Recall the steps for proving a statement "$\forall n \geq 0$: something" using mathematical induction:

(1) Clearly state the property $P(n)$, that you are using mathematical induction, and what variable you are doing induction over.

(2) Prove the base case: $P(0)$.

(3) Prove the inductive case: $P(n-1) \Rightarrow P(n)$.

In this activity, you will prove that $\forall n \geq 0$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}.$$

Answer the following questions:

- Do you believe the claim? I give you an example of it holding below. Give at least two more examples of n for which the claim holds by filling in two more rows of the table for different n.

n	$\sum_{i=0}^{n} i$	$\frac{n(n+1)}{2}$
1	$\sum_{i=0}^{1} i = 0 + 1 = 1$	$\frac{1(1+1)}{2} = \frac{1(2)}{2} = 1$

- What is the predicate $P(n)$ that you will prove holds $\forall n \geq 0$?

- What variable will you do induction over?

- What is the base case? (Don't just write $P(0)$; translate it into the specific $P(n)$ you defined above.)

- What is the inductive case? (Again, don't just write $P(n-1) \Rightarrow P(n)$; translate it into the specific $P(n)$ you defined above.)

- What is the inductive hypothesis?

Now that you have answered the above questions, you are ready to write the full proof! The three steps are labeled in the proof for you to fill in.

Proof.

(1)

(2) For the base case, we prove that _____.

(3) For the inductive case, we prove that $\forall n \geq 1$, _____.

□