Def A *set* is an unordered collection of distinct items called *elements*.

ex $D = \{0, 1, 2, 3, \dots, 9\}$ has 10 elements

bits $= \{0, 1\}$ has 2 elements

$\mathbb{Z}$ = set of all integers

$\{\dots, -2, -1, 0, 1, 2, \dots\}$

has infinite elements

$\mathbb{Q}$ = rationals

$\mathbb{R}$ = reals

$V = \{a, e, i, o, u, y\}$ has 6 elts

$A = \{-20, \pi, a\}$

Def Two sets A, B are *equal* $(A = B)$ if A and B contain exactly the same elements.

ex. $\{0, 1\} = \{1, 0\}$

Def We write $x \in S$ if x in S.

"x is an element of S"

We write $x \notin S$ if x not in S.

ex $0 \in bits = \{0, 1\}$

$2 \notin bits$

$\pi \notin \mathbb{Z}$

Def The cardinality or size of set S is the number of distinct elements of S.

$|S|$

ex $|bits| = 2$

$|\{\underbrace{\{3,4\}}, \underbrace{cat}\}| = 2$

Q Can we have a set such that (s.t.) $|S| = 0$?

Def The empty set, denoted $\{\}$ or $\emptyset$, is the set with no elements.

$|\emptyset| = 0$

$|\{\emptyset\}| = 1$

$F = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$

$|F| = 3$

Q If $A = B$, does $|A| = |B|$? T

Is the converse true? 2 min on own

If $|A| = |B|$, then $A = B$. F

Pf by counter example:

$A = \{0\}$ $B = \{1\}$

$|A| = 1$, $|B| = 1$, but $A \neq B$.

Def Set builder notation defines a set

$S = \{x : \text{a rule about } x\}$

such that

S contains the elements x s.t. the rule about x is true.

ex evens $= \{x : x \in \mathbb{Z} \text{ and } x \text{ even}\}$

"x such that x is in integers and x even"

evens $= \{x : x = 2c \text{ for } c \in \mathbb{Z}\}$

evens $= \{x : x \in \mathbb{Z} \text{ and } 2 \mid x\}$

"2 divides x"

Def A is a subset of B (denoted $A \subseteq B$) if every element of A is also in B.

ex evens $\subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

Q: $\mathbb{R} \subseteq \mathbb{Q}$? no, $\pi \in \mathbb{R}$ but $\pi \notin \mathbb{Q}$.

Q: $\emptyset \subseteq \mathbb{R}$ T

Note: $\emptyset \subseteq S$ for all sets S.

$S \subseteq S$ for all sets S.

($A \subset B$ means A is a strict subset of B, $A \subseteq B$ and $|A| < |B|$)

Note: if $A \subseteq B$, then $|A| \leq |B|$.

Q: Is the converse true?

If $|A| \leq |B|$, then $A \subseteq B$ F

claim $\{x \in \mathbb{Z} : 18 \mid x\} \subseteq \{x \in \mathbb{Z} : 6 \mid x\}$

divides

Step 1: understand claim!

The numbers divisible by 18 are contained/a part of the numbers divisible by 6.

Every number divisible by 18 is also divisible by 6.

Step 2: do some examples.

ex x	$18 \mid x$?	$6 \mid x$?
18	T	T
4	F	F
	T	F

Pf Want to show $\{x \in \mathbb{Z} : 18 | x\} \subseteq \{x \in \mathbb{Z} : 6 | x\}$
which is to say, if $a \in \{x \in \mathbb{Z} : 18 | x\}$,
then $a \in \{x \in \mathbb{Z} : 6 | x\}$ by def. of $\subseteq$.
assume $\underline{a \in \{x \in \mathbb{Z} : 18 | x\}}$.

$a = 18c$ for $c \in \mathbb{Z}$	def. of divisibility by 18
$a = 6 \cdot 3 \cdot c$	factoring
$a = 6 \cdot k$ for $k \in \mathbb{Z}$	because 3c is an integer, because int · int = int
$6 \mid a$	def. of divisibility

$a \in \{x \in \mathbb{Z} : 6 | x\}$

$\square$

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Sets review

recall $\mathbb{Z}$ = set of all integers = $\{\ldots,-2,-1,0,1,\ldots\}$

$2 \in \mathbb{Z}$ $\quad 1.5 \notin \mathbb{Z}$

$\{2,4\} \subseteq \mathbb{Z}$ $\quad \{x \in \mathbb{Z} : 2 \mid x\} \subseteq \mathbb{Z}$

evens

is $2 \subseteq \mathbb{Z}$ ← F

"2 subset of the integers"

$\{2\} \subseteq \mathbb{Z}$ $\quad$ T

"the set containing 2 is a subset of the integers"

Def $A \cup B$ "A union B" is $\{x : x \in A$ or $x \in B\}$

note that elements $x \in A$ and $x \in B$ are in $A \cup B$.

ex $\{2,4,6\} \cup \{2,3,4\} = \{2,3,4,6\}$

evens $\cup$ odds $= \mathbb{Z}$

$$\mathbb{R}^{\geq 0} \cup \mathbb{R}^{\leq 0} = \mathbb{R}$$

reals ≥ 0 reals ≤ 0

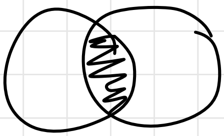
$$A \cup \emptyset = A$$

for all sets A

$$A \cup A = A$$

$\cancel{2 \cup \{1,3\} =}$ ✗ $\{2\} \cup \{1,3\} = \{1,2,3\}$

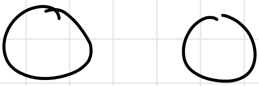
Def $A \cap B$ "A intersect B" $\{x : x \in A \text{ and } x \in B\}$

A  B

$$B \cap A = A \cap B$$

not disjoint

$\{2,4,6\} \cap \{2,3,4\} = \{2,4\}$ evens   odds



evens $\cap$ odds $= \emptyset$ disjoint

$A \cap \emptyset = \emptyset$ disjoint

for all sets A

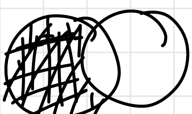
$$A \cap A = A$$

$$\mathbb{R}^{\geq 0} \cap \mathbb{R}^{\leq 0} = \{0\}$$

Def Sets A, B are disjoint if $A \cap B = \emptyset$.

Are $\mathbb{R}^{\geq 0}$ and $\mathbb{R}^{\leq 0}$ disjoint? no

Def $A - B$ or $A \setminus B$ "A minus B" $\{x : x \in A$ and $x \notin B\}$

A  B ex $\{2,4,6\} - \{2,3,4\} = \{6\}$

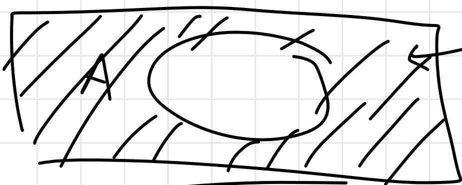
$\{2,3,4\} - \{2,4,6\} = \{3\}$

evens − odds = evens

$A - B \subseteq A$

$A - \emptyset = A$ ~~compliment~~

Def $\bar{A}$ or $\sim A$ "A complement" $\{x : x \notin A\}$



Universe
U

ex $\overline{\{2,4,6\}} = \{0,1,3,5,7,8,9\}$

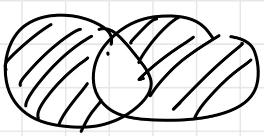
if U is $\{x \in \mathbb{Z} : 0 \le x \le 9\}$

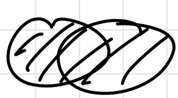
$\overline{\{2,4,6\}} = \{\dots,-2,-1,0,1,3,5,7,8,9,10,\dots\}$

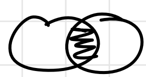
if U is $\mathbb{Z}$

Def $A \oplus B$ "A exclusive or"

$(A \cup B) - (A \cap B)$

A  B





$0 = 2k$
for $k \in \mathbb{Z}$
$0 = 2(0)$

2 divides x
↓

9 divides x
↓

claim $\overset{A}{\{x \in \mathbb{Z} : 2 \mid x\}} \cap \overset{B}{\{x \in \mathbb{Z} : 9 \mid x\}}$

$\subseteq \underset{C}{\{x \in \mathbb{Z} : 6 \mid x\}}$

[if a number is divisible by 2 and 9, then it is divisible by 6.]

$A \cap B \subseteq C$ if $x \in A \cap B$, then $x \in C$.

$P \subseteq Q$ if $y \in P$ then $y \in Q$

examples

x	$x \in A \cap B$	$x \in C$
6	F ($x \notin B$)	T
0	T	T
	T	F

what a counter example would look like

Proof Assume $x \in A \cap B$. Want to show $x \in C$.

Statement	reason
$x \in A$ and $x \in B$	def. of $\cap$
$2 \mid x$ and $9 \mid x$	def. of A, B

$x = 2c$ and $x = 9d$ for integers c, d def. of divisibility

$x = 18e$ for $e \in \mathbb{Z}$

$x = 6 \cdot 3 \cdot e$ factoring

$x = 6f$ for $f \in \mathbb{Z}$ $f = 3 \cdot e \in \mathbb{Z}$ by int. int = int

$6 \mid x$ by def. of divisibility

$x \in C$