Examples of propositions:

- for ints n, $n(n+1)^2$ is even
- for ints n, if n^2 even, then n even
- for $x, y \in \mathbb{R}$, if $x \in \mathbb{Q}$, $y \in \mathbb{Q}$, then $xy \in \mathbb{Q}$
- $\sqrt{2} \notin \mathbb{Q}$

In proof, we've done:

assume n is even.

$n = 2c$ for $c \in \mathbb{Z}$ ...

$n \in \mathbb{Z}$, $y \in \mathbb{Z}$

$ny \in \mathbb{Z}$

$\sqrt{2}$ rational

$\vdots$

some prop. that is false (contradiction)

We can construct <u>compound propositions</u> out of smaller propositions

Propositions that can't be broken down are <u>atomic</u> propositions.

Syntax vs. Semantics

Syntax → gramatically correct (for a given language)

Semantics → meaning of a gramatically correct statement

$1 \subseteq \mathbb{Z}$ gramatically incorrect → ✗

$1 \in \mathbb{Z}$ T → 1 is an integer

$1 \notin \mathbb{Z}$ F → 1 is not an integer

Let p, q be propositions.

example: (p = "2 is even", q = "$\sqrt{2}$ is rational")

natural language	syntax in discrete math	informal semantics
p and q	$p \wedge q$	T iff both p, q T
p or q	$p \vee q$	T iff ≥1 p, q T
not p	$\neg p$	T iff p is F
if p, then q	$p \Rightarrow q$	T iff when p T, q T
p if and only if q	$p \Leftrightarrow q$	T iff p, q match

p exclusive or q $p \oplus q$ T iff p, q mismatch

formal semantics

p	q	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

p		q		$p \wedge q$
T	2 is even	T	3 is odd	T
T	2 is even	F	4 is odd	F
F	1 is even	T	3 is odd	F
F	3 is even	F	2 is odd	F

if/then: $p \Rightarrow q$

true if p "forces" q

false if p doesn't "force" q

$p \Rightarrow q$ is false when the promise that p forces q is false

when p is T and q is F ↙ when is this a lie?

<u>ex</u> If $\underbrace{\text{it rains}}_{q}$, then the $\underbrace{\text{grass is wet}}_{q}$.

p rains	q grass wet	if rains, then grass wet $p \Rightarrow q$
T	T	T
→ T	F	F
F	T	T
F	F	T

If p, then q can also be written as:

- p implies q
- p is a sufficient condition for q
- p only if q
- p whenever q
- q is necessary for p