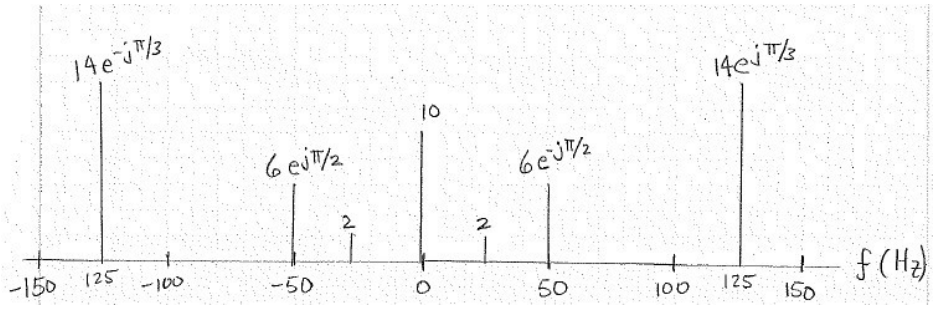
Coverage: Line Spectra

Instructions

- Complete each of the problems by hand (with calculator assistance if necessary.)

Problems

(1) Consider the line spectrum plotted below for the signal $x(t)$. Write $x(t)$ as a sum of cosine functions.



(2) Given the signal

$$x(t) = 12\cos(1000\pi t - \pi/4) + 6\cos(1500\pi t + \pi/3) - 3\cos(500\pi t) + 8$$

Sketch the line spectrum, and write the complex amplitude and phase associated with each line on the plot. Use Hz for the x-axis.

(3) Do an internet search to determine the DTMF frequencies for the digits 3, 5, and 0. Draw the line spectrum of the signal generated when you press each of these digits on your phone. (Draw one spectrum plot for each digit, three (3) plots total.)

(4) Amplitude modulation (AM, the same as AM radio) modulates a message signal $m(t)$, i.e. voice, by multiplying it by a relatively high frequency sinusoid:

$$x(t) = m(t)\cos(\omega_0 t).$$

(a) Let $m(t) = 10 + 8\sin(\pi t - \pi/3)$ with $\omega_0 = 13\pi$ rad/s. Express $x(t)$ in the form

$$x(t) = A_1\cos(\omega_1 t + \varphi_1) + A_2\cos(\omega_2 t + \varphi_2) + A_3\cos(\omega_3 t + \varphi_3)$$

with $\omega_1 < \omega_2 < \omega_3$, and find the A_n, ω_n, and φ_n.

(b) Draw the line spectrum of $x(t)$ with the frequency axis in rad/s.