



GRP 17: Euchred?

Devan Pearson - 22KGX1

Liam Harper-McCabbe - 22VPYX

Christian Nainer - 21CSN2

Charlie Guarasci - 22DSN2

Course Modelling Project

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Abstract

This modeling project aims to model whether or not a euchre can occur in the popular card game Euchre. Based on the outcome of the first three tricks in a hand of euchre, the trump suit, and the team that called trump, our model will analyze game play of tricks 4 and 5 to see if a Euchre takes place.

Propositions

- Called Trump Q_i : This is true when team X has called trump. Subscript indicates which player called trump

$$- Q_1 \vee Q_2$$

- Trick $T_{i,j}$: This is true when player i is on team x and has won the trick j.

$$- C_{1,v,n} \vee C_{3,v,n}$$

- Win W_t : This is true when team X has won ≥ 3 tricks t.

$$- W_3 \vee W_4 \vee W_5$$

- Euchre E : This is true when $W \neq Q$

$$- W_t \wedge \neg Q_i$$

- Player P_i : This is true when player i is on team X and holds the winning card for trick T

$$- P_1 \vee P_3$$

- Card $C_{t,v,n}$: This is true when a card of type t(trump/off suit led/off suit)(1-Trump, 2-off suit led, 3-off suit) and rank(1-6 for off suit 9 through Ace and 1-7 for Trump 9 through right Bauer) v is winning a trick n(4 or 5).

$$- \text{Ex. } C_{1,7,4} \vee C_{3,2,4} \vee C_{1,2,4} \vee C_{1,4,4}$$

- Lead L_i : This is true when team X won trick 3 or 4, i corresponds to the player, j corresponds to the trick

$$- L_{1,3} \vee L_{3,3}$$

- Suit $S_{i,j}$: This is true when a card played is off suit, i corresponds to whether or not it was lead(1 = lead, 2 = not led), j corresponds to whether or not it's the same suit that was led(1 = same, 2 = not).

$$- S_{1,1} \vee S_{2,1} \vee S_{2,2}$$

- Trump B : This is true when a card played is of trump.

1: P, Q, R	premises
2: S	assumption
3: $((P \wedge \neg Q) \vee (\neg P \wedge R)) \wedge ((P \wedge \neg Q) \vee (\neg P \wedge R))$	assumption
4: S	hyp 2
5: $((P \wedge \neg Q) \vee (\neg P \wedge R)) \wedge ((P \wedge \neg Q) \vee (\neg P \wedge R)) \rightarrow S$	\rightarrow intro 3-4
6: $S \rightarrow ((P \wedge \neg Q) \vee (\neg P \wedge R)) \wedge ((P \wedge \neg Q) \vee (\neg P \wedge R)) \rightarrow S$	\rightarrow intro 2-5

Figure 1: A euchre can only take place if the team that did not call trump wins the round.

1: $(P \vee R) \rightarrow T, (Q \vee S) \rightarrow T$	premises
2: $(P \vee R) \vee (Q \vee S)$	assumption
3: $P \vee R$	assumption
4: T	\rightarrow elim 1.1,3
5: $Q \vee S$	assumption
6: T	\rightarrow elim 1.2,5
7: T	\vee elim 2,3-4,5-6
8: $(P \vee R) \vee (Q \vee S) \rightarrow T$	\rightarrow intro 2-7

Figure 2: A team can only win a trick if their player holds the winning card.

Constraints

- A euchre can only take place if a team wins a round and also did not call trump:

$$- (W \wedge \neg Q) \vee (\neg W \wedge Q)$$

- A team can only win a round if they win ≥ 3 tricks T_i :

$$- (t_1 \wedge t_2 \wedge t_3) \vee (t_1 \wedge t_2 \wedge t_4) \vee (t_1 \wedge t_2 \wedge t_3 \wedge t_4 \wedge t_5) \vee \dots etc$$

- A team can only win a trick if their player holds the winning card:

$$- (P_1 \vee P_3) \vee (P_2 \vee P_4)$$

- A card is only winning if it is the possible highest trump or the Highest suited card if no trump is played

$$- C_{1,v} \vee (\neg C_{1,v} \vee C_{2,v})$$

- Only a player who leads can determine the Suit:

$$- L_{i,j} \text{ Player } i \text{ leads trick } j$$

1: $P(x) \rightarrow ((T(x) \wedge Q(x)) \vee (T(x) \wedge \neg S(x)))$, $P(x)$	premises
2: $(T(x) \wedge Q(x)) \vee (T(x) \wedge \neg S(x))$	\rightarrow elim 1.1,1.2
3: $T(x) \wedge Q(x)$	assumption
4: $T(x)$	\wedge elim 3
5: $T(x) \wedge \neg S(x)$	assumption
6: $T(x)$	\wedge elim 5
7: $T(x)$	\vee elim 2,3-4,5-6
8: $T(x) \wedge P(x)$	\wedge intro 7,1.2

Figure 3: A card played is valid only if the card follows suit or a players hand does not contain an on suit card.

Model Exploration: Explanation

Our initial interpretation for modeling the game of euchre was to examine the state of the game after a hand had been played, with conditions for who called trump, played each card to determine who had won a hand that already happened. However, we pivoted away from this concept to make it more challenging; we looked at the results of the first 3 tricks, then determined whether or not a euchre can occur based on the information of the hand. Once we had all our propositions and constraints in our Python file, we were able to write an example theory to explore random hands and their results. Our example theory included functions for dealing out hands and randomizing the results of the first 3 tricks to provide a new set of initial conditions every run, testing our constraints against each set.

Model Exploration: Results

Now with working test code, we could successfully run multiple test cases to verify our models ability to determine whether a euchre has occurred in a variety of cases.

Case 1 - No Euchre Occurs

In this case, the conditions were:

TeamX Hands:

- Player 1: Nine of Diamonds, Jack of Hearts
- Player 3: Ten of Spades, Queen of Spades

TeamY Hands:

- Player 2: Ten of Hearts, Ten of Diamonds
- Player 4: Nine of Clubs, Ace of Diamonds

Initial State:

- TeamX had won 3 tricks
- TeamX called trump

- Trump Suit: Spades
- Player 3 led Trick 4

The play proceeded with:

1. Player 3 leading the Ten of Spades,
2. Player 4 playing Nine of Clubs,
3. Player 1 playing Nine of Diamonds
4. Player 2 playing Ten of Hearts.
5. Player 3 won this trick with the Ten of Spades (trump). In Trick 5, Player 3 led with Queen of Spades and ultimately won that trick as well.

```
Team Hands:  
Player1: Nine of Diamonds, Jack of Hearts  
Player3: Ten of Spades, Queen of Spades  
  
Teamy Hands:  
Player2: Ten of Hearts, Ten of Diamonds  
Player4: Nine of Clubs, Ace of Diamonds  
  
Initial Tricks:  
TeamX has won 3 tricks.  
TeamY has won 8 tricks.  
  
Called: TeamX called trump  
Trump Suit: Spades  
Trick 4 Lead: Player3  
  
Play:  
  
Player3 plays Ten of Spades as the lead card.  
Player4 plays Nine of Clubs  
Player1 plays Nine of Diamonds.  
Player2 plays Ten of Hearts.  
  
Trick 4 Winner: Player3 from TeamX with Ten of Spades.  
  
Trick 5 Lead: Player3  
  
Player3 plays Queen of Spades.  
Player4 plays Ace of Diamonds.  
Player1 plays Jack of Hearts.  
Player2 plays Ten of Diamonds.  
  
Trick 5 Winner: Player3 from TeamX with Queen of Spades.  
Win instance created with trick7: True, Trump, True, 4, 11  
teamX won 5 tricks; W is True.  
win = C<<-baubaus.core.CustomNF object at 0x181657db>, (-W & C)<:-baubaus.core.CustomNF object at 0x181335f8be>  
Euchre proposition! ->baubaus.core.CustomNF object at 0x181a2f15de)  
  
Final Trick results:  
TeamX has won 5 tricks.  
TeamY has won 9 tricks.  
  
M (TeamX won 3+ tricks): Prop(M)  
C (TeamX called trump): Prop(C)  
A Euchre did not take place.  
  
compiled successfully
```

Figure 4: Euchre did not occur

As shown in the output screenshot above, we can see that the model successfully determined that a euchre would not occur because the team that called trump won that hand.

Case 2 - Euchre Occurs

TeamX Hands:

- Player 1: King of Hearts, Ace of Spades
- Player 3: Nine of Clubs, Nine of Diamonds

TeamY Hands:

- Player 2: Nine of Hearts, Nine of Spades
- Player 4: Ten of Hearts, Jack of Spades

Initial State:

- TeamX had won 1 trick
- TeamY had won 2 tricks
- TeamY called trump
- Trump Suit: Spades
- Player 1 led Trick 4

The play sequence showed Player 1 leading with King of Hearts, followed by matching suit plays. Player 1 won Trick 4 with the King of Hearts. However, in Trick 5, despite Player 1 leading with Ace of Spades, Player 4 won with the Jack of Spades (trump).

```

TeamX Hands:
  Player1: King of Hearts, Ace of Spades
  Player3: Nine of Clubs, Nine of Diamonds
TeamY Hands:
  Player2: Nine of Hearts, Nine of Spades
  Player4: Ten of Hearts, Jack of Spades
Initial Tricks:
  TeamX has won 1 tricks.
  TeamY has won 2 tricks.
Called: TeamY called trump
Trump Suit: Spades
Trick 4 Lead: Player1
Play:
  Player1 plays King of Hearts as the lead card.
  Player2 plays Nine of Hearts.
  Player3 plays Nine of Clubs.
  Player4 plays Ten of Hearts.
Trick 4 Winner: Player1 from TeamX with King of Hearts.
Trick 4 Lead: Player1
  Player1 plays Ace of Spades.
  Player2 plays Nine of Spades.
  Player3 plays Nine of Diamonds.
  Player4 plays Jack of Spades.
Trick 5 Winner: Player4 from TeamY with Jack of Spades.
Win instance created with tricks: [False, False, True, 1, 0]
TeamX won 2 tricks. Win: False
Euchre condition: (W & ~C): <bauhaus.core.CustomWF object at 0x104cc8896>, (~W & C): <bauhaus.core.CustomWF object at 0x1043f570b>
Euchre proposition: <bauhaus.core.CustomWF object at 0x104c9f156>
Final trick results:
  TeamX has won 2 tricks.
  TeamY has won 3 tricks.
W (TeamX won 3e tricks): <bauhaus.core.CustomWF object at 0x104cc5f30b>
C (TeamX called trump): <bauhaus.core.CustomWF object at 0x104cf0596>
A Euchre took place!
compiled successfully
Example solution: (Prop(C): True, Prop(Euchre): True, Prop(W3): True, Prop(RightBower(teamX, player3, card2)): True, Prop(LeftBower(teamY, player2, card1)): True, Prop(LeftBower(teamX, player3, card1)): True, Prop(RightBower(teamX, player3, card1)): True, Prop(RightBower(teamY, player2, card1)): True, Prop(LeftBower(teamY, player4, card1)): True, Prop(RightBower(teamY, player4, card1)): True, Prop(LeftBower(teamY, player4, card2)): True, Prop(RightBower(teamY, player2, card2)): True, Prop(LeftBower(teamY, player2, card2)): True, Prop(RightBower(teamY, player4, card2)): True, Prop(LeftBower(teamX, player1, card1)): True, Prop(LeftBower(teamX, player1, card2)): True, Prop(LeftBower(teamX, player3, card2)): True, Prop(RightBower(teamX, player1, card2)): True)

```

Figure 5: Euchre occurred

As shown in the screenshot above, this test case demonstrates our model's ability to correctly identify a euchre situation where the team calling trump fails to win three tricks.

Conclusion:

Based on this, our model can successfully track and validate card plays according to euchre rules, correctly implement trump suit hierarchy, properly count tricks won by each team, and putting it all together, accurately determine when a euchre occurs based on the relationship between who called trump and who won the majority of tricks.

First-Order Extension

Predicate logic could allow for a more expressive analysis of the state of the game. By using quantifiers, we could potentially model situations that hold

for any trick. For example if **there exists** a player on the non trump calling team who has won 3 tricks, a euchre has already occurred. Or if **there exists** a player on the non trump calling team who has won two tricks, a euchre can occur. Below are some predicates we could use.

$C(a, b, x, y)$ as an effective way to represent cards in a player's hand, where a and b are card 1 and card 2 rank, while x and y are the suit.

Or where a trick is won by Team X $W(t)$ with t being the trick number. As in:

A = 1,2,3,4,5

W = 1,3,4

That is, team X won tricks 1, 3, and 4.