CSCI 432/532, Spring 2025
Problem Session 10

1. Below is the start of a proof that the language AcceptGriz $:= \{\langle M \rangle \mid M \text{ accepts the string } \texttt{GRIZ}\}$ is undecidable.

Solution: For the sake of argument, suppose there is an algorithm DecideAcceptGriz that correctly decides the language AcceptGriz. Then we can solve the halting problem as follows:

```
DecideHalt(⟨M, w⟩):
    Encode the following Turing machine M′:
        M′(x):
            ⟨⟨ignore the input string x⟩⟩
            run M on input w
            ⟨⟨ignore the output of M⟩⟩
            return True
    if DecideAcceptGriz(⟨M′⟩)
        return True
    else
        return False
```

■

   (a) List the four distinct Turing machines that are at play in this reduction and describe their role.

   (b) Prove that this reduction is correct by proving both
   - if M halts on input ws, then DecideHalt halts on $(\langle M, w \rangle)$, and
   - if DecideHalt halts on $(\langle M, w \rangle)$, then M halts on input ws.

2. Prove that each of the following languages is undecidable.

   (a) AlwayReject $= \{\langle M \rangle : \text{Reject}(M) = \Sigma^*\}$

   (b) AlwayHalt $= \{\langle M \rangle : \text{Halt}(M) = \Sigma^*\}$

   (c) AlwayDiverge $= \{\langle M \rangle : \text{Diverge}(M) = \Sigma^*\}$