Vehicle System Dynamics

ISSN: 0042-3114 (Print) 1744-5159 (Online) Journal homepage: https://www.tandfonline.com/loi/nvsd20

Improving vehicle dynamics by active rear wheel steering systems

Marlene Kreutz , Martin Horn & Josef Zehetner

To cite this article: Marlene Kreutz , Martin Horn & Josef Zehetner (2009) Improving vehicle dynamics by active rear wheel steering systems, Vehicle System Dynamics, 47:12, 1551-1564, DOI: 10.1080/00423110802691507

To link to this article: https://doi.org/10.1080/00423110802691507

Published online: 06 Nov 2009.

Vehicle System Dynamics
Vol. 47, No. 12, December 2009, 1551–1564

Improving vehicle dynamics by active rear wheel steering systems

Marlene Kreutz[a]*, Martin Horn[b] and Josef Zehetner[c]

[a] *Institute for Automation and Control, Graz University of Technology, Graz, Austria;* [b] *Institute for Smart System Technologies, Klagenfurt University, Klagenfurt, Austria;* [c] *AVL List GmbH, Graz, Austria*

(*Received 10 January 2008; final version received 10 December 2008*)

This article presents two design strategies for an active rear wheel steering control system. The first method is a standard design procedure based on the well-known single track model. The aim of the feedback loop is to track a reference yaw rate in order to improve the handling behaviour. Unfortunately, a reasonable specification of the reference yaw rate proves to be a nontrivial task. A second approach avoids this drawback. The structure of the controller is regarded as a virtual mass-spring-damper system with adjustable parameters. Due to the high abstraction level of this method, the controller parameters can be tuned intuitively. Experiments with a prototype vehicle illustrate the effectiveness of the two proposed methodologies.

Keywords: vehicle dynamics; active rear wheel steering; model based design; road holding ability

1. Introduction

During the past few decades much research has been focused on the development of driver assistance systems. This considerable interest is mainly due to the fact that driver assistance systems represent an effective way to improve the comfort and safety of modern passenger cars. The advancing technology of mechatronic systems provides potential to the active control of chassis and driveline.

In principle a driver can influence vehicle dynamics in three ways: by accelerating, braking and steering [1]. Active systems for accelerating and braking offer assistance in critical driving situations. In contrast to these well-known technologies, active steering systems intervene over the entire dynamic range of a car.

Four wheel steering systems have been the subject of numerous investigations and are still being studied as a means of improving manoeuvrability and stability of vehicles. In [2], possible concepts for the control of active steering systems are discussed. They are categorised into feedforward and feedback approaches. In [3], feedforward control to minimise swing out

*Corresponding author. Email: marlene.kreutz@tugraz.at

ISSN 0042-3114 print/ISSN 1744-5159 online

DOI: 10.1080/00423110802691507
http://www.informaworld.com

of the rear end of cars with long wheelbase at low speed is outlined. In [4], the rear wheel steering angle is computed by an adaptive feedforward law depending on the yaw rate. A neural network feedforward approach for active four wheel steering is introduced in [5]. Yamamoto [6] proposes three strategies that combine feedforward and (velocity dependent) feedback concepts. In the nonlinear range of tyre adhesion two additional control categories, namely driving/braking- and vertical-load-control are used. These concepts go far beyond the scope of the present article where only the rear steering angle is used as control input. The concept of robust decoupling of yaw motions and lateral dynamics of cars with active front or four wheel steering is investigated in [2,7]. The resulting control law is robust with respect to varying vehicle mass, inertia and tyre–road contact characteristics at constant speed. Jia [8] introduces a robust control algorithm with decoupling performance for four wheel steering vehicles under velocity varying motion. In [9,10] the problem of active steering is addressed by applying $\mathcal{H}_\infty$-control, and You and Jeong [11] use mixed $\mathcal{H}_\infty/\mathcal{H}_2$-synthesis based on linear matrix inequality theory for steering angles and individual wheel torque control. Adaptive schemes to cope with model uncertainties, e.g. tyre and road characteristics, are considered in [12,13]. Other contributions come from fuzzy-logic based control design. A vehicle speed dependent fuzzy-logic-LQR-design is carried out in [14]. In [15] a fuzzy model of a vehicle based on nonlinear tyre characteristics is derived. Theoretical aspects concerning vehicle stability under consideration of inevitable dead times in active steering systems are discussed in [16].

The work presented in this article proposes two unrelated approaches for the control of an active rear wheel steering system that intend to facilitate the handling of a car during dynamic driving manoeuvres. The first controller is based on classical ideas of control engineering. The second approach uses completely new ideas. The potential of these concepts is analysed by simulation as well as by experiment using a prototype vehicle.

The remainder of the article is organised as follows. Section 2 contains performance specifications for the closed loop system. Section 3 is concerned with a classical model-based controller design approach as well as an alternative concept that allows intuitive controller tuning. Experimental results are presented in Section 4, and conclusions are drawn in Section 5.

2. Design specifications

A Mercedes E-class 4-matic was chosen as a prototype car [17]. Due to the innovative chassis design, this car shows excellent handling behaviour. Therefore, it is a challenging task to improve its dynamics by an active rear wheel steering system.

By virtue of the well-tuned passive vehicle, i.e. the vehicle without active rear wheel steering, it is reasonable not to influence the established self-steering behaviour. Therefore the proposed controllers will not stabilise the vehicle during steady state cornering. The main objective of the active steering system is to enhance the stability[1] of the car during dynamic driving manoeuvres like, e.g., step steer or lane changes.

According to [1], the driveability of a car depends strongly on the step response characteristics of the yaw rate

$$r := \frac{\mathrm{d}\psi}{\mathrm{d}t}, \tag{1}$$

where ψ represents the yaw angle of the vehicle. To guarantee that even inexperienced drivers are able to control the vehicle, the rise time and the overshoot of the step response should be sufficiently small. Therefore, it is a reasonable task of the active rear wheel steering system to reduce both characteristic parameters.

3. Controller design

In this section, two possible approaches to the design of control laws for active rear wheel steering systems are presented. The first method is a classical control strategy based on the well-known linear single track model. The main idea of the concept is to design a standard feedback system to force the yaw rate to track a given reference signal in a desired manner [18]. A major drawback of this model based approach is the adequate choice of the reference yaw rate such that the self-steering behaviour of the vehicle remains unchanged. This fact leads to an alternative control concept, which is not based on a mathematical model of the system and does not require a reference yaw rate.

3.1. *Model based synthesis*

3.1.1. *Single track model*

The essential properties of the lateral dynamics of a car can be captured by the so-called linear bicycle model [19]. The model description is derived by lumping together the wheels of each axle in the vehicle's centreline. We suppose that the centre of gravity is situated on the road surface, and therefore pitch and roll motions can be neglected. Small sideslip angles justify the presumption of linear tyre characteristics with the corresponding cornering stiffnesses[2] c_f and c_r. Assuming sufficiently small steering angles δ_f, δ_r and body sideslip angle β yield the second order model for constant vehicle speed v:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \mathbf{x} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \boldsymbol{\delta}, \tag{2}$$

where

$$\mathbf{x} = \begin{bmatrix} \beta \\ r \end{bmatrix} \quad \text{and} \quad \boldsymbol{\delta} = \begin{bmatrix} \delta_\mathrm{f} \\ \delta_\mathrm{r} \end{bmatrix}. \tag{3}$$

The parameters of the model (2) can be computed as

$$\begin{aligned} a_{11} &= -\frac{c_\mathrm{f} + c_\mathrm{r}}{mv}, \quad & a_{12} &= -1 + \frac{c_\mathrm{r} l_\mathrm{r} - c_\mathrm{f} l_\mathrm{f}}{mv^2}, \\ a_{21} &= \frac{c_\mathrm{r} l_\mathrm{r} - c_\mathrm{f} l_\mathrm{f}}{J_\mathrm{z}}, \quad & a_{22} &= -\frac{c_\mathrm{r} l_\mathrm{r}^2 + c_\mathrm{f} l_\mathrm{f}^2}{J_\mathrm{z} v} \end{aligned} \tag{4}$$

and

$$\begin{aligned} b_{11} &= \frac{c_\mathrm{f}}{mv}, \quad & b_{12} &= \frac{c_\mathrm{r}}{mv}, \\ b_{21} &= \frac{c_\mathrm{f} l_\mathrm{f}}{J_\mathrm{z}}, \quad & b_{22} &= -\frac{c_\mathrm{r} l_\mathrm{r}}{J_\mathrm{z}}. \end{aligned} \tag{5}$$

The vehicle mass is denoted by m, J_z is the moment of inertia with respect to a perpendicular axis through the centre of gravity. The distances of the particular axes from the centre of gravity are l_f and l_r. Numerical values for the prototype vehicle are given in Appendix 1.

Using transfer functions, the multi-input-multi-output system (2) takes the form

$$r(s) = \mathbf{P}(s)\boldsymbol{\delta}(s) = \begin{bmatrix} P_{21}(s) & P_{22}(s) \end{bmatrix} \boldsymbol{\delta}(s). \tag{6}$$

In (6) $r(s)$ and $\boldsymbol{\delta}(s)$ denote the Laplace transforms of $r(t)$ and $\boldsymbol{\delta}(t)$, respectively.

3.1.2. *Design procedure*

With regard to the implementation in a prototype vehicle the control law should be as simple and clear-structured as possible. As a prerequisite for a time-invariant control strategy, a constant vehicle speed of 100 km/h is assumed. This assumption is justified due to the fact that the eigenvalue configuration of model (2) is a representative of vehicle speeds ranging from 50 km/h to 200 km/h (see Figure 1).

Introducing the transfer function

$$D(s) = -\frac{P_{21}(s)}{P_{22}(s)}, \tag{7}$$

(see Figure 2) which compensates for the influence of δ_f on r, the system (6) can be treated as a single-input-single-output plant. The corresponding plant transfer function is $P_{22}(s)$. Note that the compensation is not supposed to be exact in view of parameter uncertainties, e.g. variations in vehicle speed. However, it significantly reduces coupling of the front wheel steering angle and the yaw rate. Due to the internal stability of the resulting feedback loop the stability is guaranteed even in the case of unexact compensation.

As mentioned in Section 2, the step response characteristics of the yaw rate play a significant role concerning the driveability of the car. Following the ideas of the linear algebraic method

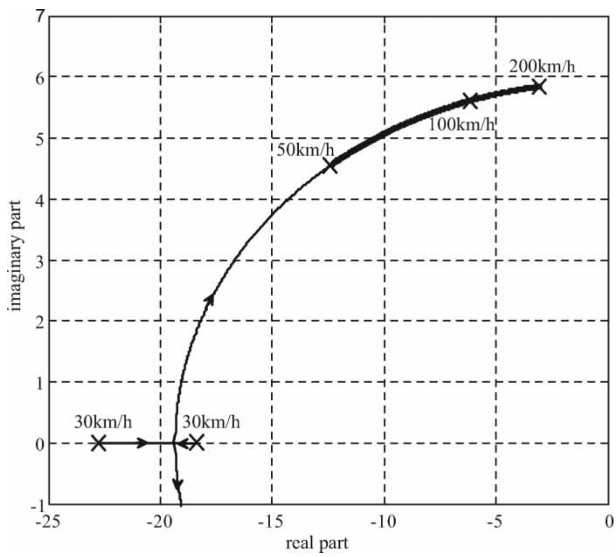

Figure 1. Root locus for different vehicle speeds.

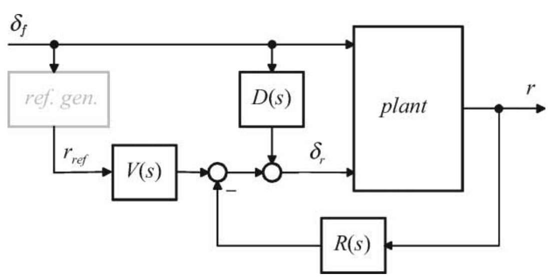

Figure 2. Structure of the feedback loop.

[20] an overall transfer function

$$T(s) = \frac{r(s)}{\delta_{\mathrm{r}}(s)},$$

which meets desired properties, is chosen. In the present case, the design specifications can be expressed in terms of the rise time t_{r} and the overshoot, so that $T(s)$ can be well approximated by a system with a dominant pair of complex-conjugate poles, i.e.

$$T(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}.$$

The specifications

$$t_{\mathrm{r}} = 0.1\,s \quad \text{and} \quad \text{overshoot} = 0\%$$

yield the parameters

$$\omega_n = 28.5 \quad \text{and} \quad \xi = 0.9.$$

Requiring a vanishing steady-state error for constant reference signals leads to an integrating controller, represented by the two transfer functions $R(s)$ and $V(s)$. Based on the configuration of the feedback system (see Figure 2) the required compensator parameters can be computed by solving linear algebraic equations. As shown in Figure 2 the structure requires a reference yaw rate r_{ref}. The computation of an appropriate reference signal is a nontrivial task [21]. In case of stationary driving manoeuvres r_{ref} should represent the steady state yaw rate of the passive vehicle with the applied front steering angle, so that the rear wheel steering system does not change the stationary driving behaviour (see Section 2). As a consequence, the rear wheel steering actuates only during dynamical manoeuvres. In steady state it is inactive, i.e. $\delta_{\mathrm{r}} = 0$.

Thus, the reference yaw rate can be provided by either a characteristic diagram or a (non-linear) mathematical model that represents the properties of the passive vehicle. Generation of the yaw rate is symbolised by a grey block (ref. gen.) connecting δ_{f} and r_{ref} (see Figure 2). The appropriate choice of r_{ref} guarantees that the the control inputs δ_{f} and δ_{r} are not counteracting.

3.2. *Alternative concept*

The model based controller design (Section 3.1.2) requires a well-defined time-invariant vehicle model, which of course cannot be guaranteed during typical driving conditions. Therefore, the controller has to feature robustness properties to meet the specified requirements even under rapidly varying vehicle parameters. Besides, the generation of an appropriate reference yaw rate is not always possible in a satisfactory manner. This is due to unknown variations of vehicle- and vehicle-road-interaction-parameters. To simplify the controller design (respectively, the controller tuning) it might be favourable to abandon the model based approach. In the present case the feedback structure shown in Figure 3 is proposed. This alternative configuration has the advantage that no reference yaw rate is required. Furthermore the controller design is not based on a mathematical model of the vehicle. By interpreting the controller as a simple mechanical system the task of tuning the controller gets more intuitive. Therefore satisfying controller settings can be found in a shorter period of time.

To motivate the proposed controller concept some preliminary reflections have to be carried out. Basically, the wheels of the rear axle can be steered in the opposite or in the same direction as the front wheels. A rear steering angle in the same direction as the front steering angle makes the car behave like one with a longer wheelbase, and therefore causes a weaker yaw motion with

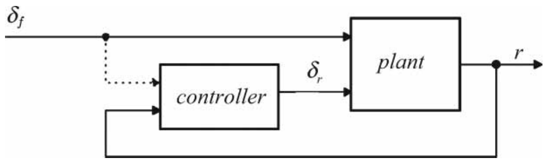

Figure 3. Modified structure of the feedback loop.

less overshoot compared with the passive car. This steering strategy facilitates the handling of the car at higher vehicle speeds significantly.

By steering the rear wheels in the opposite direction, the vehicle reacts like a car with reduced wheelbase which causes a yaw motion with a larger overshoot of the yaw rate. Therefore the reaction of the car to a steering wheel input is faster. Countersteering at the rear axle at high vehicle speeds thus may lead to critical driving situations and has to be handled with care.

3.2.1. *Mass-spring-damper structure*

Based on the considerations above and the claim not to change the self steering behaviour of the vehicle, a desired step steer characteristic of the rear steering angle shown in Figure 4 can be specified. The idea of the alternative controller concept is to make the vehicle's rear axle behave as illustrated in Figure 4. Therefore a 'mass-spring-damper system' as shown in Figure 5 with the mass η_1, the spring constant $\tilde{\kappa}_1$ and the damper constant $\tilde{\vartheta}_1$ is introduced. The rear wheel steering angle δ_{r} is made proportional to the travel of the mass λ_1 measured from the equilibrium position. The excitation ρ_1 of the system has to be carried out based on the vehicle response. On the assumption of a linear spring characteristic and a velocity proportional

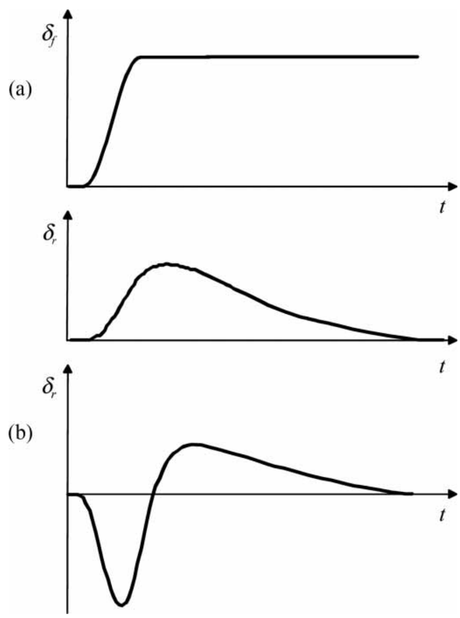

Figure 4. Step steer characteristic.

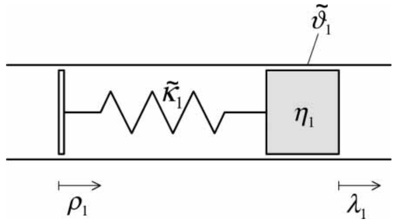

Figure 5. Mass-spring-damper system.

damper the dynamics of the system can be modelled by the second order differential equation

$$\eta_1 \frac{d^2\lambda_1}{dt^2} = -\tilde{\vartheta}_1 \frac{d\lambda_1}{dt} - \tilde{\kappa}_1(\lambda_1 - \rho_1). \tag{8}$$

Introducing the normalised parameters

$$\kappa_1 = \frac{\tilde{\kappa}_1}{\eta_1} \quad \text{and} \quad \vartheta_1 = \frac{\tilde{\vartheta}_1}{\eta_1} \tag{9}$$

the system's input–output-behaviour can be described by the transfer function

$$M_1(s) = \frac{\lambda_1(s)}{\rho_1(s)} = \frac{\kappa_1}{s^2 + \vartheta_1 s + \kappa_1}. \tag{10}$$

To guarantee that the mass – respectively, the rear steering angle – is at rest during stationary driving conditions a suitable input for the mass-spring-damper system has to be applied. One reasonable input candidate is the yaw acceleration. As the yaw rate can easily be measured, it seems obvious to calculate this quantity by derivation of the yaw rate with respect to time that can be implemented by a DT_1-element

$$\rho_1(s) = \frac{s}{1 + s/\varepsilon} r(s) = \Delta(s) r(s). \tag{11}$$

The positive real valued constant ε has to be chosen sufficiently large. The corresponding control loop is shown in Figure 6. As a consequence to the rear steering angle characteristic shown in Figure 4a the overshoot of the yaw rate can be reduced. Unfortunately this positive effect is accompanied by an increase of the rise time, which might be regarded as an undesirable deterioration of the vehicle response. This possible drawback can be avoided by steering the rear wheels in the opposite direction for a short period of time as shown in Figure 4b to achieve a better responsiveness of the car.

In principle, various approaches to obtain the desired rear steering characteristic are conceivable. The structure shown in Figure 7 turns out to be favourable with respect to filtering the

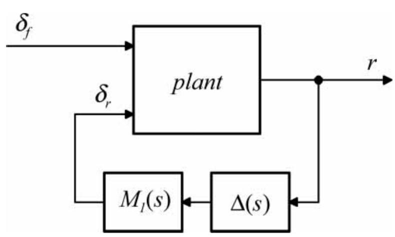

Figure 6. Structure of the closed loop system.

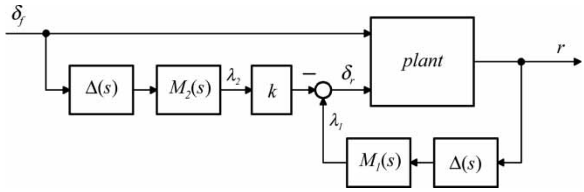

Figure 7. Extended structure of the closed loop system.

input signal. The original system (see Figure 6) is augmented by an additional mass-spring-damper system with unchanged mass and spring constant but a smaller normalised damper constant ϑ_2 which can be modelled by the transfer function

$$M_2(s) = \frac{\lambda_2(s)}{\rho_2(s)} = \frac{\kappa_1}{s^2 + \vartheta_2 s + \kappa_1}. \tag{12}$$

According to Figure 7, the rear steering angle now is computed by subtracting the weighted output signal of the additional mass-spring-damper system λ_2 from the output signal of the original system λ_1. The weighting factor k defines the degree of countersteering at the rear axle. The parameters ϑ_1 and ϑ_2 serve as tuning factors. The effect of k and ϑ_2 on the controller output signal is illustrated in Figure 8. Typically, the parameter k ranges between 0 and 5. The damper ratio ϑ_2/ϑ_1 takes values within the bounds 1/7 and 1/3. To achieve the fastest possible vehicle response the derivation with respect to time of the front steering angle δ_f is chosen as input signal ρ_2 for the second mass-spring-damper system.[3] Following (11) the input quantity

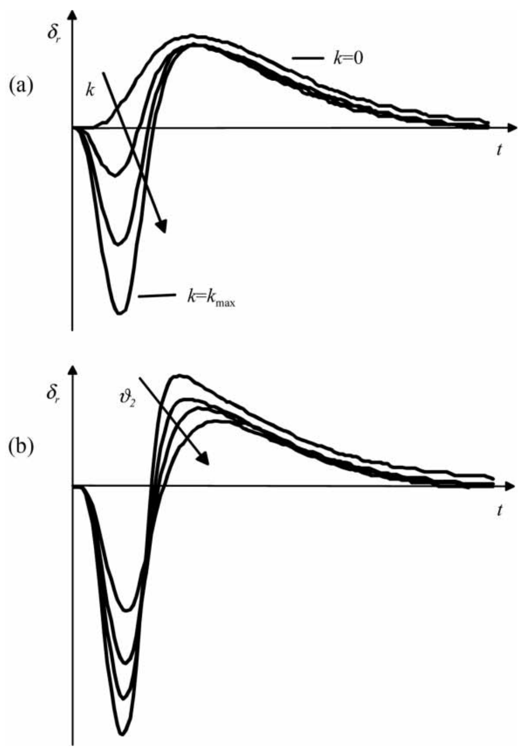

Figure 8. Variation of the tuning factors (a) k and (b) ϑ_2.

can be computed as

$$\rho_2(s) = \frac{s}{1 + s/\varepsilon}\delta_{\mathrm{f}}(s) = \Delta(s)\delta_{\mathrm{f}}(s). \tag{13}$$

3.2.2. *Controller tuning*

The controller parameters to be tuned are the spring constant κ_1, the damper constants ϑ_1 and ϑ_2 and the weighting factor *k*. Due to the concise structure of the control law, the parameters can be interpreted in a physical sense. Therefore, basic settings can easily be found either by simulation or by experiment. The subsequent fine tuning can be achieved intuitively and requires no background in control theory. A standardised procedure for test drivers to systematically adjust the controller based on lane change manoeuvres was developed.

To gain more insight into the potentials of the mass-spring-damper control law, the root locus of the closed loop system is shown in Figures 9 and 10. In Figure 9, the spring constant is kept constant at a basic setting $\kappa_1 = 5000$, whereas the value for the damper constant varies in a range $\vartheta_1 = [\vartheta_{1\,\min} \;\; \vartheta_{1\,\max}] = [200 \;\; 2000]$. Figure 10 shows the root locus for a basic setting of $\vartheta_1 = 1000$ and a parameter interval of $\kappa_1 = [\kappa_{1\,\min} \;\; \kappa_{1\,\max}] = [500 \;\; 10{,}000]$. In both pictures the relevant part of the root locus related to the parameter intervals is depicted in black.

A major advantage of the concept is that the driving behaviour of the vehicle can be individually adapted to the driver's demands. A possible scenario is that the driver can choose from a number of parameter presettings (sportive, comfortable etc.).

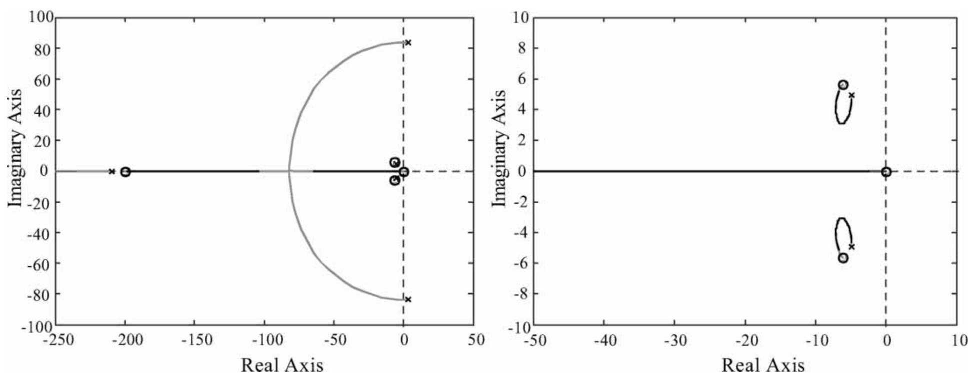

Figure 9. Root locus of the closed loop system varying the damper constant ϑ_1.

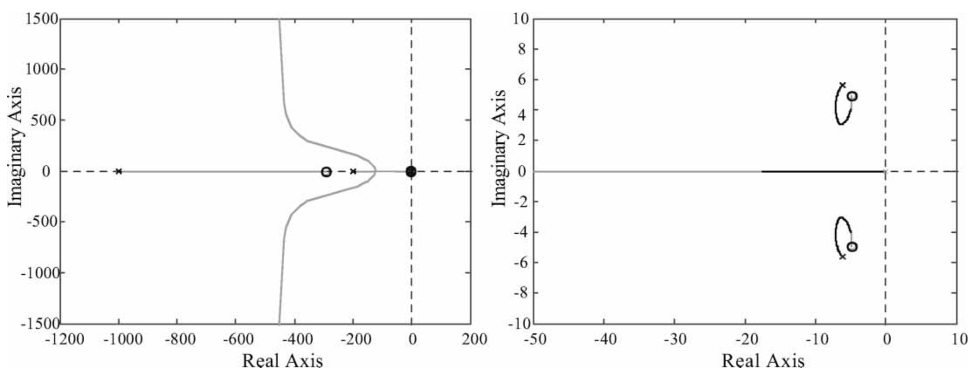

Figure 10. Root locus of the closed loop system varying the spring constant κ_1.

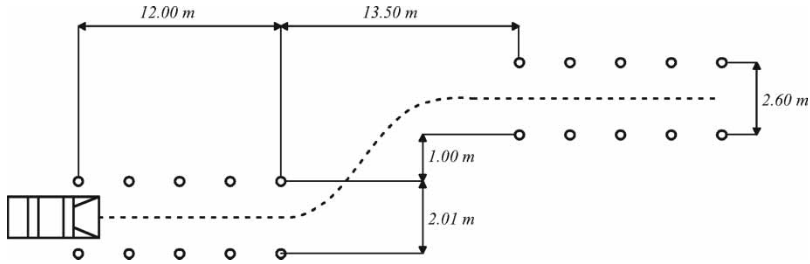

Figure 11. Single lane change test track.

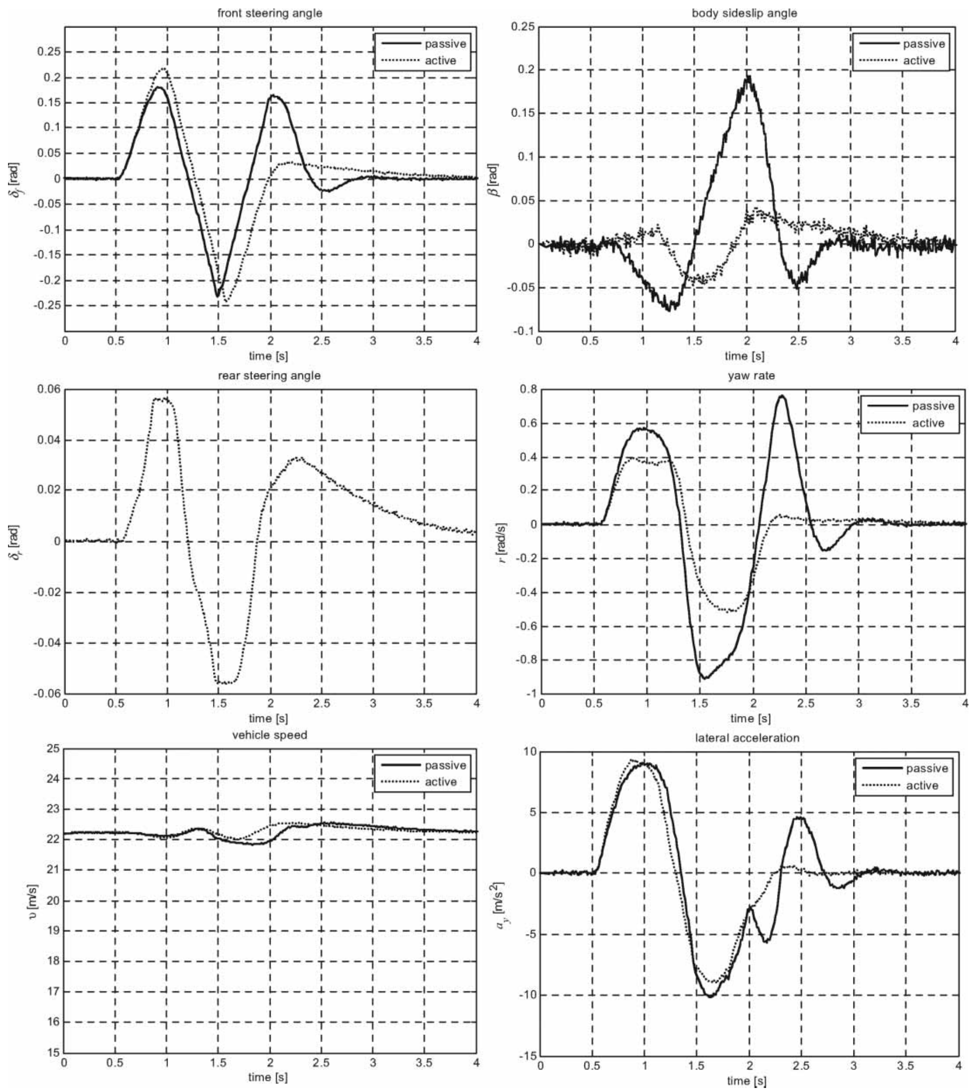

Figure 12. Single lane change using the model based controller.

4. Results

The two different approaches for active rear wheel steering (see Sections 3.1 and 3.2) were tested in the prototype vehicle equipped with an electromechanical actuator developed by MAGNA STEYR Fahrzeugtechnik, Graz. The controller was implemented on a dSPACE MicroAutoBox with a sampling time of 10 ms. The body sideslip angle was measured by an optical sensor.

In order to show a significant improvement of the road holding ability, a severe single lane change manoeuvre with a constant vehicle speed of 80 km/h through a well-defined test track was carried out. The dimensions of the test track according to [22] are shown in Figure 11.

Figures 12 and 13 show the front and the rear steering angles, vehicle speed, lateral acceleration, yaw rate and body sideslip angle during the test manoeuvre. Both approaches show a

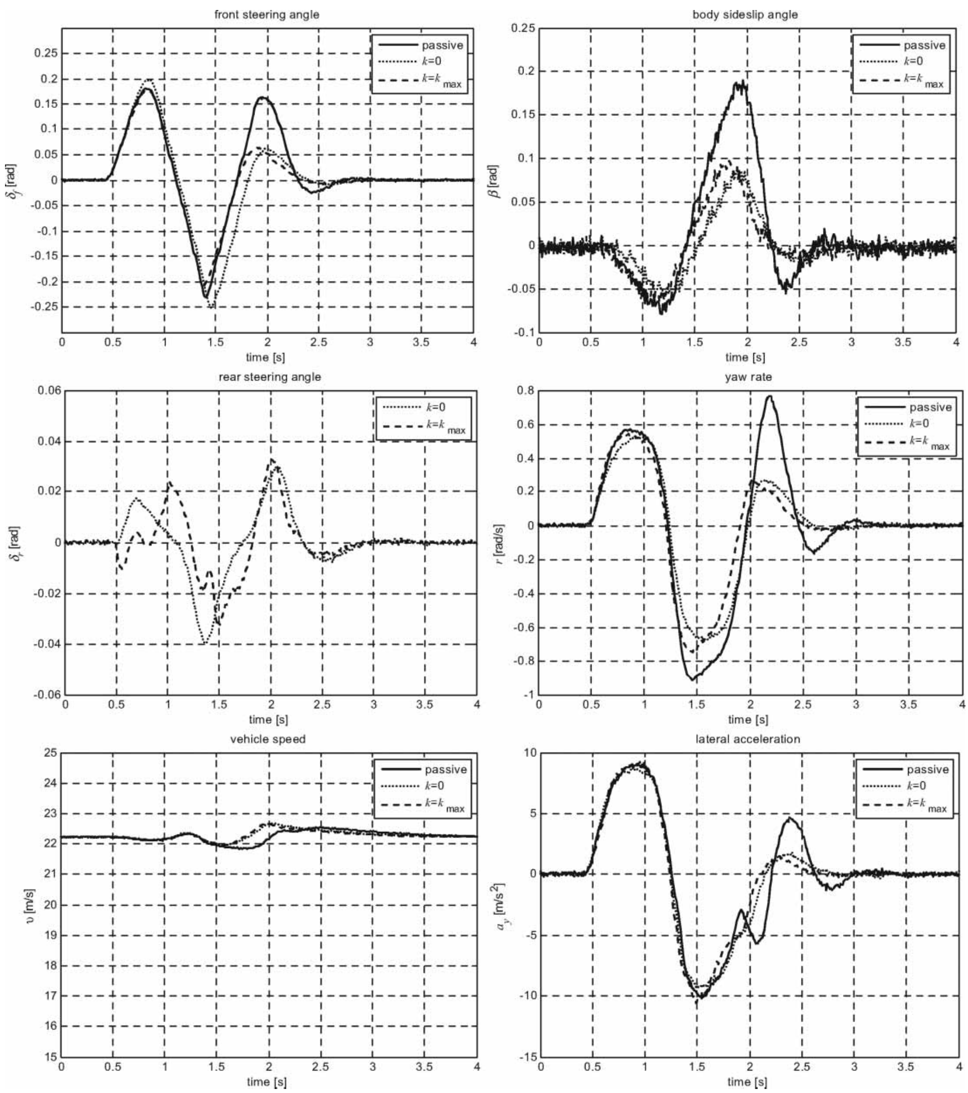

Figure 13. Single lane change using the mass-spring-damper system.

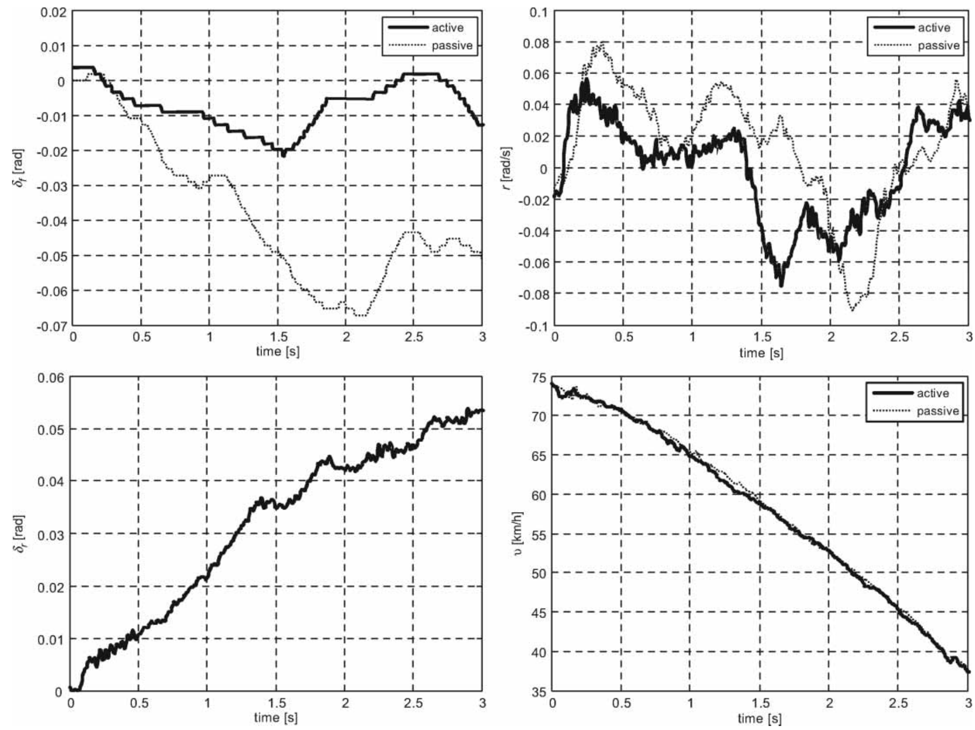

Figure 14. μ-split braking manoeuvre using the model based controller.

noticeable enhancement of the handling behaviour that can be seen especially when the driver stabilises the car in the new lane. Compared with the passive vehicle, i.e. the vehicle without active rear steering, the yaw rate as well as the body sideslip angle are clearly reduced. So the driver's effort to avoid skidding of the car by countersteering is decreased significantly. Note that a properly tuned model based controller can achieve similar performance as the mass-spring-damper concept. A drawback of the model based approach is that the online parameter tuning is difficult. Besides, an exact bicycle model for the vehicle is required. Therefore from a practical point of view the mass-spring-damper concept seems to be favourable.

Due to the integral part of the model based controller, the vehicle features an excellent behaviour in the so-called μ-split braking situation. The control mechanism compensates for the undesirable yaw rate that arises from braking on a road with different adhesive friction values between the right and the left side of the vehicle. Figure 14 shows data of a braking manoeuvre on wet tiles, respectively, dry tarmac starting with a vehicle speed of 75 km/h. In contrast to the passive vehicle the required corrective steering action of the active vehicle is shifted from the front to the rear wheels. So the handling of the vehicle can be facilitated considerably.

5. Conclusions

In this article we have outlined two approaches to the control of an active rear wheel steering system. An essential prerequisite for a successful application of the model based concept is an accurate parameterisation of the single track model. This fact is among others due to the required specification of the reference yaw rate r_{ref}. In the present case, the proposed control strategy proved to be very efficient in the μ-split braking situation. A drawback of the concept, however, is that the adaption to different car types requires considerable effort.

The second approach that is based on a mass-spring-damper structure overcomes the mentioned drawbacks. In addition, it permits intuitive controller tuning and thus provides portability to different vehicle types. In contrast to dynamical manoeuvres (e.g. lane changes) the handling of the vehicle in μ-split braking situations cannot be improved drastically. To achieve similar results as the model based control system, additional measures have to be taken. During μ-split braking situations, which can be detected from online measurements, the mass-spring-damper system has to be replaced by a standard proportional–integral controller.

Considering the driving comfort, the 'straight line driving' behaviour of the vehicle plays a major role. Due to its integral part, the model based controller shows undesirable control activity, e.g. forced by bumpy roads. In contrast, the mass-spring-damper structure exhibits satisfactory 'straight line' performance. Using springs and dampers with nonlinear characteristics the straight line driving behaviour can even be improved.

Acknowledgements

The authors gratefully acknowledge the support of MAGNA STEYR Fahrzeugtechnik, Graz.

Notes

1. Note that stability is not used in the sense of control technology but in the sense of directional stability.
2. Index f denotes the *front* axle, whereas index r denotes the *rear* axle.
3. All quantities lag the front steering angle.

References

[1] H. Bauer, *Kraftfahrtechnisches Taschenbuch: 25. Auflage*, Robert Bosch GmbH, Wiesbaden, 2003.
[2] J. Ackermann and W. Sienel, *Robust yaw damping of cars with front and rear wheel steering*, IEEE Trans. Control Syst. Technol. 1(1) (1993), pp. 15–20.
[3] K. Adachi, K. Ito, and T. Fujishiro, *Study on a new four-wheel-steering control method at low speeds-front-end path memorizing method*, Eighth International Conference on Automotive Electronics, 1991, London, pp. 214–218.
[4] Y. Wang and M. Nagai, *Integrated control of four-wheel-steer and yaw moment to improve dynamic stability margin*, Proceedings of the 35th Conference on Decision and Control, Kobe, Japan, 1996, pp. 1783–1784.
[5] S. Rezeka, *A feedforward neural network for the automatic control of a four-wheel-steering passenger car*, Proceedings of the American Control Conference, 1995, Seattle, WA, pp. 2292–2296.
[6] M. Yamamoto, *Active Control Strategy for Improved Handling and Stability*, SAE Trans. 911902 100(6) (1991), pp. 1638–1648.
[7] J. Ackermann, *Robust control prevents car skidding, 1996 Bode Prize Lecture*, IEEE Control Syst. Mag. 17(3) (1991), pp. 23–31.
[8] Y. Jia, *Robust control with decoupling performance for steering and traction of 4WS vehicles under velocity-varying motion*, IEEE Trans. Control Syst. Tech. 8(3) (2000), pp. 554–569.
[9] S. Mammar and D. Koenig, *Vehicle handling improvement by active steering*, Veh. Syst. Dyn. 38(3) (2002), pp. 211–242.
[10] X. Gao, B.D. McVey, and R.L. Tokar, *Robust controller design of four wheel steering system using μ-synthesis techniques*, Proceedings of the 34th Conference on Decision and Control, 1995, New Orleans, LA, pp. 875–882.
[11] S.-S. You and S.-K. Jeong, *Vehicle dynamics and control synthesis for four-wheel steering passenger cars*, Proc. Inst. Mech. Eng. D, 1998, pp. 449–461.
[12] S. Brennan and A. Alleyne, *Driver assisted yaw rate control*, Proceedings of the American Control Conference, 1999, San Diego, CA, pp. 1697–1701.
[13] A. Halanay, A. Ionita, and V. Rasvan, *Stability and maneuverability analysis of vehicle with four wheel steering system*, Proceedings of the 3rd IEEE Conference on Control Applications, 1994, Glasgow, pp. 385–390.
[14] A. Shaout, M.A. Jarrah, H. Al-Araji, and K. Al-Tells, *A nonlinear optimal four wheel steering controller*, Proceedings of the 43rd IEEE Midwest Symposium on Circuits and Systems, 2000, Lansing, MI, pp. 1426–1429.

[15] A.B. Will and M. Teixeira, *Four wheel steering control system design using fuzzy models*, Proceedings of the 1997 IEEE International Conference on Control Applications, 1997, Hartford, CT, pp. 73–78.
[16] H. Harada, *Control strategy of active rear wheel steering in consideration of system delay and dead times*, JSAE Rev. 16 (1995), pp. 171–177.
[17] S. Eitzinger, B. Zemla, M. Kreutz, M. Horn, and A. Riepl, *Simulation-based development of an active system to improve stability and agility of passenger vehicles*, Proceedings of the 7th International Symposium on Advanced Vehicle Control (AVEC'04), 2004, Arnhem, pp. 169–174.
[18] M. Kreutz and M. Horn, *Regelungskonzept für eine aktive Hinterach-slenkung*, SSRP 2005–14, Steirisches Seminar über Regelung-stechnik und Prozessautomatisierung, Schloss Retzhof/Leibnitz, 2005, pp. 3–15.
[19] P. Riekert and T. Schunck, *Zur Fahrmechanik des gummibereiften kraftfahrzeugs*, Ingenieur Arch. 11 (1940), pp. 210–224.
[20] C.-T. Chen, *Analog and Digital Control System Design: Transfer-Functions, State-Space, and Algebraic Methods*, Saunders College Publishing, 1993.
[21] N. Stamer, *Ermittlung optimaler PKW-Querdynamik und ihre Realisierung durch Allradlenkung*, VDI-Verlag, Düsseldorf, 1997.
[22] ISO 3888-2, *Passenger cars – Test track for a severe lane-change manoeuvre – Obstacle avoidance*, ISO, 2002.
[23] A. Zomotor, *Fahrwerktechnik: Fahrverhalten*, Vogel Verlag, Würzburg, 1987.
[24] M. Mitschke and H. Wallentowitz, *Dynamik der Kraftfahrzeuge: 4. Auflage*, Springer-Verlag, Berlin, Heidelberg, 2004.

Appendix 1. Nominal vehicle data for the prototype vehicle

m	1725 kg
J_z	1750 kgm^2
l_f	1.30 m
l_r	1.46 m
c_f	9631 N/rad
c_r	14194 N/rad
v	100 km/h