CSCI 332, Fall 2024

Homework 4

Due before class on Tuesday, October 1, 2024–that is, due at 9:30am Mountain Time

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Submission Requirements

- Type or clearly hand-write your solutions into a PDF format so that they are legible and professional. Submit your PDF on Gradescope.
- Do not submit your first draft. Type or clearly re-write your solutions for your final submission.
- Use Gradescope to assign problems to the correct page(s) in your solution. If you do not do this correctly, we will ask you to resubmit.
- You may work with a group of up to three students and submit ***one single document*** for the group. Just be sure to list all group members at the top of the document. When submitting a group assignment to Gradescope, only one student needs to upload the document; just be sure to select your groupmates when you do so.

Academic Integrity

Remember, you may access ***any*** resource in preparing your solution to the homework. However, you ***must***

- write your solutions in your own words, and
- credit every resource you use (for example: "Bob Smith helped me on this problem. He took this course at UM in Fall 2020"; "I found a solution to a problem similar to this one in the lecture notes for a different course, found at this link: www.profzeno.com/agreatclass/lecture10"; "I asked ChatGPT how to solve part (c)"; "I put my solution for part (c) into ChatGPT to check that it was correct and it caught a missing case.") If you use the provided LaTeX template, you can use the `sources` environment for this. Ask if you need help!

Grading

Remember, submitted homeworks are graded for completeness, not correctness. Correctness is evaluated using homework quizzes.

Each submitted problem will be graded out of six points according to the following rubric:

- Does the solution address the correct problem?
- Does the solution make a reasonable attempt at solving the problem, even if not fully correct?
- Is the presentation neat?
- Is the explanation clear?

- Does the solution list collaborators or sources, or state that the student did not use any collaborators or outside resources?
- Is the solution written in the student's own voice (not copied directly from an outside resource)?

1. Suppose you know that an algorithm has a best-case runtime that is $\Theta(n \log n)$. For each of the following, decide whether it is true, false, or could be true or false. Give a short explanation for your answer.

   - The algorithm's worst-case runtime is $\Omega(n \log n)$.
   - The algorithm's worst-case runtime is $\Omega(n^3)$.
   - The algorithm's worst-case runtime is $O(n^2)$.
   - The algorithm's worst-case runtime is $O(n \log n)$.
   - The algorithm's best-case runtime is $O(n^3)$.

   Here are some examples to help you.

   Solution: Suppose you know that an algorithm has a best-case runtime that is $\Theta(n \log n)$. Which of the following are always true?

   - The algorithm's worst-case runtime is $\Omega(n)$.

   Since the best-case runtime is $\Theta(n \log n)$, we know that in the best case, the algorithm takes $\Omega(n \log n)$ steps. This means that in the worst case it also takes $\Omega(n \log n)$ steps, since the worst case must take at least as many steps as the best case. And finally, if a function is $\Omega(n \log n)$, then it is also $\Omega(n)$, so this statement is true true.

   - The algorithm's worst-case runtime is $O(n^2)$.

   This statement could be true or it could be false. For example, consider an algorithm with best-case runtime of $\Theta(n \log n)$ but worst-case runtime of $\Theta(n^3)$.

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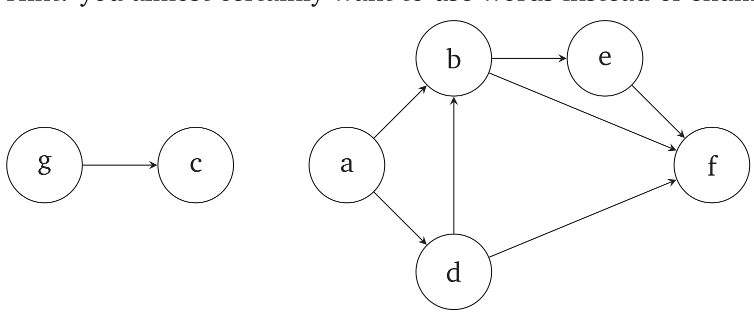
2. (This is somewhat similar to problem 1 from Chapter 3)

   Give all of the topological orderings of the following seven-node directed acyclic graph? Hint: you almost certainly want to use words instead of enumerating all of them!



3. (This is problem 5 from Chapter 3)

   A binary tree is a rooted tree in which each node has at most two children. Show by induction that in any binary tree the number of nodes with two children is exactly one less than the number of leaves.

   To get credit for this problem, you must follow the boilerplate format from lecture. That is, given that we want to prove that all Y have quality Z, your proof must follow this format:

- Let x be an arbitrary Y.
- Suppose that for all w less than/smaller than x, quality Z holds.
- There are (at least) two cases:
  - Case 1: non inductive case, aka, base case. You can prove directly that the theorem holds. (Note that there could be more than one of these!)
  - Case 2: inductive case. You must use the inductive hypothesis to show that the theorem holds. (Note that ther could be more than one of these!)
- Because the cases were exhaustive, x has quality Z. Because x was an arbitrary Y, every Y has quality Z.

4. (This is problem 2 from Chapter 4, part a)

   For the following two statement, decide whether it is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

   Suppose we are given an instance of the Shortest s-t Path Problem on a directed graph G. We assume that all edge costs are positive and distinct. Let P be a minimum-cost s-t path for this instance. Now suppose we replace each edge cost c_e by its square, c_e^2, thereby creating a new instance of the problem with the same graph but different costs.

   True or false? P must still be a minimum-cost s-t path for this new instance.