ECE180

PS1

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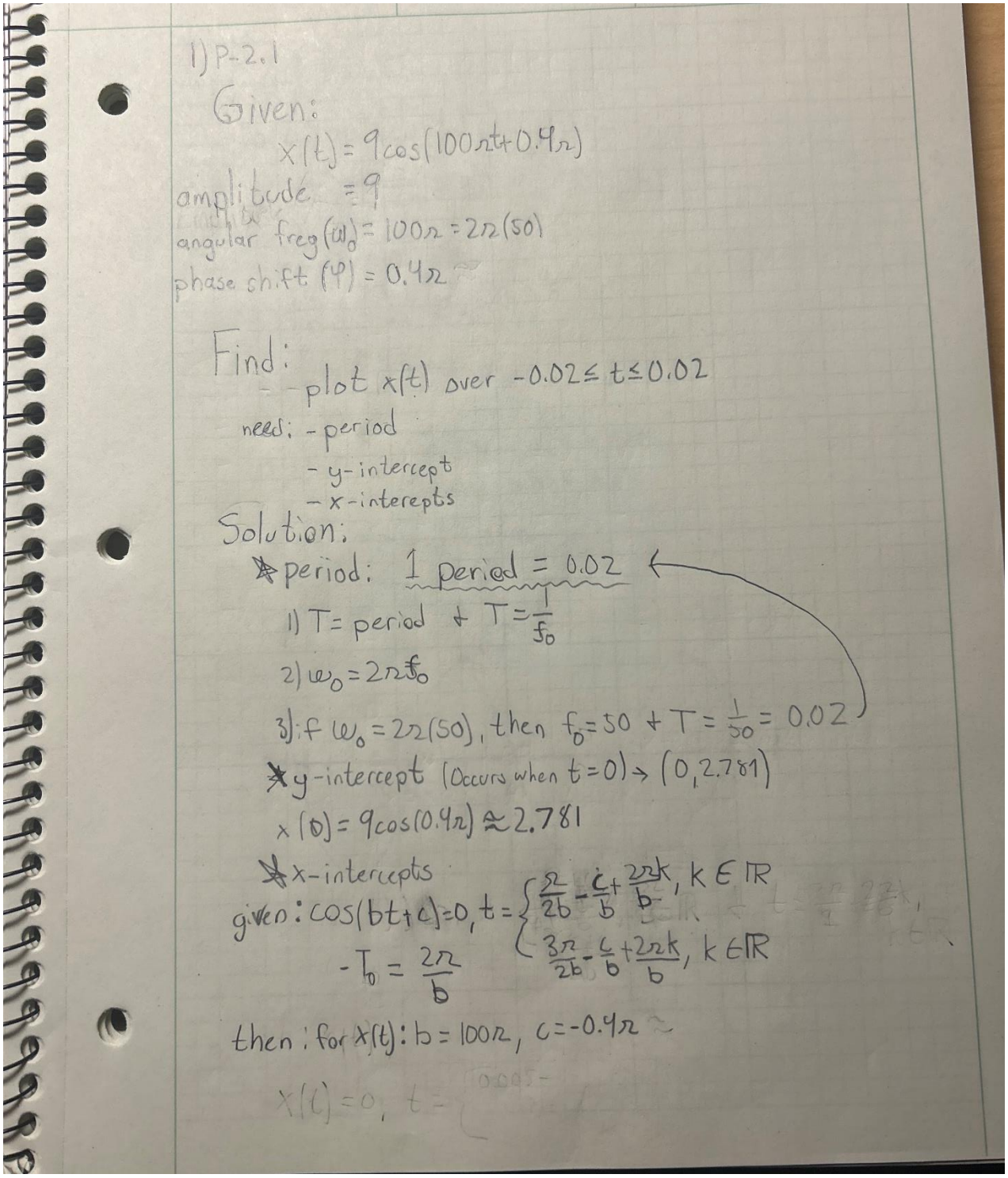
Problem 1:


1) P-2.1
Given:
$x(t) = 9\cos(100\pi t + 0.4\pi)$
amplitude = 9
angular freq (ω_0) = $100\pi = 2\pi(50)$
phase shift (φ) = 0.4π
Find:
- plot $x(t)$ over $-0.02 \le t \le 0.02$
need: - period
- y-intercept
- x-interepts
Solution:
* period: 1 period = 0.02
1) T = period & $T = \frac{1}{f_0}$
2) $\omega_0 = 2\pi f_0$
3) if $\omega_0 = 2\pi(50)$, then $f_0 = 50$ & $T = \frac{1}{50} = 0.02$
* y-intercept (Occurs when $t = 0$) → $(0, 2.781)$
$x(0) = 9\cos(0.4\pi) \approx 2.781$
* x-intercepts
given: $\cos(bt + c) = 0$, $t = \begin{cases} \frac{\pi}{2b} - \frac{c}{b} + \frac{2\pi k}{b}, k \in \mathbb{R} \\ \frac{3\pi}{2b} - \frac{c}{b} + \frac{2\pi k}{b}, k \in \mathbb{R} \end{cases}$
$T_0 = \frac{2\pi}{b}$
then: for $x(t)$: $b = 100\pi$, $c = -0.4\pi$
$x(t) = 0$, $t =$

Problem 1:

1) P-2.1 (cont.)

Solution: (cont.)

$\frac{2\pi k}{b} = \frac{2\pi k}{100\pi} = \frac{k}{50} = 0.02k$

$\frac{\pi}{2b} = \frac{\pi}{2 \cdot 100\pi} = \frac{1}{200} = 0.005$

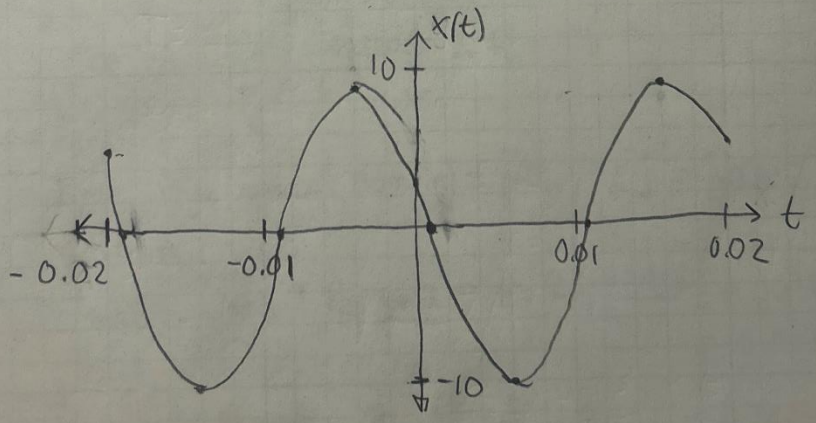
$\frac{3\pi}{2b} = \frac{3\pi}{200\pi} = \frac{3}{200} = 0.015$

$\frac{a}{b} = \frac{0.4\pi}{100\pi} = 0.004$

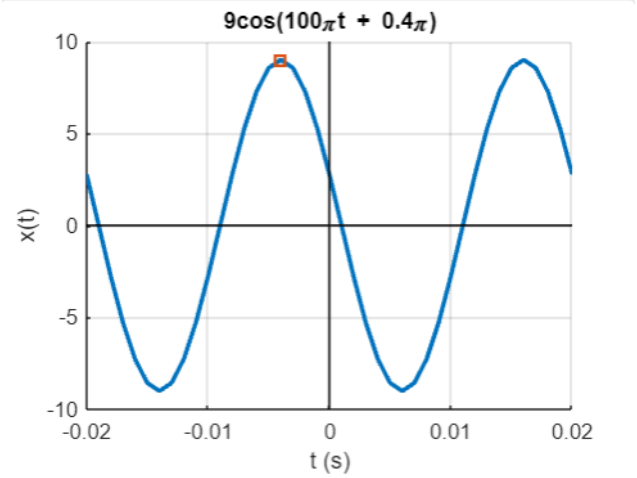
So...

$$x(t) = 0, \; t \begin{cases} 0.001 + 0.02k, & k \in \mathbb{R} \\ 0.011 + 0.02k, & k \in \mathbb{R} \end{cases}$$

I then applied the same math and concepts to $x(t) = 9$ and $x(t) = -9$.



Problem 1:



Variables / Set up

```
% Time set up
tStart = -0.02
tFinal = tStart * -1
tStep = 10e-4


tt_s = tStart: tStep: tFinal;


% parameters of the sinusoidal signal
A = 9                       % Amplitude
f_Hz = 50                   % Frequency
phi_rad = 0.4 * pi          % phase shift
```

Computation

```
T = 1/ f_Hz
x = A*cos(2*pi*f_Hz*tt_s + phi_rad);
timeOfMax = ((-phi_rad * T) / (2*pi))
```

Plotting

```
hold on ; grid on;
plot (tt_s, x, 'LineWidth', 2);
plot (timeOfMax, A,'square', 'LineWidth', 2)


xlabel('t (s)')
ylabel('x(t)')
title('9cos(100\pit + 0.4\pi)')
line([0,0], ylim, 'Color', 'k', 'LineWidth', 0.5);
line(xlim, [0,0], 'Color', 'k', 'LineWidth', 0.5);
```

Problem 2:

P.S. 1 - Problem 2

Given:

$x(t) = 25\cos(2\pi(200)t - 0.75\pi)$

amplitude = 25

angular freq $(\omega_0) = 2\pi(200)$

phase shift $(\varphi) = -\frac{3}{4}\pi$

Find:

- plot of $x(t) = 25\cos(2\pi(200) - 0.75\pi)$
  - period (T_0)
  - y-intercept
  - x-intercepts
  - maxima & minima
  - time range (two cycles)

Solution

- period $= T_0 = 0.005$ sec or 5ms

  $\omega_0 = 2\pi f_0$, $\omega_0 = 2\pi(200)$, $\therefore f_0 = 200$

  $\frac{1}{f_0} = T_0 \quad \therefore \quad T_0 = \frac{1}{200} = 0.005$

- Time range (selected domain) $= [-T_0, T_0] = [-0.005, 0.005]$

- maxima

  $t_{max} = \frac{-\varphi T_0}{2\pi} + T_0 K, \; K \in \mathbb{R} \qquad t = \frac{0.75\pi \cdot 0.005}{2\pi} = 0.001875$

  $t = -0.003125$

- minima

  minima occur half a period after a maxima (if the sinusoid isn't translated vertically)

  $t_{min} = \frac{-\varphi t_0}{2\pi} \pm \frac{T_0}{2} + T_0 K, \; K \in \mathbb{R}$

  $t_{min} = \frac{0.75\pi \cdot 0.005}{2\pi} \pm \frac{0.005}{2} = -0.000625$

  $t = 0.004375$

Problem 2:

- x intercepts

these occur at $\frac{1}{4}T_0$ and $\frac{3}{4}T_0$ (if the sinusoid is not translate vertically)

so

$t_{x\text{-intercept}} = \frac{0.75\pi \cdot 0.005}{2\pi} + \frac{T_0}{4} + T_0 k,\ k \in \mathbb{R}$

$t = 0.00315, -0.001875$

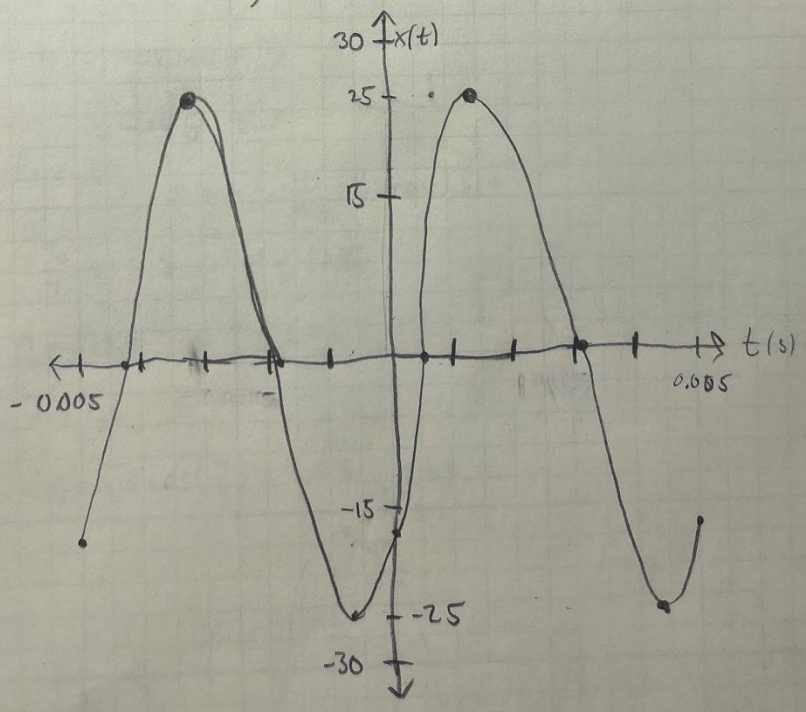
and $t_{x\text{-intercept}} = \frac{0.75\pi \cdot 0.005}{2\pi} + \frac{3T_0}{4} + T_0 k,\ k \in \mathbb{R}$

$t = 0.000625, -0.004375$

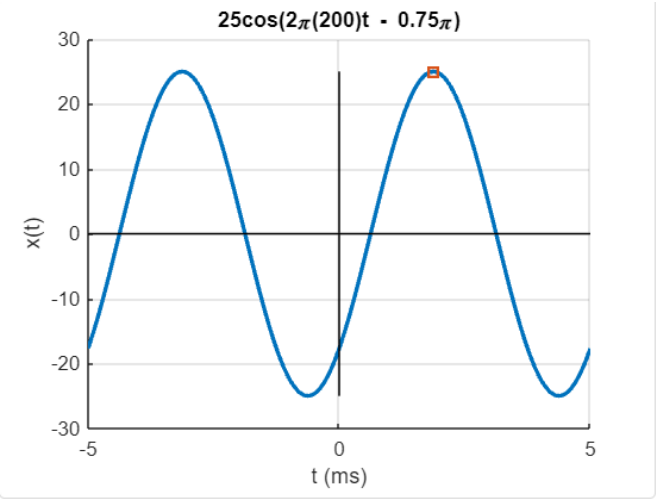
- y-intercept

$x(0) = 25\cos(25\pi(200)(0) - 0.75\pi)$

$= 25\cos\left(-\frac{3\pi}{4}\right) = 25 \cdot \frac{\sqrt{2}}{2} = \frac{25\sqrt{2}}{2} \simeq -17.67$



Problem 2:



Variables / Set up

```
% Time set up
tStart = -0.005
tFinal = tStart * -1
tStep = 10e-5


tt_s = tStart: tStep: tFinal;


% parameters of the sinusoidal signal
A = 25                      % Amplitude
f_Hz = 200                  % Frequency
phi_rad = -0.75 * pi        % phase shift
```

Computation

```
T = 1/ f_Hz
x = A*cos(2*pi*f_Hz*tt_s + phi_rad);
timeOfMax = ((-phi_rad * T) / (2*pi)) * 1e3
```

Plotting

```
hold on ; grid on;
plot (tt_s * 1e3, x, 'LineWidth', 2);
plot (timeOfMax, A,'square', 'LineWidth', 2)


xlabel('t (ms)')
ylabel('x(t)')
title('25cos(2\pi(200)t - 0.75\pi)')
line([0,0], ylim, 'Color', 'k', 'LineWidth', 0.5);
line(xlim, [0,0], 'Color', 'k', 'LineWidth', 0.5);
```

Problem 3:

P.S 1 - Problem 3

Given:

$x(t) = \sqrt{19}\cos(2.1^6 \pi t + 0.25\pi)$

amplitude $= \sqrt{19} \approx 4.36$ | $\omega_0 = 2.1^6 \pi = 2\pi(42.88)$ | $\varphi = \frac{\pi}{4}$

Find

Plot of $x(t)$, maxima, minima, x-intercepts, y intercept, period, time range

Period $= T_0 = \frac{2\pi}{\omega_0} = \frac{1}{42.88} \approx 0.02331$ sec $= 23.31$ ms

Time Range $= [-23.31, 23.31]$

Maxima $= \frac{-\varphi T_0}{2\pi} \pm T_0 k, k \in \mathbb{R}$

$= \frac{-\frac{\pi}{4} \cdot 23.31}{2\pi} \pm 23.31 = -2.9, 20.4$

minima $=$ maxima $\pm \frac{T_0}{2}$

$= -14.5, 8.7$

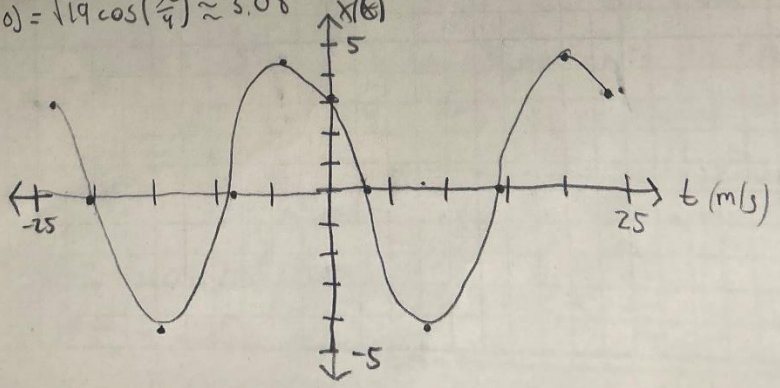
x-intercepts $=$ maxima $+ \frac{T_0}{4}$ & maxima $+ \frac{3T_0}{4}$

$\frac{T_0}{4} \approx 5.8$ $\frac{3T_0}{4} \simeq 17.5$

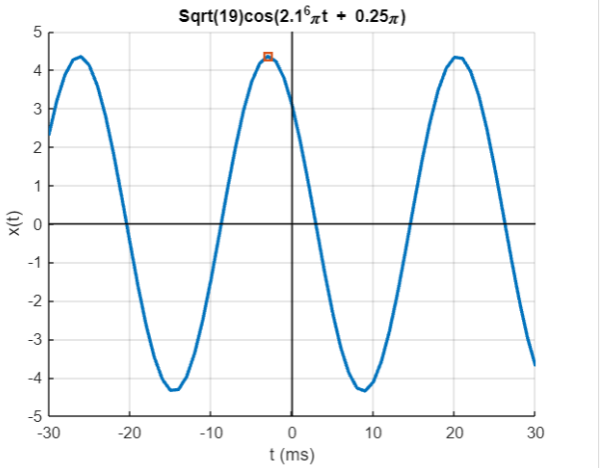
$t = -20.4, 2.9$ $t = -8.7, 14.6$

y-intercept

$x(0) = \sqrt{19}\cos\left(\frac{\pi}{4}\right) \approx 3.08$



Problem 3:



Variables / Set up

```
% Time set up
tStart = -0.03
tFinal = tStart * -1
tStep = 10e-4


tt_s = tStart: tStep: tFinal;


% parameters of the sinusoidal signal
A = sqrt(19)              % Amplitude
f_Hz = 2.1^6 / 2          % Frequency
phi_rad = 0.25 * pi       % phase shift
```

Computation

```
T = 1/ f_Hz
x = A*cos(2*pi*f_Hz*tt_s + phi_rad);
timeOfMax = ((-phi_rad * T) / (2*pi)) * 1e3
```

Plotting

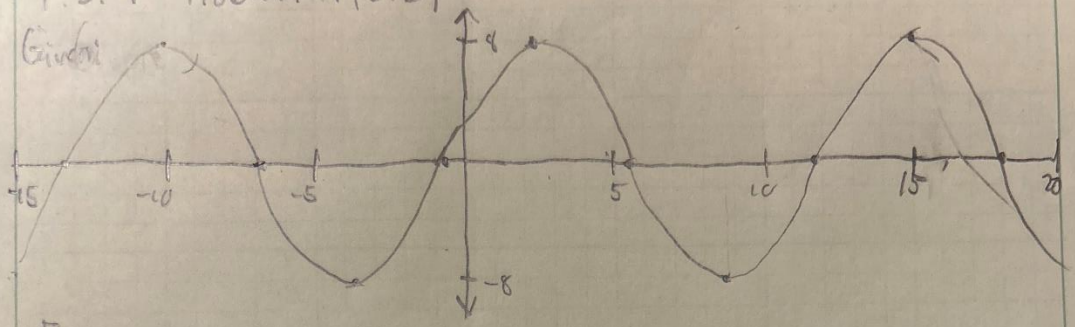
```
hold on ; grid on;
plot (tt_s * 1e3, x, 'LineWidth', 2);
plot (timeOfMax, A,'square', 'LineWidth', 2)


xlabel('t (ms)')
ylabel('x(t)')
title('Sqrt(19)cos(2.1^6\pit + 0.25\pi)')
line([0,0], ylim, 'Color', 'k', 'LineWidth', 0.5);
line(xlim, [0,0], 'Color', 'k', 'LineWidth', 0.5);
```

Problem 4:

P.S. 1 - Problem 4 (2.2)

Given:



Find: Amplitude (A), Angular Freq (ω_0), Phase shift

Solution

- $Amp = \frac{t_{max} - t_{min}}{2} = \frac{8-(-8)}{2} = 8 = A$
- $\omega_0 = 80(2\pi)$

between $t=-10$ and $t=15$ there are two cycles

$f_0 = \frac{2 \text{ periods}}{25 ms} = \frac{2 \text{ periods}}{0.025 s} = 80 \frac{1}{s}$

$\omega_0 = 2\pi f_0 = 2\pi(80)$

- $\varphi = -\frac{2\pi}{5}$ or $\frac{4\pi}{5}$

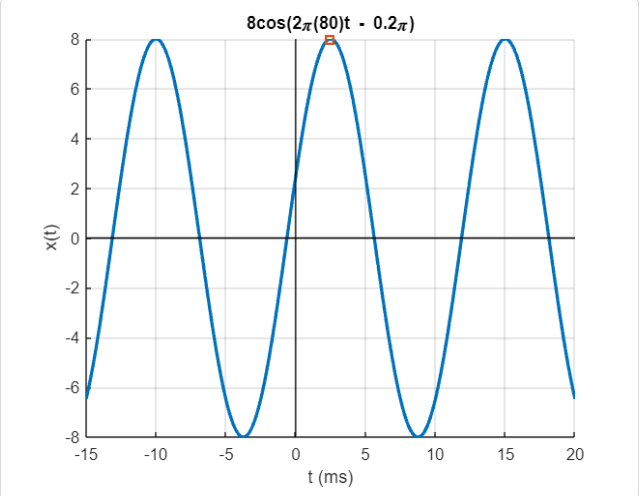
$T_0 = \frac{1}{f_0} = \frac{1}{80} = 0.0125 s = 12.5 ms$

Since 1 period equals 12.5 ms, given the maxima at -10ms, the next maxima is at 2.5 ms, meaning $\frac{2.5}{12.5}$ $(\frac{1}{5})$ of a period stands between the nearest maxima and the y-axis. Since the peak is after $t=0$, it is delayed $2\pi(\frac{1}{5})$ or advanced $2\pi(\frac{4}{5})$

$$x(t) = 8\cos\left(2\pi(80)t - \frac{2\pi}{5}\right)$$
$$= 8\cos\left(2\pi(80)t + \frac{8\pi}{5}\right)$$

$A = 8, \omega_0 = 2\pi(80) = 160\pi, \varphi = \frac{-2\pi}{5}, \frac{8\pi}{5}$

Problem 4:



Variables / Set up

```
% Time set up
tStart = -0.015
tFinal = 0.02
tStep = 10e-5


tt_s = tStart: tStep: tFinal;


% parameters of the sinusoidal signal
A = 8                       % Amplitude
f_Hz = 80                   % Frequency
phi_rad = -2*pi/5           % phase shift
```

Computation

```
T = 1/ f_Hz
x = A*cos(2*pi*f_Hz*tt_s + phi_rad);
timeOfMax = ((-phi_rad * T) / (2*pi)) * 1e3
```

Plotting

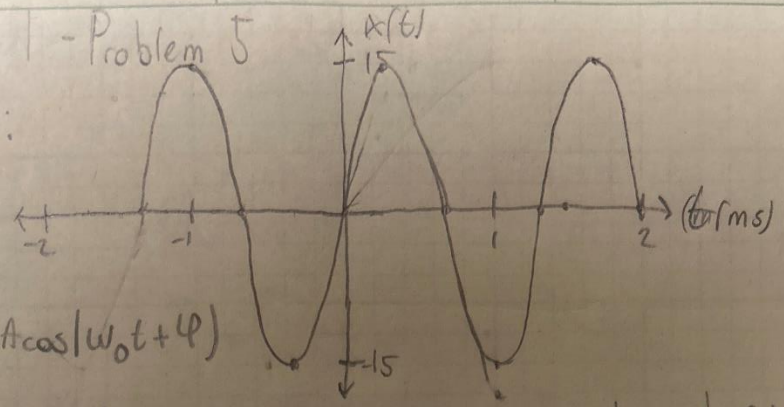
```
hold on ; grid on;
plot (tt_s * 1e3, x, 'LineWidth', 2);
plot (timeOfMax, A,'square', 'LineWidth', 2)


xlabel('t (ms)')
ylabel('x(t)')
title('8cos(2\pi(80)t - 0.2\pi)')
line([0,0], ylim, 'Color', 'k', 'LineWidth', 0.5);
line(xlim, [0,0], 'Color', 'k', 'LineWidth', 0.5);
```

Problem 5:

P.S. 1 - Problem 5

Given:



$x(t) = A\cos(\omega_0 t + \varphi)$

Find: amplitude(A), angular freq(ω_0), phase shift(φ)

Solution

- $A = \frac{|t_{max}| + |t_{min}|}{2} = \frac{|15| + |-15|}{2} = 15$

- $\omega_0 = 2\pi(750)$

between t=-1, a maxima, and t=1 a minima there are one and a half p... So..

$f_0 = \frac{1.5 \text{ periods}}{2 \text{ ms}} = \frac{1.5 \text{ periods}}{0.002 \text{ s}} = 750$

$\therefore \omega_0 = 2\pi(750)$

- $\varphi = -\frac{\pi}{2}$ or $\frac{3\pi}{2}$

since $[-1, 1]$ has 1.5 periods we know that $[-1, 0]$ (half the domain contains 0.75 periods we know the sinusoid is shift $\frac{3\pi}{2}$ or $\frac{\pi}{2}$. since the nearest maxima is to the right of the y-axis, it is a delay of $\frac{\pi}{2}$ or advancement of $\frac{3\pi}{2}$

$x(t) = 15\cos(2\pi(750) - \frac{\pi}{2})$ $A = 15$, $\omega_0 = 2\pi(750)$

or $x(t) = 15\cos(2\pi(750) + \frac{3\pi}{2})$ $\varphi = -\frac{\pi}{2}$ or $\frac{3\pi}{2}$

Problem 5:

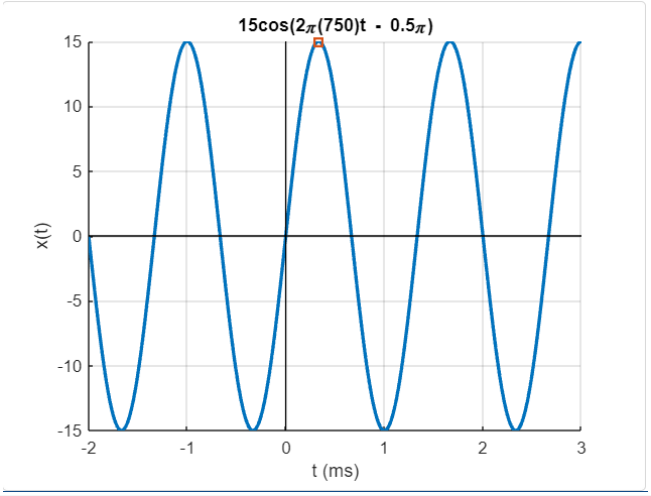
Variables / Set up

```
% Time set up
tStart = -2 * 1e-3
tFinal = 3 * 1e-3
tStep = 10e-6


tt_s = tStart: tStep: tFinal;


% parameters of the sinusoidal signal
A = 15                      % Amplitude
f_Hz = 750                  % Frequency

phi_rad = -pi/2             % phase shift
```



Computation

```
T = 1/ f_Hz
x = A*cos(2*pi*f_Hz*tt_s + phi_rad);
timeOfMax = ((-phi_rad * T) / (2*pi)) * 1e3
```

Plotting

```
hold on ; grid on;
plot (tt_s * 1e3, x, 'LineWidth', 2);
plot (timeOfMax, A,'square', 'LineWidth', 2)


xlabel('t (ms)')
ylabel('x(t)')
title('15cos(2\pi(750)t - 0.5\pi)')
line([0,0], ylim, 'Color', 'k', 'LineWidth', 0.5);
line(xlim, [0,0], 'Color', 'k', 'LineWidth', 0.5);
```