- Turn in your group's worksheet
- Sit wherever you want for lecture

Big O

Def Let $f, g : \mathbb{R}^{\geq 0} \to \mathbb{R}^{\geq 0}$. We say that f is $O(g)$ if $\exists\, c > 0,\ n_0 \geq 0$ s.t.

$$\forall n \geq n_0 : f(n) \leq c \cdot g(n).$$

We also write $f = O(g)$ to mean f is $O(g)$.

Why O? "order" of a function.

ex $f(n) = 3n^2 + 2$ is $O(n^2)$.

Proof We must give $c > 0,\ n_0 \geq 0$ s.t.

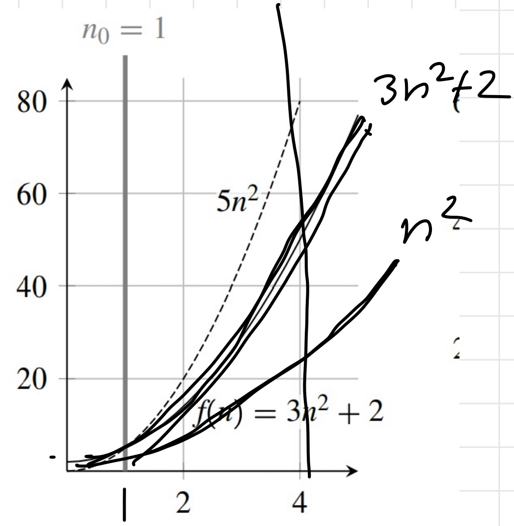
$$\forall n \geq n_0 : f(n) \leq c \cdot n^2.$$

Note that $\forall n \geq 1$, $2n^2 \geq 2$, so

$$\rightarrow \forall n \geq 1 : f(n) = 3n^2 + 2 \leq 3n^2 + 2n^2 = 5n^2.$$

So we can choose $c = 5$, $n_0 = 1$

and we have $\forall n \geq \underset{\substack{\uparrow \\ n_0}}{1} : f(n) \leq \underset{\substack{\uparrow \\ c}}{5} \cdot n^2$



(a) $f(n)$ is $O(n^2)$ with $c = 5$ and $n_0 = 1$.

$f(n) \le 5 \cdot n^2$

$3n^2 + 2 = O(n^2)$

$\forall n \ge n_0 \; 3n^2 + 2 \le cn^2$

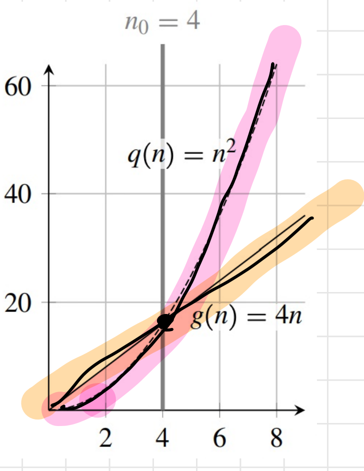
$n_0 = 4$

ex $g(n) = 4n$ is $O(n^2)$. T

Proof We must give $c > 0$, $n_0 \ge 0$ s.t.

$4n \le c \cdot n^2$ for all $n \ge n_0$.

Note that $4n$ and n^2 cross exactly one time, at $n = 4$. So we can pick $n_0 = 4$, $c = 1$.



$$\forall \underset{\substack{\uparrow \\ n_0}}{n \ge 4} : 4n \le \underset{\substack{\uparrow \\ c}}{1} \cdot n^2$$

Q Is $4n = O(4n^2)$? yes

Is $3n^2 + 2 = O(\frac{1}{2}n^2)$ yes

we prefer $3n^2+2 = O(n^2)$ — the simplest form in big O.

$n^2 = O(n^3)$
but $n^2 \neq n^3$

another ex

n^3 is not $O(n^2)$. T

f is $O(g)$ if $\exists\, c > 0,\ n_0 \geq 0$ s.t.

$\forall n \geq n_0 : f(n) \leq c \cdot g(n)$.

Proof we WTS $\forall\, c > 0,\ n_0 \geq 0,\ \exists\, n \geq n_0$:

$n^3 > c \cdot n^2$.