LAB 3 PRELAB: PART 3

Case 1: A sphere of radius R centered at $c = (c_x, c_y, c_z)$

b is the point on the boundary of the sphere that is closest to $O_i(q)$

- all points on the boundary of the sphere are a distance R away from c, and in the direction of the vector $O_i(q) - c$

$$b = c + R \cdot \frac{O_i(q) - c}{\|O_i(q) - c\|}$$

$$O_i(q) - b = O_i(q) - c - R \frac{O_i(q) - c}{\|O_i(q) - c\|}$$

$$\boxed{O_i(q) - b = (O_i(q) - c)\left(1 - R \cdot \frac{1}{\|O_i(q) - c\|}\right)}$$

$$\|O_i(q) - b\| = \|O_i(q) - c\|\left(1 - R \frac{1}{\|O_i(q) - c\|}\right)$$

$$\boxed{\|O_i(q) - b\| = \|O_i(q) - c\| - R}$$

Case 2: A cylinder of infinite height centered at $c = (c_x, c_y)$ with radius R and axis parallel to the z_0 axis.

b is a point on the boundary of the cylinder that is closest to $O_i(q)$

$$b = C + R\,\frac{O_i(q) - C}{\|O_i(q) - C\|} \quad \text{where} \quad C = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix}, \quad C_z = O_i(q)_z$$

and $O_i(q) - C = O_i(q)_{x,y,z} - C_{x,y,z}$ (ie: component-wise subtraction)

$$b = \begin{bmatrix} C_x \\ C_y \\ O_i(q)_z \end{bmatrix} + R\left(\frac{[O_i(q)_x \vdots O_i(q)_y \vdots O_i(q)_z]^T - [C_x \vdots C_y \vdots C_z]^T}{\|[O_i(q)_x \vdots O_i(q)_y \vdots O_i(q)_z]^T - [C_x \vdots C_y \vdots C_z]^T\|}\right)$$

$$O_i(q) - b = \begin{bmatrix} O_i(q)_x \\ O_i(q)_y \\ O_i(q)_z \end{bmatrix} - \begin{bmatrix} C_x \\ C_y \\ O_i(q)_z \end{bmatrix} - R\left(\left(\begin{bmatrix} O_i(q)_x \\ O_i(q)_y \\ O_i(q)_z \end{bmatrix} - \begin{bmatrix} C_x \\ C_y \\ O_i(q)_z \end{bmatrix}\right) \cdot \frac{1}{\|O_i(q)_{x,y} - C\|}\right)$$

$$\boxed{O_i(q) - b = \begin{bmatrix} O_i(q)_x - C_x \\ O_i(q)_y - C_y \\ 0 \end{bmatrix}\left(1 - R \cdot \frac{1}{\|O_i(q)_{x,y} - C\|}\right)}$$

$$\|O_i(q) - b\| = \|O_i(q)_{x,y} - C\| \cdot \left(1 - R \cdot \frac{1}{\|O_i(q)_{x,y} - C\|}\right)$$

$$\boxed{\|O_i(q) - b\| = \|O_i(q)_{x,y} - C\| - R}$$