EE-559 – Deep learning

4.1. DAG networks

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https://fleuret.org/ee559/

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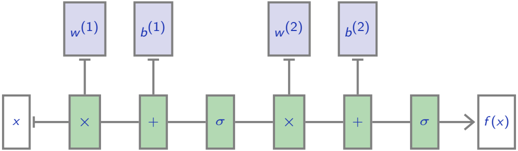
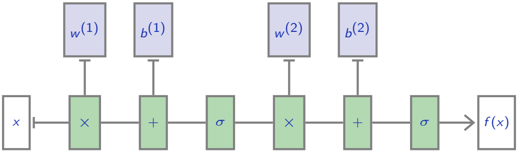
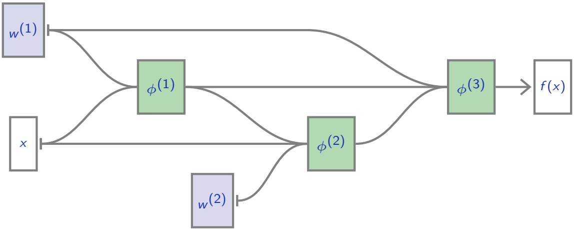
Everything we have seen for an MLP



Everything we have seen for an MLP



can be generalized to an arbitrary "Directed Acyclic Graph" (DAG) of operators



Remember that we use tensorial notation.

If $(a_1, \dots, a_Q) = \phi(b_1, \dots, b_R)$, we have

$$\left[\frac{\partial a}{\partial b}\right] = J_\phi = \begin{pmatrix} \frac{\partial a_1}{\partial b_1} & \dots & \frac{\partial a_1}{\partial b_R} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_Q}{\partial b_1} & \dots & \frac{\partial a_Q}{\partial b_R} \end{pmatrix}.$$

This notation does not specify at which point this is computed. It will always be for the forward-pass activations.

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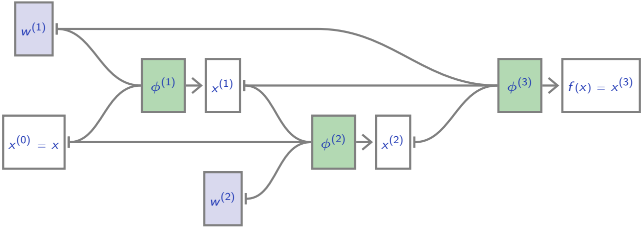
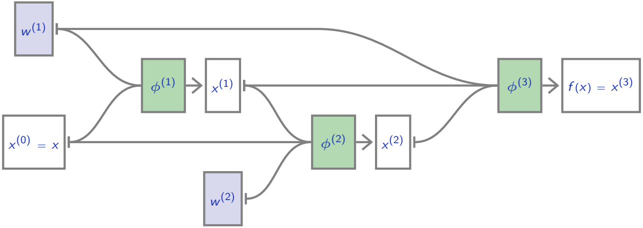
Also, if $(a_1, \dots, a_Q) = \phi(b_1, \dots, b_R, c_1, \dots, c_S)$, we use

$$\left[\frac{\partial a}{\partial c}\right] = J_{\phi|c} = \begin{pmatrix} \frac{\partial a_1}{\partial c_1} & \dots & \frac{\partial a_1}{\partial c_S} \\ \vdots & \ddots & \vdots \\ \frac{\partial a_Q}{\partial c_1} & \dots & \frac{\partial a_Q}{\partial c_S} \end{pmatrix}.$$

Forward pass

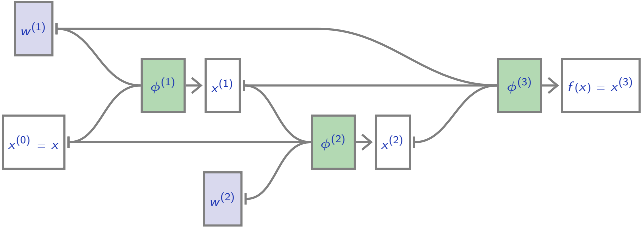


Forward pass



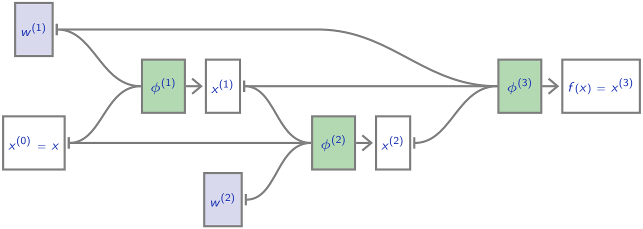
$$x^{(0)} = x$$

Forward pass



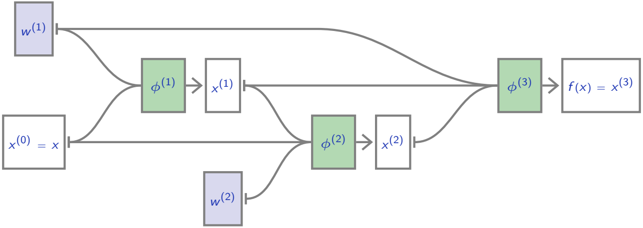
$$x^{(0)} = x$$
$$x^{(1)} = \phi^{(1)}(x^{(0)}; w^{(1)})$$

Forward pass



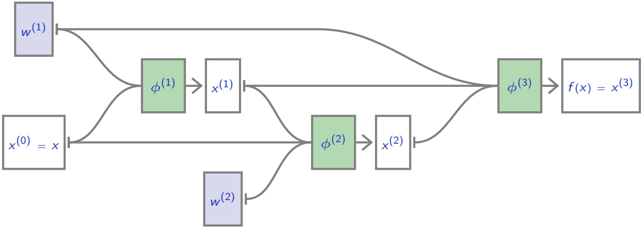
$$x^{(0)} = x$$
$$x^{(1)} = \phi^{(1)}(x^{(0)}; w^{(1)})$$
$$x^{(2)} = \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)})$$

Forward pass



$$
\begin{aligned}
x^{(0)} &= x \\
x^{(1)} &= \phi^{(1)}(x^{(0)}; w^{(1)}) \\
x^{(2)} &= \phi^{(2)}(x^{(0)}, x^{(1)}; w^{(2)}) \\
f(x) = x^{(3)} &= \phi^{(3)}(x^{(1)}, x^{(2)}; w^{(1)})
\end{aligned}
$$

Backward pass, derivatives w.r.t activations



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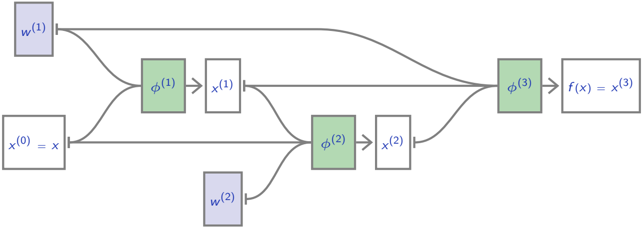

$$\left[\frac{\partial \ell}{\partial x^{(2)}}\right] = \left[\frac{\partial x^{(3)}}{\partial x^{(2)}}\right] \left[\frac{\partial \ell}{\partial x^{(3)}}\right] = J_{\phi^{(3)}|x^{(2)}} \left[\frac{\partial \ell}{\partial x^{(3)}}\right]$$

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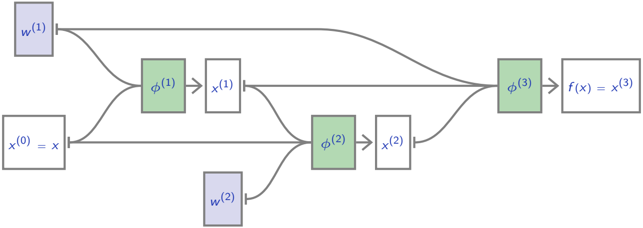

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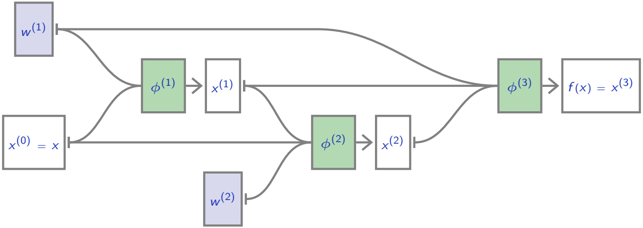

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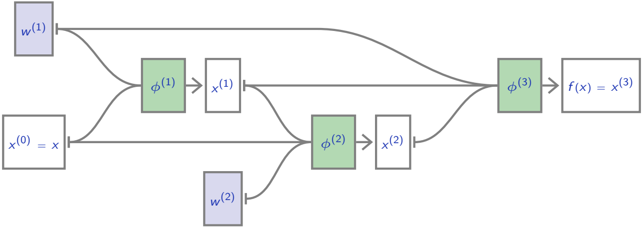
Backward pass, derivatives w.r.t parameters



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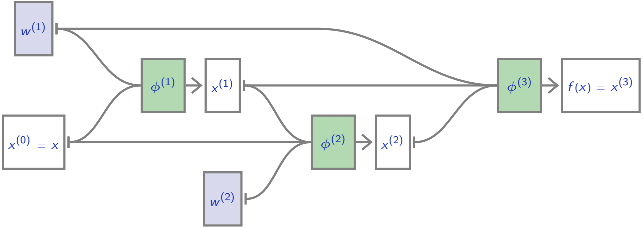

$$\left[\frac{\partial \ell}{\partial w^{(1)}}\right] = \left[\frac{\partial x^{(1)}}{\partial w^{(1)}}\right]\left[\frac{\partial \ell}{\partial x^{(1)}}\right] + \left[\frac{\partial x^{(3)}}{\partial w^{(1)}}\right]\left[\frac{\partial \ell}{\partial x^{(3)}}\right] = J_{\phi^{(1)}|w^{(1)}}\left[\frac{\partial \ell}{\partial x^{(1)}}\right] + J_{\phi^{(3)}|w^{(1)}}\left[\frac{\partial \ell}{\partial x^{(3)}}\right]$$

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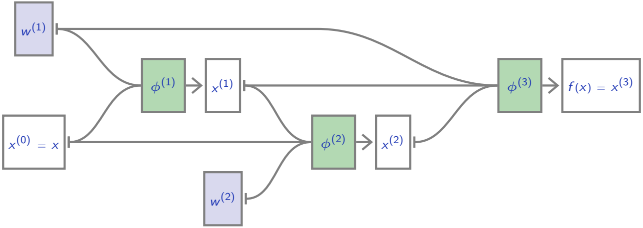

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So if we have a library of "tensor operators", and implementations of

$$
\begin{aligned}
(x_1, \dots, x_d, w) &\mapsto \phi(x_1, \dots, x_d; w) \\
\forall c,\ (x_1, \dots, x_d, w) &\mapsto J_{\phi|x_c}(x_1, \dots, x_d; w) \\
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we can build an arbitrary directed acyclic graph with these operators at the nodes, compute the response of the resulting mapping, and compute its gradient with back-prop.

Writing from scratch a large neural network is complex and error-prone.

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Multiple frameworks provide libraries of tensor operators and mechanisms to combine them into DAGs and automatically differentiate them.

	Language(s)	**License**	**Main backer**
PyTorch	**Python**	BSD	Facebook
Caffe2	C++, Python	Apache	Facebook
TensorFlow	Python, C++	Apache	Google
MXNet	Python, C++, R, Scala	Apache	Amazon
CNTK	Python, C++	MIT	Microsoft
Torch	Lua	BSD	Facebook
Theano	Python	BSD	U. of Montreal
Caffe	C++	BSD 2 clauses	U. of CA, Berkeley

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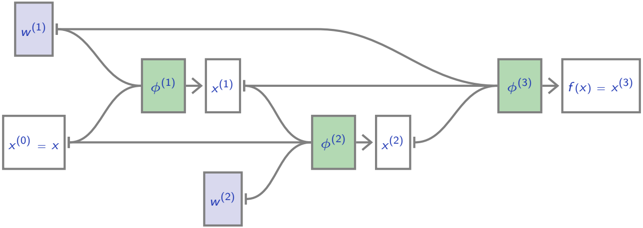
One approach is to define the nodes and edges of such a DAG statically (Torch, TensorFlow, Caffe, Theano, etc.)

In TensorFlow, to run a forward/backward pass on



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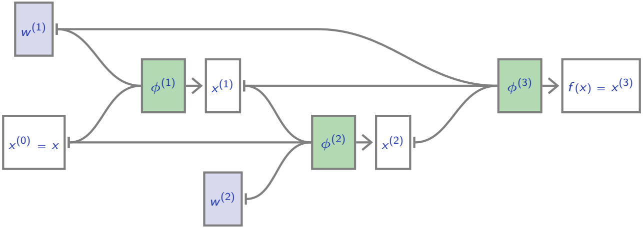

$$\phi^{(1)}\left(x^{(0)}; w^{(1)}\right) = w^{(1)} x^{(0)}$$

$$\phi^{(2)}\left(x^{(0)}, x^{(1)}; w^{(2)}\right) = x^{(0)} + w^{(2)} x^{(1)}$$

$$\phi^{(3)}\left(x^{(1)}, x^{(2)}; w^{(1)}\right) = w^{(1)}\left(x^{(1)} + x^{(2)}\right)$$

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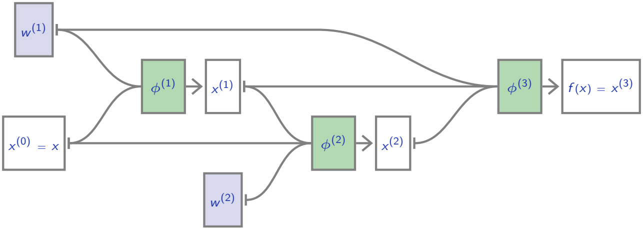

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$$\phi^{(3)}\left(x^{(1)}, x^{(2)}; w^{(1)}\right) = w^{(1)}\left(x^{(1)} + x^{(2)}\right)$$

```
w1 = tf.Variable(tf.random_normal([5, 5]))
w2 = tf.Variable(tf.random_normal([5, 5]))
x = tf.Variable(tf.random_normal([5, 1]))
x0 = x
x1 = tf.matmul(w1, x0)
x2 = x0 + tf.matmul(w2, x1)
x3 = tf.matmul(w1, x1 + x2)
q = tf.norm(x3)

gw1, gw2 = tf.gradients(q, [w1, w2])

with tf.Session() as sess:
    sess.run(tf.global_variables_initializer())
    _gw1, _gw2 = sess.run([gw1, gw2])
```

Weight sharing

In our generalized DAG formulation, we have in particular implicitly allowed the same parameters to modulate different parts of the processing.

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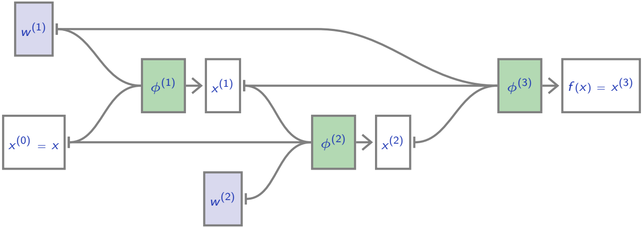
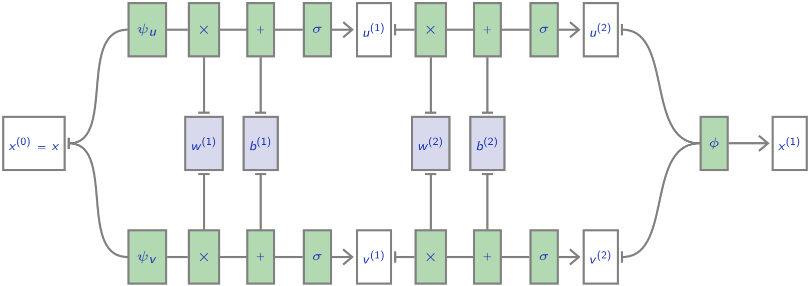
For instance $w^{(1)}$ in our example parametrizes both $\phi^{(1)}$ and $\phi^{(3)}$.



This is called **weight sharing**.

Weight sharing allows in particular to build **siamese networks** where a full sub-network is replicated several times.



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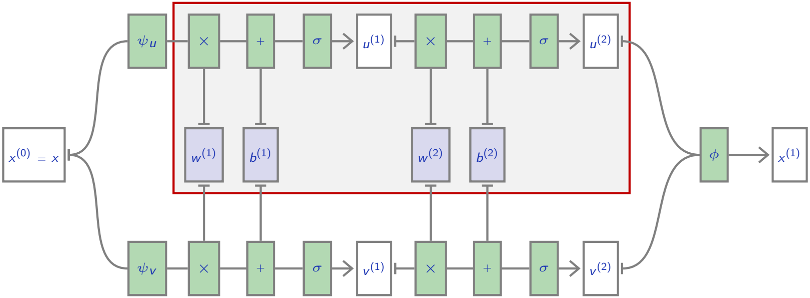

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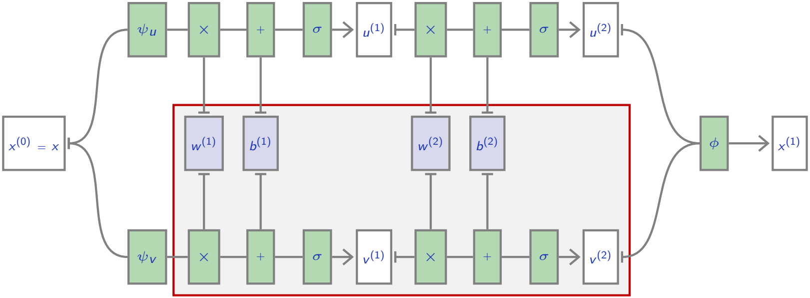

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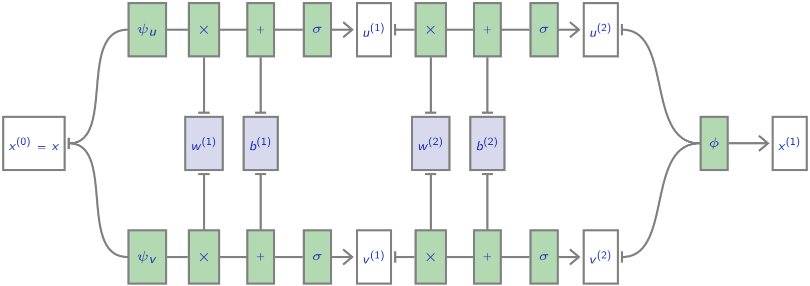

The end