

## Examples of propositions:

- for ints  $n$ ,  $n(n+1)^2$  is even
- for ints  $n$ , if  $n^2$  even, then  $n$  even
- for  $x, y \in \mathbb{R}$ , if  $x \in \mathbb{Q}$ ,  $y \in \mathbb{Q}$ , then  $xy \in \mathbb{Q}$
- $\sqrt{2} \notin \mathbb{Q}$

In proof, we've done:

assume  $n$  is even.

$$n = 2c \text{ for } c \in \mathbb{Z}$$

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$$n \in \mathbb{Z}, y \in \mathbb{Z}$$

$$ny \in \mathbb{Z}$$

$\sqrt{2}$  rational

$\vdots$

some prop. that is false (contradiction)

We can construct compound propositions out of smaller propositions

Propositions that can't be broken down are atomic propositions.

Syntax vs. Semantics

↓  
gramatically correct  
(for a given language)

↘ meaning of a  
gramatically correct  
statement

$1 \in \mathbb{Z}$  gramatically incorrect  $\rightarrow$  X

$1 \in \mathbb{Z}$  T  $\longrightarrow$  1 is an integer

$1 \notin \mathbb{Z}$  F  $\longrightarrow$  1 is not an integer

Let  $p, q$  be propositions.

example: ( $p$  = "2 is even",  $q$  = " $\sqrt{2}$  is rational")

<u>natural language</u>	<u>syntax in discrete math</u>	<u>informal semantics</u>
$p$ and $q$	$p \wedge q$	$\rightarrow$ T iff both $p, q$ T
$p$ or $q$	$p \vee q$	T iff $\exists 1$ $p, q$ T
not $p$	$\neg p$	T iff $p$ is F
if $p$ , then $q$	$p \Rightarrow q$	$\rightarrow$ T iff whenever $p$ T, $q$ T
$p$ if and only if $q$	$p \Leftrightarrow q$	T iff $p, q$ match

$p$  exclusive or  $q$      $p \oplus q$     T iff  $p, q$  mismatch

## formal Semantics

$p$	$q$	$p \wedge q$	$p \vee q$	$\neg p$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	T	F	T	T	F
T	F	F	T	F	F	F	T
F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F

$p$

T 2 is even  
T 2 is even  
F 1 is even  
F 3 is even

$q$

T 3 is odd  
F 4 is odd  
T 3 is odd  
F 2 is odd

$p \wedge q$

T  
F  
F  
F

if/then:  $p \Rightarrow q$

true if  $p$  "forces"  $q$

false if  $p$  doesn't "force"  $q$

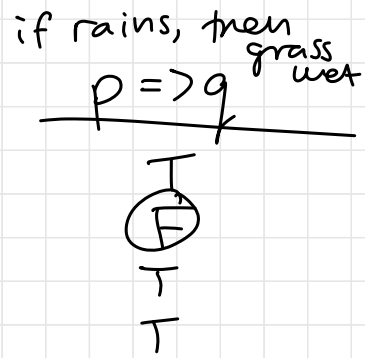
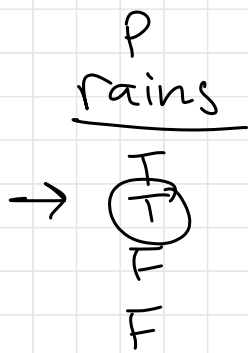
$p \Rightarrow q$  is false when the promise that  $p$  forces  $q$  is false

when  $p$  is T and  $q$  is F     $\swarrow$  <sup>men is this a tie?</sup>

ex If it rains, then the grass is wet.

$p$

$q$



If  $p$ , then  $q$  can also be written as:

- $p$  implies  $q$
- $p$  is a sufficient condition for  $q$
- $p$  only if  $q$
- $p$  whenever  $q$
- $q$  is necessary for  $p$