ECE180

PS2

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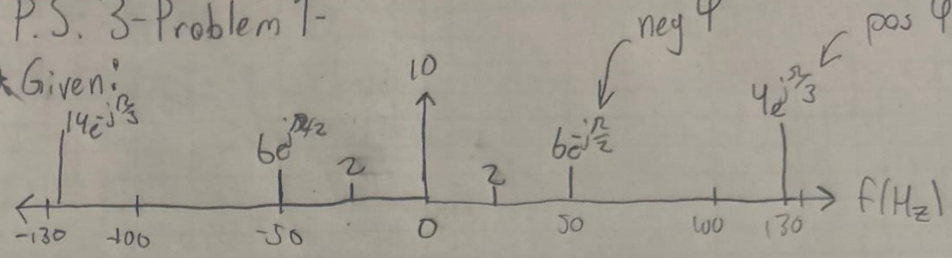
P.S. 3- Problem 1-

* Given:



* Find: signal x(t) as a sum of cosine functions

* Solution:

$14e^{j\frac{\pi}{3}} \Rightarrow A = 28$ @ 50Hz | $6e^{-j} = A = 12$ @ -50Hz

$$\boxed{x(t) = 12\cos(2\pi(50) - \tfrac{\pi}{2}) + 28\cos(2\pi(130) + \tfrac{\pi}{3}) + 4\cos(2\pi(25)}$$

2 @ -25 + 25 = A = 4 @ 25 Hz

$4 = 4e^{j0} = 4\cos(\omega_0 + 0)$

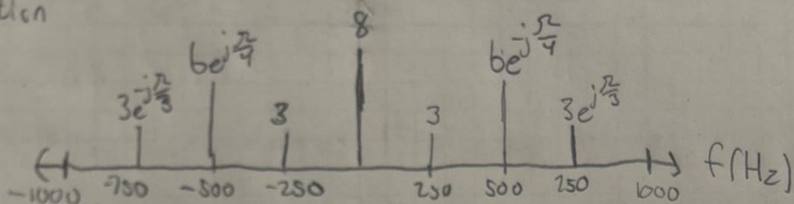
$\omega_0 = 25(2\pi)$

P.S 3-Problem 2

Given: $x(t) = 12\cos(1000\pi t - \pi/4) + 6\cos(1500\pi t + \pi/3)$
$-3\cos(500\pi t) + 8$

Find: The line spectrum for $x(t)$

Solution



$+8 \rightarrow DC = 8$

$+12\cos(1000\pi t - \frac{\pi}{4}) = 12\cos(2\pi(500)t - \frac{\pi}{4}) = \frac{12}{2}e^{-j\frac{\pi}{4}}$ at 500 Hz

$+\frac{12}{2}e^{j\frac{\pi}{4}}$ at -500 Hz

$+6\cos(1500\pi t + \pi/3) = 6\cos(2\pi(750)t + \frac{\pi}{3}) = \frac{6}{2}e^{j\frac{\pi}{3}}$ at 750 Hz

$\frac{6}{2}e^{-j\frac{\pi}{3}}$ at -750 Hz

$+3\cos(500\pi t) = 3\cos(250(2\pi)t) = \frac{3}{2}e^{j0} = 3e^{0} = 3$ at 250 + -250

P.S. 3-Problem 3

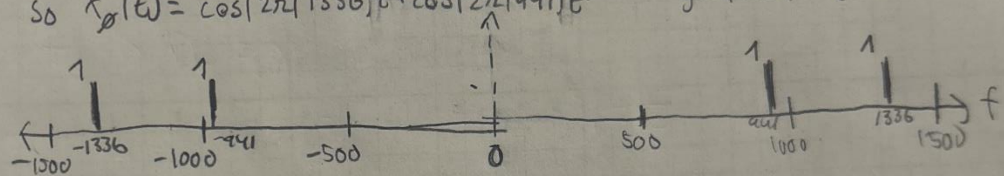
Given: The digits 0,3, & 5 on a phone,

Find: The DTMF freq. for 0,3, & 5 and each of their line spectrums.

Solution

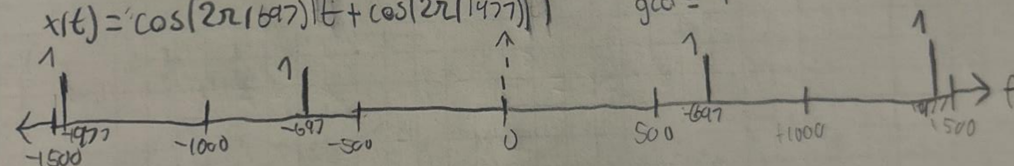
#Ø is a combination of 1336Hz and 941Hz

So $x_{\emptyset}(t) = \cos(2\pi(1336)t + \cos(2\pi(941)t$ $\gcd(1336, 941) = 1$



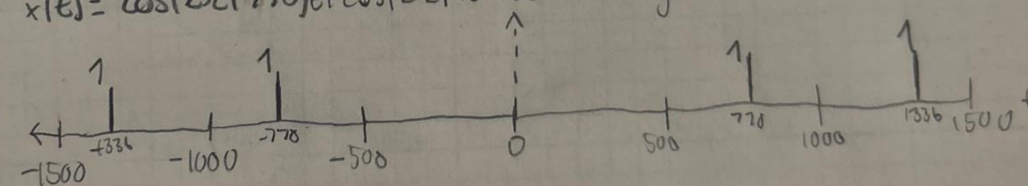
#3 is a combination of 697Hz and 1477Hz

$x(t) = \cos(2\pi(697))t + \cos(2\pi(1477))$ gcd = 1



#5 is a combination of 1336Hz and 770Hz

$x(t) = \cos(2\pi(770)t + \cos(2\pi(1336)t$ gcd = 2



P.S. 3- Problem 4

Given: $x(t) = m(t)\cos(\omega_0 t)$

$m(t) = 10 + 8\sin(\pi t - \pi/3)$, w/ $\omega_0 = 13\pi$ rad/s

Find: $x(t)$ in the form

$x(t) = A_1\cos(\omega_1 t + \varphi_1) + A_2\cos(\omega_2 t + \varphi_2) + A_3\cos(\omega_3 t + \varphi_3)$

with $\omega_1 < \omega_2 < \omega_3$ find A_n, ω_n and φ_n

Draw the line spectrum of $x(t)$

Solution:

$$x(t) = \left(10 + 8\sin(\pi t - \tfrac{\pi}{3})\right)\cos(13\pi t)$$

$$= 10\cos(2\pi(6.5))t + \underbrace{8\sin(\pi t - \tfrac{\pi}{3})\cos(2\pi(6.5)t)}$$

$$8\sin(2\pi(\tfrac{1}{2})t - \tfrac{\pi}{3})\cos(2\pi(7.5)t)$$

$$= 8\cdot\left(\frac{e^{j(\pi t - \frac{\pi}{3})} - e^{-j(\pi t - \frac{\pi}{3})}}{2j}\right)\left(\frac{e^{j(13\pi t)} + e^{-j(13\pi t)}}{2}\right)$$

$$= 2e^{j14\pi t}e^{-j\frac{\pi}{3}} + 2e^{-j12\pi t}e^{-j\frac{\pi}{3}} + 2e^{j12\pi t}e^{j\frac{\pi}{3}} + 2e^{-j14\pi t}e^{j\frac{\pi}{3}}$$

$$= 4\cos(2\pi(7) - \tfrac{\pi}{3}) + 4\cos(2\pi(6) - \tfrac{\pi}{3})$$

$$x(t) = 10\cos(2\pi(6.5)t) + 4\cos(2\pi(7) - \tfrac{\pi}{3}) + 4\cos(2\pi(6) - \tfrac{\pi}{3})$$