Discrete Structures (CSCI 246)

Homework 1

Purpose & Goals

The following problems provide practice relating to:

- direct proofs, proof by cases, and proofs by counter-example,
- mathematical definitions (rational, absolute value, etc.), and
- the problem solving process.

Submission Requirements

- **Type or clearly hand-write your solutions** into a pdf format so that they are legible and professional. Submit your pdf to Gradescope. **Illegible, non-pdf, or emailed solutions will not be graded.**
- Each problem should start on a new page of the document. When you submit to Gradescope, associate each page of your submission with the correct problem number. Please post in Discord if you are having any trouble using Gradescope.
- Try to model your formatting off of the proofs from lecture and/or the textbook.
- Submit to Gradescope early and often so that last-minute technical problems don't cause you any issues. Only the latest submission is kept. Per the syllabus, assignments submitted within 24 hours of the due date will receive a 25% penalty and assignments submitted within 48 hours will receive a 50% penalty. After that, no points are possible.

Academic Integrity

- You may work with your peers to solve the problems, but **you must construct your write-up in your own words on your own.**
- Do not search the web for solutions or hints, use generative AI (such as ChatGPT) to get hints or problem solutions, orpost the problem set, or otherwise violate the course collaboration policy or the MSU student code of conduct.
- Violations (regardless of intent) will be reported to the Dean of Students, per the MSU student code of conduct, and you will receive a 0 on the assignment.

Tips

- Answer each problem to the best of your ability. Partial credit is your friend!
- Read the hints for where to find relevant examples for each problem.
- Refer to the problem solving and homework tips guide.
- It is not a badge of honor to say that you spent 5 hours on a single problem or 15 hours on a single assignment. Use your time wisely and get help (see "How to Get Help" below).

How to Get Help

When you are stuck and need a little or big push, **please ask for help!**

- Timebox your effort for each problem so that you don't spend your life on the problem sets. (See the problem solving tips guide for how to do this effectively.)
- Post in Discord. If you're following the timebox guide, you can post the exact statement that you produced after spending 20 minutes being stuck.
- Come to office hours or visit the CS Student Success Center.

1. Problem 1 (10 points)

   Consider the following proposition: $\{a \in \mathbb{Z} : 3|a\} \cap \{b \in \mathbb{Z} : 10|b\} \subseteq \{a \in \mathbb{Z} : 6|a\} \cap \{b \in \mathbb{Z} : 21|b\}$.

   Give a proof in two-column format to show that it is true.

   Hint: We saw proofs involving sets and divisibility in the first full lecture on sets. Try to understand those proofs before attempting this one. Don't forget to apply the problem solving process by making sure you understand all notation and trying the claim out on examples first to get a sense for the result.

   Grading Notes. While a detailed rubric cannot be provided in advance as it gives away the solution details, the following is a general idea of how points are distributed for this problem. If you can get at least partway, we give partial credit where we can.

   (8) **Correctness**. If your proof is not correct, this is where you'll get docked.
      (6) Regardless of how you formulate your proof, somewhere you'll need certain facts without which your proof wouldn't work. For example, if it weren't true that the sum of two integers is an integer, would your proof fail? If so, then that is a fact I need to see stated somewhere.
      (2) The order of these facts must make sense, so that you're not inferring something before you have all the facts to infer it. E.g., you cannot use the fact that sum of two integers is an integer if you don't already know that you have two integers to begin with.

   (2) **Communication**. We need to see a mix of mathematical notation and intuition in the column format with the notation on the left and the reasoning on the right. If you skip too many steps at once, or we cannot follow your proof, or if your solution is overly wordy or confusing, this is where you'll get docked.

   Note that if you incorrectly think the statement is false when it is true or vice-versa, partial credit will be sparse. Try to check that you have the right claim before proceeding too far.

2. Problem 2 (5 points)

   Draw the Venn diagrams for the following, for one point each:

   (a) $A \oplus B$
   (b) $A - C \cap \overline{(C - B)}$
   (c) $\overline{(C \cap B)} \cup \overline{A}$
   (d) $(C) \cap \overline{(A \cup B)}$
   (e) $(B \cap C) \cup \overline{A \cap B}$

3. Problem 3 (10 points)

Give a direct proof in two-column format that for any sets A, B, C, $(B-A)\cap(C-A) = (B\cap C)-A$.

Hint: We saw direct proofs involving set properties in the second full lecture on sets. Try to understand those proofs before attempting this one. Don't forget to apply the problem solving process by making sure you understand all notation and trying the claim out on examples first to get a sense for the result.

Hint: Recall that, given two sets X and Y, to prove that $X = Y$, it is almost always easier to split it into two proofs: one that $X \subseteq Y$ and the other that $Y \subseteq X$. While there is a way to prove this without doing so, you are less likely to miss steps and explanations (and thus points) if you *do* split it apart. Up to you.

Grading Notes. While a detailed rubric cannot be provided in advance as it gives away the solution details, the following is a general idea of how points are distributed for this problem. If you can get at least partway, we give partial credit where we can.

(8) **Correctness**. If your proof is not correct, this is where you'll get docked.

- (6) Regardless of how you formulate your proof, somewhere you'll need certain facts without which your proof wouldn't work. For example, if it weren't true that the sum of two integers is an integer, would your proof fail? If so, then that is a fact I need to see stated somewhere.
- (2) The order of these facts must make sense, so that you're not inferring something before you have all the facts to infer it. E.g., you cannot use the fact that sum of two integers is an integer if you don't already know that you have two integers to begin with.

(2) **Communication**. We need to see a mix of mathematical notation and intuition in the column format with the notation on the left and the reasoning on the right. If you skip too many steps at once, or we cannot follow your proof, or if your solution is overly wordy or confusing, this is where you'll get docked.