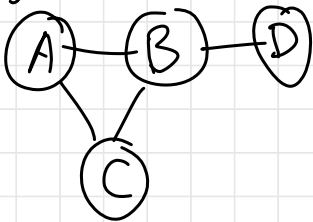


Intro to Graphs

Def An undirected graph $G = (V, E)$ is a non-empty set of vertices (nodes) V and a set $E = \{ \{u, v\} : u, v \in V \}$ of edges joining pairs of nodes.

ex \textcircled{A} $V = \{A\}$
 $E = \emptyset$

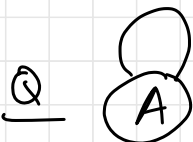
$\textcircled{A} - \textcircled{B}$ $V = \{A, B\}$
 $E = \{ \{A, B\} \}$

G_3 :
 $V = \{A, B, C, D\}$
 $E = \{ \{A, B\}, \{B, D\}, \{B, C\}, \{A, C\} \}$

$\textcircled{A} \quad \textcircled{B}$ $V = \{A, B\}$
 $E = \emptyset$

non-ex

$\textcircled{A} -$ all edges need 2 endpoints



is this a graph? yes.

$$V = \{A\}$$
$$E = \{ \{A, A\} \}$$
$$= \{ \{A\} \}$$

real-world examples

- Facebook friends

nodes: people

edge: 2 people are Facebook friends

- blood relationships

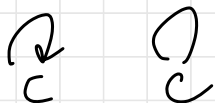
alice — bob
 / Catherine

Q what property (or properties) would a mathematical relation need to have to be represented as an undirected graph?

ideas: symmetric $a \leftrightarrow b$ $a - b$

reflexive $a \rightarrow a$

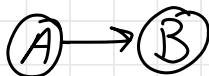
self loops are equivalent when directed or undirected



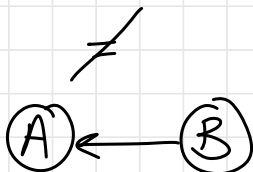
Def A directed graph $G = (V, E)$ has a set of vertices V and edges $E \subseteq V \times V = \{(u, v) : u, v \in V\}$ so that edges are directed from one vertex to another.

Note: relations ^{on a single set} and directed graphs are the same!

ex (A)



$$V = \{A, B\}$$
$$E = \{(A, B)\}$$



$$V = \{A, B\}$$
$$E = \{(B, A)\}$$

ordered pair
tuple
list
array

undirected:



$$E = \{\{A, B\}\}$$

set

real-world example

Twitter followers

Def A graph is simple if it contains no parallel edges or self-loops.

parallel edges: (A) → (B)



note that (A) → (B) has no parallel edges
(A, B) ≠ (B, A)

self-loops: (A)



Example 11.3: Self-loops and parallel edges.

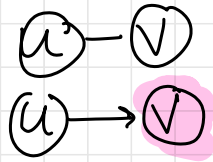
Suppose that we construct a graph to model each of the following phenomena. In which settings do self-loops or parallel edges make sense?

- 1 A social network: nodes correspond to people; (undirected) edges represent friendships.
- 2 The web: nodes correspond to web pages; (directed) edges represent links.
- 3 The flight network for a commercial airline: nodes correspond to airports; (directed) edges denote flights scheduled by the airline in the next month.
- 4 The email network at a college: nodes correspond to students; there is a (directed) edge $\langle u, v \rangle$ if u has sent at least one email to v within the last year.

	self-loops	parallel edges
Social network	no	no
The web	yes	yes
Flight network	no	yes
Email network	yes	no

Def let $e = \{u, v\}$ or (u, v)

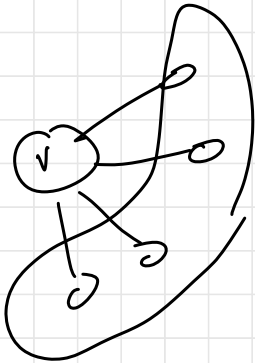
- nodes u, v are adjacent or neighbors



- in a directed graph, v is an out-neighbor of u and u is an in-neighbor of v
- u, v are endpoints of e
- u, v are incident to e

let v be a node in a simple undirected graph.

$$\begin{aligned} \text{degree}(v) &= \deg(v) = d(v) = \# \text{ of neighbors of } v \\ &= \left| \{ u \in V : \underbrace{\{v, u\}}_{\text{or } \{u, v\}} \in E \} \right| \end{aligned}$$



$$\deg(v) = 4$$

for directed graphs, $\text{indeg}(v) = \# \text{ of in-neighbors of } v$

$\text{outdeg}(v) = \# \text{ of out-neighbors of } v$

