ECE180

PS2

Luis Antonio
Hernandez Aguirre

P.S. 2 - Problem 1 - P.A.1

Given

a) $z = 0+j2$ b) $z = -3-j4$ c) $z = (1,1)$ d) $z = (0,1)$

Find: the polar form of a-d

Solutions:

a) $r = \sqrt{x^2+y^2}$ | $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ | $\boxed{0+j2 = 2e^{j\frac{\pi}{2}}}$

$r = \sqrt{0+4}$ | $\tan^{-1}\left(\frac{2}{0}\right) = \frac{\pi}{2}$

$r = 2$

b) $r = \sqrt{(-3)^2+(-4)^2}$ | $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ | $\boxed{-3-j4 = 5e^{-j2.21}}$

$r = \sqrt{9+16}$ | $\theta = -2.21$

$r = \sqrt{25} = 5$

c) $r = \sqrt{1^2+1^2}$ | $\theta = \tan^{-1}(1)$ | $\boxed{(1,1) = \sqrt{2}e^{j\frac{\pi}{4}}}$

$r = \sqrt{2} \approx 1.4$ | $\theta = \frac{\pi}{4}$

d) $r = \sqrt{0^2+1^2}$ | $\theta = \tan^{-1}(0)$ | $1e^{j0} = 1e^{0} = 1 \cdot 1$

$r = \sqrt{1} = 1$ | $\theta = 0$ | $\boxed{(0,1) = 1}$

P.S 1 - Problem 2 - ~~P~~-A.2

Given:

a) $\sqrt{2}e^{j(\frac{3\pi}{4})}$ b) $3e^{-j(\frac{\pi}{2})}$ c) $1.6\angle(\pi/6)$ d) $7\angle 7\pi$

Find: the rectangular form of a-d

Solution:

a) $x+jy = r\cos(\theta) + jr\sin(\theta)$

$= \sqrt{2}\cos(\frac{3\pi}{4}) + j\sqrt{2}\sin(\frac{3\pi}{4})$

$= \sqrt{2} \cdot -\frac{\sqrt{2}}{2} + j\sqrt{2} \cdot \frac{1}{\sqrt{2}}$

$\boxed{\sqrt{2}e^{j\frac{3\pi}{4}} = -1 + j}$

b) $3\cos(-\frac{\pi}{2}) + j3\sin(-\frac{\pi}{2})$

$= 3 \cdot 0 + j \cdot 3 \cdot -1$

$\boxed{3e^{-j(\frac{\pi}{2})} = -3j}$

c) $1.6\cos(\frac{\pi}{6}) + j1.6\sin(\frac{\pi}{6})$

$= 1.6 \cdot \frac{\sqrt{3}}{2} + j \cdot 1.6 \cdot \frac{1}{2}$

$\boxed{1.6\angle(\frac{\pi}{6}) = 1.39 + j0.8}$

d) $7\pi = \pi$

$7\cos(\pi) + j7\sin(\pi)$

$7 \cdot -1 + j \cdot 7 \cdot 0$

$\boxed{7\angle 7\pi = -7}$

P.S 2 - Problem 3 - P.A.3 (a)(b)(c)

Given:

a) j^3 b) j^{2n} c) $e^{j(\pi+2\pi m)}$

Find: the rectangular form.

Solutions:

a) $j = \sqrt{-1} \therefore j^3 = (\sqrt{-1})^3 = \sqrt{-1}\cdot\sqrt{-1}\cdot\sqrt{-1} = (\sqrt{-1})^2 \cdot \sqrt{-1} = -1 \cdot \sqrt{-1}$

$= -\sqrt{-1} = -j$ $\boxed{j^3 = -j}$

b) $j^{2n} = (j^2)^n = (\sqrt{-1}\cdot\sqrt{-1})^n = \sqrt{1}^n = 1^n = 1$

$\boxed{j^{2n} = 1}$

c) $e^{j(\pi+2\pi m)} = e^{j\pi + j2\pi m} = e^{j\pi} \cdot e^{j2\pi m}$

$e^{j\pi}$

$= \cos(\pi) + j\sin(\pi)$

$= -1 + j\cdot 0)$

$= -1$

$e^{j2\pi m}$

$\cos(2\pi m) + j\sin(2\pi m)$

$-\cos(m) + j\sin(m)$

$= -\cos m + j\sin(m)$

$-1(-\cos(m) + j\sin(m))$

$= \cos m - j\sin m$

$$\boxed{e^{j(\pi+2\pi m} = -\cos(m) - j\sin(m)}$$

P.S. 1 - Problem 4 - P.A.6

Given: $z = e^{j\frac{9\pi}{8}} + e^{-j\frac{5\pi}{8}} + e^{j\frac{13\pi}{8}}$

Find: z in a simplied version

Solution:

$e^{j\frac{9\pi}{3}} = e^{j3\pi} = \cos(3\pi) + j\sin(3\pi) = -1 + j0 = -1$

$e^{-j\frac{5\pi}{8}} = \cos(-\frac{5\pi}{8}) + j\sin(-\frac{5\pi}{8}) = -0.38 + j0.92$

$e^{j\frac{13\pi}{8}} = \cos(\frac{13\pi}{8}) + j\sin(\frac{13\pi}{8}) = 0.38 - j0.92$

$-1 - 0.38 - j0.92 + 0.38 - j0.92 = -1 - j1.84$

$r = \sqrt{(-1)^2 + (1.84)^2} = 2.09$

$\theta = \tan\left(\frac{1.84}{1}\right) + \pi$

$\theta = 4.21$

$\boxed{2.09e^{j4.21}}$

P.S. 2 - Problem 5 - A-2.4

Given:

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$ $\quad \cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots$ $\quad \sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}$

Find: Proof of Euler's formula

Solution:

$$e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} \cdots$$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$$= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} \cdots$$

$$= 1 + i\underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\right)}_{\sin(x)} - \frac{x^2}{2!} + \frac{x^4}{4!}$$

$\cos(x)$

P.S. 2 - Problem 6 - P.2.8

Given: Provided Code

Find: x(t) and it's plot

Solution:

$tt = -0.15 : dt : 0.15$

From this we can tell that the time range is $[-0.15, 0.15]$

$zz = 15e^{j(2\pi f_0 t + 0.875)}$, $f_0 = 7$

$= 15e^{j\left(2\pi(7)t + \frac{49\pi}{14}\right)}$

$x(t) = 15\cos\left(2\pi(7)t + \frac{49\pi}{4}\right)$

amp = 15

$F_0 = 7$

$T_0 = \frac{1}{7} = 1$ period

$\varphi = \frac{49\pi}{4} = 48\pi \cdot \frac{1}{4}\pi$

maxima $= \frac{-\varphi T_0}{2\pi} = \frac{-49\pi \cdot 1}{4 \cdot 2\pi \cdot 7} = -\frac{49}{56} \pm \frac{k}{7}, k \in \mathbb{R}$

$t = 0.125, -0.01786$

minima $= \frac{-\varphi T_0}{2\pi} + \frac{T_0}{2} = -\frac{49\pi \cdot 1}{4 \cdot 2\pi \cdot 7} + \frac{1}{7 \cdot 2} = \frac{-49}{56} + \frac{4}{56} = -\frac{45}{56} \pm \frac{k}{7}, k \in \mathbb{R}$

x-intercepts $= \frac{-\varphi T_0}{2\pi} + \frac{T_0}{4}, \; -\frac{\varphi T_0}{2\pi} + \frac{3T_0}{4}$

$t = -\frac{49}{56} + \frac{1}{7 \cdot 4} = -\frac{47}{56} \pm \frac{k}{7}, k \in \mathbb{R}$

$t = -0.125, 0.01786$

$t = -\frac{49}{56} + \frac{3}{7 \cdot 4} = -\frac{43}{56} \pm \frac{k}{7}, k \in \mathbb{R}$

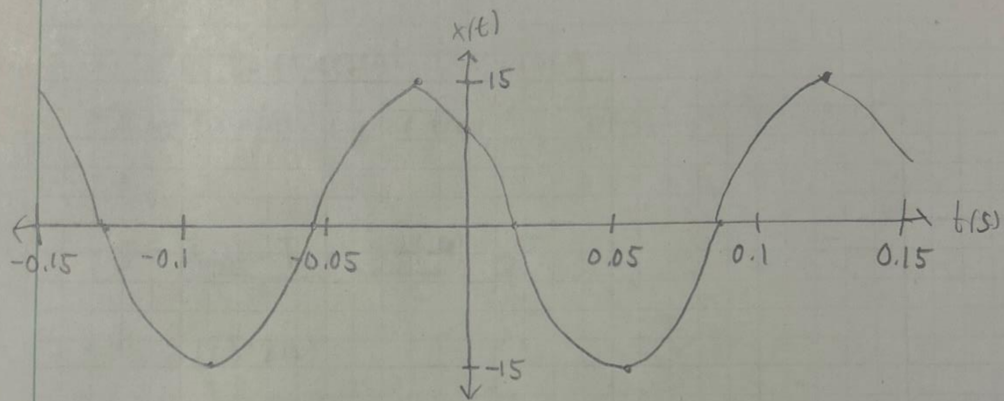
$t = -0.05357, 0.08929$

P.S.2 - Problem 6(cont)

Solution:

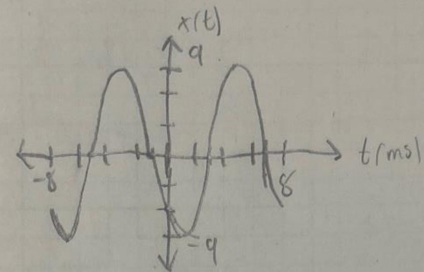
y-intercept

$x(0) = 15\cos\left(\frac{49\pi}{4}\right) = 10.61$



P.S. 2 - Problem 7 - P-2.9

Given:



Find $x(t)$ to represent the graph

-Amplitude (A), angular freq (ω_0), phase shift (φ)

Solution:

$-A = \frac{T_{max} - T_{min}}{2} = \frac{9-(-9)}{2} = 9$

$-\omega_0 = 125(2\pi)$

1 phase between $T = -3$ms, 5ms

$f_0 = \frac{1 \text{ period}}{8\text{ms}} = \frac{1 \text{ period}}{0.008 \text{ sec}} = 125$

$\omega_0 = 2\pi f_0 = 2\pi(125)$

$-\varphi = \frac{3\pi}{4}$

There is $\frac{1}{2}$ period between $t=-3$ms and $t=1$ms

so the phase was shifted $\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{2\pi}{1} = \frac{3\pi}{4}$ to the left. So it is $+\frac{3\pi}{4}$

$$\boxed{\begin{gathered} x(t) = 9\cos(2\pi(125) + \tfrac{3\pi}{4}) \\ A = 9,\ \omega_0 = 2\pi(125),\ \varphi = \tfrac{3\pi}{4} \end{gathered}}$$

P.S 2 Problem 8

Given: ae^{jb} & $a+jb$

Find: Proof $ae^{jb} \neq a+jb$

Solution:

$$ae^{jb} = a\cos(b) + ja\sin(b)$$

$$a+jb = \sqrt{a^2+b^2}\, e^{j\tan^{-1}\left(\frac{b}{a}\right)}$$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$\boxed{\begin{gathered}\sqrt{a^2+b^2}\, e^{j\tan^{-1}\left(\frac{b}{a}\right)} \neq ae^{jb} \\ a\cos(b) + ja\sin(b) \neq a+jb\end{gathered}}$$