Measuring the Rotation Period of the Sun

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I. Introduction

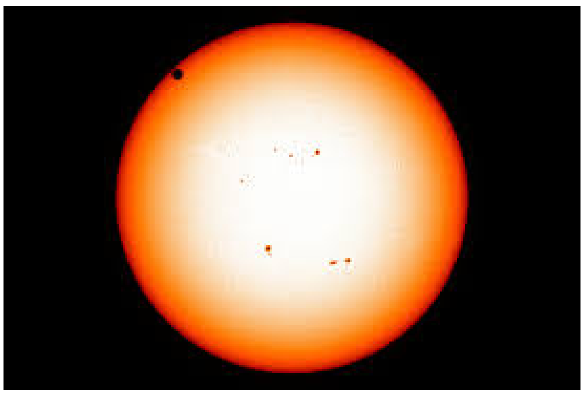

FIG. 1. An image of the Sun with sunspots, indicating increased magnetic activity. Image credit: NASA.

Purpose

The purposes of this experimentation were (i) to estimate the rotation period of the Sun, (ii) to simulate a professional environment in terms of group coordination, collaboration, and data collection, (iii) to understand the importance of taking measurements with associated uncertainties, and (iv) to understand how those uncertainties propagate throughout various calculations.

The Sun

Astrophysics is a broad, expanding branch of physics concerning the study of planets, galaxies, and other astronomical bodies beyond Earth. Despite the search for other Earth-like planets and the endeavor to understand the Universe, the Sun remains the most important heavenly body necessary for the Solar System, providing warmth, light, and a gravitational beacon around which the planets traverse. Further, understanding the Sun aids in building a foundation of general knowledge for a more broad perspective of stars like the Sun.

Figure 1 is an image of the Photosphere of the Sun, the first of its outer layers, which contains light and dark specks commonly called sunspots or solar spots[1]. The Photosphere, however, is only one of six layers of the Sun, with three interior layers and three exterior ones. On the interior, the Sun contains a dense *Core* where the nuclear fusion of hydrogen atoms produces helium atoms and all of the Sun's energy. This energy is then projected radially outwards through the *Radiative Zone* to the *Convection Zone*. The Convection Zone recycles plasma that has cooled and brings hot plasma back to the surface, as shown in Figure 2. This allows for the Sun's seemingly perpetual radiation of light and thermal energy.

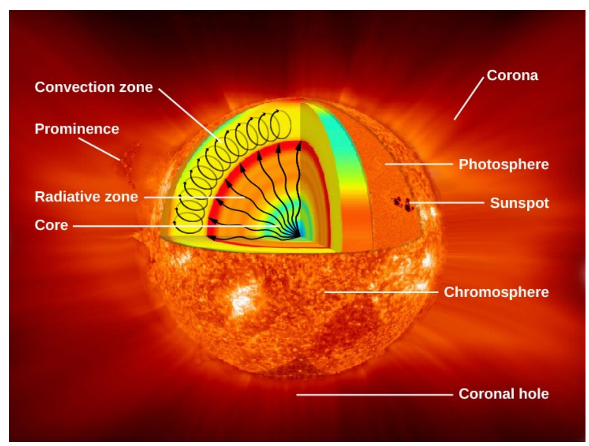

FIG. 2. This is the structure of the Sun. It contains six layers: three inner layers and three outer layers. It is important to note that this is not the case for all stars. Image credit: NASA/Goddard.

Moreover, because the surface of the Sun is plasma, it is not solid like the surface of Earth. This causes

[1] Fraknoi,A., Morrison, D., & Wolff, S. C. (2016). The Sun: A Garden-Variety Star. *Astronomy.* (1^{st} ed.). Houston, Texas: Rice University.

the Sun's rotation period to be different for different latitudes, called differential rotation.

The exterior structure consists of the *Photosphere*, the *Chromosphere*, and the *Corona*. The Photosphere is the part of the sun visible to the human eye due to its increased opacity. This is also the part of the Sun in which sunspots form. The Chromosphere lies directly above it, with a greater temperature, and the Corona composes the last of the Sun's outer atmosphere, also being the hottest.

The Photosphere, Sunspots, and the Solar Cycle

Each layer of the Sun has special characteristics. The Core has nuclear fusion and the Corona has coronal mass ejections. The Photosphere's special contents are sunspots. Sunspots are dark spots that appear on the surface of the Sun due to an increase in magnetic activity. Because plasma is a collection of highly ionized gas particles, and because moving charges generate magnetic fields, the totality of the magnetic fields induced by plasma produces the Sun's overall magnetic field. When changes in the magnetic field occur, these changes produce sunspots. Sunspots contain dark and light spots, called the umbra and penumbra, respectively. The umbra has a dark coloration due to its decreased temperature compared to the penumbra and the surrounding plasma.

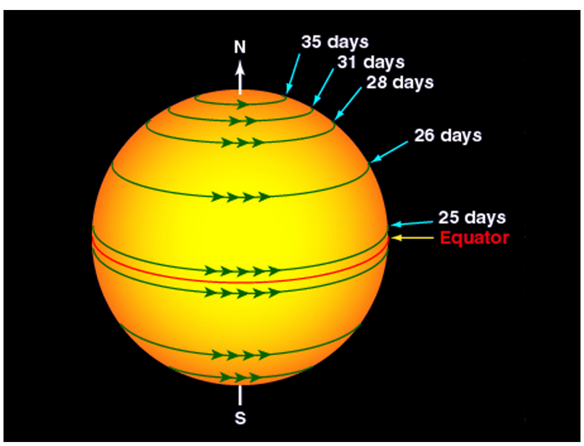

FIG. 3. Due to differential rotation, the Sun rotates at a faster speed about its equator. This speed decreases as the latitude approaches the poles.

Because the Sun's rotation period is different for different latitudes, this means that sunspots at different latitudes will have different rotational speeds. The speeds are greatest at the equator and decrease as the latitude approaches the poles due to differential rotation. As sunspots move, they do not change their appearance, but they do move closer to the equator as time progresses, which makes them suitable candidates for measuring the Sun's rotation period.

The magnetic activity and the formation and movement of sunspots is primarily due to the solar cycle, which is an eleven year period in which the Sun's magnetic poles change direction. This suggests that every twenty-two years, the Sun returns to the original magnetic orientation. The eleven year period consists of a minimum period and a maximum period. The minimum period occurs when the Sun's magnetic activity is low, resulting in a low number of sunspots. During a maximum period, the Sun's magnetic activity is high, and so is the number of sunspots. Currently, the Sun is in an abnormally low minimum period, but this does not suggest any negative consequences about the Sun.

II. Data Acquisition

Materials

The materials used to perform this experimentation are as follows: a meter stick with a tolerance of ($\pm$ 0.5 cm), a whiteboard, Sun images provided by the instructor, a projector, Adobe Photoshop, and Matlab.

Procedure and Methods

The data that were used for analysis were images that were taken years ago by NASA's SOHO (Solar and Heliospheric Observatory). The purpose for using these photos instead of obtaining them is due primarily due to the fact that the solar cycle is in a period of minimum magnetic activity, which means a minimum amount of sunspots for observation, but also because of time constraints.

The methods for measuring the rotation per day for spots A, B, and C were the same, the procedure with which spot A was measured is slightly different than the procedure that was used for both spots B and C.

Procedure for Spot A

Spot A was measured by projecting the Sun images onto the whiteboard, onto which the grid-lines of the Sun were traced. The longitude lines were drawn free-handedly and the latitude lines were drawn with the aid of the meter stick. This was accomplished with the presumption that the drawn grid lines would line up with each image, which was not the case. Then, with each day, starting from the first day scrolling to the last day (a total of eleven days), the spot was drawn onto the whiteboard so as to cover the umbra of the spot.

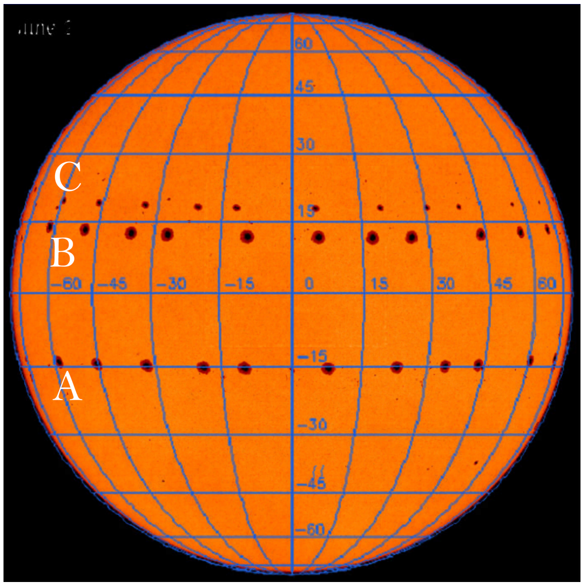

FIG. 4. Layered images of the Sun rotation data. Image credit: Kadri B. M. Nizam.

Furthermore, instead of estimating the uncertainty for each spot by sight, the meter stick was used to measure the width of the spot and the width of the interval in which the spot was present. In doing so, a ratio was made using the width between longitudes at the latitude of the sunspot and the width of the spot, to determine how many subdivisions for each section, providing a more reliable estimate. Then, the positions of the spots were estimated, and the mean of the difference between two consecutive days was taken. Using the means and the estimated uncertainties for each spot, the rotation period of the Sun was calculated for each spot by converting degrees per day to days.

Procedure for Spots B and C

Spots B and C were also measured using a projection onto the whiteboard, following the same procedure as for spot A up until marking the longitudes and latitudes. For spots B and C, the images were layered on top of each other into one image so that the longitudes and latitudes would be uniform, providing consistency and better precision, using Adobe Photoshop. Afterwards, the methods for measuring the positions and uncertainties of spots B and C were the same as those used for spot A.

III. Analysis

Calculating the Rotation Period

In order to calculate the rotation period of the Sun, Matlab was used. The data and uncertainty for each day was made into their own vectors, and the difference between consecutive days was obtained as such:

$$x_i = \{\mathrm{Day}_{i+1} - \mathrm{Day}_i\}_{i=1}^{11},$$

where each difference is measured in degrees per day. In order to account for the Earth's rotation around the Sun in the same direction as the Sun's rotation, one degree per day was added to each *difference* of each measurement, as opposed to adding it to each measurement. This corrects for the appearance of the Sun moving slower than it actually is. Then, to propagate the errors for each day, the root of the sum of the squares of the uncertainties for the ith difference was calculated, utilizing quadrature to neither overestimate or underestimate the uncertainty,

$$\delta q_i = \sqrt{(\delta \mathrm{Day}_{i+1})^2 + (\delta \mathrm{Day}_i)^2}$$

where δq_i is the uncertainty for the ith difference, $\delta \mathrm{Day}_i$ is the uncertainty for the current day, and $\delta \mathrm{Day}_{i+1}$ is the uncertainty for following day. Afterwards, weighted averages were used to calculate the average rotation and the overall uncertainty for each day[2] as such:

$$x_{wav} = \frac{\sum_i w_i x_i}{\sum_i w_i},$$

where $w_i = 1/(\delta {q_i}^2)$. Then, 360 was divided by a vector that contained the average of each difference and the margin of error. This was

[2] Taylor, J. R. (1997). Weighted Averages. *Introduction to Error Analysis.* (2^{nd} ed.). Sausalito, California: University Science Books.

done to see the average amount of days for the rotation period and the upper and lower bounds as well. Then, the best estimates were rounded to the nearest day and the bound with the greatest magnitude was used as the uncertainty for the best estimation possible. The values obtained for spots A, B, and C were 25(± 1) days (n = 11), 25(± 1) days (n = 11), and 27(± 1) days (n = 11), respectively, where n is the number of measurements. The data is given in Table 1 in the appendix.

Estimating Uncertainties

Because uncertainties are inevitable and give measurements meaning, it is always important to account for them. In both procedures, there were uncertainties in the drawn grid lines, the drawn spots, and the identification of the position of the spots[3]. The meter stick, however, aided in quantifying the uncertainty for each spot and each fifteen degree interval for the longitudes. As per the previous section, a ratio was made in order to determine the number of subdivisions of each interval, and the amount of degrees determined in one subdivision was used as the uncertainty such that

$$n = \frac{w_I}{w_d},$$

where n is the number of subdivisions, w_I is the width of the longitude interval in centimeters, and w_d is the width of the drawn dot in centimeters. The number of subdivisions was rounded down so as to ensure that the number of degrees per division was the greatest number for a good estimate of the uncertainty. This number then divided fifteen (15/n) so as to determine the number of degrees per subdivision. Because the Sun is a sphere, this process was repeated for each dot and longitude interval to account for differing uncertainties[4]. Further, symmetry was assumed, so the uncertainties calculated on one half of the zero longitude line were applied to the same intervals on the other half.

IV. Discussion

The purposes of this experimentation were to estimate the rotation period of the Sun, to simulate a professional environment, to understand the importance of taking measurements with associated uncertainties, and to understand how those uncertainties propagate throughout various calculations. The data that that was calculated for spots A, B, and C (25(± 1) days (n = 11), 25(± 1) days (n = 11), and 27(± 1) days (n = 11), respectively) are consistent because each measurement lies within the margin of error of the other measurements. This suggests that the measurements do not differ significantly in a statistical manner. The latitudes of spots A, B, and C were determined to be -15°, 12°, and 17°, respectively. Although the uncertainties were not directly estimated for the latitudes, as they were for the longitudes, a good estimate for an overall margin of error would be -20° to 20° for the latitudes of the points. This now allows for a comparison of the data to actual measured values. According to a paper published in 1974 by P. A. Gilman, from the latitudes of -20° to 20°, the period of the Sun's rotation varies from about 25 days to 27 days[5].This suggests that the obtained data is not only consistent with each other, but it is also consistent with measured values. For spots A and B, the period of rotation was measured to be the same for both spots. This is consistent, as both spots were close to the 15° latitude line. For spot C, it is also consistent that C's rotation period is slightly larger than both spot's A and B because of differential rotation.

V. Conclusion

The rotation period for the Sun varies at different latitudes, called differential rotation, due to the surface of the Sun behaving as a liquid. The rotation period of the Sun was obtained from measuring three sunspots A, B, and C with rotation periods of 25(± 1) days, 25(± 1) days, and 27(± 1) days, respectively, between the latitudes of -20° and 20°. These values are consistent not only statistically, but also physically because the actual values of rotation at those latitudes varies from 25 days to 27 days, which is within the margin of error of every mea-

[3] Even though in the procedure for spots B and C used a layered image for better precision, there is still uncertainty present due to the drawing of "imperfect" lines.

[4] From the zero longitude line, the uncertainties increase as the longitude increases in either direction due to the curvature of the Sun

[5] Gilman, P. A. (1974). Solar rotation. *In: Annual review of astronomy and astrophysics. Volume 12. (A75-13476 03-90) Palo Alto, Calif., Annual Reviews, Inc., 1974, p. 47-70.* 10.1146/annurev.aa.12.090174.000403.

surement.

VI. References

1. Fraknoi,A., Morrison, D., & Wolff, S. C. (2016). The Sun: A Garden-Variety Star. *Astronomy.* (1st ed.). Houston, Texas: Rice University.

2. Gilman, P. A. (1974). Solar rotation. *In: Annual review of astronomy and astrophysics. Volume 12. (A75-13476 03-90) Palo Alto, Calif., Annual Reviews, Inc., 1974, p. 47-70.* 10.1146/annurev.aa.12.090174.000403.

3. Taylor, J. R. (1997). Weighted Averages. *Introduction to Error Analysis.* (2nd ed.). Sausalito, California: University Science Books.

VII. Appendix

A.1

Position	Spot A (°)	Spot B (°)	Spot C (°)
1	-58 ± 4	-62 ± 3	-59 ± 2
2	-45 ± 4	-49 ± 2	-49 ± 2
3	-32 ± 2	-36 ± 1	-34 ± 1
4	-19 ± 1	-27 ± 1	-21 ± 1
5	-10 ± 1	-9 ± 1	-12 ± 1
6	8 ± 1	6 ± 1	5 ± 1
7	24 ± 1	17 ± 1	20 ± 1
8	35 ± 2	26 ± 1	31 ± 1
9	44 ± 2	44 ± 1	39 ± 1
10	61 ± 4	57 ± 2	56 ± 2
11	74 ± 4	70 ± 3	60 ± 3

TABLE I. The longitudes of spots A, B, and C.

A.2

```
%% Data

A = [-58 -45 -32 -19 -10 8 24 35 44 61 74];
Aerr = [4 4 2 1 1 1 1 2 2 4 4];
B = [-62 -49 -36 -27 -9 6 17 26 44 57 70];
Berr = [3 2 1 1 1 1 1 1 1 2 3];
C = [-59 -46 -34 -21 -12 5 20 31 39 56 69];
Cerr = [2 2 1 1 1 1 1 1 1 2 3];

A1 = diff(A) + 1; % All differences have units of degrees per day
B1 = diff(B) + 1; % There is one less element for these three
C1 = diff(C) + 1; % Add one to each element to account for Earth's orbit

% The best approx. of true errors of the differences in days
Aerr1 = [4*sqrt(2) 2*sqrt(5) sqrt(5) sqrt(2) sqrt(2) sqrt(2) sqrt(5) 2*sqrt(2) 2*sqrt(5) 4*sqrt(2)];
Berr1 = [sqrt(13) sqrt(5) sqrt(2) sqrt(2) sqrt(2) sqrt(2) sqrt(2) sqrt(2) sqrt(5) sqrt(13)];
Cerr1 = [2*sqrt(2) sqrt(5) sqrt(2) sqrt(2) sqrt(2) sqrt(2) sqrt(2) sqrt(2) sqrt(5) sqrt(13)];
```

```
% weights for weighted average
w1 = 1./((Aerr1).^2);
w2 = 1./((Berr1).^2);
w3 = 1./((Cerr1).^2);

% weighted average of degrees/day
A_avg = (A1*w1')/sum(w1);
B_avg = (B1*w2')/sum(w2);
C_avg = (C1*w3')/sum(w3);

% the best approx. of true uncertainty of the average
Asigma = 1/sqrt(sum(w1));
Bsigma = 1/sqrt(sum(w2));
Csigma = 1/sqrt(sum(w3));

% The best measure including the uncertainty bounds
x = 360./[(A_avg - Asigma) A_avg (A_avg + Asigma)];
y = 360./[(B_avg - Bsigma) B_avg (B_avg + Bsigma)];
z = 360./[(C_avg - Csigma) C_avg (C_avg + Csigma)];

% Compute difference between the best value and the upper bound
% Then the difference between the best value and the lower bound
% The maximum of the two will be taken as the uncertainty
e1 = max(abs(diff(x)));
e2 = max(abs(diff(y)));
e3 = max(abs(diff(z)));
```