Lab 4 Prelab: Part 2

1. Repulsion upward from the workspace plane, parallel to the x_0-y_0 plane, with $z_0 = 32\,mm$

Using the textbook's notation:

$$F_{i,rep} = \begin{cases} \eta \left(\frac{1}{\rho(q)} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2(q)} \nabla \rho(q) & \text{if } \rho(q) < \rho_0 \\ 0 & \text{otherwise} \end{cases}$$

where $\rho(q) = \|q-b\|$ if the set is convex and b is the point on the boundary closest to q, and $\nabla \rho(q) = \frac{q-b}{\|q-b\|}$

Let a point b on the workspace plane be represented as follows:

$$b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \begin{bmatrix} O_i(q)_x \\ O_i(q)_y \\ 0 \end{bmatrix}$$

then $\rho(q)$ is as follows:

$$\rho(q) = \left\| \begin{bmatrix} O_i(q)_x - O_i(q)_x \\ O_i(q)_y - O_i(q)_y \\ O_i(q)_z - 0 \end{bmatrix} \right\| = \left\| \begin{bmatrix} 0 \\ 0 \\ O_i(q)_z \end{bmatrix} \right\|$$

and $\nabla \rho(q)$ is as follows:

$$\nabla \rho(q) = \frac{O_i(q) - b}{\|O_i(q) - b\|} = \frac{O_i(q)_z}{\|O_i(q)_z\|}$$

So

$$F_{i,rep} = \begin{cases} \eta_i \left(\frac{1}{\|O_i(q)_z\|} - \frac{1}{\rho_0} \right) \times \frac{1}{\|O_i(q)_z\|^2} \wedge \frac{O_i(q)_z}{\|O_i(q)_z\|} & \text{if } O_i(q)_z \leq \rho_0 \\ \emptyset & \text{otherwise} \end{cases}$$

2. Repulsion from a cylinder of finite length where the bottom of the cylinder lies on the x_0-y_0 plane and the height of the cylinder is h

Assume that the cylinder is centered at

$$C = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} \text{ where } C_z = \begin{cases} O_i(q)_z & \text{if } O_i(q)_z \le h \\ h & \text{otherwise} \end{cases}$$

Then from Lab 3's prelab, we know:

$$b = \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \text{ where}$$

$$b_x = C_x + R\left(\frac{O_i(q)_x - C_x}{\|O_i(q)_{xy} - C_{xy}\|}\right); \quad b_y = C_y + R\left(\frac{O_i(q)_y - C_y}{\|O_i(q)_{xy} - C_{xy}\|}\right); \quad b_z = C_z \text{ as defined above}$$

Using the textbook's notation:

$$F_{i,rep} = \begin{cases} \eta\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(q)} \nabla \rho(q) & \text{if } \rho(q) < \rho_0 \\ 0 & \text{otherwise} \end{cases}$$

where $\rho(q) = \|q-b\|$ if the set is convex and b is the point on the boundary closest to q, and $\nabla\rho(q) = \frac{q-b}{\|q-b\|}$

$$\rho(q) = \|O_i(q) - b\|$$

$$\nabla\rho(q) = \frac{O_i(q) - b}{\|O_i(q) - b\|}$$

So

$$F_{i,rep} = \begin{cases} \eta_i\left(\frac{1}{\|O_i(q) - b\|} - \frac{1}{\rho_0}\right) \times \frac{1}{\|O_i(q) - b\|^2} \times \frac{O_i(q) - b}{\|O_i(q) - b\|} & \text{if } \|O_i(q) - b\| \le \rho_0 \\ 0 & \text{otherwise.} \end{cases}$$