Last time:

- proofs
- direct proofs
- disproof by counter example

Def Let n, m be integers. n is divisible by m, if there exists int. k such that $n = mk$

ex

Is 10 divisible by 5?

yes, because $10 = 5 \cdot 2$

$$\underset{n}{10} = \underset{m}{5} \cdot \underset{k}{2}$$

Is 11 divisible by 5?

$n = 11$, $m = 5$. $11 = 5 \cdot k$ ← 2.2 not integer!

no, because there is no k such that $11 = 5k$

Is -5 divisible by 5?

Choose $k = -1$? $-5 = 5 \cdot (-1)$

Is 0 divisible by 2? yes!

Choose $k=0$. $\underset{n}{0} = \underset{m}{2} \cdot \underset{k}{0}$

Def If n is divisible by m, we also say m <u>divides</u> n.

$m \mid n$ "m divides n"

$2 \mid 10$ but $10 \nmid 2$

"2 divides 10" "10 does not divide 2"

($x = y$ $x \neq y$)

"x equals y" "x does not equal y"

(4.12)

Claim Let n be an integer. Then $n \cdot (n+1)^2$ is even.

Step 1: understand claim

terms: even: divisible by 2

if n is divisible by 2,

then $n = 2k$ for an integer k

why is 4 even? $4 = 2 \cdot \underset{k}{2}$

why is 10 even? $10 = 2 \cdot \underset{k}{5}$

step 2: exampces

n	$n(n+1)^2$	$n(n+1)^2$ is even?
0	$0(1)^2 = 0$	T
3	$3(4)^2 = 3 \cdot 16 = 48$	T
-2	$-2(-1)^2 = -2$	T

we ~~know~~: even # times anything is even.

easy special case:

n is even.

n is odd. then n+1 is even.

<u>Proof</u> Consider 2 cases.

Case 1: n is even.

<u>statements</u>	<u>reasoning</u>
$n = 2c$ for int. c	by def. of even
$n(n+1)^2 = 2c(n+1)^2$	by substitution
$\underline{c(n+1)^2}$ is an int.	sum, product of ints ~~ie~~ is int

$n(n+1)^2$ is even

we gave a way to write it as $2k$ for integer k

k is $c(n+1)^2$

case 2: n is odd.

statement	reasoning
$n+1$ is even	n is odd
$n+1 = 2c$ for integer c	def. of even
$n(n+1)^2 = n(2c)^2$ $= n4c^2$	by substitution, algebra
$2nc^2$ is integer	because product of integers is int
$n(n+1)^2$ is even	def. of even ($2nc^2 = k$)

Since n is either even or odd, and in either case $n(n+1)^2$ is even, the claim holds.

$\square$

$$n(n+1)^2 = 4nc^2$$
$$= 2(2nc^2)$$

want:

$n(n+1)^2 = 2k$, k int