4.12

**Claim** Let $n$ be any int. Then $n \cdot (n+1)^2$ is even.

terms: integer ✓
even → div. by 2
→ $x/2$ is an integer

**ex**

| $n$ | $n(n+1)^2$ | is $n(n+1)^2$ even? |
|---|---|---|
| 0 | $0(1) = 0$ | T |
| 3 | $3(3+1)^2 = 48$ | T |
| $-2$ | $-2(-2+1)^2 = -2$ | T |

easy special cases:

$n$ is even. $n$ times anything is even.
$n$ is odd. so $n+1$ is even.

wait! that covers everything.

**Proof** Consider two cases.

Case 1: $n$ is even.

| statement | reasoning |
|---|---|
| $n = 2c$ for int $c$ | by def. of even |

$n(n+1)^2 = 2c(n+1)^2$ by subs.

$c(n+1)^2$ is an int. sums, prods of ints are ints

$n(n+1)^2$ is even we gave a way to write it as $2k$ for int. $k$ (it is $c(n+1)^2$)

Case 2: $n$ is odd.

| Statement | reasoning |
|---|---|
| $n+1$ is even | $n$ is odd |
| $n+1 = 2c$ for int $c$ | def. of even |
| $n(n+1)^2 = n(2c)^2$ $= 2n\,2c^2$ | subs., algebra |
| $n2c^2$ is int. | sums, prods of ints are ints |
| $n(n+1)^2$ is even | by def. of even |

Since $n$ is either even or odd, and in both $n(n+1)^2$ is even, $n(n+1)^2$ is even. □