1

$g: \mathbb{Z} \to \mathbb{Z}$ defined by $g(a) = 1$

Domain $= \mathbb{Z}$
codomain $= \mathbb{Z}$
range $: 1$

1) $\forall a \in \mathbb{Z}$ $g(a)$ is defined as $1$

2) To show $\forall a \in \mathbb{Z}$, $g(a)$ does not produce 2 diff. outputs.
WTS that if $g(a) = a$ and $g(a) = b$ then $a = b$.

| statement | reason |
|---|---|
| $a = 1$ | by def of $g$ |
| $b = 1$ | by def of $g$ |
| $a = b$ | substitution. |

3) Notice $1 \in \mathbb{Z}$, there for for all $a \in \mathbb{Z}$ $g(a) \in \mathbb{Z}$
because this function satisfies these 3 properties, it is a function, based on the definition of function.

$\square$

$p: \mathbb{Z}^{\geq 0} \to \mathbb{Z}^{\geq 0}$ $\quad p(x) = x - 1$

THIS IS NOT A FUNCTION. IT VIOLATES RULE 3. PROOF BY COUNTEREXAMPLE

LET $x = 0$, WHICH IS IN THE DOMAIN.

$p(0) = (0) - 1 = -1$

$-1$ IS NOT IN THE CODOMAIN $\mathbb{Z}^{\geq 0}$, VIOLATING PROPERTY 3.

Ex $f: \mathbb{R} \to \mathbb{R}^{\geq 0}$ defined by $f(x) =$ the number

$f: \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined $f(x) = \{(x, y) : x + 3y - 4\}$

domain : $\mathbb{R}$

comain : $\mathbb{R} \times \mathbb{R}$

$y = -\frac{1}{3}x + \frac{4}{3}$

Range : $\{(x, y) : x + 3y = 4\}$

Proof

1) $\forall x \in \mathbb{R}$ $f(x)$ is defined as $-\frac{1}{3}x + \frac{4}{3}$

2) $\forall x \in \mathbb{R}$ $f(x)$ does no have 2 different outputs

WTS $f(x) = a$ & $f(x) = b$ then $a = b$

Suppose $f(x) = a$ & $f(x) = b$

$a = -\frac{1}{3}x + \frac{4}{3}$ & $b = -\frac{1}{3}x + \frac{4}{3}$ , def of f

$a = b$ Substitution

3) $\forall x \in \mathbb{R}$, $f(x) \in \mathbb{R} \times \mathbb{R}$

$f(x) = -\frac{1}{3}x + \frac{4}{3}$ because addition & product of $\mathbb{R}$ gives $\mathbb{R}$

$\{(x, y) : x + 3y = 4\} \subseteq \mathbb{R}^2$ given Range

$f: \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $f(x) = \{(x, y) : x + 3y = 4\}$

domain: $\mathbb{Z}$ codomain: $\mathbb{Z} \times \mathbb{Z}$

This is not a function, because violates (3)

Let $x = 2$, which is an int

$f(2) = \{(2, \frac{2}{3}) : (2) + 3(\frac{2}{3}) = 4\}$

$f(x) = (2, \frac{2}{3}) \notin \mathbb{Z} \times \mathbb{Z}$

2) $E: \mathbb{Z} \to \{T, F\}$ defined by $E(x) \begin{cases} T \text{ is even} \\ F \text{ is odd} \end{cases}$

Domain: $\mathbb{Z}$ codomain: all Even numbers $(2n)$

1) all ints are either odd or Even $E(x)$ is defined for all $\mathbb{Z}$
2) for each $x \in E$, all ints are either even or odd but not both
3) $\forall x \in \mathbb{Z}, E(x) \in \mathbb{Z}$
   $E(x) = \{T, F\}$, for all ints

note from Lucy: this is good, but if we wanted to be very formal, here is how we could do it:

2) we want to show that $\forall x \in \mathbb{Z}$, E does not produce 2 diff. outputs. To do this, we show that if $E(a) = Y$ and $E(a) = Z$, then $a = b$.

we prove using cases.

case 1: Let $Y = T$. Then a is even, so Z is also T. So $Y = Z$.

case 2: Let $Y = F$. then a is odd, so Z is also F. So $Y = Z$.

Since Y is either T or F, the cases are exhaustive.