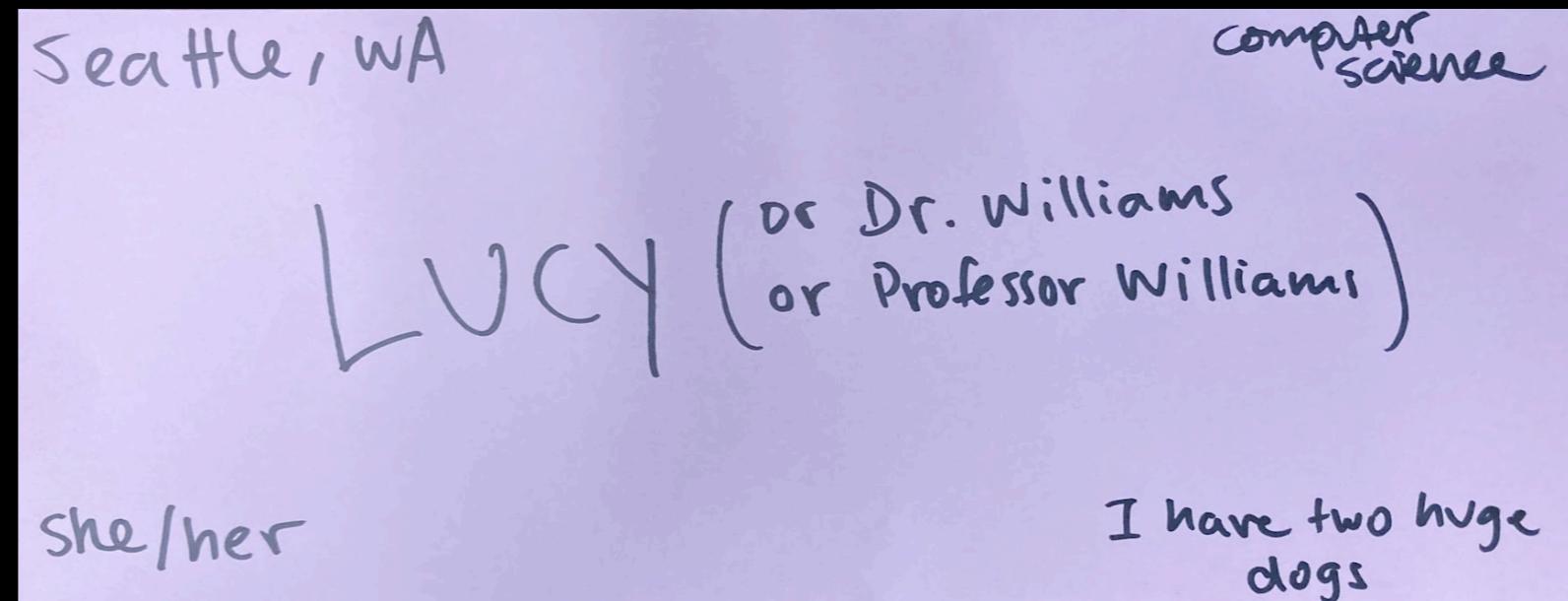


CSCI 332: ADVANCED ALGORITHMS & DATA STRUCTURES

INSTRUCTOR: LUCIA WILLIAMS

After you sit down, please fold your paper hot dog style and write:

- ▶ What you'd like to be called
- ▶ Your hometown
- ▶ Your pronouns
- ▶ Your major/concentration
- ▶ A fun fact about you



Introduce yourself to your neighbors!

Algorithm definition

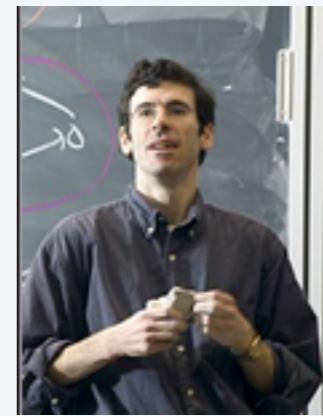
Algorithm definition

“ An *algorithm* is a finite, definite, effective procedure,
with some input and some output. ”

— Donald Knuth



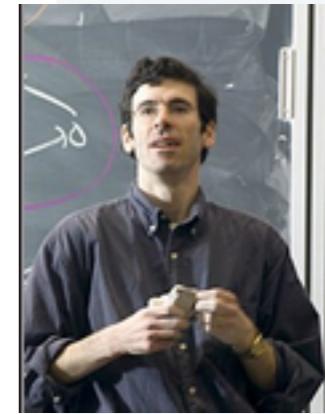
But...



But...

“Algorithmic problems form the heart of computer science, but they rarely arrive as cleanly packaged, mathematically precise questions. Rather, they tend to come bundled together with lots of messy, application-specific detail, some of it essential, some of it extraneous.”

— Kleinberg & Tardos



CSCI 232 vs. CSCI 332

What were the focuses of CSCI 232?

CSCI 232 vs. CSCI 332

CSCI 232 vs. CSCI 332

CSCI 232. Implementation and consumption of classic algorithms.

CSCI 232 vs. CSCI 332

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- Fundamental data structures (arrays, stacks, queues, etc.).

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- Sorting.

CSCI 232 vs. CSCI 332

CSCI 232. Implementation and consumption of classic algorithms.

- Fundamental data structures (arrays, stacks, queues, etc.).
- Sorting.
- Searching.

CSCI 232 vs. CSCI 332

CSCI 232. Implementation and consumption of classic algorithms.

- Fundamental data structures (arrays, stacks, queues, etc.).
- Sorting.
- Searching.
- Graph algorithms.

CSCI 232 vs. CSCI 332

CSCI 232. Implementation and consumption of classic algorithms.

- Fundamental data structures (arrays, stacks, queues, etc.).
- Sorting.
- Searching.
- Graph algorithms.
- String processing.

CSCI 232 vs. CSCI 332

CSCI 232. Implementation and consumption of classic algorithms.

- Fundamental data structures (arrays, stacks, queues, etc.).
- Sorting.
- Searching.
- Graph algorithms.
- String processing.
- Compression.

```
private static void sort(double[] a, int lo, int hi) {  
    if (hi <= lo) return;  
    int lt = lo, gt = hi;  
    int i = lo;  
    while (i <= gt) {  
        if      (a[i] < a[lo]) swap(a, lt++, i++);  
        else if (a[i] > a[lo]) swap(a, i, gt--);  
        else                i++;  
    }  
  
    sort(a, lo, lt - 1);  
    sort(a, gt + 1, hi);  
}
```

Emphasizes critical thinking, problem-solving, and code.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.
- Dynamic programming.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.
- Dynamic programming.
- Duality.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.
- Dynamic programming.
- Duality.
- Data structures.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.
- Dynamic programming.
- Duality.
- Data structures.
- Intractability.

CSCI 232 vs. CSCI 332

COS 423. Design and analysis of algorithms.

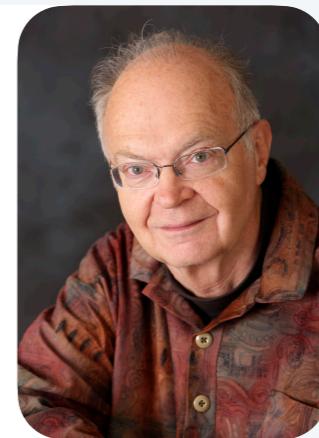
- Finding computational problems in the real world.
- Greed.
- Divide-and-conquer.
- Dynamic programming.
- Duality.
- Data structures.
- Intractability.

$$\begin{aligned} \sum_{i=1}^n \sum_{j=i+1}^n \frac{2}{j-i-1} &= 2 \sum_{i=1}^n \sum_{j=2}^{n-i+1} \frac{1}{j} \\ &\leq 2n \sum_{j=1}^n \frac{1}{j} \\ &\sim 2n \int_{x=1}^n \frac{1}{x} dx \\ &= 2n \ln n \end{aligned}$$

Emphasizes critical thinking, problem-solving, and both open-ended problems and rigorous analysis.

Why study algorithms?

*“ Algorithms are the life-blood of computer science...
the common denominator that underlies and unifies the
different branches. ” — Donald Knuth*



Why study algorithms?

Internet. Web search, packet routing, distributed file sharing, ...

Biology. Human genome project, protein folding, ...

Computers. Circuit layout, databases, caching, networking, compilers, ...

Computer graphics. Movies, video games, virtual reality, ...

Security. Cell phones, e-commerce, voting machines, ...

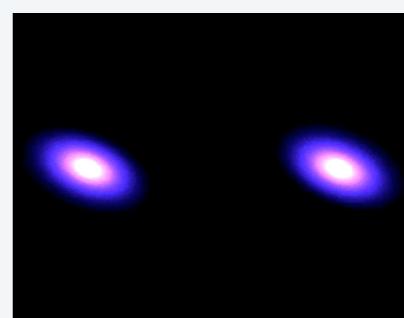
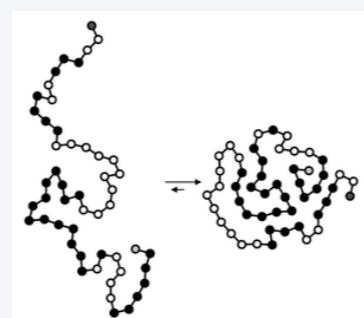
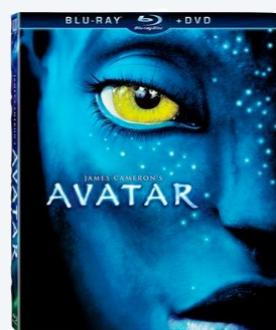
Multimedia. MP3, JPG, DivX, HDTV, face recognition, ...

Social networks. Recommendations, news feeds, advertisements, ...

Physics. Particle collision simulation, n -body simulation, ...

:

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YAHOO![®]
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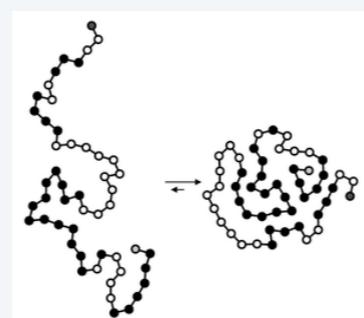
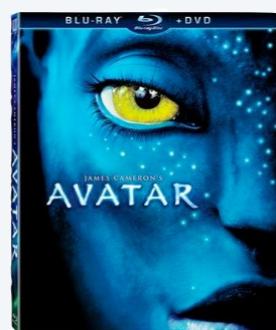
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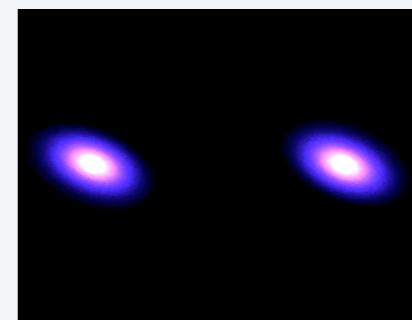
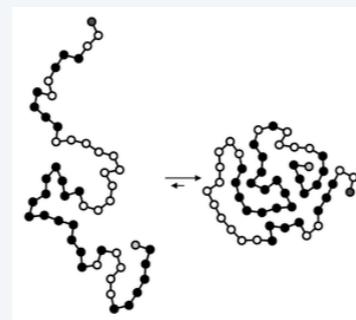
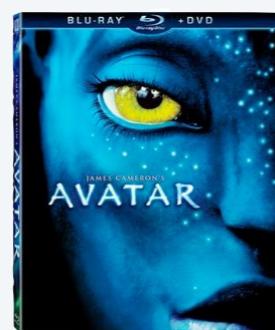
Security. Cell phones, e-commerce, voting machines, ...

Multimedia. MP3, JPG, DivX, HDTV, face recognition, ...

Social networks. Recommendations, news feeds, advertisements, ...

Physics. Particle collision simulation, n -body simulation, ...

•
•
•



We emphasize algorithms and techniques that are useful in practice.

Course logistics

In table groups, try to complete the syllabus quiz. Some of the questions are open-ended and may not have one single answer!

If your group comes up with a question you can't answer (not necessarily one on the quiz), post it in #questions in Discord.

Matching med-school students to hospitals



How to match? What should we think about when designing an algorithm for this problem?

Matching med-school students to hospitals

Given:

- * a set of preferences among hospitals and med-school students

	favorite ↓	least favorite ↓	
	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

	favorite ↓	least favorite ↓	
	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

- * a matching of hospitals to students

$$\{ A-Z, B-Y, C-X \}$$

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	favorite ↓	least favorite ↓	
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Chicago	Xavier	Yolanda	Zeus

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	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

- * a matching of hospitals to students

$$\{ A-Z, B-Y, C-X \}$$

With your table group, give at least two *measurable* criterion for a “good” matching.

Matching med-school students to hospitals

Goal. Given a set of preferences among hospitals and med-school students, design a **self-reinforcing** admissions process.



Matching med-school students to hospitals

Goal. Given a set of preferences among hospitals and med-school students, design a **self-reinforcing** admissions process.

Unstable pair. Hospital h and student s form an **unstable pair** if both:

- h prefers s to one of its admitted students.
- s prefers h to assigned hospital.



Matching med-school students to hospitals

Goal. Given a set of preferences among hospitals and med-school students, design a **self-reinforcing** admissions process.

Unstable pair. Hospital h and student s form an **unstable pair** if both:

- h prefers s to one of its admitted students.
- s prefers h to assigned hospital.

Stable assignment. Assignment with no unstable pairs.

- Natural and desirable condition.
- Individual self-interest prevents any hospital–student side deal.



Stable matching problem: input

Input. A set of n hospitals H and a set of n students S .



one student per hospital (for now)

Stable matching problem: input

Input. A set of n hospitals H and a set of n students S .

- Each hospital $h \in H$ ranks students.

one student per hospital (for now)

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

Stable matching problem: input

Input. A set of n hospitals H and a set of n students S .

- Each hospital $h \in H$ ranks students.
- Each student $s \in S$ ranks hospitals.

one student per hospital (for now)

favorite
↓
least favorite
↓

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

hospitals' preference lists

favorite
↓
least favorite
↓

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

students' preference lists

Stable matching problem: output

Def. A set $M \subseteq H \times S$ is a **matching** if and only if:

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a perfect matching $M = \{ A-Z, B-Y, C-X \}$

Stable matching problem: output

Def. A set $M \subseteq H \times S$ is a **matching** if and only if:

- Each hospital $h \in H$ appears in at most one pair of M .
- Each student $s \in S$ appears in at most one pair of M .

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
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- Each hospital $h \in H$ appears in at most one pair of M .
- Each student $s \in S$ appears in at most one pair of M .

Def. A matching M is **perfect** if $|M| = |H| = |S| = n$.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a perfect matching $M = \{ A-Z, B-Y, C-X \}$

Unstable pair

Def. Given a perfect matching M , hospital h and student s form an **unstable pair** if both:

- h prefers s to matched student.
- s prefers h to matched hospital.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
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Zeus	Atlanta	Boston	Chicago

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	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
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A-Y is an unstable pair for matching $M = \{ A-Z, B-Y, C-X \}$

Unstable pair

Def. Given a perfect matching M , hospital h and student s form an **unstable pair** if both:

- h prefers s to matched student.
- s prefers h to matched hospital.

Key point. An unstable pair $h-s$ could each improve by joint action.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

A-Y is an unstable pair for matching $M = \{ A-Z, B-Y, C-X \}$

On your own, think about...

Which pair is unstable in the matching { A-X, B-Z, C-Y } ?

1. A-Y.
2. B-X.
3. B-Z.
4. None of the above.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
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Which pair is unstable in the matching { A-X, B-Z, C-Y } ?

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2. B-X.
3. B-Z.
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	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

B-X is an unstable pair

Stable matching problem

Def. A **stable matching** is a perfect matching with no unstable pairs.

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a stable matching $M = \{ A-X, B-Y, C-Z \}$

Stable matching problem

Def. A **stable matching** is a perfect matching with no unstable pairs.

Stable matching problem. Given the preference lists of n hospitals and n students, find a stable matching (if one exists).

	1 st	2 nd	3 rd
Atlanta	Xavier	Yolanda	Zeus
Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a stable matching $M = \{ A-X, B-Y, C-Z \}$

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Boston	Yolanda	Xavier	Zeus
Chicago	Xavier	Yolanda	Zeus

	1 st	2 nd	3 rd
Xavier	Boston	Atlanta	Chicago
Yolanda	Atlanta	Boston	Chicago
Zeus	Atlanta	Boston	Chicago

a stable matching $M = \{ A-X, B-Y, C-Z \}$

Do you see any potential issues with using Stable Matching to solve the med student to hospital matching problem?

Do stable matchings always exist?

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1 st	2 nd	3 rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

Do stable matchings always exist?

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1 st	2 nd	3 rd	
A	B	C	D	$A-B, C-D$
B	C	A	D	$A-C, B-D$
C	A	B	D	$A-D, B-C$
D	A	B	C	

Do stable matchings always exist?

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1 st	2 nd	3 rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

$A-B, C-D \Rightarrow B-C$ unstable

$A-C, B-D \Rightarrow A-B$ unstable

$A-D, B-C \Rightarrow A-C$ unstable

Observation. Stable matchings need not exist.

Do stable matchings always exist?

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1 st	2 nd	3 rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

$A-B, C-D \Rightarrow B-C$ unstable

$A-C, B-D \Rightarrow A-B$ unstable

$A-D, B-C \Rightarrow A-C$ unstable

Observation. Stable matchings need not exist.

What about for our version of stable matching?

Gale–Shapley deferred acceptance algorithm

An intuitive method that **guarantees** to find a stable matching.



GALE–SHAPLEY (*preference lists for hospitals and students*)

INITIALIZE M to empty matching.

WHILE (some hospital h is unmatched and hasn't proposed to every student)

$s \leftarrow$ first student on h 's list to whom h has not yet proposed.

IF (s is unmatched)

Add $h-s$ to matching M .

ELSE IF (s prefers h to current partner h')

Replace $h'-s$ with $h-s$ in matching M .

ELSE

s rejects h .

RETURN stable matching M .

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

We enter the while loop.
How many valid first
steps are there?

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Atlanta proposes to ????

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Atlanta proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Wayne

Wayne accepts
(since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Yolanda

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Yolanda

**Yolanda accepts
(since previously unmatched)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Chicago proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

What happens?

Chicago proposes to Wayne

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Chicago proposes to Wayne

**Wayne accepts
(and renounces Atlanta)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Atlanta proposes to Val

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Atlanta proposes to Val

Val accepts
(since previously unmatched)

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Val

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Val

**Val rejects
(since she prefers Atlanta)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Yolanda

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Yolanda

**Yolanda accepts
(and renounces Boston)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Wayne

**Wayne rejects
(since he prefers Chicago)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Val

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Val

**Val rejects
(since she prefers Atlanta)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Boston proposes to Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Boston proposes to Xavier

**Xavier accepts
(since previously unmatched)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

El Paso proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Wayne

**Wayne rejects
(since he prefers Chicago)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

El Paso proposes to Yolanda

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

El Paso proposes to Yolanda

**Yolanda accepts
(and renounces Detroit)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Xavier

**Xavier rejects
(since he prefers Boston)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Wayne

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Wayne

**Wayne rejects
(since he prefers Chicago)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

Detroit proposes to Zeus

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta

Detroit proposes to Zeus

**Zeus accepts
(since previously unmatched)**

Gale–Shapley demo

hospitals' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Atlanta	Wayne	Val	Yolanda	Zeus	Xavier
Boston	Yolanda	Wayne	Val	Xavier	Zeus
Chicago	Wayne	Zeus	Xavier	Yolanda	Val
Detroit	Val	Yolanda	Xavier	Wayne	Zeus
El Paso	Wayne	Yolanda	Val	Zeus	Xavier

**STOP
(stable matching)**

students' preference lists

	1 st	2 nd	3 rd	4 th	5 th
Val	El Paso	Atlanta	Boston	Detroit	Chicago
Wayne	Chicago	Boston	Detroit	Atlanta	El Paso
Xavier	Boston	Chicago	Detroit	El Paso	Atlanta
Yolanda	Atlanta	El Paso	Detroit	Chicago	Boston
Zeus	Detroit	Boston	El Paso	Chicago	Atlanta