CSCI 332, Fall 2024

Homework 8

Due before class on Tuesday, October 29, 2024–that is, due at 9:30am Mountain Time

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Submission Requirements

- Type or clearly hand-write your solutions into a PDF format so that they are legible and professional. Submit your PDF on Gradescope.
- Do not submit your first draft. Type or clearly re-write your solutions for your final submission.
- Use Gradescope to assign problems to the correct page(s) in your solution. If you do not do this correctly, we will ask you to resubmit.
- You may work with a group of up to three students and submit ***one single document*** for the group. Just be sure to list all group members at the top of the document. When submitting a group assignment to Gradescope, only one student needs to upload the document; just be sure to select your groupmates when you do so.

Academic Integrity

Remember, you may access ***any*** resource in preparing your solution to the homework. However, you ***must***

- write your solutions in your own words, and
- credit every resource you use (for example: "Bob Smith helped me on this problem. He took this course at UM in Fall 2020"; "I found a solution to a problem similar to this one in the lecture notes for a different course, found at this link: www.profzeno.com/agreatclass/lecture10"; "I asked ChatGPT how to solve part (c)"; "I put my solution for part (c) into ChatGPT to check that it was correct and it caught a missing case.") If you use the provided LaTeX template, you can use the `sources` environment for this. Ask if you need help!

Grading

Remember, submitted homeworks are graded for completeness, not correctness. Correctness is evaluated using homework quizzes.

Each submitted problem will be graded out of six points according to the following rubric:

- Does the solution address the correct problem?
- Does the solution make a reasonable attempt at solving the problem, even if not fully correct?
- Is the presentation neat?
- Is the explanation clear?

- Does the solution list collaborators or sources, or state that the student did not use any collaborators or outside resources?
- Is the solution written in the student's own voice (not copied directly from an outside resource)?

1. Recall the problem of finding the number of inversions from class: we are given a sequence of n numbers $a_1, a_2, \ldots, a_n$, which we assume are all distinct, and we define an inversion to be a pair $i < j$ such that $a_i > a_j$.

   One reason we considered the problem of counting inversions was as a measure of how different two orderings are. However, we might feel that this measure is too sensitive. Let's call a pair a *significant inversion* if $i < j$ and $a_i < 2a_j$. Given an $O(n \log n)$ algorithm to count the number of significant inversions between two orderings.

2. We can write an algorithm to compute the factorial of a positive integer recursively as follows:

```
fact(n):
  if n = 1:
      Return 1
  else:
      Return n· fact(n−1)
```

   (a) Write a recurrence relation for the runtime of the algorithm above.

   (b) Draw the recurrence tree for this recurrence relation, and use it to make a guess about the closed-form solution.

   (c) Use proof by induction to prove that your closed-form solution is valid. Hint: follow the example we did in class!