Section 9. Neural Network 1 : XOR 문제, Backpropagation

I. XOR 문제

· 하나의 logistic regression unit으로는 절대 풀 수 없음.

X_1	X_2	XOR
0	0	0
0	1	1
1	0	1
1	1	0
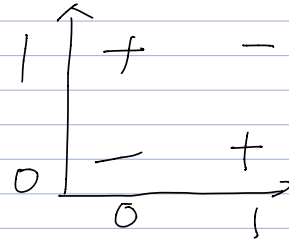
$$x_1, x_2 \xrightarrow{wx+b} \boxed{sigmoid} \longrightarrow y_1 \qquad W = \begin{bmatrix} 5 \\ 5 \end{bmatrix} \quad b = 8$$

$$y_1, y_2 \longrightarrow \boxed{} \longrightarrow \bar{y} \qquad W = \begin{bmatrix} -11 \\ -11 \end{bmatrix} \quad b = 6$$

$$x_1, x_2 \longrightarrow \boxed{} \longrightarrow y_2 \qquad W = \begin{bmatrix} -7 \\ -7 \end{bmatrix} \quad b = 3$$

$x_1 = 0, x_2 = 0$ 인 경우 $\quad [0 \ 0]\begin{bmatrix} 5 \\ 5 \end{bmatrix} - 8 = -8 \quad y_1 = S(-8) \doteq 0$

$$[0 \ 0]\begin{bmatrix} -7 \\ -7 \end{bmatrix} + 3 = 3 \quad y_2 = S(3) \doteq 1$$

$$[0 \ 1]\begin{bmatrix} -11 \\ -11 \end{bmatrix} + 6 = -5 \quad \bar{y} = S(-5) \doteq 0$$

<Forward propagation>

$$x_1, x_2 \longrightarrow \boxed{} \xrightarrow{S} \boxed{} \xrightarrow{S} \bar{y} \quad \xRightarrow{NN} \quad x \longrightarrow \boxed{} \xrightarrow{S} \boxed{} \xrightarrow{S} \bar{F}$$
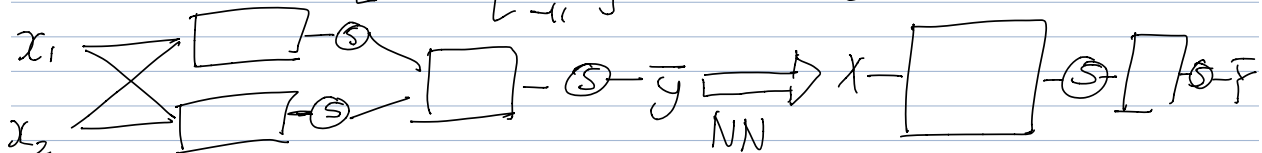
$$W_1 = \begin{bmatrix} 5 & -7 \\ 5 & -7 \end{bmatrix} \quad b_1 = [-8 \ 3]$$

$$W_2 = \begin{bmatrix} -11 \\ -11 \end{bmatrix} \quad b_2 = 6$$

$$K(x) = sigmoid(XW_1 + B_1)$$
$$\bar{F} = H(x) = sigmoid(K(x)W_2 + b_2)$$

II. 미분

1. $\dfrac{d}{dx} f(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
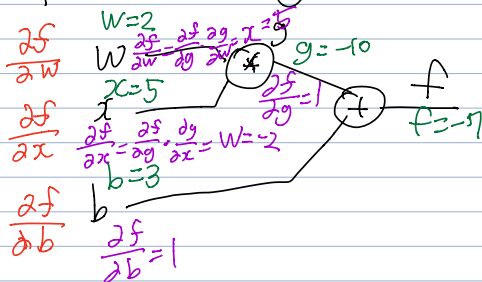
2. partial derivative

$f(x, y) = xy \qquad \dfrac{\partial f}{\partial x} = y \qquad \dfrac{\partial f}{\partial y} = x$
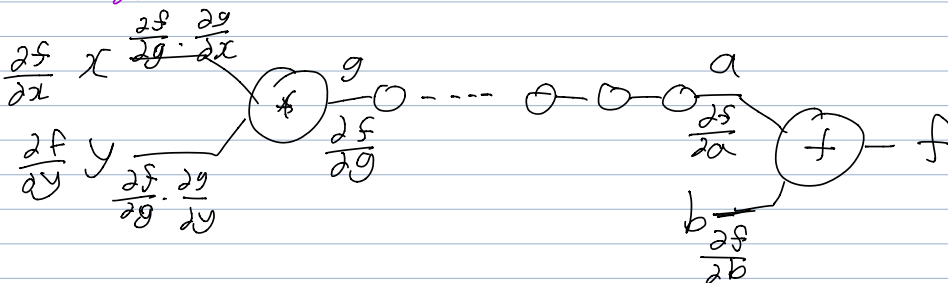
3. Chain Rule

$f(g(x))$에서 $\quad \dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial g} \dfrac{\partial g}{\partial x}$

III. 딥네트워크 학습 (backpropagation)

$f = wx + b \qquad g = wx \qquad f = g + b \qquad \dfrac{\partial g}{\partial w} = x \quad \dfrac{\partial g}{\partial x} = w \quad \dfrac{\partial f}{\partial g} = 1 \quad \dfrac{\partial f}{\partial b} = 1$

1) forward $(w = -2, x = 5, b = -3)$

2) backward

$\dfrac{\partial f}{\partial w}$

$\dfrac{\partial f}{\partial x}$

$\dfrac{\partial f}{\partial b}$

$w = 2$

$\dfrac{\partial f}{\partial w} = \dfrac{\partial f}{\partial g} \dfrac{\partial g}{\partial w} = x = 5$

$x = 5$

$g = -10$

$\dfrac{\partial f}{\partial g} = 1$

$\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial g} \cdot \dfrac{\partial g}{\partial x} = w = -2$

$b = 3$

$\dfrac{\partial f}{\partial b} = 1$

$f = -7$

$\dfrac{\partial f}{\partial x} \quad x \quad \dfrac{\partial f}{\partial g} \cdot \dfrac{\partial g}{\partial x}$

$\dfrac{\partial f}{\partial y} \quad y \quad \dfrac{\partial f}{\partial g} \cdot \dfrac{\partial g}{\partial y}$

g

$\dfrac{\partial f}{\partial g}$

a

$\dfrac{\partial f}{\partial a}$

f

$b \quad \dfrac{\partial f}{\partial b}$

$g(z) = \dfrac{1}{1 + e^{-z}}$

$z \ \text{—}\ \boxed{*-1} \ \text{—}\ \boxed{exp} \ \text{—}\ \boxed{+1} \ \text{—}\ \boxed{\tfrac{1}{x}} \ \text{—}\ g$