Examples of propositions:

$\boxed{\text{for ints } n \qquad n(n+1)^2 \text{ is even}}$

for ints n — if n^2 even, then n even

for $x, y \in \mathbb{R}$ — if $x \in \mathbb{Q}$, $y \in \mathbb{Q}$ then $xy \in \mathbb{Q}$

$\sqrt{2}$ is not rational

In proofs, we've done

n even $\Rightarrow$ $n = 2c$ for $c \in \mathbb{Z}$ $\Rightarrow \cdots$

("implies that")

$nx, ny \in \mathbb{Z} \Rightarrow nxny \in \mathbb{Z}$

$\sqrt{2}$ rational $\Rightarrow \cdots \Rightarrow$ false (contradiction)

we can construct compound prop. out of smaller (atomic) prop.

can't be broken down

if $\underbrace{\boxed{\overset{p}{\text{n is integer}}}}$ then $\boxed{n(n+1)^2 \text{ even}}$ (q)

Syntax vs. Semantics

- Syntax ↳ grammatically correct (for a given language)
- Semantics ↳ meaning of a grammatically correct sentence or statement

let p, q be prop.

natural lang	syntax	informal semantics
p and q	$p \wedge q$	T iff both p, q T
p or q	$p \vee q$	T iff ≥ 1 of p, q T
not p	$\neg p$	T iff p is false
if p then q	$p \Rightarrow q$ ($p \rightarrow q$)	T iff when p T, q T
p if and only if q	$p \Leftrightarrow q$	T iff p, q match
p exclusive or q	$p \oplus q$	T iff p, q mismatch

formal semantics (truth table)

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

p			q	$p \wedge q$
T	2 is even	and	T 3 is odd	T
T	2 is even	and	F 4 is odd	F
F	1 is even	and	T 3 is odd	F
F	3 is even	and	F 2 is odd	F

if / then

true iff p "forces" q (false if p doesn't force q)
it's a promise that
whenever p T, q also T
so $p \Rightarrow q$ is F when that promise is broken
That is, when p is T and q is F, $p \Rightarrow q$ is F

ex If it rains then the grass is wet.

when is a lie?

p rains	q grass wet	rain $\Rightarrow$ grass wet
T	T	T
T	F	F
F	T	T
F	F	T

if p then q can also be written as:

- q whenever p
- q is necessary for p
- p only if q
- p is a sufficient condition for q
- whenever p also q
- p implies q

Def 2 propositions are logically equivalent iff their truth tables are the same

p	$\neg p$	$\neg\neg p$
T	F	T
F	T	F

$$p \equiv \neg\neg p$$

Def prop p is <u>satisfiable</u> iff its truth table has at least one T. That is, it's true under at least one truth assignment.

Def A prop. is a <u>tautology</u> iff every row of the truth table is T

ex