EE-559 – Deep learning

6.5. Residual networks

François Fleuret

https://fleuret.org/ee559/

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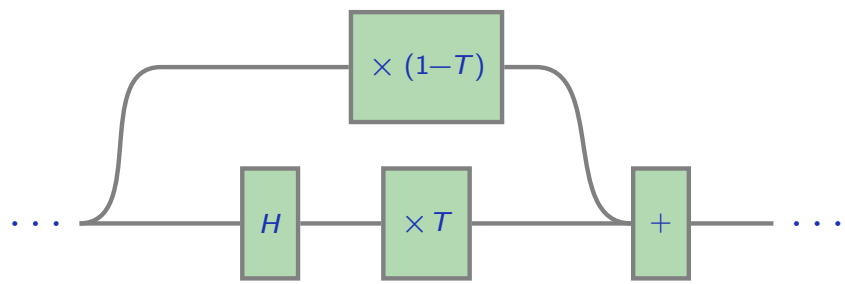
The "Highway networks" by Srivastava et al. (2015) use the idea of gating developed for recurrent units. It replaces a standard non-linear layer

$$y = H(x; W_H)$$

with a layer that includes a "gated" pass-through

$$y = T(x; W_T)H(x; W_H) + (1 - T(x; W_T))x$$

where $T(x; W_T) \in [0, 1]$ modulates how much the signal should be transformed.



This technique allowed them to train networks with up to 100 layers.

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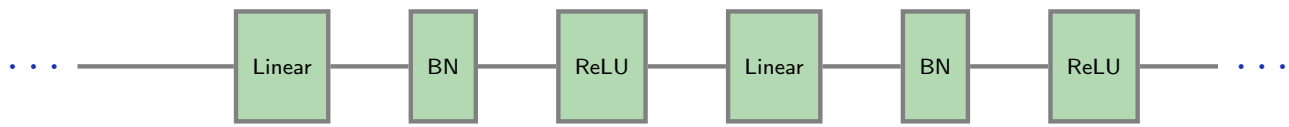

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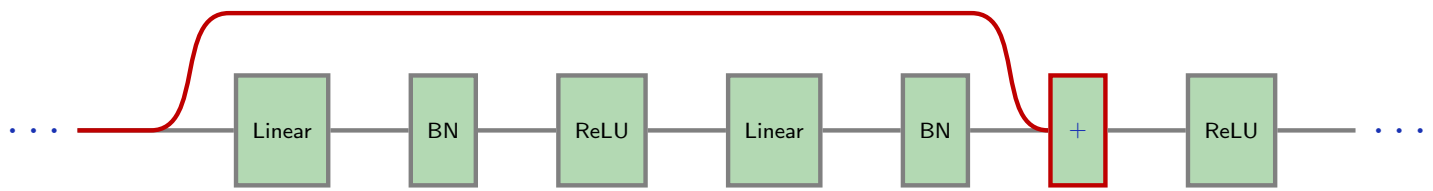

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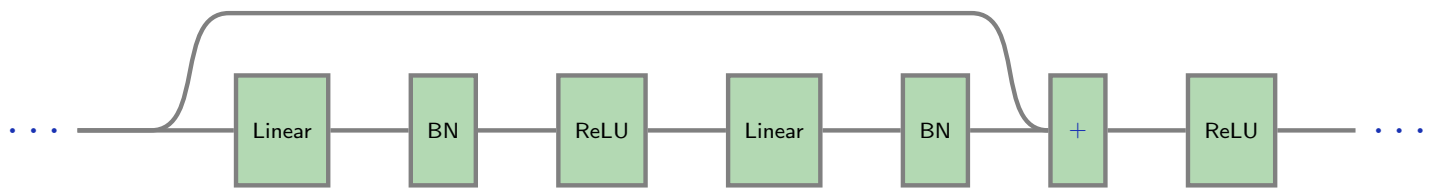
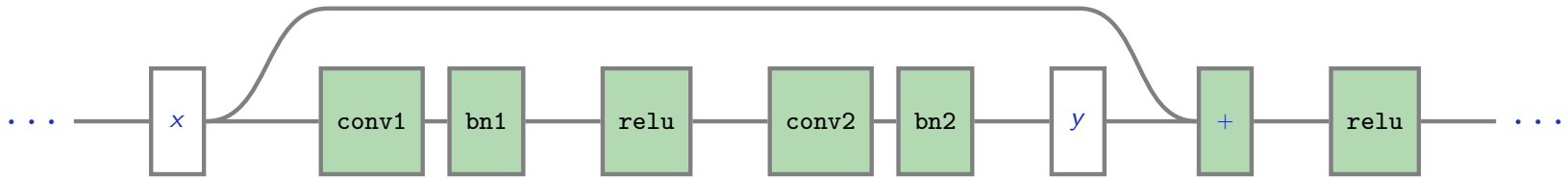
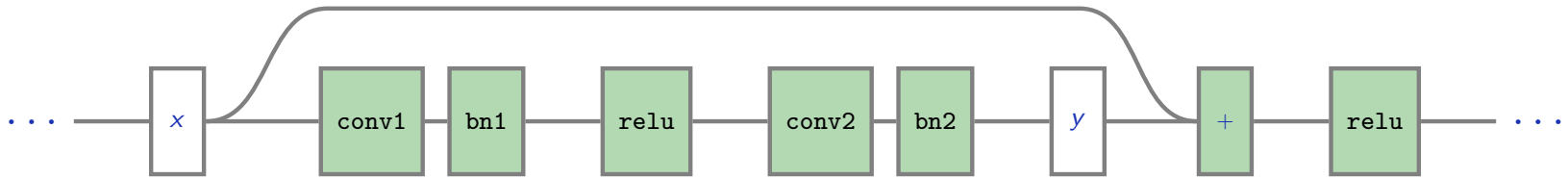


Thanks to this structure, the parameters are optimized to learn a **residual**, that is the difference between the value before the block and the one needed after.

We can implement such a network for MNIST, composed of:

- A first convolution layer `conv0` with kernels 1×1 to convert the tensor from $1 \times 28 \times 28$ to `nb_channels` $\times 28 \times 28$,
- a series of `ResBlock`s, each composed of two convolution layers and two batch normalization layers, that maintains the tensor size unchanged,
- an average poling layer `avg` that produces an output of size `nb_channels` $\times 1 \times 1$,
- a fully connected layer `fc` to make the final prediction.


. . .
x
conv1
bn1
relu
conv2
bn2
y
+
relu
. . .




```
class ResBlock(nn.Module):
    def __init__(self, nb_channels, kernel_size):
        super(ResBlock, self).__init__()

        self.conv1 = nn.Conv2d(nb_channels, nb_channels, kernel_size,
                               padding = (kernel_size-1)//2)
        self.bn1 = nn.BatchNorm2d(nb_channels)

        self.conv2 = nn.Conv2d(nb_channels, nb_channels, kernel_size,
                               padding = (kernel_size-1)//2)
        self.bn2 = nn.BatchNorm2d(nb_channels)

    def forward(self, x):
        y = self.bn1(self.conv1(x))
        y = F.relu(y)
        y = self.bn2(self.conv2(y))
        y += x
        y = F.relu(y)
        return y
```

```
class ResNet(nn.Module):
    def __init__(self, nb_channels, kernel_size, nb_blocks):
        super(ResNet, self).__init__()

        self.conv0 = nn.Conv2d(1, nb_channels, kernel_size = 1)

        self.resblocks = nn.Sequential(
            # A bit of fancy Python
            *(ResBlock(nb_channels, kernel_size) for _ in range(nb_blocks))
        )

        self.avg = nn.AvgPool2d(kernel_size = 28)
        self.fc = nn.Linear(nb_channels, 10)

    def forward(self, x):
        x = F.relu(self.conv0(x))
        x = self.resblocks(x)
        x = F.relu(self.avg(x))
        x = x.view(x.size(0), -1)
        x = self.fc(x)
        return x
```

With 25 residual blocks, 16 channels, and convolution kernels of size 3×3, we get the following structure, with $117,802$ parameters.

```
ResNet(
  (conv0): Conv2d(1, 16, kernel_size=(1, 1), stride=(1, 1))
  (resblocks): Sequential(
    (0): ResBlock(
      (conv1): Conv2d(16, 16, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
      (bn1): BatchNorm2d(16, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
      (conv2): Conv2d(16, 16, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
      (bn2): BatchNorm2d(16, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
    )
    /.../
    (24): ResBlock(
      (conv1): Conv2d(16, 16, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
      (bn1): BatchNorm2d(16, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
      (conv2): Conv2d(16, 16, kernel_size=(3, 3), stride=(1, 1), padding=(1, 1))
      (bn2): BatchNorm2d(16, eps=1e-05, momentum=0.1, affine=True, track_running_stats=True)
    )
  )
  (avg): AvgPool2d(kernel_size=28, stride=28, padding=0)
  (fc): Linear(in_features=16, out_features=10, bias=True)
)
```

A technical point for a more general use of a residual architecture is to deal with convolution layers that change the activation map sizes or numbers of channels.

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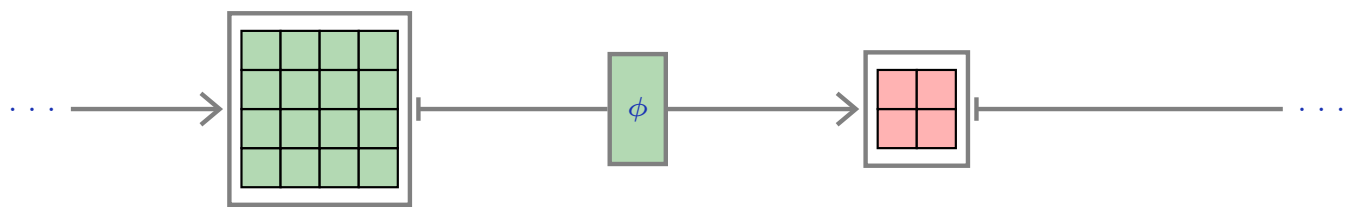
He et al. (2015) only consider:

- reducing the activation map size by a factor 2,
- increasing the number of channels.

To reduce the activation map size by a factor 2, the identity pass-trough extracts 1/4 of the activations over a regular grid (*i.e.* with a stride of 2),



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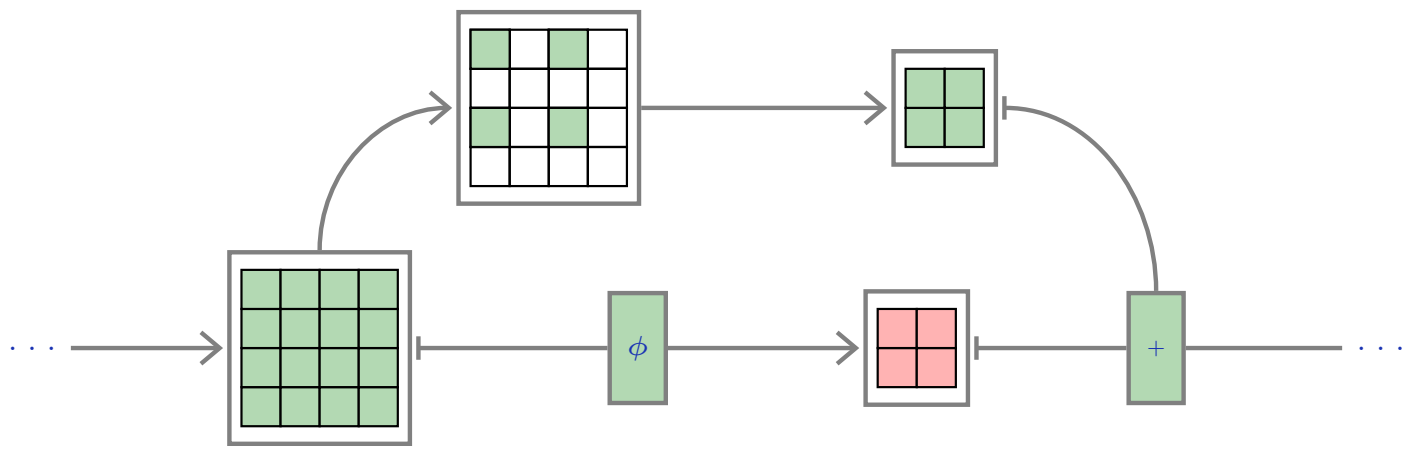


To increase the number of channels from C to C', they propose to either:

- pad the original value with $C' - C$ zeros, which amounts to adding as many zeroed channels, or
- use C' convolutions with a $1 \times 1 \times C$ filter, which corresponds to applying the same fully-connected linear model $\mathbb{R}^C \to \mathbb{R}^{C'}$ at every location.

Finally, He et al.'s residual networks are fully convolutional, which means they have no fully connected layers. We will come back to this.

Their one-before last layer is a per-channel global average pooling that outputs a $1d$ tensor, fed into a single fully-connected layer.

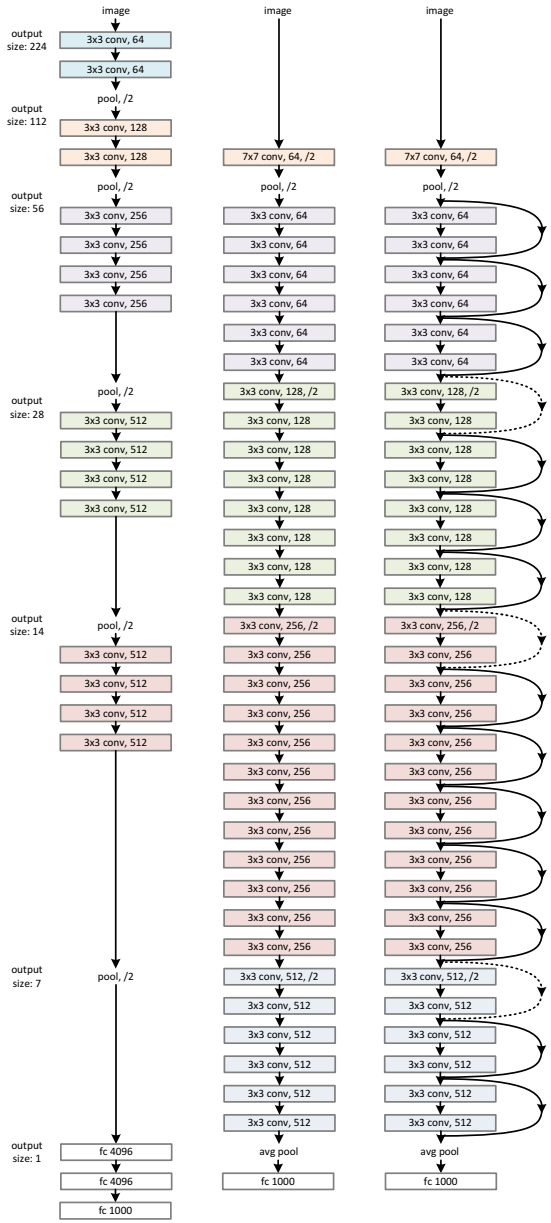


image
output size: 224
3x3 conv, 64
3x3 conv, 64
pool, /2
output size: 112
3x3 conv, 128
3x3 conv, 128
pool, /2
output size: 56
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
pool, /2
output size: 28
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
pool, /2
output size: 14
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
pool, /2
output size: 7
fc 4096
fc 4096
fc 1000
output size: 1
image
7x7 conv, 64, /2
pool, /2
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 128, /2
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 256, /2
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 512, /2
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
avg pool
fc 1000
image
7x7 conv, 64, /2
pool, /2
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 64
3x3 conv, 128, /2
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 128
3x3 conv, 256, /2
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 256
3x3 conv, 512, /2
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
3x3 conv, 512
avg pool
fc 1000

(He et al., 2015)

Performance on ImageNet.

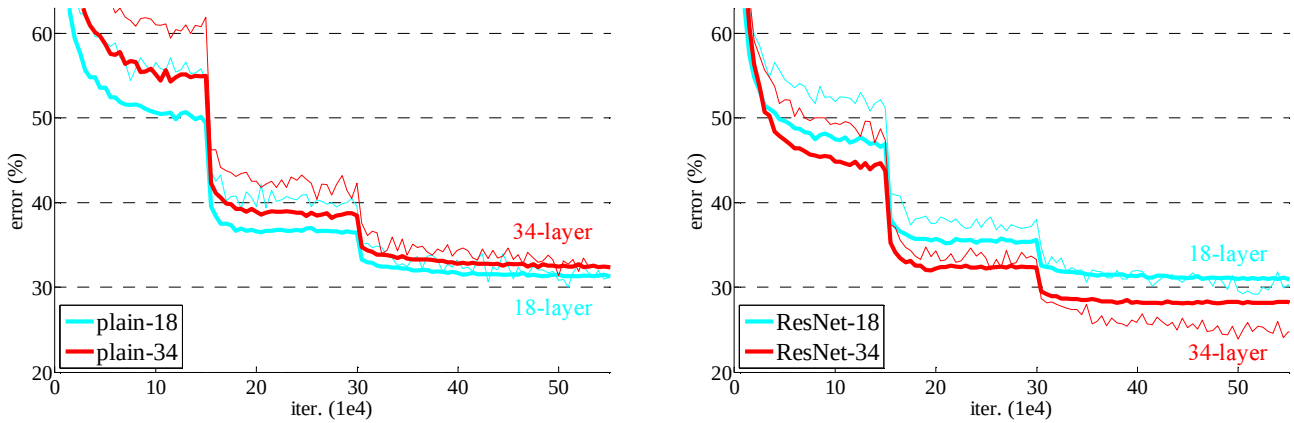


Figure 4. Training on **ImageNet**. Thin curves denote training error, and bold curves denote validation error of the center crops. Left: plain networks of 18 and 34 layers. Right: ResNets of 18 and 34 layers. In this plot, the residual networks have no extra parameter compared to their plain counterparts.

(He et al., 2015)

Veit et al. (2016) interpret a residual network as an ensemble, which explains in part its stability.

E.g., with three blocks we have

$$\begin{aligned} x_1 &= x_0 + f_1(x_0) \\ x_2 &= x_1 + f_2(x_1) \\ x_3 &= x_2 + f_3(x_2) \end{aligned}$$

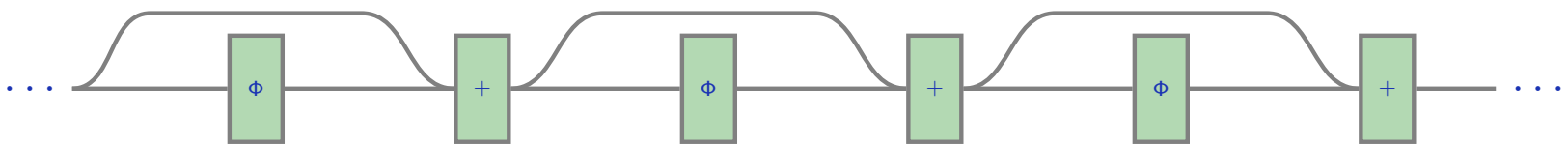
hence there are four "paths":

$$\begin{aligned} x_3 &= x_2 + f_3(x_2) \\ &= x_1 + f_2(x_1) + f_3(x_1 + f_2(x_1)) \\ &= \underbrace{x_0} + \underbrace{f_1(x_0)} + \underbrace{f_2(x_0 + f_1(x_0))} + \underbrace{f_3(x_0 + f_1(x_0) + f_2(x_0 + f_1(x_0)))}. \end{aligned}$$

Veit et al. show that (1) performance reduction correlates with the number of paths removed from the ensemble, not with the number of blocks removed, (2) only gradients through shallow paths matter during train.

An extension of the residual network, is the **stochastic depth** network.

> "Stochastic depth aims to shrink the depth of a network during training, while keeping it unchanged during testing. We can achieve this goal by randomly dropping entire ResBlocks during training and bypassing their transformations through skip connections."
>
> (Huang et al., 2016)



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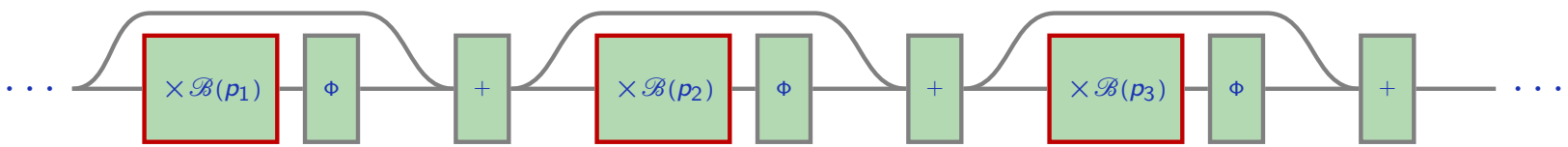

Shattered Gradient

Balduzzi et al. (2017) points out that depth "shatters" the relation between the input and the gradient wrt the input, and that Resnets mitigate this effect.

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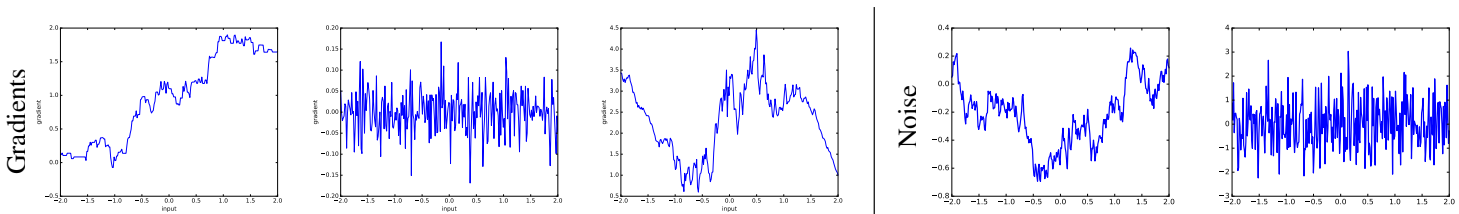

(a) 1-layer feedforward. (b) 24-layer feedforward. (c) 50-layer resnet. (d) Brown noise. (e) White noise.

(Balduzzi et al., 2017)

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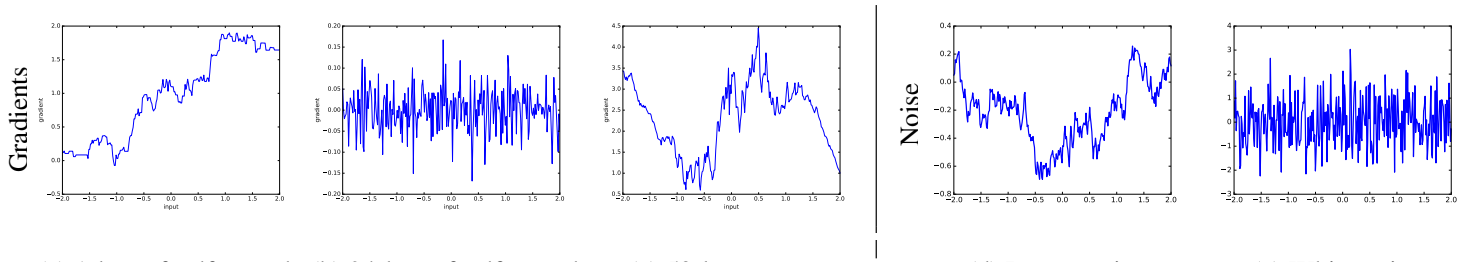

(a) 1-layer feedforward. (b) 24-layer feedforward. (c) 50-layer resnet. (d) Brown noise. (e) White noise.

(Balduzzi et al., 2017)

Since linear networks avoid this problem, they suggest to combine CReLU with a **Looks Linear initialization** that makes the network linear initially.

Let $\sigma(x) = \max(0, x)$, and

$$\Phi : \mathbb{R}^D \to \mathbb{R}^{2D}$$

the CReLU non-linearity, *i.e.*

$$\forall x \in \mathbb{R}^D,\ q = 1, \ldots, D, \begin{cases} \Phi(x)_{2q-1} & = \sigma(x_q), \\ \Phi(x)_{2q} & = \sigma(-x_q) \end{cases}$$

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and a weight matrix $\tilde{W} \in \mathbb{R}^{D' \times 2D}$ such that

$$\forall j = 1, \ldots, D', q = 1, \ldots, D,\ \tilde{W}_{j,2q-1} = -\tilde{W}_{j,2q} = W_{j,q}.$$

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So two neighboring columns of $\Phi(x)$ are the $\sigma(\cdot)$ and $\sigma(-\cdot)$ of a column of x, and two neighboring columns of $\tilde{W}$ are a column of W and its opposite.

From this we get, $\forall i = 1, \ldots, B,\ j = 1, \ldots, D'$:

$$\begin{aligned}
\left(\tilde{W}\Phi(x)\right)_j &= \sum_{k=1}^{2D} \tilde{W}_{j,k}\Phi(x)_k \\
&= \sum_{q=1}^{D} \tilde{W}_{j,2q-1}\Phi(x)_{2q-1} + \tilde{W}_{j,2q}\Phi(x)_{2q} \\
&= \sum_{q=1}^{D} W_{j,q}\sigma(x_q) - W_{j,q}\sigma(-x_q) \\
&= \sum_{q=1}^{D} W_{j,q}x_q \\
&= (Wx)_j \, .
\end{aligned}$$

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&= \sum_{q=1}^{D} W_{j,q}\sigma(x_q) - W_{j,q}\sigma(-x_q) \\
&= \sum_{q=1}^{D} W_{j,q}x_q \\
&= (Wx)_j \, .
\end{aligned}$$

Hence

$$\forall x,\ \tilde{W}\Phi(x) = Wx$$

and doing this in every layer results in a linear network.

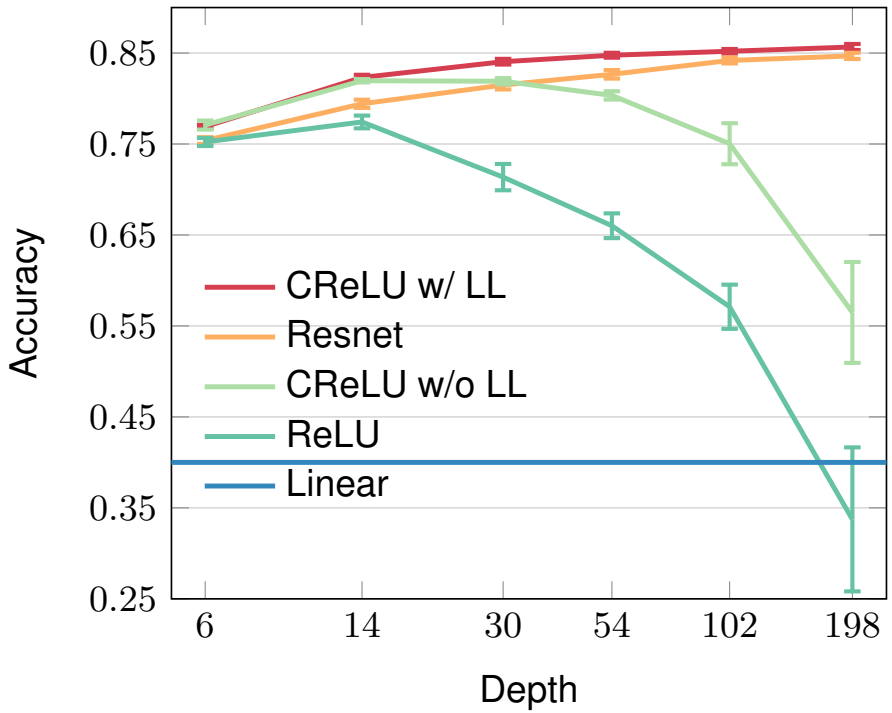

Figure 6: **CIFAR-10 test accuracy**. Comparison of test accuracy between networks of different depths with and without LL initialization.

(Balduzzi et al., 2017)

We can summarize the techniques which have enabled the training of very deep architectures:

- rectifiers to prevent the gradient from vanishing during the backward pass,
- dropout to force a distributed representation,
- batch normalization to dynamically maintain the statistics of activations,
- identity pass-through to keep a structured gradient and distribute representation,
- smart initialization to put the gradient in a good regime.

The end

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