

1. (2pt) Compute the entropy in bits for each of the following random variables:
  - (i) Pixel values in an image whose possible grey values are all the integers from 0 to 255 with uniform probability.
  - (ii) Humans classified according to whether they are, or are not, mammals.
  - (iii) Gender in a tri-sexed insect population whose three genders occur with probabilities  $1/4$ ,  $1/4$ , and  $1/2$ .
  - (iv) A population of persons classified by whether they are older, or not older, than the population's median age.

2. (3pt) Let  $p(x, y)$  be as shown in the table below.

$X \backslash Y$	0	1	2
0	$1/12$	$1/6$	$1/12$
1	$1/6$	$1/6$	$1/6$
2	0	$1/12$	$1/12$

Find

- (a) (0.5pt)  $H(X)$ ,  $H(Y)$ ,
  - (b) (0.5pt)  $H(X, Y)$
  - (c) (1pt)  $H(Y|X)$
  - (d) (0.5pt)  $I(X; Y)$
  - (e) (0.5pt) Draw a Venn diagram for the quantities in (a) through (d)
3. (2pt) We have a dataset in the following table where A, B denote attributes and Y denotes labels. We want to build a decision tree to classify them according to Y.

Y	A	B
-	1	0
-	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

Which attribute should be selected for the next split? Give your explanation.

- 1) A
  - 2) B
  - 3) A or B (tie)
  - 4) Neither
4. (3pt) Consider a stock market with the state space  $S = \{\text{Up}, \text{Steady}, \text{Down}\}$ . The transition probability matrix  $P$  is given as follows:

$$P = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.2 & 0.5 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

where  $P_{ij}$  represents the probability of transitioning from state  $i$  to state  $j$ .

- (a) Given that today is Up, what is the probability that it will be Down three days from now?

- (b) Suppose today there is a 40% chance of being Up, a 30% chance of being Steady, and a 30% chance of being Down. What is the market state distribution for tomorrow?
- (c) Suppose today there is a 40% chance of being Up, a 30% chance of being Steady, and a 30% chance of being Down. What is the market state distribution three days from today?