

My Notebook

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Chapter 1

Mathematical and Physics Background

1.1 Universal Topics

1.1.1 Scientific Notation and International Standards

SI Notation

The *International System of Units (SI)* is the modern form of the metric system and is the most widely used system of measurement. The SI system uses a set of base units and derived units to define the quantities used in scientific and engineering calculations. The base units are defined by the *International System of Units (SI)* as follows:

Universal Standards

- Meter (m) - length, a fundamental dimension
- Kilogram (kg) - mass, a fundamental dimension
- Second (s) - time, a fundamental dimension
- Ampere (A) - electric current, rate of flow of electric charge (Coulombs per second), a derivative function with respect to time
- Kelvin (K) - temperature, a fundamental dimension
- Mole (mol) - amount of substance, a fundamental dimension
- Candela (cd) - luminous intensity, power per unit solid angle, a derived dimension (power divided by angle)

Derived Units

- Newton (N) - force, mass times acceleration ($kg \cdot m/s^2$)
- Joule (J) - energy, force times distance ($N \cdot m = kg \cdot m^2/s^2$)
- Watt (W) - power, energy per unit time ($J/s = kg \cdot m^2/s^3$)
- Pascal (Pa) - pressure, force per unit area ($N/m^2 = kg/(m \cdot s^2)$)
- Ohm (Ω) - electric resistance, voltage per unit current ($V/A = kg \cdot m^2/(s^3 \cdot A^2)$)
- Volt (V) - electric potential, work per unit charge ($J/C = kg \cdot m^2/(s^3 \cdot A)$)
- Farad (F) - capacitance, charge per unit voltage ($C/V = s^4 \cdot A^2/(kg \cdot m^2)$)

- Henry (H) - inductance, magnetic flux per unit current ($Wb/A = kg \cdot m^2/(s^2 \cdot A^2)$)
- Siemens (S) - electrical conductance, reciprocal of resistance ($1/\Omega = s^3 \cdot A^2/(kg \cdot m^2)$)
- Weber (Wb) - magnetic flux, magnetic field times area ($T \cdot m^2 = kg \cdot m^2/(s^2 \cdot A)$)
- Tesla (T) - magnetic field, magnetic flux per unit area ($Wb/m^2 = kg/(s^2 \cdot A)$)
- Coulomb (C) - electric charge, current times time ($A \cdot s$)
- Lumen (lm) - luminous flux, light power ($cd \cdot sr$)
- Lux (lx) - illuminance, light power per unit area ($lm/m^2 = cd \cdot sr/m^2$)

Prefixes

Prefix	Factor
Yotta (Y)	10^{24}
Zetta (Z)	10^{21}
Exa (E)	10^{18}
Peta (P)	10^{15}
Tera (T)	10^{12}
Giga (G)	10^9
Mega (M)	10^6
Kilo (k)	10^3
Hecto (h)	10^2
Deca (da)	10^1
Deci (d)	10^{-1}
Centi (c)	10^{-2}
Milli (m)	10^{-3}

Table 1.1: SI Prefixes

Relevant Mathematical, Physical and Chemical Constants

Constant	Value	Unit/Field
Mathematical Constants		
Pi (π)	3.14159265358979323846	Geometry
Euler's number (e)	≈ 2.71828	Calculus
Golden ratio (ϕ)	≈ 1.61803	Algebra
Imaginary unit (i)	$\sqrt{-1}$	Complex numbers
Euler–Mascheroni constant (γ)	≈ 0.57721	Number theory
Physical Constants		
Speed of light in vacuum (c)	$\approx 3.00 \times 10^8$	m/s
Planck's constant (h)	$\approx 6.63 \times 10^{-34}$	J s
Reduced Planck's constant (\hbar)	$\approx 1.05 \times 10^{-34}$	J s
Gravitational constant (G)	$\approx 6.67 \times 10^{-11}$	$m^3 \text{ kg}^{-1} \text{ s}^{-2}$
Electron charge (e)	$\approx 1.60 \times 10^{-19}$	C
Chemical Constants		
Avogadro's number (N_A)	$\approx 6.02 \times 10^{23}$	mol^{-1}
Boltzmann's constant (k_B)	$\approx 1.38 \times 10^{-23}$	J/K
Gas constant (R)	≈ 8.31	J/(mol K)
Faraday's constant (F)	≈ 96485	C/mol
Stefan-Boltzmann constant (σ)	$\approx 5.67 \times 10^{-8}$	$W/(m^2 \text{ K}^4)$

Table 1.2: Important Mathematical, Physical and Chemical Constants

Standards Reference

ISO (International Organization for Standardization):

- ISO 9001: Quality Management Systems
- ISO 14001: Environmental Management Systems
- ISO 27001: Information Security Management
- ISO 31000: Risk Management
- ISO 80000-2: Mathematical signs and symbols to be used in the natural sciences and technology
- ISO 8601: Representation of dates and times
- ISO 3166-1: Country codes
- ISO/IEC 5218: Codes for the representation of human sexes
- ISO/IEC 15418: GS1 Application Identifiers and ASC MH10 Data Identifiers and maintenance
- ISO/IEC 15459: Unique identifiers
- ISO/IEC 15459-4: Individual transport units
- ISO/IEC 15459-6: Groupings
- ISO/IEC 19762-1: Harmonized vocabulary
- ISO/IEC 19762-4: RFID for Item Management
- ISO/IEC 20248: Symbolology data carrier identification

DIN (Deutsches Institut für Normung):

- DIN 476: Paper sizes (now replaced by ISO 216)
- DIN 72552: Terminal designations in automotive electrical systems
- DIN 82079: Preparation of instructions for use
- DIN 931: Hexagon head bolts with shank
- DIN 933: Hexagon head bolts with thread up to head
- DIN 912: Hexagon socket head cap screws
- DIN 7985: Cross recessed pan head screws

IEC (International Electrotechnical Commission):

- IEC 60038: Standard voltages
- IEC 60529: Degrees of protection provided by enclosures (IP Code)
- IEC 60950: Safety of information technology equipment
- IEC 61000: Electromagnetic compatibility (EMC)

IEEE (Institute of Electrical and Electronics Engineers):

- IEEE 802.3: Ethernet
- IEEE 802.11: Wireless Networking (Wi-Fi)
- IEEE 754: Floating-Point Arithmetic
- IEEE 1588: Precision Time Protocol
- IEEE 802.15.1: Bluetooth
- IEEE 802.16: Broadband Wireless Access (WiMAX)

NBR/NR (Norma Brasileira Regulamentadora):

- NR 10: Safety in Electrical Installations and Services
- NR 12: Safety in Work with Machines and Equipment
- NBR 5410: Electrical Installations in Low Voltage
- NR 35: Work at Height
- NR 17: Ergonomics
- NR 18: Working Conditions and Environment in the Construction Industry
- NBR 10126: Technical Drawing
- NBR 14565: Procedures for data communication network within commercial buildings
- NBR IEC 60079-11: Electrical apparatus for explosive gas atmospheres
- NBR 8403: Application of lines in technical drawings

Error Tolerances

Tolerance	Field	Impact
± 0.1 mm	Hand Tools Manufacturing	Impacts tool precision and fit
$\pm 5\%$	Power Generation Voltage	Impacts system performance and safety
$\pm 1\%$	Power Transmission Voltage	Impacts system performance and safety
$\pm 0.5\%$	Power Distribution Voltage	Impacts system performance and safety
$\pm 1\%$	Electronics (Resistance, Capacitance, Inductance)	Impacts circuit performance
$\pm 0.5\%$	Electronics (Voltage)	Impacts circuit performance
$\pm 0.1\%$	Electronics (Grounding)	Impacts system safety and performance
$\pm 1^\circ\text{C}$	Electronics (Temperature)	Impacts component lifespan and performance
± 0.5 ULP	Computational Algebra (Floating Point)	Impacts accuracy of calculations

Table 1.3: Acceptable Error Tolerances and Their Impacts

Error Tolerances for Multimeters and Ammeters

Residential Circuits:

- AC Voltage: $\pm 1.5\%$ - Impacts safety and device performance
- DC Voltage: $\pm 0.5\%$ - Impacts safety and device performance
- AC Current: $\pm 2.5\%$ - Impacts safety and device performance
- DC Current: $\pm 2.0\%$ - Impacts safety and device performance
- Resistance: $\pm 0.5\%$ - Impacts safety and device performance
- Continuity: $\pm 0.1\%$ - Impacts safety and device performance
- Capacitance: $\pm 1.0\%$ - Impacts safety and device performance

Industrial Circuits:

- AC Voltage: $\pm 1.0\%$ - Impacts safety, device performance, and costs
- DC Voltage: $\pm 0.5\%$ - Impacts safety, device performance, and costs
- AC Current: $\pm 1.5\%$ - Impacts safety, device performance, and costs
- DC Current: $\pm 1.0\%$ - Impacts safety, device performance, and costs
- Resistance: $\pm 0.5\%$ - Impacts safety, device performance, and costs
- Continuity: $\pm 0.1\%$ - Impacts safety, device performance, and costs
- Capacitance: $\pm 1.0\%$ - Impacts safety, device performance, and costs

Chapter 2

Electrical Circuits I

2.1 Resistors

2.1.1 Resistor Notation - Algebraic Definition

Definition: Resistor A resistor is a two-terminal passive electrical component that implements electrical resistance as a circuit element.



Definition: Algebraic Definition The algebraic definition of a resistor is given by Ohm's Law, which states that the current through a conductor between two points is directly proportional to the voltage across the two points.

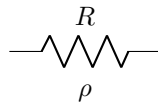
$$D : v \longrightarrow i \quad \text{where} \quad v = i \cdot R \quad V = i \cdot R \quad (2.1)$$

- Resistance as a function of voltage and current
- Unit

$$\frac{V}{A} = \Omega$$

2.1.2 Resistor Notation - Geometric Definition

Definition: Geometric Definition The geometric definition of a resistor is given by the resistor's physical properties, such as its length, cross-sectional area, and resistivity.



$$R = \rho \cdot \frac{l}{A} \quad (2.2)$$

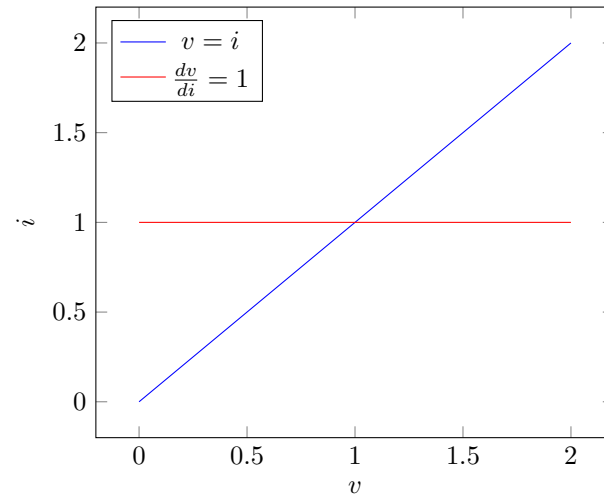
where:

- R is the resistance of the resistor
- ρ is the resistivity of the material
- l is the length of the resistor
- A is the cross-sectional area of the resistor

Geometrical Interpretation of Resistance as Relationship between Voltage and Current

Let's consider the following diagram, which represents a resistor with a voltage source connected to it. The voltage source creates an electric field within the resistor, which causes the free electrons to move in the direction of the electric field. This movement of electrons creates a current flow through the resistor.

Resistance and voltage derivative of a non-electrical variable



2.2 Circuit Analysis

2.2.1 Kirchhoff's Laws

Definition: Kirchhoff's Laws Kirchhoff's Laws are two fundamental laws that govern the behavior of electrical circuits.

There is two things to consider: devices and topology restrictions.

- Kirchhoff's Current Law (KCL)
- Kirchhoff's Voltage Law (KVL)
- Devices: resistors, capacitors, inductors, etc.

Kirchhoff's Current Law - KCL

Definition: Kirchhoff's Current Law Kirchhoff's Current Law states that the sum of currents entering a node is equal to the sum of currents leaving a node.

$$\sum_{i=1}^n i_{\text{in}} = \sum_{i=1}^n i_{\text{out}} \quad (2.3)$$

The circuit diagrams above illustrate *Kirchhoff's Current Law*(KCL), which states that the algebraic sum of currents entering a node (or a junction) equals zero. This principle is a consequence of the conservation of electric charge.

In the first diagram, we see positive charges (+) flowing into a node. Each arrow represents a current path, and the label i_{in}^- indicates that these are incoming currents. In the second diagram, we see negative charges (−) flowing out of a node, which represent electrons moving out of the node, creating a current.

The arrows again indicate the direction of the current flow, and the label i_{out}^- indicates that these are outgoing currents. According to KCL, the sum of the incoming currents should equal the sum of the outgoing currents at a node, which is visually represented in these diagrams.

The graph below illustrates the concept of Kirchhoff's Current Law, showing the algebraic sum of currents entering a node equal to zero in the passive convention.

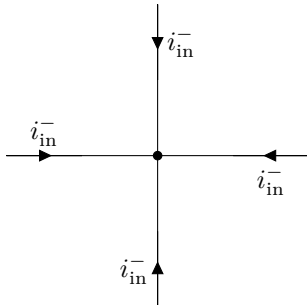


Figure 2.1: Positive charges (+) flowing into the node

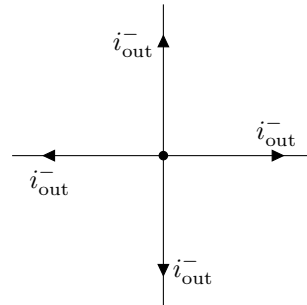


Figure 2.2: Negative charges (−) flowing out of the node

Kirchhoff's Voltage Law - KVL

Definition: Kirchhoff's Voltage Law Kirchhoff's Voltage Law states that the sum of voltages around a closed loop is equal to zero.

At any closed loop, the sum of the voltage sources is equal to the sum of the voltage drops.

$$\sum_{i=1}^n v_{\text{in}} = \sum_{i=1}^n v_{\text{out}} \quad (2.4)$$

The circuit diagram above represents a node with negative charges flowing out. This is a visualization of Kirchhoff's Current Law, which states that the algebraic sum of currents entering a node (or a junction) equals zero. In this case, the negative charges represent electrons moving out of the node, which creates a current.

The arrows indicate the direction of the current flow. Each arrow represents a current path, and the label i_{out}^- indicates that these are outgoing currents. This diagram is a useful tool for understanding the behavior of electrical circuits, particularly the principle of conservation of electric charge, which is the basis of *Kirchhoff's Current Law*.

Sum of voltages around a closed loop is equal to zero independent of the direction of the loop.

$$\begin{aligned} V_g &= V_1 + V_2 + V_3 \\ &= i_g \cdot R_1 + i_g \cdot R_2 + i_g \cdot R_3 \\ &= i_g \cdot (R_1 + R_2 + R_3) \\ &= i_g \cdot R_{\text{eq}} \end{aligned} \quad (2.5)$$

$$\begin{aligned} V_g &= -V_1 - V_2 - V_3 \\ \therefore R_{eq}^+ &\equiv R_{eq}^- \end{aligned} \quad (2.6)$$

Figure 2.3: Circuit from + to -

Figure 2.4: Circuit from - to +

Example I. Kirchhoff's Laws in a Series Circuit with Resistors

Consider a simple series circuit with a DC power source and two resistors, as shown below, **assuming the passive convention**:

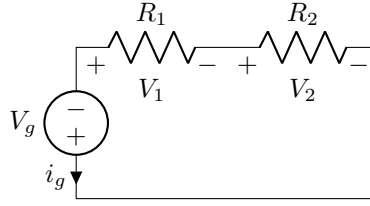


Figure 2.5: Series circuit with a DC power source and two resistors

According to Kirchhoff's Voltage Law (KVL), the sum of the voltages around any closed loop in the circuit is equal to zero. For this circuit, the equation is:

$$V_g = V_1 + V_2 \quad (2.7)$$

According to Ohm's law, the voltage across a resistor is equal to the current through the resistor times the resistance of the resistor. So we can write:

$$V_1 = i_g \cdot R_1 \quad (2.8)$$

$$V_2 = i_g \cdot R_2 \quad (2.9)$$

Substituting these equations into the KVL equation, we get:

$$V_g = i_g \cdot R_1 + i_g \cdot R_2 = i_g \cdot (R_1 + R_2) \quad (2.10)$$

This equation tells us that the voltage of the power source is equal to the current through the circuit times the total resistance of the circuit.

According to Kirchhoff's Current Law (KCL), the sum of the currents entering a node (or a junction) equals the sum of the currents leaving the node. For this circuit, the equation is:

$$i_g = i_1 = i_2 \quad (2.11)$$

This equation tells us that the current through the power source is equal to the current through each resistor, which is a characteristic of series circuits.

It's important to note that the direction of the loop does not affect the application of KVL. Whether we move in the direction of the current (from + to -) or against it (from - to +), the sum of the voltages around the loop is still zero. This is true even in the presence of a power source or a load, where the power is negative or positive, respectively.

Example II. Proof that the direction of the loop does not affect the application of KVL

Consider a simple series circuit with a DC power source and two resistors, as shown below, **assuming the passive convention**:

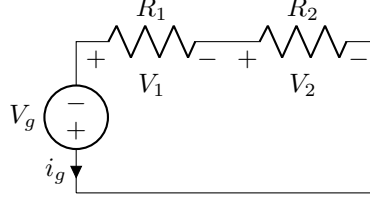


Figure 2.6: Series circuit with a DC power source and two resistors

Given the values:

- Voltage of the power source, $V_g = 12V$
- Resistance of the first resistor, $R_1 = 2\Omega$
- Resistance of the second resistor, $R_2 = 3\Omega$
- Current through the circuit, $i_g = 2A$

We can calculate the voltage across each resistor using **Ohm's law**, $V = i \cdot R$:

$$V_1 = i_g \cdot R_1 = 2A \cdot 2\Omega = 4V \quad (2.12)$$

$$V_2 = i_g \cdot R_2 = 2A \cdot 3\Omega = 6V \quad (2.13)$$

Now we can apply **KVL** in the direction of the current (from + to -):

$$V_g - V_1 - V_2 = 0 \quad (2.14)$$

Substituting the given values:

$$12V - 4V - 6V = 2V \neq 0 \quad (2.15)$$

This is not consistent with KVL. The issue here is that the current i_g is not correct. The current through the circuit should be equal to the voltage of the power source divided by the total resistance of the circuit, according to Ohm's law. So, the correct current is:

$$i_g = \frac{V_g}{R_1 + R_2} = \frac{12V}{2\Omega + 3\Omega} = 2.4A \quad (2.16)$$

Substituting this current into the voltages across the resistors:

$$V_1 = i_g \cdot R_1 = 2.4A \cdot 2\Omega = 4.8V \quad (2.17)$$

$$V_2 = i_g \cdot R_2 = 2.4A \cdot 3\Omega = 7.2V \quad (2.18)$$

Substituting these voltages into the KVL equation:

$$12V - 4.8V - 7.2V = 0V \quad (2.19)$$

Now, the sum of the voltages around the loop is equal to zero, which is consistent with KVL. So, the direction of the loop does not affect the application of KVL, as long as we follow the passive sign convention and use the correct current.