

Name _____

CSCI 332, Fall 2025

Quiz 8

1. (5 points) In this problem, you will need to write the formal recursive definition for the subproblem (step two of our four steps for writing dynamic programming algorithms) for the following problem.

The problem. Your neighborhood playground has a set of monkey bars with n bars. Since you are an adult (suck it kids), you can reach up to 3 bars ahead from the current bar you are hanging on. How many different ways are there to reach the n th bar, assuming you start from the first bar?

To solve this problem, we first write the English definition of the subproblem.

1. **The subproblem.** Let $C(i)$ be the number of different ways to reach the i th bar.

Next, we think about the recursive relationship between subproblems. You notice that to reach the i th bar, you could have come from either the $(i - 1)$ st bar, the $(i - 2)$ nd bar, or the $(i - 3)$ rd bar, meaning that the number of ways to reach that bar is the sum of the ways to reach each of those bars. Using this observation, fill in the formal recursive definition for $C(i)$ below.

2. **The recursive definition.** The recursive relationship can be expressed as: (fill in both recursive case(s) and base case(s). You may not need every line provided!)

$$C(i) = \begin{cases} \text{_____} & \text{if } i \text{ _____} \\ \text{_____} & \text{if } i \text{ _____} \end{cases}$$

2. (5 points) In this problem, you will need to write algorithm pseudocode (step four of our four steps for writing dynamic programming algorithms) for the following problem.

The problem You need to visit your grandma at most every other weekend. (You can visit on consecutive weekends if you want to, but you don't have to.) When you visit her, she always makes her signature spinach cookies, and to be polite, you must eat all of the cookies she makes. However, she makes a different number of cookies each weekend. If you know that $\text{cookies}[1..n]$ is a length n array indicating the number of cookies she makes on the i th weekend, write an algorithm to determine the minimum number of cookies you can get by visiting her on some subset of n weekends. *Note: on the n th weekend, your grandma is hosting a birthday party and will not be making spinach cookies, so $\text{cookies}[n]$ will always be 0.*

To solve this problem, we first write the English definition of the subproblems.

1. **The subproblem.** Let $\text{MinCookies}(i)$ be the minimum number of cookies you can eat, assuming you visit on the i th weekend.

Next, we think about the recursive relationship between subproblems. You notice that if you visit on the i th weekend, you must add $\text{cookies}[i]$ to the minimum number of cookies you would eat by visiting on either the $(i - 1)$ st or $(i - 2)$ nd weekend.

2. The recursive definition.

$$\text{MinCookies}(i) = \begin{cases} \text{cookies}[1] & \text{if } i = 1 \\ \text{cookies}[2] & \text{if } i = 2 \\ \min(\text{cookies}[i] + \text{MinCookies}(i-1), \text{cookies}[i] + \text{MinCookies}(i-2)) & \text{if } i > 2 \end{cases}$$

3. How to memoize. We can use an array of length n to store the values of $\text{MinCookies}(i)$ and fill it in increasing order.

4. Write the algorithm. Using the above information, finish the pseudocode to find the minimum amount of spinach cookies you need to eat over the n weekends, given $\text{cookies}[1..n]$.

```
GrandmaCookies( $\text{cookies}[1..n]$ ):  
    Let array  $MC$  be an array of length  $n$   
    # do we have any base cases?  
  
    # how should we fill in the rest of the array?  
  
    # what should we return to get the final answer?
```