Determination of Attenuation Coefficients

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I. Introduction

Purpose

The purpose of this experimentation is to (i) determine the attenuation coefficients for lead and aluminium for the gamma ray sources of Cobalt-60 (Co-60) and Cesium-137 (Cs-137) and (ii) to compare the measured values of the attenuation coefficients with the the theoretical values.

Radiation and Radioactive Decay

There are three primary forms of natural radioactive decay: alpha decay (α), beta decay (β), and gamma decay (γ). Alpha decay occurs when a radioisotope emits a particle that is equivalent to a doubly ionized Helium atom, i.e., an atom with two protons, two neutrons, and no electrons. Beta decay is normally associated with the loss of an electron and an increase in atomic number; however, although this is the end result, this raises the question of how the loss of an electron affects the number of protons. What actually occurs to produce the ejected electron is the decay of a neutron into a proton, an electron, and an antineutrino[1]. The last of the primary types of decay is gamma decay. Gamma decay is unique compared to alpha and beta decay in that the atomic mass and the atomic number are conserved, at the expense of an ejected high energy photon. Gamma decay typically occurs after alpha and beta decay, since both decays leave the nucleus in a high energy state. Thus, gamma decay occurs to bring the nucleus to a lower energy state.

How Radiation Interacts with Matter

Due to physical differences, alpha, beta, and gamma particles interact with matter differently. Alpha particles don't have very strong penetrating power because of the size of the nucleus. Thus, alpha particles can be stopped by one or two sheets of paper, as a visualization. This results from the interatomic distance between cellulose molecules being smaller than the atomic radius of the helium nucleus. Beta particles have much more penetrating power because beta particles are electrons, which are orders of magnitude smaller than helium nuclei. As a consequence, beta radiation can be stopped by materials with similar interatomic distances to that of aluminum. Gamma rays are the most energetic of the basic types of radioactive decay. The penetration power of gamma radiation is the greatest of the three mentioned. Gamma rays are high energy photons, which is why they have the greatest penetration power of the three. Einstein's Theory of General Relativity provides information that photons are massless, yet contain momentum and energy. So, because gamma rays are high energy photons, lead is required to stop them. Figure 1 provides a summary of the varying degrees of interaction between the radioactive decay processes and matter.

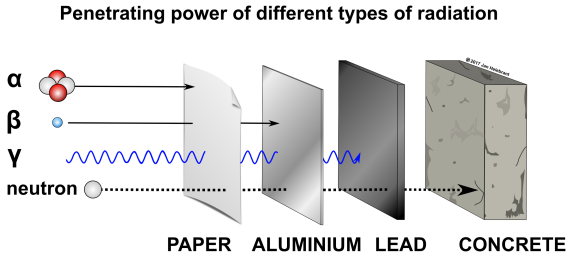

FIG. 1. The varying degrees of interaction of the different types of radioactive decay.

Radiation and Safety

Radiation and radioactive decay are commonly perceived as dangerous phenomena, which is a good perception to have; over-exposure to any type of radiation will result in adverse effects to the human

[1] Berkeley Labs. (2009, February). *Neutron beta decays.* http://www.particleadventure.org/npe.html.

body. Among the three types of radiation described, beta and gamma radiation are the most dangerous. Alpha particles can be stopped by human skin, but beta and gamma particles can penetrate skin and damage cells. Thus, safety protocols must be taken whenever working with radioactive materials.

There are three important considerations concerning radiation protocols: time, distance, and barriers. Time spent around radioactive materials must be minimized to reduce exposure to such materials. Further, because radioactive decay processes follow the inverse-square law in terms of distance (i.e., for a detector that is one meter away and a detector that is two meters away from a radioactive source, the detector that is two meters away detects four times less radiation on average than the detector that is one meter away), it is best to maintain a large distance away from a radioactive source. Lastly, should one spend long amounts of time around radioactive materials, then the proper protection must be used in order to minimize exposure. For alpha, beta, and gamma radiation, lead is always the safest barrier to use. Although it was not mentioned, neutron radiation has greater penetration power than gamma rays; however, it can be stopped by water. In the absence of water, though, concrete is used to stop neutrons.

How Gamma Rays Interact with Matter

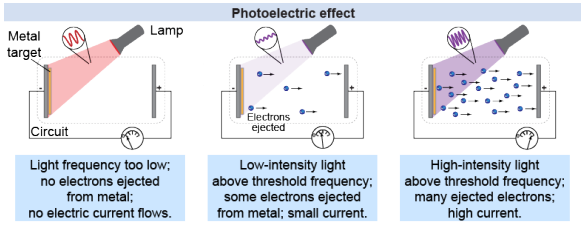

FIG. 2. The mystery surrounding the photoelectric effect was first solved by Einstein in 1905.

More specifically, gamma rays interact with matter via three phenomena: the photoelectric effect, the Compton effect, and pair production. The photoelectric effect is an effect when a gamma ray behaves as a particle and completely transfers its energy to an electron on a surface with free electrons (e.g., a metal). As a result, the electron that absorbed the photon's energy is ejected from the surface of the metal, where the total energy is conserved such that the energy imparted by the photon is equal to the energy required to eject the electron from the surface plus the kinetic energy of the electron. Figure 2 shows other consequences of the photoelectric effect. The Compton effect is another effect in which photons behave as particles, imparting only part of its energy to an electron compared to all of its energy for the photoelectric effect. When a gamma ray scatters an electron via Compton scattering, the photon imparts some of its energy to the electron and as a result, the electron contains kinetic energy and the scattered photon contains less energy than before it struck the electron, resulting in a shorter wavelength than before, as depicted in Figure 3. Pair production is the process in which the gamma ray is converted into an electron and positron pair, which is generally referred to as a particle-antiparticle pair. During this process, the energy of the gamma ray is completely converted into the formation of the particles, where the excess energy is given to the par-

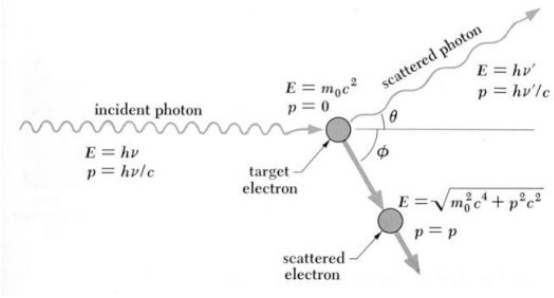

FIG. 3. The phenomenon of Compton Scattering.

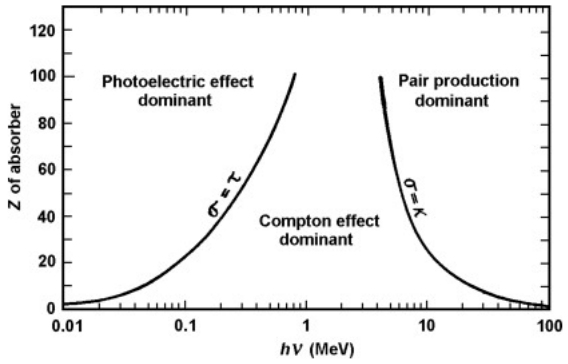

FIG. 4. The y-axis represents the atomic number of the gamma-ray absorber and the x-axis represents the energy of the gamma ray.

ticles as kinetic energy. When gamma rays interact with matter, the factors that determine which phenomenon will predominate are the identity of the gamma-ray absorber (the atomic number) and the energy of the incoming gamma ray, as depicted in Figure 4. Because the attenuators under investigation are lead and aluminum, and the gamma ray sources are Co-60 and Cs-137, then according to Figure 4, the effects that should primarily be observed fall in the Compton Scattering region. The only exception is that for lead and Cs-137, where both photoelectric and Compton effects will dominate.

In order to determine the ability of a material to stop radiation, the attenuation coefficient for the specific material and for the specific gamma ray source must be known and the Geiger Muller (GM) counter must be further away from the source. The attenuation coefficient provides information on how the intensity of gamma rays change as they pass through a given material. The general formula for calculating the attenuation coefficient for an attenuator (absorber) and a gamma ray source is

$$\mu = \rho\sigma Z, \tag{1}$$

where μ is the attenuation coefficient in inverse centimeters, ρ is the density of the attenuator in atoms per cubic centimeter, σ is the cross-section of the gamma ray source in square centimeters, and Z is the atomic number of the attenuator, which is unitless. The cross-section of a gamma ray source is a quantity that provides information on the probability of the gamma ray interacting with matter and the atomic number is the identifying number of an element, due to the number of protons present in the nucleus. Together, these quantities define the attenuation coefficient; the attenuation coefficient provides a direct means of understanding how well an attenuator stops radiation. The greater the value of μ for a given attenuator and gamma ray source, the better the material is at stopping and reducing the amount of radiation.

The reason why the GM counter must be farther away from the source is related to the amount of scattered gamma rays. The presence of scattered gamma rays would provide incorrect data because the effect of a scattered gamma ray that is detected by a GM counter is the same as that of an unscattered gamma ray. So, the detection of scattered gamma rays must be minimized so that true unscattered rays can be detected by the GM counter. Further, to measure μ in terms of inverse centimeters, the following relation must be used:

$$I = I_0 e^{-\mu x}, \tag{2}$$

where I is the intensity, I_0 is the initial intensity, and x is the thickness of the attenuator in centimeters. When the natural logarithm of equation (2) is taken, then the intensities are unitless and so, the attenuation coefficient must have units of inverse centimeters to maintain unitless dimensions.

$$\ln(I) = \ln(I_0) - \mu x. \tag{3}$$

II. Data Acquisition

Materials

FIG. 5. The experimental set up for the determination of attenuation coefficients.

The materials used in this experimentation, from left to right according to Figure 5, are the attenuators of different thicknesses (the box), the USB connector, the ST360 Counter, radioisotopes (Co-60 and Cs-137), the GM 35 Probe, the GMS 35 Stand, the source tray, the BNC cable, and a laptop with the appropriate Spectrum Techniques software not present in the figure.

Procedure and Methods

In order to determine the attenuation coefficients for Co-60, the GM counter was run at its optimum voltage for twenty runs, each at 60 seconds per run. The source was placed on the slot furthest from the top of the stand, with the label down, and the attenuators were place in the slot directly above the source. This process was repeated 10 times for the aluminum attenuators (thicknesses in inches

of 0.125, 0.100, 0.090, 0.080, 0.063, 0.050, 0.040, 0.032, 0.025, and 0.020), and four times for the lead attenuators (thicknesses in inches of 0.250, 0.125, 0.064, and 0.032). This process was repeated for the Cs-137 source.

III. Analysis

In order to analyze the data, equation (1) was used to calculate the theoretical values of the attenuation coefficients for lead and aluminum. Because two gamma ray sources were used, there were a total of four theoretical values for the attenuation coefficients: two for the Co-60 source and two for the Cs-137 source. The theoretical values for the attenuation coefficient for the Co-60 source are 0.5866 cm^{-1} for lead and 0.1716 cm^{-1} for aluminum. For the Cs-137 source, the theoretical values are 0.7457 cm^{-1} for lead and 0.2163 cm^{-1} for aluminum. After calculating the theoretical values, the data was analyzed by first obtaining the weighted mean of the count data for each of the attenuators because there were 20 runs for each attenuator. Then, the data was linearized and Monte Carlo fitting, using 10,000 randomly distributed points, was used to determine the attenuation coefficient. Because the Monte Carlo fits returned values for the slope, and followed a normal distribution, the arithmetic mean of the slopes was taken in order to determine the attenuation coefficient. The uncertainty, though, for the attenuation coefficient was obtained by applying proper error propagation to equation (3), such that

$$\mu = \frac{\ln(I_0) - \ln(I)}{x}$$

$$\delta\mu = \sqrt{\left(\frac{\partial\mu}{\partial x}\delta x\right)^2 + \left(\frac{\partial\mu}{\partial I}\delta I\right)^2}$$

$$= \left| -\frac{\delta I}{I} \right|,$$

where the uncertainty in the thickness δx is zero because exact values were provided. The resulting plots for the Monte Carlo fits are provided in Figure 6.

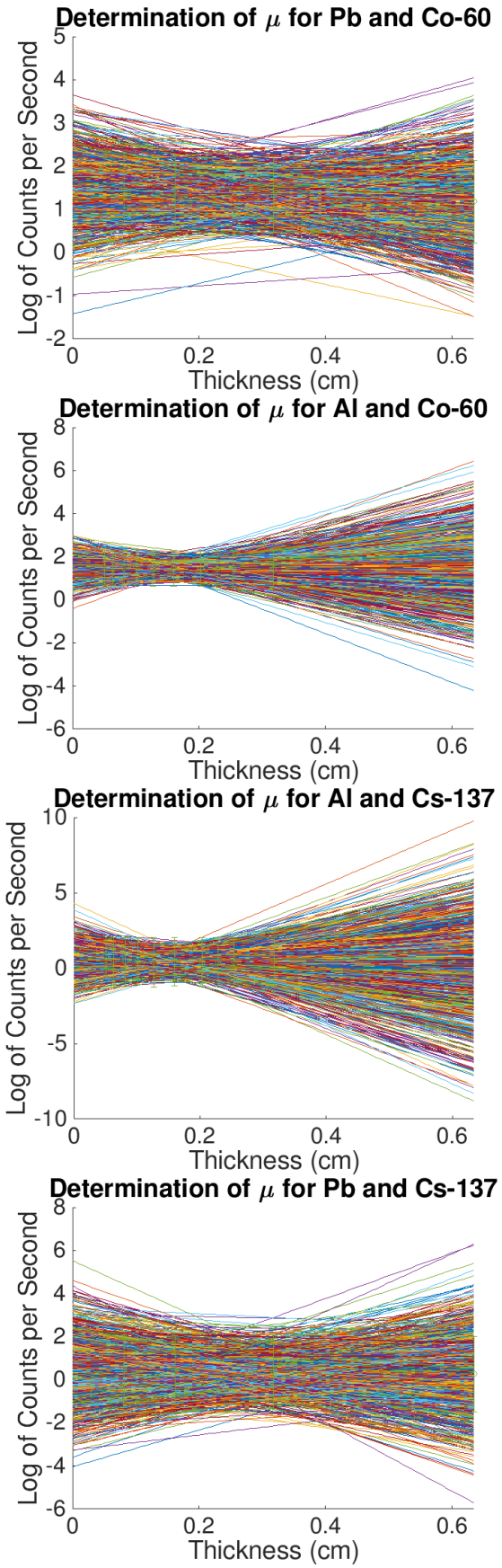

FIG. 6. The Monte Carlo approximations for the attenuation coefficients.

The measured values obtained for the attenuation coefficient for the Co-60 source are 0.5807 ± 0.0146 cm^{-1} for lead and 0.1861 ± 0.0137 cm^{-1} for aluminum. The measured vales for Cs-137 are 0.3360 ± 0.0276 cm^{-1} for lead and 0.7553 ± 0.0264 cm^{-1} for aluminum.

IV. Discussion, Including Uncertainties

The measured values for the attenuation coefficient for the Co-60 source agree very well with the calculated values. The discrepancy for the lead coefficient is 0.0059, being two and a half times smaller than the uncertainty. The discrepancy for the aluminum coefficient is 0.0145, which is 0.0008 cm^{-1} greater than the uncertainty. Even though the theoretical value lies just outside of the measured values for the aluminum coefficient, it is accurate to conclude that the measured value is consistent with the theoretical value because the discrepancy is less than twice the uncertainty (i.e., the discrepancy lies between one and two standard deviations, being closer to one standard deviation). This further suggests that the methods of measurement were not only consistent, but also adequate. Thus, the attenuation coefficients for Co-60 accurately describe that lead is a better attenuator than aluminum. For the Cs-137 source, however, the values are poor, and are not only inconsistent with the theoretical values, but the measured value for aluminum looks like that for lead and the same is true for lead. The discrepancy for the aluminum coefficient is 0.5390 cm^{-1}, which is 20 times greater than the uncertainty for the aluminum coefficient, and the discrepancy for the lead coefficient is 0.4097 cm^{-1}, which is 15 times greater than the uncertainty for the lead coefficient.

In this experimentation, the primary sources of uncertainty were the dead time of the GM counter, potential background radiation, the natural uncertainty in the activity of the radioisotopes, and the distance from the detector. These uncertainties have a direct effect on the counts observed. The dead time causes uncertainty because when the GM counter is quenched, then the counter is unable to detect an electric pulse. Of course, this is an instrumental limitation, which cannot be minimized. The background potentially contributes extra counts to the GM and can be reduced by subtracting potential background radiation from collected data. Because activities can not be measured precisely, there will always be a natural uncertainty with the activity of a radioisotope, which too cannot be minimized. Further, if the GM counter was not at a sufficient distance from the source, than there could be a greater number of counts detected; conversely, perhaps the GM counter was not close enough in order to detect a sufficient amount of counts due to the inverse-square law.

For the case of the Cs-137 with lead, the reason for the large discrepancy in both quantities is likely due to the fact that the expected behavior for lead is both the photoelectric effect and the Compton effect and that perhaps the GM counter was too close to the detector. This would cause an increased number of detected counts on average, resulting in a smaller value for the lead coefficient. For the case of the aluminum, the results are so strange that it is difficult to determine the cause of the large discrepancy. It could be due to the distance needing to be farther away, from the detector. However, this reasoning would not be consistent with the results because the large attenuation coefficient for aluminum suggests that aluminum is a good attenuator, meaning that the source should be closer to the detector, which should not be. Further, the results are strange because the dominant effect, according to Figure 4, is the Compton effect, so there should not be any other effects present. Lastly, because the experimental set up for both was exactly the same, there should not have been any other factors to consider.

V. Conclusion

To conclude, the attenuation coefficients for the Co-60 source and the lead and aluminum attenuators agreed very well with the theoretical values. The theoretical values were 0.5866 cm^{-1} for lead and 0.1716 cm^{-1} for aluminum and the measured values were 0.5807 ± 0.0146 cm^{-1} for lead and 0.1861 ± 0.0137 cm^{-1} for aluminum. The discrepancy for the lead coefficient was 0.0059, two and a half times less than the uncertainty, and the discrepancy for the aluminum coefficient was 0.0145, which lies between one and two standard deviations from the mean. The attenuation coefficients for the Cs-137 source and the attenuators had very poor agreements with the theoretical values. The theoretical values were 0.7457 cm^{-1} for lead and 0.2163 cm^{-1} for aluminum and the measured values were 0.3360 ± 0.0276 cm^{-1} for lead and 0.7553 ± 0.0264 cm^{-1} for aluminum. The discrepancy for lead was 20 times the uncertainty and the discrepancy for aluminum was 15 times the uncertainty. The cause of error for the aluminum

attenuator can not accurately be discerned, but for the lead, the errors are most likely due to the presence of the photoelectric and Compton effects and that there was not enough distance between the source and the detector.

VI. Sources

1. Berkeley Labs. (2009, February). *Neutron beta decays*. http://www.particleadventure.org/npe.html.