Goals this week:

- language is regular ⇄ language is automatic (DFA recognizing language)

  exists regular expression

  wed

- method to show that a language is not regular

  today

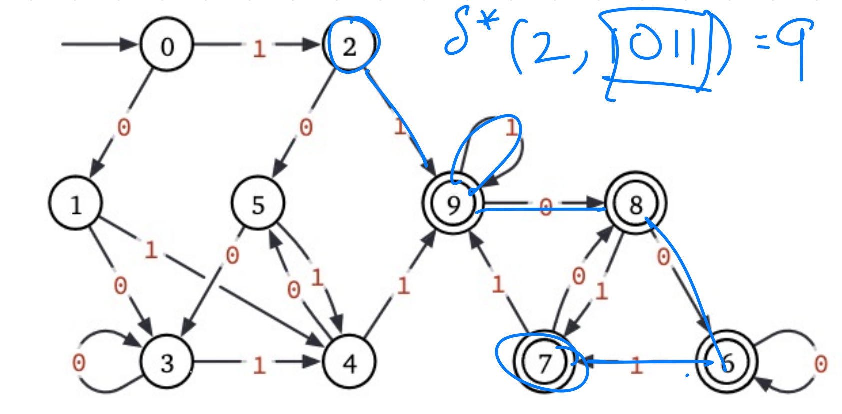
warmup

This DFA accepts all strings containing either 00 or 11 as a substring.

Are any states equivalent?

$\delta(2,1) = 9$

$\delta^*(2, \boxed{011}) = 9$



Def extended transition function

$\delta^* : Q \times \Sigma^*$

↑ any string over alphabet Σ

$$\delta^*(q, w) = \begin{cases} q & \text{if } w = \varepsilon \\ \delta^*(\delta(q,a), x) & \text{if } w = ax \end{cases}$$

$\delta^*(2, 11001) = 7$

Def States p, q are distinguishable iff for some string w

$\delta^*(p, w) \in A$ and $\delta^*(q, w) \notin A$

p, q or vice versa

are $2, 9$ distinguishable?

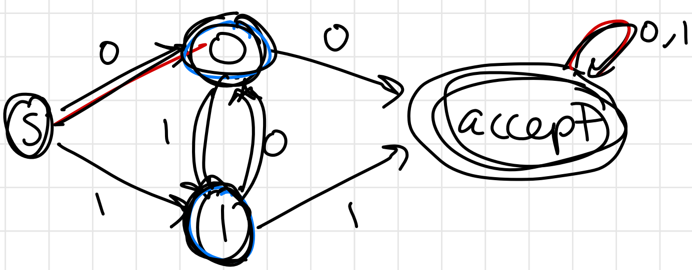
$w = 0$

$\delta^*(2, 0) = 5 \notin A$

$\delta^*(9, 0) = 8 \in A$

what about $8, 9$? not distinguishable.

Is this the smallest DFA for L?



w

s	accept	are distinguished by	0
s	0		0
0	1		0
0	accept		1
1	accept		0
s	1		1

note: $\varepsilon \in \Sigma^*$

Two strings x and y are distinguishable with respect to language L iff for some z

$xz \in L$ and $yz \notin L$
or vice versa

ex $L = (0+1)^*(00+11)(0+1)^*$

are 0 and 1 distinguishable?
$\downarrow$ x, $\downarrow$ y

need a z. how about $z = 0$

$xz = 00 \in L$

$yz = 10 \notin L$

Def A fooling set for L is a set of strings that are all mutually distinguishable.

ex $\boxed{F = \{\varepsilon, 0, 1, 00\}}$ is a fooling set for $\boxed{L = (0+1)^*(00+11)(0+1)^*}$

x	y	z	
ε	0	0	$\varepsilon 0 = 0 \notin L$, $00 \in L$
ε	1	1	$\varepsilon 1 = 1 \notin L$, $11 \in L$
→ ε	00	0	$\varepsilon 0 = 0 \notin L$, $000 \in L$

0	1	0
0	00	
1	00	

For any language L,

min # of states in DFA recognizing L = max # of strings in a fooling set for L

$4 = 4$

$L = \{ 0^n 1^n : n \geq 0 \}$

String w, w^n means $\underbrace{w \cdot w \cdot \cdots \cdot w}_{n \text{ times}}$

$0011 = 0^2 1^2 \in L$

$011 \notin L$

$0^1 1^2$

0^*

let $F = \{ \varepsilon, 0, 00, 000, \ldots \} = \{ 0^n : n \geq 0 \}$

F is a fooling set for L.

x	y	z	
ε	0	1	$\varepsilon 1 = 1 \notin L$, $0^1 1^1 \in L$
ε	00	11	$\varepsilon 11 = 11 \notin L$, $0011 \in L$
0	00	$\vdots$	
00	000		
$\vdots$	$\vdots$		

let $x = 0^i$ and $y = 0^j$ for $i \geq 0$, $j \geq 0$, $i \neq j$.

Let $z = 1^i$.

$xz = 0^i 1^i \in L$.

$yz = 0^j 1^i \notin L \quad i \neq j$

Theorem: $L = \{0^n 1^n : n \geq 0\}$ is not regular.

Proof:

Theorem: L = palindromes = $\{ w : w = rev(w) \}$ is not regular.

Proof: