9}{10}+\frac{13}{20}\right] \Rightarrow \boxed{\int_0^1 \frac{1}{1+x^2}\,dx \approx \frac{31}{40} = 0.775}$

actual value:

$I=\int_0^1 \frac{1}{1+x^2}\,dx = \arctan(x)\Big|_0^1 = \frac{\pi}{4} \approx 0.785$; let $T=0.775$

$\Rightarrow \boxed{I-T \approx 0.0104}$, $f'(x)=\frac{-2x}{(1+x^2)^2}$

$f''(x)=\frac{6x^2-2}{(1+x^2)^3} \Rightarrow \max_{0\le x\le 1}\left(|f''(x)|\right) = |f''(0)| = 2$

$|I-T| = \left|\frac{b-a}{12}\right|\left|h^2 f''(\xi)\right| = \frac{1}{48}|f''(\xi)| \le \frac{2}{48} = \frac{1}{24}$

upper bound

$\Rightarrow \boxed{|I-T| \le \frac{1}{24} \approx 0.0417}$

5.1.12) $\int_{-1}^{2}\sin(x)\,dx$, $a=-1$, $b=2$ $\quad h=0.01$

$f'(x)=\cos(x)$ $\quad f''(x)=-\sin(x) \Rightarrow \max_{-1\le x\le 2}|f''(x)|=1$

$$\boxed{|I-T| = \left|\frac{b-a}{12}h^2 f''(\xi)\right| = \left|\frac{(0.01)^2}{4}f''(\xi)\right| \le 2.5\times10^{-5}}$$

5.1.24) $\int_a^{a+h} f(x)\,dx = hf(a)+\frac{h^2}{2}f'(a)+\frac{h^3}{3!}f''(a)+\dots$ (24.1)

(c) By (24.1): $\int_a^{a+h}f(x)\,dx = hf(a)+\frac{h^2}{2}f'(\xi)$ for some $\xi\in(a,a+h)$

$\Rightarrow$ $\boxed{\text{error term: } \frac{h^2}{2}f'(\xi) \text{ for some } \xi\in(a,a+h)}$

$\Rightarrow \left[\int_a^{x_1} f(x)\,dx - hf(a)\right] + \left[\int_{x_1}^{x_2} f(x)\,dx - hf(x_1)\right] + \dots + \left[\int_{x_{n-1}}^{b} f(x)\,dx - hf(x_{n-1})\right]$

$= \frac{h^2}{2}f'(\gamma_1)+\frac{h^2}{2}f'(\gamma_2)+\dots+\frac{h^2}{2}f'(\gamma_n) = \frac{h}{2}\left[hnf'(\xi)\right]$, $\xi\in(a,b)$

$\Rightarrow$ $\boxed{\text{general rule: } \frac{h}{2}(b-a)f'(\xi) \text{ for some } \xi\in(a,b)}$

(d) By (24.1): $\int_a^{a+h} f(x)\,dx = hf(a)+\frac{h^2}{2}f'(a)+\frac{h^3}{6}f''(\xi)$, $\xi\in(a,a+h)$

$\Rightarrow$ $\boxed{\text{error term: } \frac{h^3}{6}f''(\xi) \text{ for some } \xi\in(a,a+h)}$

$\Rightarrow \sum_{k=1}^{n}\left[\int_{x_{k-1}}^{x_k} f(x)\,dx - hf(x_{k-1}) - \frac{h^2}{2}f'(x_{k-1})\right] = \frac{h^3}{6}\sum_{k=1}^{n} f''(\gamma_k)$

$= \frac{h^2}{6}\left[nhf''(\xi)\right]$, $\xi\in(a,b)$ $\quad nh=(b-a)$

$\Rightarrow$ $\boxed{\text{general rule: } \frac{h^2}{6}(b-a)f''(\xi) \text{ for some } \xi\in(a,b)}$