Functions

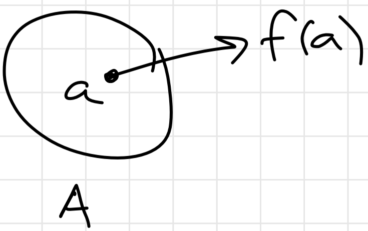
Def Let A, B be sets.

$f: A \to B$ is a function if f assigns to each $a \in A$ a single value $b \in B$, denoted $f(a)$.

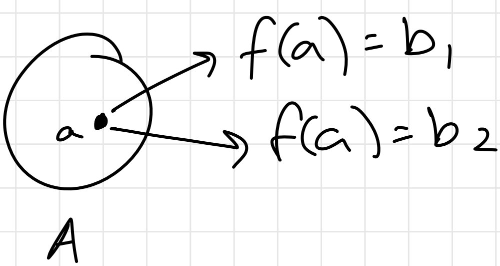
"f from A to B"

Equivalently, f has 3 properties:

1) for each $a \in A$, $f(a)$ is defined.

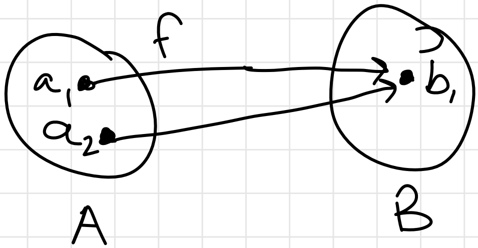


2) For each $a \in A$, $f(a)$ does not produce 2 different outputs.



only if $b_1 = b_2$

3) for each $a \in A$, $f(a) \in B$.



Come up for another potential function when done!

$f: A \to B$

A is the <u>domain</u> of f

B is the <u>codomain</u> of f

The <u>range</u> of f is $\{f(a) : a \in A\}$

range $\subseteq$ codomain

let $A = \{1, 2, 3\}$
let $B = \{x, y\}$

$a \in A$	$b \in B$
1	$x = f(1)$
2	$y = f(2)$
3	$x = f(3)$

Props:

(1) $\forall a \in A$, $f(a)$ is defined ✓

(2) $\forall a \in A$, $f(a)$ does not produce 2 diff. outputs ✓

(3) $\forall a \in A$, $f(a) \in B$ ✓

- exactly 1 row for every element of A
- some elements of B can have zero rows, or elements of B can have multiple rows

ex $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$.

domain: $\mathbb{R}$
codomain: $\mathbb{R}$
range: $\mathbb{R}^{\geq 0}$ (reals greater than or equal to 0)

Intuitive "proof" of 3 properties:

(1) $\forall x \in \mathbb{R}$, $f(x) = x^2$ ✓

(2) $\forall x \in \mathbb{R}$, $f(x) = x^2$, a single value

(3) $\forall x \in \mathbb{R}$, $f(x) \in \mathbb{R}$, because $x^2 \in \mathbb{R}$

ex $f: \mathbb{R} \to \mathbb{R}^{<0}$, $f(x) = x^2$

f is not a function.

Violates (3). Consider $2 \in \mathbb{R}$. $f(2) = 4 \notin \mathbb{R}^{<0}$

ex $s: \mathbb{Z} \to \mathbb{Z}$ defined by $s(x) = x+1$

"successor function"

domain, codomain: $\mathbb{Z}$

range: $\mathbb{Z}$

<u>claim</u> $s: \mathbb{Z} \to \mathbb{Z}$ is a function.

<u>Proof</u> We prove all 3 properties.

1) $\forall x \in \mathbb{Z}$, $s(x)$ is defined as $x+1$.

2) To show $\forall x \in \mathbb{Z}$, $s(x)$ does not produce 2 diff. outputs, we show that if $s(x) = a$ and $s(x) = b$, then $a = b$.

Assume $s(x) = a$ and $s(x) = b$.

$a = x+1$, $b = x+1$ def. of s

$a = b$ substitution

3) WTS (want to show) $\forall x \in \mathbb{Z}$, $s(x) \in \mathbb{Z}$.

$s(x) = x+1$, which is an integer because int + int = int.