



$$\begin{aligned}
 1. \quad m(ax) &= \frac{1}{N} \sum_{i=1}^N ax_i \\
 &= \frac{1}{N} \left( \sum_{i=1}^N a + b \sum_{i=1}^N x_i \right) \\
 &= \frac{1}{N} \left( Na + b \sum_{i=1}^N x_i \right) = a + b \left( \frac{1}{N} \sum_{i=1}^N x_i \right) \\
 &= a + bm(x)
 \end{aligned}$$

$$\begin{aligned}
 2) \quad \text{cov}(x, x) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(x_i - m(x)) \\
 &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))^2 \\
 &= s^2
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \text{cov}(x, a+bx) &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))(a + bx_i - a - bm(x)) \\
 &= \frac{1}{N} \sum_{i=1}^N (x_i - m(x))b(x_i - m(x)) \\
 &= \frac{b}{N} \sum_{i=1}^N (x_i - m(x))(x_i - m(x)) \\
 &= b(\text{cov}(x, x))
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \text{cov}(a+bx, a+by) &= \frac{1}{N} \sum_{i=1}^N ((a+bx_i) - (a+bm(x)))((a+by_i) - (a+bm(y))) \\
 &= \frac{1}{N} \sum_{i=1}^N (b(x_i - m(x)))(b(y_i - m(y))) \\
 &= \frac{b^2}{N} \sum_{i=1}^N (x_i - m(x))(y_i - m(y)) \\
 &= b^2 \text{cov}(x, y)
 \end{aligned}$$

5) if  $b > 0$ , the order of the values stays the same, so  $\text{med}(a+bx) = a + b(\text{med}(x))$   
 Scaling/shifting doesn't affect median.  
 but  $\text{IQR}(a+bx) \neq a + b(\text{IQR}(x))$  because scaling by  $b$  will stretch the IQR of  $a+bx$ .

b)  $x = [0, 2]$   $m(x) = 1$   $m(x^2) = 2$   
 $m(x)^2 = 1$  not equal

$m(\sqrt{x}) = 2$   $\sqrt{m(x)} = \sqrt{1}$   $2 \neq \sqrt{1}$   
 $x = [1, 4]$   $m(x) = 5$