Luís Henrique Cruz — Lista 3

3) a) $\sim(p \vee \sim q)$

$\sim p \wedge \sim\sim q$

$\sim p \wedge q$

b) $\sim(\sim p \wedge q)$

$\sim\sim p \vee \sim q$

$p \vee \sim q$

c) $\sim(\sim p \vee \sim q)$

$\sim\sim p \wedge \sim\sim q$

$p \wedge q$

d) $(p \vee q) \wedge \sim p$

$(\sim p \wedge p) \vee (q \wedge \sim p)$

$\bot \vee (q \wedge \sim p)$

$q \wedge \sim p$

e) $(p \to q) \wedge (\sim p \to q)$

$(\sim p \vee q) \wedge (p \vee q)$

$q \wedge q$

q

4) $p \wedge (p \to q) \wedge (p \to \sim q)$

$p \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

$p \wedge \sim p \vee \sim p$

$p \wedge \sim p = \bot$

4) DEMONSTRAR $p \to q \iff ((p \wedge p) \wedge (p \wedge p)) \wedge (q \wedge q)$

$\sim((\sim p \vee \sim p) \vee \sim(\sim p \vee \sim p)) \vee \sim(\sim q \vee \sim q)$

$\sim(\sim(\sim p) \vee \sim(\sim p)) \vee \sim(\sim q)$

$\sim(p \vee p) \vee q$

$p \wedge q$

$\sim p \vee q \iff p \to q$

5) a) $p \wedge \sim p \Rightarrow q$

$\bot \to q$

$\sim \bot \vee q$

$\top \vee q \iff q$

b) $\sim p \to p \iff p$

$p \vee p \iff p$

$p \iff p$

$(\sim p \vee p) \wedge (p \vee \sim p)$

$p \wedge p$

p

c) $p \to q \wedge q \iff p \to q$

$(p \wedge q) \to (\sim p \vee q)$

$p \wedge q \iff \sim p \vee q$

$(p \wedge q) \to (p \vee q) \wedge (\sim p \vee q) \to (p \vee q)$

$\sim(p \vee q) \vee (\sim p \wedge q) \vee \sim(\sim p \vee q) \vee (p \vee q)$

c) $p \to p \wedge q \iff p \to q$

$\sim p \vee (p \wedge q)$

$(\sim p \vee p) \wedge (\sim p \vee q)$

$T \wedge (\sim p \vee q)$

$\sim p \vee q \iff q \wedge \sim p$

d) $(p \to q) \to q \iff p \vee q$

$\sim(\sim p \vee q) \vee q$

$p \wedge \sim q \vee q$

$p \vee T$

$T \iff p \vee q$

$p \to (p \vee q) \wedge (p \vee q) \to p$

$\sim p \vee (p \vee q) \wedge (\sim p \wedge \sim q) \vee p$

$T \vee q \wedge \sim p \wedge \sim q \vee p$

$T \vee \underbrace{q \wedge \sim q}_{\perp} \wedge \underbrace{\sim p \vee p}_{T}$

$T \vee \perp \wedge T$

$\iff$

T

e) $(p \to r) \vee (q \to r) \iff p \wedge q \to r$

$(\sim p \vee r) \vee (\sim q \vee r)$

$\sim p \vee \sim q \vee r \iff r \wedge \sim(p \wedge q)$

$r \wedge \sim p \vee \sim q$

f) $(p \to q) \wedge (p \to r) \iff p \to q \wedge r$

$(\sim p \vee q) \wedge (\sim p \vee r)$

$\sim p \vee q \wedge r$

$p \wedge q \to r$

g)

$p \downarrow q \iff (((p \uparrow (q \uparrow q)) \uparrow ((p \uparrow p) \uparrow (q \uparrow q))) \uparrow ((p \uparrow (q \uparrow q)) \uparrow ((p \uparrow p) \uparrow (q \uparrow q))))$

$(((p \uparrow p) \uparrow (q \uparrow q)) \uparrow ((p \uparrow p) \uparrow (q \uparrow q)))$

$((p \uparrow p) \uparrow (q \uparrow q)) \uparrow ((p \uparrow p) \uparrow (q \uparrow q))$

$\sim(p \vee q) \vee \sim(p \vee q)$

$\sim(p \vee q)$

$\sim p \wedge \sim q \iff \sim p \wedge \sim q$

4) a)

p	→	q	= ψ
V	V	V	
V	F	F	
F	V	V	
F	V	F	

FNC(ψ) = (p ∨ ~q)

b)

p	→	~	p	= ψ
V	F	F	V	
F	V	V	F	

FNC(ψ) = p

c)

p	↔	~	p	= ψ
V	F	F	V	
F	F	V	F	

FNC(ψ) = (p) ∧ (~p)

d)

p	V	~	p	= ψ
V	V	F	V	
F	V	V	F	

FNC(ψ) = não há

e) p ↑ q

~	p	∧	~	q
F	V	F	F	V
F	V	F	V	F
V	F	F	F	V
V	F	V	V	F

FNC(ψ) = (p ∨ q) ∧ (p ∨ ~q) ∧ (~p ∨ q)

f) p ↑ p = ψ

FNC(ψ) = p

~	p	V	~	p
F	V	F	F	V
V	F	V	V	F

g) p ↑ ~p = ψ

~	p	V	p
F	V	V	V
V	F	V	F

FNC(ψ) = não há

h) p ↓ q = ψ

~	p	∧	~	q
F	V	F	F	V
F	V	F	V	F
V	F	F	F	V
V	F	V	V	F

FNC(ψ) = (p ∨ q) ∧ (p ∨ ~q) ∧ (~p ∨ q)

i) (p ∧ ~p) ↓ (q ∧ ~q) = ψ

(~	p	V	p)	∧	(~	q	V	q)
F	V	V	V	V	F	V	V	V
F	V	V	V	V	V	F	V	F
V	F	V	F	V	F	V	V	V
V	F	V	F	V	V	F	V	F

FNC(ψ) = não há

i) $(\sim p \wedge q) \veebar q = \psi$

p	q	$\sim p$	$\sim p \wedge q$	$(\sim p \wedge q) \veebar q$
V	V	F	F	V
V	F	F	F	F
F	V	V	V	V
F	F	V	F	F

$FNC(\psi) = (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

k) $(p \barwedge q) \leftrightarrow p = \psi$

p	q	$p \barwedge q$	$(p \barwedge q) \leftrightarrow p$
V	V	F	F
V	F	V	V
F	V	V	V
F	F	V	F

$FNC(\psi) = (p \vee q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

L) $\sim p \barwedge (q \veebar p) = \psi$

p	q	$\sim p$	$q \veebar p$	$\sim p \barwedge (q \veebar p)$
V	V	F	F	V
V	F	F	V	F
F	V	V	V	F
F	F	V	F	F

$FNC(\psi) = (\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$

m) $p \barwedge \sim (q \veebar r) = \psi$

p	q	r	$q \veebar r$	$\sim (q \veebar r)$	$p \barwedge \sim (q \veebar r)$
V	V	V	F	V	F
V	V	F	V	F	V
V	F	V	V	F	V
V	F	F	F	V	F
F	V	V	F	V	V
F	V	F	V	F	V
F	F	V	V	F	V
F	F	F	F	V	F

$FNC(\psi) = (p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee r)$

N) $(\sim(\sim p \barwedge \sim q)) \barwedge (r \rightarrow \sim r) = \psi$

p	q	r	$\sim p$	$\sim q$	$\sim p \barwedge \sim q$	$\sim(\sim p \barwedge \sim q)$	$r \rightarrow \sim r$	$(\sim(\sim p \barwedge \sim q)) \barwedge (r \rightarrow \sim r)$
V	V	V	F	F	V	F	F	V
V	V	F	F	F	V	F	V	F
V	F	V	F	V	F	V	F	V
V	F	F	F	V	F	V	V	F
F	V	V	V	F	F	V	F	V
F	V	F	V	F	F	V	V	F
F	F	V	V	V	V	F	F	V
F	F	F	V	V	V	F	V	F

$FNC(\psi) = (\sim p \vee q \vee r) \wedge (\sim p \vee \sim q \vee r) \wedge (r \vee \sim p \vee \sim q) \wedge (p \vee q \vee \sim r) \wedge (p \vee \sim q \vee \sim r)$

8) a) $\sim(\sim p \vee \sim q) = \psi$

p	∧	q
V	V	V
V	F	F
F	F	V
F	F	F

$FND(\psi) = (p \wedge q)$

b) $\sim (p \rightarrow q) = \psi$

p	→	q	
F	V	V	V
V	V	F	F
F	F	V	V
F	F	V	F

$FND(\psi) = (p \wedge \sim q)$

c) $(p \rightarrow q) \wedge \sim p = \psi$

p	→	q	∧	~	p
V	V	V	F	F	V
F	V	V	V	V	F
V	F	F	F	F	V
F	V	F	V	V	F

$FND(\psi) = (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

d) $\sim (p \vee q) = \psi$

p	∨	q	
V	V	V	F
V	V	F	F
V	V	V	F
F	F	F	V

$FND(\psi) = (\sim p \wedge \sim q)$

e) $(p \rightarrow q) \vee \sim p = \psi$

p	→	q	∨	~	p
V	V	V	V	F	V
F	V	V	V	V	F
V	F	V	F	F	V
F	V	F	V	V	F

$FND(\psi) = (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

f) $\sim (p \wedge q) = \psi$

p	∧	q	
F	V	V	V
V	V	F	F
V	F	F	V
F	F	F	F

$FND(\psi) = (p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

g) $p \vee \sim p = \psi$

p	∨	~	p
V	V	F	V
F	V	V	F

$FND(\psi) = (p) \vee (\sim p)$

h) $p \leftrightarrow \sim p = \psi$

p	↔	~	p
V	F	F	V
F	F	V	F

$FND(\psi) =$ não há

i) $p \uparrow q = \psi$

~p	∨	~q	
F	V	F	V
F	V	V	F
V	F	V	V
V	F	V	F

$FND(\psi) = (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

j) $p \downarrow q = \psi$

~	p	∧	~	q
F	V	F	F	V
F	V	F	V	F
V	F	F	F	V
V	F	V	V	F

FND(ψ) = $(\sim p \wedge \sim q)$

k) $p \bar{\vee} p = \psi$

~	p	∨	~	p
F	V	F	F	V
V	F	V	V	F

FND(ψ): $\sim p$

l) $p \bar{\vee} \sim p = \psi$

~	p	∨	p
F	V	V	V
V	F	V	F

FND(ψ) = $(p) \vee (\sim p)$