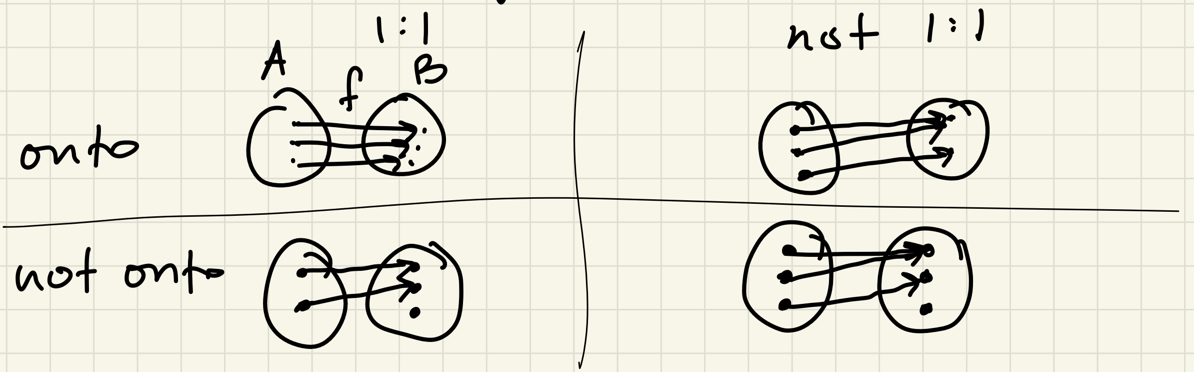
<u>Def</u> A function $f: A \to B$ is

1. onto (surjective) if
$\forall b \in B \ \exists a \in A : f(a) = b$
$\equiv \forall b \in B$, something in A maps to it
$\equiv \forall b \in B$, b shows up in ≥ 1 row of table
$\equiv$ codomain = range

2. one-to-one (injective) if
1:1
$\forall a_1, a_2 \in A \quad a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$
$\equiv \forall b \in B$, at most 1 thing in A maps to it
$\equiv \forall b \in B$, b shows up in ≤ 1 row of table

3. a bijection if both onto and 1:1
$\forall b \in B$, exactly 1 elt. of A maps to it.



How do we prove that f onto / 1:1?

onto

WTS $\forall b \in B \ \exists a \in A : f(a) = b$.

$\equiv$ if $b \in B$ then $\exists a \in A : f(a) = b$.

Step 1: suppose that $b \in B$.

Step 2: Show that $\exists a \in A : f(a) = b$ by constructing a s.t. $f(a) = b$.

ex recall $s : \mathbb{Z} \to \mathbb{Z}$, $s(x) = x + 1$.

Claim: S is onto.

$s(x) = x + 1$. have $b \in$. how did I get it?

$b - 1$.

proof let $b \in \mathbb{Z}$. we need to show that $\exists a \in \mathbb{Z} : s(a) = b$. Consider $a = b - 1$. $a \in \mathbb{Z}$ and $s(a) = b - 1 + 1 = b$, as needed. $\square$

This is an example of proof by construction.

not onto:

WTS $\neg(\forall b \in B \ \exists a \in A : f(a) = b)$

$\equiv \exists b \in B \ \forall a \in A \ f(a) \neq b$

construct $b \in B$ s.t. nothing in A maps to it.

ex $f: \mathbb{R} \to \mathbb{R}$ $f(x) = x^2$ not onto.

Proof consider $b = -1 \in \mathbb{R}$

$\forall a \in \mathbb{R} : f(a) = a^2$ def of f

$\forall a \in \mathbb{R} : f(a) \geq 0$ prop. of ^{2}

$\forall a \in \mathbb{R} : f(a) \neq b$ $b < 0$

invalid proof that f is onto:

Let $b \in \mathbb{R}$. WTS that $\exists a \in \mathbb{R} : f(a) = b$.
Consider $a = \sqrt{b}$. Since $b \in \mathbb{R}$, $\sqrt{b} \in \mathbb{R}$.
Also, $f(a) = (\sqrt{b})^2 = b$.

next time: 1:1