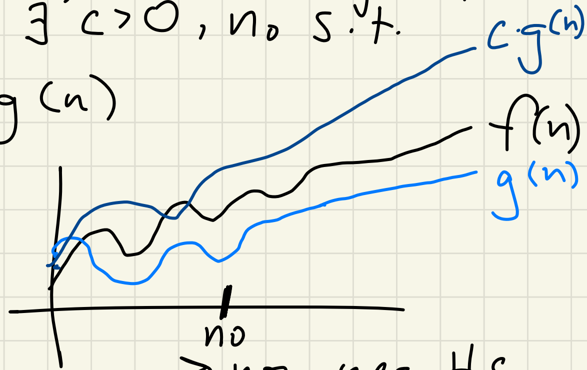
Recall

"has property that"

For functions $f(n)$, $g(n)$ $f(n) = O(g(n))$
"f is big O of g" if $\exists\ c > 0,\ n_0$ s.t.
$\rightarrow \forall n \geq n_0 : f(n) \leq c \cdot g(n)$



$\rightarrow$ non-neg #s

ex

	n_0	c
$n = O(n)$	0	3
$2n = O(n)$	0	3
$n+8 = O(n)$	4	3
$10 = O(n)$	1	10

$f(n)$ $g(n)$

Logarithms for positive real # $b \neq 1$ and real # $x > 0$, $\log_b x$ is the real number y s.t. $b^y = x$.

$\log_4 16$ means "the number we need to raise 4 to to get 16"
$= 2$

$\log_{10} 100$
$= 2$

$\log_2 8 = 3$ $\quad 2^3 = 2 \cdot 2 \cdot 2 = 8$

Lemma 6.7 Let $b > 1$ and $k \geq 0$.

$$\log_b(n^k) = O(\log n)$$

missing base... because it doesn't matter

$f(n) = O(\log n)$

ex $\log_{10} n = O(\log_2(n))$ by Lemma 6.7.

Lemma 6.7, intuitively: base, exponents in logs don't matter asymptotically.

Proof WTS $\exists c > 0, n_0 \geq 0$ s.t. $\forall n \geq n_0$:

$$\log_b(n^k) \leq c \log_a n$$

can be anything, so we'll drop later

Note that

$$\log_b(n^k) = k \log_b(n)$$

log rule: exponent

$$= k \frac{\log_a(n)}{\log_a(b)}$$

log rule: change of base

Now take $n_0 = 1$, $c = \frac{k}{\log_a b}$.

$$\forall n \geq 1, \quad \underbrace{\log_b(n^k)}_{f(n)} = \underbrace{\frac{k \log_a(n)}{\log_a(b)} \leq \overbrace{\frac{k}{\log_a(b)}}^{c} \cdot \overbrace{\log_a(n)}^{g(n)}}$$

so $\log_b(n^k) = O(\log_a n)$.

Since a could have been anything, we drop it.

<u>Q</u> $3^n = O(2^n)$? $\qquad k^n = O(\ell^n)$? $\forall k, \ell$

<u>Lemma</u> Let $b, d > 1$. If $d < b$ then $b^n \neq O(d^n)$.

<u>ex</u>
Let $d = 2$, $b = 3$. $d < b$ because $2 < 3$.

$3^n \neq O(2^n)$.

"3^n is asymptotically larger than 2^n"

This lemma tells us that the base of an exponent does matter, asymptotically.

$n^3 \neq O(n^2)$