Exercise 2.2.1)

$$\frac{\partial \sigma(a)}{\partial a} = \frac{-e^{-a}}{1+e^{-a}} = \sigma(a)(1-\sigma(a)),\ where\ \sigma()\ denotes\ the\ sigmoid\ function$$

$$\therefore\ \frac{\partial \sigma(Wx_i+b)}{\partial (Wx_i+b)} = \sigma(Wx_i+b)(1-\sigma(Wx_i+b))\ \ldots(1)$$

(a)

$$\frac{\partial L(W,b)}{\partial w} = \sum_1^N \frac{\partial (y_i - h(x_i,W,b))^2}{\partial w} = \sum_i^N \frac{\partial (y_i - \sigma(Wx_i+b))^2}{\partial w}$$

$$= \sum_i^N \frac{\partial (y_i - \sigma(Wx_i+b))^2}{\partial \sigma(Wx_i+b)} \times \frac{\partial \sigma(Wx_i+b)}{\partial w}$$

$$= \sum_i^N \frac{\partial (y_i - \sigma(Wx_i+b))^2}{\partial \sigma(Wx_i+b)} \times \frac{\partial \sigma(Wx_i+b)}{\partial (Wx_i+b)} \times \frac{\partial (Wx_i+b)}{\partial w}$$

$$= \sum_i^N -2(y_i - \sigma(Wx_i+b)) \times \frac{\partial \sigma(Wx_i+b)}{\partial (Wx_i+b)} \times x_i$$

$$= \sum_i^N -2(y_i - \sigma(Wx_i+b)) \times \sigma(Wx_i+b)(1-\sigma(Wx_i+b)) \times x_i,\ \because as\ is\ shown\ in\ (1)$$

$$= \sum_i^N -2(y_i - \frac{1}{1+e^{-(Wx_i+b)}}) \times \frac{1}{1+e^{-(Wx_i+b)}} \times (1 - \frac{1}{1+e^{-(Wx_i+b)}}) \times x_i$$

(b)

$$\frac{\partial L(W,b)}{\partial b} = \sum_1^N \frac{\partial (y_i - h(x_i,W,b))^2}{\partial b} = \sum_i^N \frac{\partial (y_i - \sigma(Wx_i+b))^2}{\partial b} = \sum_i^N \frac{\partial (y_i - \sigma(Wx_i+b))^2}{\partial \sigma(Wx_i+b)} \times \frac{\partial \sigma(Wx_i+b)}{\partial b}$$

$$= \sum_i^N \frac{\partial (y_i - \sigma(Wx_i+b))^2}{\partial \sigma(Wx_i+b)} \times \frac{\partial \sigma(Wx_i+b)}{\partial (Wx_i+b)} \times \frac{\partial (Wx_i+b)}{\partial b}$$

$$= \sum_i^N -2(y_i - \sigma(Wx_i+b)) \times \frac{\partial \sigma(Wx_i+b)}{\partial (Wx_i+b)} \times 1$$

$$= \sum_i^N -2(y_i - \sigma(Wx_i+b)) \times \sigma(Wx_i+b)(1-\sigma(Wx_i+b)) \times 1,\ \because as\ is\ shown\ in\ (1)$$

$$= \sum_i^N -2(y_i - \frac{1}{1+e^{-(Wx_i+b)}}) \times \frac{1}{1+e^{-(Wx_i+b)}} \times (1 - \frac{1}{1+e^{-(Wx_i+b)}}) \times 1$$