

## EE-559 – Deep learning

### 1.4. Tensor basics and linear regression

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- A 3d tensor can be seen as a vector of identically sized matrix (e.g. a multi-channel image),
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Compounded data structures can represent more diverse data types.

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A key specificity of PyTorch is the central role of autograd to compute derivatives of *anything!* We will come back to this.

```
>>> x = torch.empty(2, 5)
>>> x.size()
torch.Size([2, 5])
>>> x.fill_(1.125)
tensor([[ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250],
        [ 1.1250,  1.1250,  1.1250,  1.1250,  1.1250]])
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>>> x.std()
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>>> x.sum()
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In-place operations are suffixed with an underscore, and a 0d tensor can be converted back to a Python scalar with `item()`.

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Reading a coefficient also generates a 0d tensor.

```
>>> x = torch.tensor([[11., 12., 13.], [21., 22., 23.]])
>>> x[1, 2]
tensor(23.)
```

PyTorch provides operators for component-wise and vector/matrix operations.

```
>>> x = torch.tensor([ 10., 20., 30.])
>>> y = torch.tensor([ 11., 21., 31.])
>>> x + y
tensor([ 21., 41., 61.])
>>> x * y
tensor([ 110., 420., 930.])
>>> x**2
tensor([ 100., 400., 900.])
>>> m = torch.tensor([[ 0., 0., 3. ],
...                   [ 0., 2., 0. ],
...                   [ 1., 0., 0. ]])
>>> m.mv(x)
tensor([ 90., 40., 10.])
>>> m @ x
tensor([ 90., 40., 10.])
```

And as in `numpy`, the `:` symbol defines a range of values for an index and allows to slice tensors.

```
>>> import torch
>>> x = torch.empty(2, 4).random_(10)
>>> x
tensor([[8., 1., 1., 3.],
        [7., 0., 7., 5.]])
>>> x[0]
tensor([8., 1., 1., 3.])
>>> x[0, :]
tensor([8., 1., 1., 3.])
>>> x[:, 0]
tensor([8., 7.])
>>> x[:, 1:3] = -1
>>> x
tensor([[ 8., -1., -1.,  3.],
        [ 7., -1., -1.,  5.]])
```

PyTorch provides interfacing to standard linear operations, such as linear system solving or Eigen-decomposition.

```
>>> y = torch.empty(3).normal_()
>>> y
tensor([ 0.0477,  0.8834, -1.5996])
>>> m = torch.empty(3, 3).normal_()
>>> q, _ = torch.gels(y, m)
>>> torch.mm(m, q)
tensor([[ 0.0477],
        [ 0.8834],
        [-1.5996]])
```

## Example: linear regression



Given a list of points

$$(x_n, y_n) \in \mathbb{R} \times \mathbb{R}, \quad n = 1, \dots, N,$$

can we find the “best line”

$$f(x; a, b) = ax + b$$

going “through the points”

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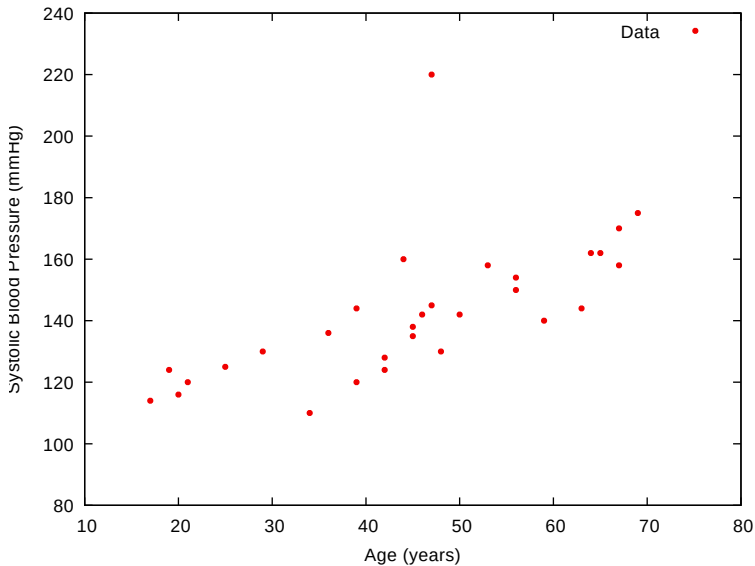
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Such a model would allow to predict the  $y$  associated to a new  $x$ , simply by calculating  $f(x; a, b)$ .

```
bash> cat systolic-blood-pressure-vs-age.dat
39 144
47 220
45 138
47 145
65 162
46 142
67 170
42 124
67 158
56 154
64 162
56 150
59 140
34 110
42 128
/.../
```



$$\underbrace{\begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \vdots \\ x_N & y_N \end{pmatrix}}_{\text{data} \in \mathbb{R}^{N \times 2}}$$

$$\underbrace{\begin{pmatrix} x_1 & 1.0 \\ x_2 & 1.0 \\ \vdots & \vdots \\ x_N & 1.0 \end{pmatrix}}_{x \in \mathbb{R}^{N \times 2}} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{\alpha \in \mathbb{R}^{2 \times 1}} \simeq \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}}_{y \in \mathbb{R}^{N \times 1}}$$

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```
import torch, numpy

data = torch.tensor(numpy.loadtxt('systolic-blood-pressure-vs-age.dat'))
nb_samples = data.size(0)

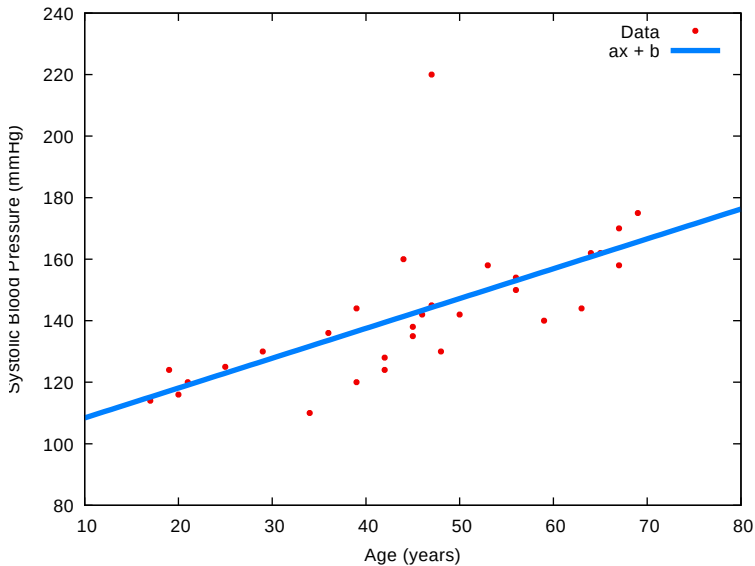
x, y = torch.empty(nb_samples, 2), torch.empty(nb_samples, 1)

x[:, 0] = data[:, 0]
x[:, 1] = 1

y[:, 0] = data[:, 1]

alpha, _ = torch.gels(y, x)

a, b = alpha[0, 0].item(), alpha[1, 0].item()
```





The end