A special compound prop. : $\neg q \Rightarrow \neg p$

The <u>contrapositive</u> of $p \Rightarrow q$.

p	q	$p \Rightarrow q$	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$	$q \Rightarrow p$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

$$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$$

Note that the converse of $p \Rightarrow q$ is not logically equiv to $p \Rightarrow q$.

$$p \Rightarrow q \not\equiv q \Rightarrow p$$

Recall proofs by contradiction.

Claim 4.18 (part of it)

If $\underbrace{n^2 \text{ is even}}_{p}$, then $\underbrace{n \text{ is even}}_{q}$.

Why did we do a proof by contradiction?
Let's try a direct proof.

Let n^2 be even. WTS n is even.

$n^2 = 2c$ for $c \in \mathbb{Z}$ def. of even

$$\left.\begin{aligned} n &= \sqrt{2c} \\ n &= \frac{2c}{n} \end{aligned}\right\}$$ we don't have any facts about these

n is even

Claim If n^2 is even, then n is even q

p

1. For contradiction, suppose $\neg(p \Rightarrow q)$
2. $\neg(p \Rightarrow q) \equiv p \wedge \neg q$
3. direct proof that $\neg q \Rightarrow \neg p$
4. established that $\neg p \wedge p$
5. noted that $\neg p \wedge p$ is a contradiction
6. $\neg(p \Rightarrow q)$ is false, so $p \Rightarrow q$ is true

Proof For contradiction, suppose the claim (1) is false. That is, suppose that n^2 is even (2) p but n is odd. $\rightarrow \neg q$

$$n = 2k+1 \text{ for } k \in \mathbb{Z}$$
$$n^2 = (2k+1)^2$$
$$n^2 = 4k^2 + 4k + 1$$
$$n^2 = 2(2k^2 + 2k) + 1$$
$$n^2 = 2c + 1 \text{ for } c \in \mathbb{Z}$$

(3) direct proof that $\neg q \Rightarrow \neg p$

n^2 is odd, $\neg p$ $\rightarrow$ $\neg p \wedge p$ (4)

(5) This contradicts that n^2 is even. So our initial assumption that n is odd is false. So the initial claim is true. (6) $\square$

Note that (3) was a direct proof of the contrapositive.

For this claim, we can give a shorter proof.

p, q, r.

$(p \wedge q) \Rightarrow r$ is claim.

Is $\neg r \Rightarrow \neg(p \wedge q)$ the contrapositive?

Let me rewrite $p \wedge q$ as z.

$z \Rightarrow r$

$\neg r \Rightarrow \neg z$

$\neg r \Rightarrow \neg(p \wedge q)$

Claim If n^2 is even, then n is even.

Proof We will prove the contrapositive.
That is, if n is odd, then n^2 is odd.

$n = 2k+1$ for $k \in \mathbb{Z}$ def. odd

$n^2 = (2k+1)^2$

$n^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$

$n = 2c+1$ for $c \in \mathbb{Z}$ $c = 2k^2 + 2k$
prod., sum of ints is int

n^2 is odd

$\square$

Note you can only use contrapositive proofs on if-then statements.
$(p \Rightarrow q)$

Sometimes a direct proof is easier/simpler.
Sometimes not.

Proposition Suppose $x \in \mathbb{Z}$. If $\underbrace{7x+9 \text{ is even}}_{p}$, then $\underbrace{x \text{ is odd}}_{q}$.

Proof (direct) Suppose $7x+9$ is even.
WTS x is odd.

$7x + 9 = 2c$ for $c \in \mathbb{Z}$ def. of even

$x = 2c - 6x - 9$	algebra
$x = 2c - 6x - 2\cdot 5 + 1$	rewriting -9
$x = 2(c - 3x - 5) + 1$	factoring
$x = 2k + 1$ for $k \in \mathbb{Z}$	sums, prods of ints are int
x odd	def. of odd

$\square$

Proof (by contrapositive) $\neg q \Rightarrow \neg p$

We prove the contrapositive. That is, if x is even, then $7x+9$ is odd.
Suppose x is even. WTS that $7x+9$ is odd.

$7x$ is even	prod. of any even int, int is even
$7x+9$ is odd	sum of even, odd is odd

$\square$

(4.17)

Claim Suppose $y \neq 0$. If $\overbrace{\boxed{x/y \text{ is irrational}}}^{p}$, then $\underbrace{\boxed{x \text{ is irrational}}}_{q}$ or $\underbrace{\boxed{y \text{ is irrational}}}_{r}$.

$$p \Rightarrow (q \vee r)$$

Contrapositive $\neg(q \vee r) \Rightarrow \neg p \equiv (\neg q \wedge \neg r) \Rightarrow \neg p$

If x, y rational, then x/y is rational.

Done, by HW 1 problem 1.