Linear algebra homework rework 2025/09/29

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Problem.4

$$\begin{vmatrix} x_1 & y & \cdots & y \\ z & x_2 & \cdots & y \\ \vdots & \vdots & \ddots & \vdots \\ z & z & \cdots & x_n \end{vmatrix} = \begin{vmatrix} x_1 & y & \cdots & y \\ z & x_2 & \cdots & y \\ \vdots & \vdots & \ddots & \vdots \\ z & z & \cdots & y+(x_n-y) \end{vmatrix} = (x_n-y)\cdot D_{n-1} + \begin{vmatrix} x_1 & y & y & \cdots & y & y \\ z & x_2 & y & \cdots & y & y \\ z & z & x_3 & \cdots & y & y \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ z & z & z & \cdots & x_{n-1} & y \\ z & z & z & \cdots & z & y \end{vmatrix}$$

$$= (x_n-y)\cdot D_{n-1} + \begin{vmatrix} x_1 & y & y & \cdots & y & y \\ z-x_1 & x_2-y & 0 & \cdots & 0 & 0 \\ z-x_1 & z-y & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ z-x_1 & z-y & z-y & \cdots & x_{n-1}-y & 0 \\ z-x_1 & z-y & z-y & \cdots & z-y & 0 \end{vmatrix}_n$$

$$= (x_n-y)\cdot D_{n-1} + (-1)^{n+1}y \begin{vmatrix} z-x_1 & x_2-y & 0 & \cdots & 0 \\ z-x_1 & z-y & x_3-y & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z-x_1 & z-y & z-y & \cdots & x_{n-1}-y \\ z-x_1 & z-y & z-y & \cdots & z-y \end{vmatrix}_{n-1}$$

$$= (x_n-y)\cdot D_{n-1} + (-1)^{n+1}y \begin{vmatrix} z-x_1 & x_2-y & 0 & \cdots & 0 \\ 0 & z-x_2 & x_3-y & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & x_{n-1}-y \\ 0 & 0 & 0 & \cdots & z-x_{n-1} \end{vmatrix}_{n-1}$$

$$= (x_n-y)\cdot D_{n-1} + (-1)^{n+1}y \cdot \prod_{i=1}^{n-1}(z-x_i)$$

Hence $D_n = (x_n - y) \cdot + y \cdot \prod_{i=1}^{n-1}(x_i - z)$
Similarly, $D_n = (x_n - z) \cdot + y \cdot \prod_{i=1}^{n-1}(x_i - y)$

when $y \neq z$, $D_n = \frac{y\prod_{i=1}^{n}(x_i - z) - x\prod_{i=1}^{n}(x_i - y)}{y - z}$