W1 Homework:(2026.02.26)

Estimating the Standard Deviation of OLS estimators in R

Consider the linear regression model

$$Y = X\beta + \varepsilon, \varepsilon \sim (0, \sigma^2 I).$$

The variance of the OLS estimator is

$$Var(\hat{\beta}_{OLS}) = \sigma^2 (X'X)^{-1}.$$

In practice, we estimate it by

$$\widehat{Var}(\hat{\beta}_{OLS}) = \hat{\sigma}^2 (X'X)^{-1} = \begin{pmatrix} \hat{b}_{11} & \cdots & \hat{b}_{1p} \\ & \vdots & \\ \hat{b}_{p1} & \cdots & \hat{b}_{pp} \end{pmatrix}_{p \times p},$$

where

$$\hat{\sigma}^2 = \frac{\hat{\varepsilon}'\hat{\varepsilon}}{n-p}, \quad \hat{\varepsilon} = Y - \hat{Y}, \quad \hat{Y} = X\hat{\beta}_{OLS}$$

The estimated standard deviation of the j-th OLS coefficient $\hat{\beta}_{j,OLS}$ is

$$sd(\hat{\beta}_{j,OLS}) = \sqrt{\hat{b}_{jj}}, \quad j = 1, \ldots, p$$

Using R, compute the estimated standard deviations of the OLS estimator in two different ways:

1. Using *lm*

2. Using Matrix Operations

Verify that the two approaches give identical results.