Def A *set* is an unordered collection of distinct items called *elements*.

ex $D = \{0, 1, 2, 3, \dots, 9\}$ has 10 elements

bits $= \{0, 1\}$ has 2 elements

$\mathbb{Z}$ = set of all integers

$\{\dots, -2, -1, 0, 1, 2, \dots\}$

has infinite elements

$\mathbb{Q}$ = rationals

$\mathbb{R}$ = reals

$V = \{a, e, i, o, u, y\}$ has 6 elts

$A = \{-20, \pi, a\}$

Def Two sets A, B are *equal* $(A = B)$ if A and B contain exactly the same elements.

ex. $\{0, 1\} = \{1, 0\}$

Def We write $x \in S$ if x in S.

"x is an element of S"

We write $x \notin S$ if x not in S.

ex $0 \in \text{bits} = \{0,1\}$

$2 \notin \text{bits}$

$\pi \notin \mathbb{Z}$

Def The cardinality or size of set S is the number of distinct elements of S.

$|S|$

ex $|\text{bits}| = 2$

$|\{\underbrace{\{3,4\}}, \underbrace{\text{cat}}\}| = 2$

Q Can we have a set such that (s.t.) $|S| = 0$?

Def The empty set, denoted $\{\}$ or $\emptyset$, is the set with no elements.

$|\emptyset| = 0$

$|\{\emptyset\}| = 1$

$F = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$

$|F| = 3$

Q If $A = B$, does $|A| = |B|$? T

Is the converse true? 2 min on own

If $|A| = |B|$, then $A = B$. F

Pf by counter example:

$A = \{0\}$ $B = \{1\}$

$|A| = 1$, $|B| = 1$, but $A \neq B$.

<u>Def</u> Set builder notation defines a set

$$S = \{x : \text{a rule about } x\}$$

such that

S contains the elements x s.t. the rule about x is true.

<u>ex</u> evens $= \{x : x \in \mathbb{Z} \text{ and } x \text{ even}\}$

"x such that x is in integers and x even"

evens $= \{x : x = 2c \text{ for } c \in \mathbb{Z}\}$

evens $= \{x : x \in \mathbb{Z} \text{ and } 2 \mid x\}$

"2 divides x"

<u>Def</u> A is a subset of B (denoted $A \subseteq B$) if every element of A is also in B.

<u>ex</u> evens $\subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

Q: $\mathbb{R} \subseteq \mathbb{Q}$? no, $\pi \in \mathbb{R}$ but $\pi \notin \mathbb{Q}$.

Q: $\emptyset \subseteq \mathbb{R}$ T

Note: $\emptyset \subseteq S$ for all sets S.

$S \subseteq S$ for all sets S.

($A \subset B$ means A is a strict subset of B, $A \subseteq B$ and $|A| < |B|$)

Note: if $A \subseteq B$, then $|A| \leq |B|$.

Q: is the converse true?

If $|A| \leq |B|$, then $A \subseteq B$ F

divides

<u>claim</u> $\{x \in \mathbb{Z} : 18 \mid x\} \subseteq \{x \in \mathbb{Z} : 6 \mid x\}$

Step 1: understand claim!

The numbers divisible by 18 are contained/ a part of the numbers divisible by 6.

Every number divisible by 18 is also divisible by 6.

Step 2: do some examples.

<u>ex</u>

x	$18 \mid x$?	$6 \mid x$?
18	T	T
4	F	F
	T	F

Pf Want to show $\{x \in \mathbb{Z} : 18 | x\} \subseteq \{x \in \mathbb{Z} : 6 | x\}$
which is to say, if $a \in \{x \in \mathbb{Z} : 18 | x\}$,
then $a \in \{x \in \mathbb{Z} : 6 | x\}$ by def. of $\subseteq$.
assume $a \in \{x \in \mathbb{Z} : 18 | x\}$.

$a = 18c$ for $c \in \mathbb{Z}$	def. of divisibility by 18
$a = 6 \cdot 3 \cdot c$	factoring
$a = 6 \cdot k$ for $k \in \mathbb{Z}$	because 3c is an integer, because int · int = int
$6 \mid a$	def. of divisibility

$a \in \{x \in \mathbb{Z} : 6 | x\}$

$\square$