# Relations

CS application: relational databases

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Questions about data stored in relational databases can be posed precisely using the language of relations.

SQL (structured query language)

**Def** The **cartesian product** of two sets $A, B$ is

$$A \times B = \{(a,b) : a \in A \wedge b \in B\}$$

lists / tuples / arrays — order matters

ex

$\mathbb{R} \times \mathbb{R}$ = 2d plane, Cartesian plane

$\{red, blue\} \times \{1,2,3\} = \{(red,1), (red,2), (red,3), (blue,1), (blue,2), (blue,3)\}$

Q what is $|A \times B|$?

$|A| \cdot |B|$

$|A|, |B|$

$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

$\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3$

Def A binary relation $R$ on sets $A, B$ is a subset $R \subseteq A \times B$.

We write $(x,y) \in R$ as $xRy$

$(x,y) \notin R$ as $x \not R y$

examples

1) $R_1$ "is (blood) related to" is a binary relation on people.

Let $P$ be the set of all people

"is blood related to" is

$\{(x,y) : x \in P \wedge y \in P \wedge x$ is related to $y\}$

(serena williams, venus williams) $\in R$,

(Lucy williams, serena williams) $\notin R$,

② $<$ on $A = \{1, 2, 3, 4\}$

$< = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$1 < 2$ but $3 \not< 2$

③ Let $f: A \to B$ be a function

$\{(a, f(a)) : a \in A\} \subseteq A \times B$, so it is a relation

Q Is the converse true? Let $R$ be a binary relation on $A, B$.

$\{(x,y) : x \in A \wedge y \in B \wedge xRy\}$

$\Rightarrow f: A \to B$ s.t. $f(x) = y$ is a function

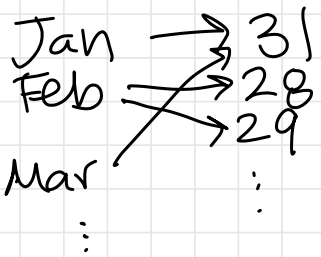
true or false

(4) let A = months, B = number of days

Relation: month, its # days

{(Jan, 31), (Feb, 28), (Feb, 29),
(Mar, 31) ... }

| | |
|---|---|
| Jan | 31 |
| Feb | 28 |
| Feb | 29 |
| Mar | 31 |
| ⋮ | ⋮ |



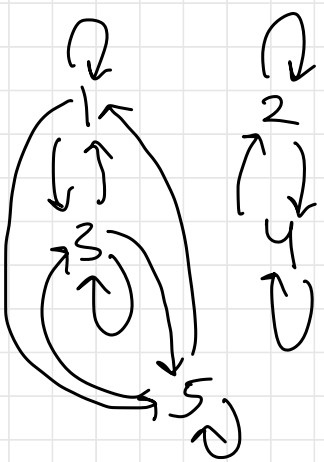

(5) $A = \{1, 2, 3, 4, 5\}$

$(1,1) \in R_2$

$(2,4) \in R_2$

$(3,2) \notin R_2$

$R_2$:




## Properties of relations

Let $R \subseteq A \times A$, so R is a relation on A

R: $a_1 \rightarrow a_2$ ↺

$a_1 \rightarrow a_3$

R is <u>reflexive</u> if $\forall a \in A : a R a$

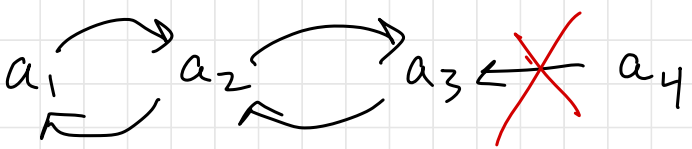
all nodes have self-loops

R is <u>irreflexive</u> if $\forall a \in A : a \not R a$

no nodes have self-loops

R is <u>symmetric</u> if

$$\forall a_1, a_2 \in A : a_1 R a_2 \Rightarrow a_2 R a_1$$

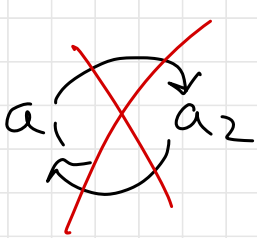


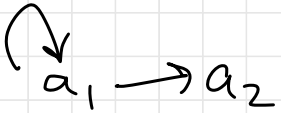

whenever we have a forward edge, we have the backward edge.

$\{(a,b),(b,c),(c,a)\}$

R is <u>anti-symmetric</u> if

$$\forall a_1, a_2 \in A : (a_1 R a_2 \wedge a_2 R a_1) \Rightarrow a_1 = a_2$$




$a \to b \to c \to a$

never have backwards edges, but self-loops okay.

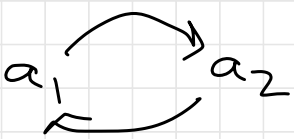
R is ~~transitive~~ if

$$\forall a,b,c \in A : (aRb \wedge bRc) \Rightarrow (aRc)$$

$a \to b \to c$ Shortcut edges always exist

~~$\forall$~~ $(aRb \wedge cRb)$

$a \to b \leftarrow c$

Q Is  transitive?