ECE 470: Lab 1 Prelab

1.

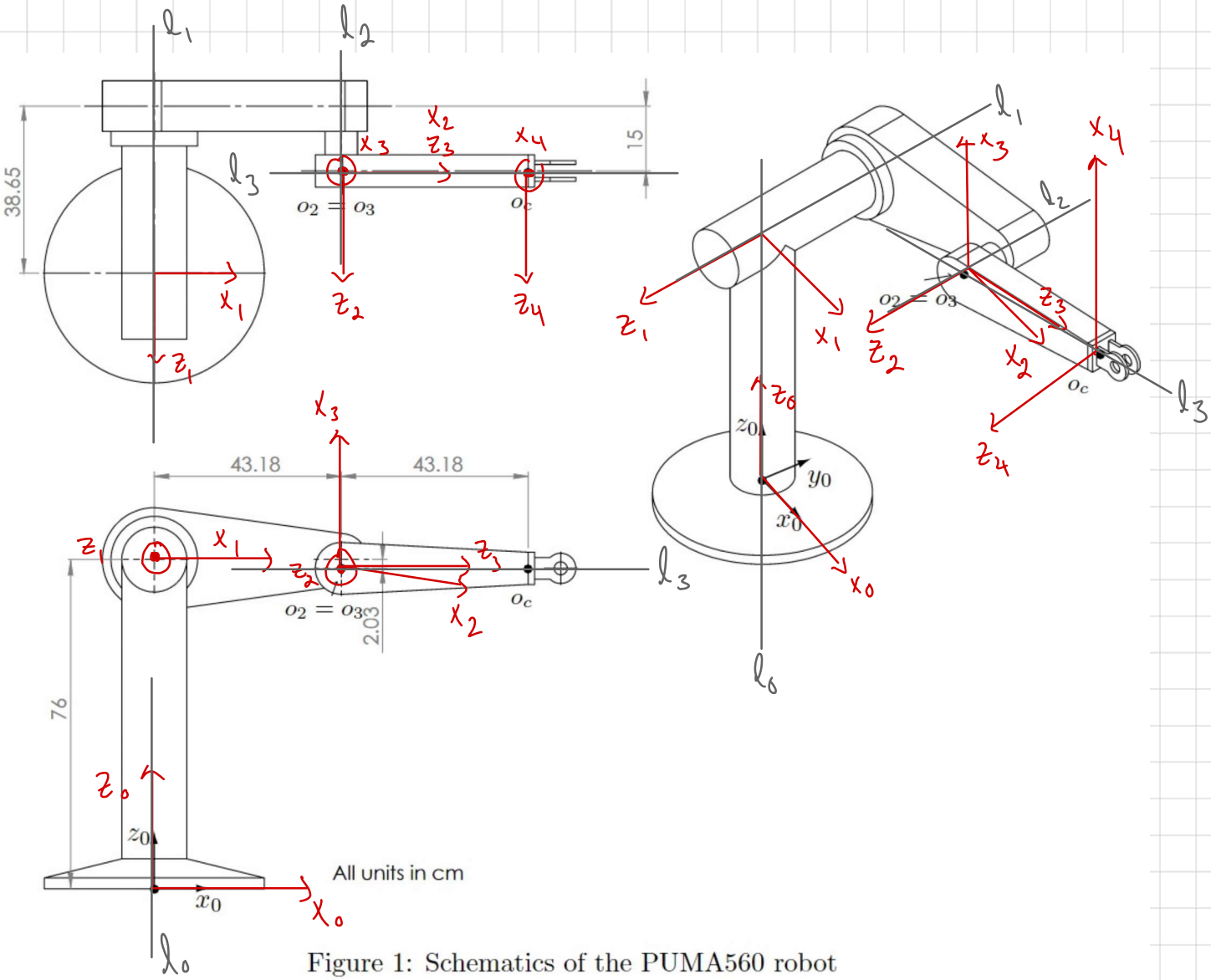
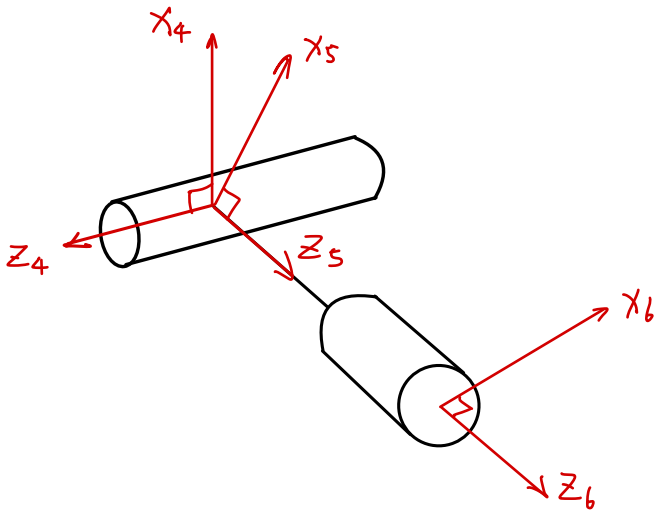


Figure 1: Schematics of the PUMA560 robot

Note that the y_i unit vectors were excluded for the sake of clarity, but can be found via the RHR from z_i to x_i

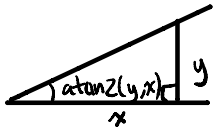


Links	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	76	θ_1
2	43.23	0	-23.65	θ_2
3	0	$\pi/2$	0	θ_3
4	0	$-\pi/2$	43.18	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	20	θ_6

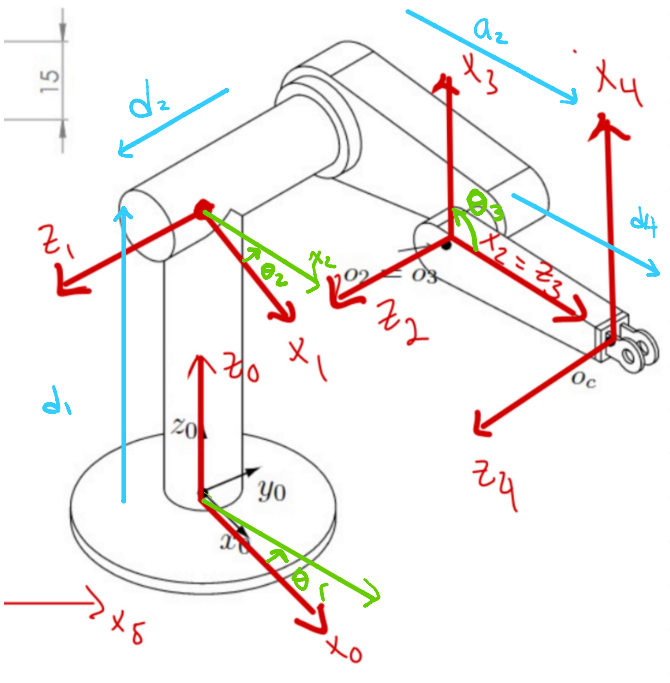
$a_2 = (43.18^2 + 2.03^2)^{0.5} = 43.23$

Lab 1 Prelab Part 3

Saturday, February 5, 2022 12:23 AM



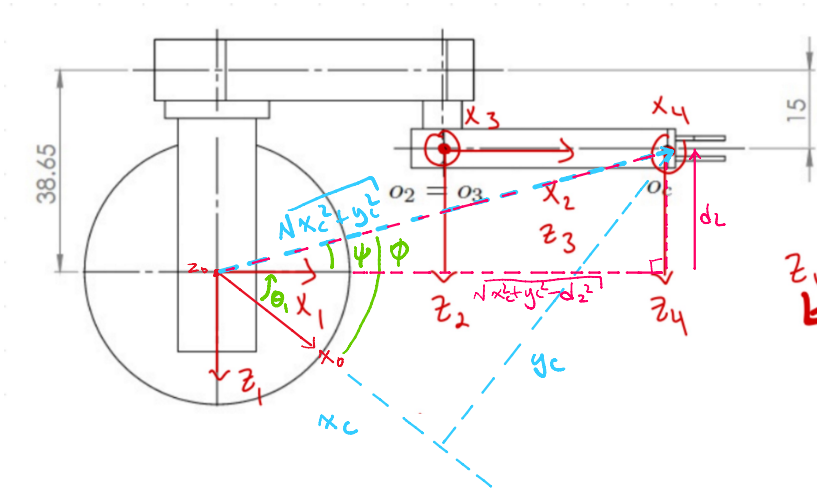
atan2 convention used in this prelab



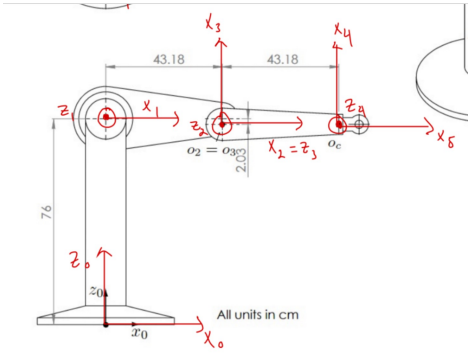
Given $\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_5 - d_6 R_6 \begin{bmatrix} 0 \\ 0 \\ d_6 \end{bmatrix} = O_c^0(\theta_1, \theta_2, \theta_3)$

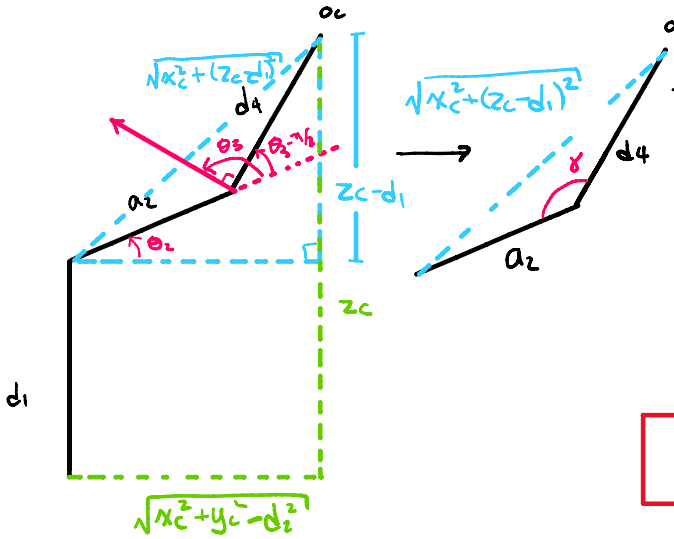
Find $(\theta_1, \theta_2, \theta_3), (q_1, q_2, q_3)$

Initial Position Kinematics



$$\theta_1 = \phi - \psi$$
$$= \text{atan2}(y_c, x_c) - \text{atan2}\left(d_2, \sqrt{x_c^2 + y_c^2 - d_2^2}\right)$$





Cosine Law: $c^2 = a^2+b^2+2ab\cos(\gamma)$

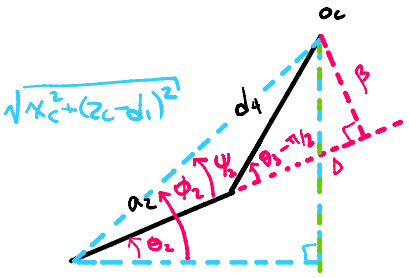
$x_c^2+y_c^2-d_2^2+(z_c-d_1)^2 = a_2^2+d_4^2+2a_2d_4\cos(\gamma)$

$\cos(\gamma) = \dfrac{x_c^2+y_c^2-d_2^2+(z_c-d_1)^2-a_2^2-d_4^2}{2a_2d_4}$

$\gamma = \pi-(\theta_3-\pi/2) \Rightarrow \cos(\gamma) = \sin(\theta_3)$

$\Rightarrow \sin(\theta_3) = \dfrac{x_c^2+y_c^2-d_2^2+(z_c-d_1)^2-a_2^2-d_4^2}{2a_2d_4} = D$

$$\theta_3 = \operatorname{atan2}(D, \pm\sqrt{1-D^2})$$



$\phi_2 = \operatorname{atan2}(z_c-d_1, \sqrt{x_c^2+y_c^2-d_2^2})$

$\beta = d_4\sin(\theta_3-\pi/2)$

$\Delta = d_4\cos(\theta_3-\pi/2)$

$\psi_2 = \operatorname{atan2}(\beta, \Delta+a_2) = \operatorname{atan2}(d_4\sin(\theta_3-\pi/2), a_2+d_4\cos(\theta_3-\pi/2))$

$= \operatorname{atan2}(-d_4\cos(\theta_3), a_2+d_4\sin(\theta_3))$

$$\theta_2 = \operatorname{atan2}(z_c-d_1, \sqrt{x_c^2+y_c^2-d_2^2}) - \operatorname{atan2}(-d_4\cos(\theta_3), a_2+d_4\sin(\theta_3))$$

$\sin(\theta_3) = D$ for greater computational stability

Inverse Orientation Kinematics

$R_6^0 = R_3^0 R_6^3 = R_d \Leftrightarrow R_6^3 (R_3^0)^T R_d$

Given: $(R_3^0)^T R_d$ find (q_4, q_5, q_6) s.t. $R_6^3(q_4,q_5,q_6) = (R_3^0)^T R_d =: M$

$$R_6^3 = \begin{bmatrix} * & * & c_{q_4}s_{q_5} \\ * & * & s_{q_4}s_{q_5} \\ -s_{q_5}c_{q_6} & s_{q_5}s_{q_6} & c_{q_5} \end{bmatrix}$$ from foward kinematics

$$M = R_6^3(q_4,q_5,q_6) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

R_6^3 corresponds to ZYZ Euler angles with $q_4 = \phi, q_5 = \theta, q_6 = \psi$

If $m_{13}^2 + m_{23}^2 \neq 0$

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$$q_4^* = \text{atan2}(m_{23}, m_{13}) \text{ or } \text{atan2}(-m_{23}, -m_{13})$$

$$q_5^* = \text{atan2}(\sqrt{1-m_{33}^2}, m_{33}) \text{ or } \text{atan2}(-\sqrt{1-m_{33}^2}, -m_{33})$$

$$q_6^* = \text{atan2}(m_{32}, -m_{31}) \text{ or } \text{atan2}(-m_{32}, m_{31})$$

where $m_{13}^2 + m_{23}^2 = 0$ is a singularity where there is no inverse kinematic solution exists.