Mancala games - Topics in Mathemathics and Artificial Intelligence

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The family of mancala games offers opportunities for new research in both Mathematics and Artificial Intelligence. To illustrate this, we will present an overview of long-time known and more recent results on mancala games. Although board-games researchers may be familiar with mancala games, the general properties of this family of board games will be explained as far as they are relevant to the understanding of the results. The results mainly reflect the mathematical characteristics of mancala, although computer science also played a significant role in the research presented. Next to this, research in the field of Artificial Intelligence is included. The paper then concludes with a discussion of new research opportunities in both Mathematics and Artificial Intelligence. These opportunities are not restricted to pure theoretical research but include issues that allow interdisciplinary co-operation of applied research.

Mancala games

Mancala games are board games that are played almost all over the world in many variations. It is impossible to describe all variations here. For a detailed overview see the books by Murray (1952) and Russ (2000). So far, all mancala games appear to share the following properties:

a) The games are played on a board with a number of *pits*, usually arranged in two or more *rows*. Sometimes additional pits are used that we will call *stores*.
b) The games are played with a collection of equal *counters* (stones, seeds, coins, or shells).
c) Players own pits, not counters. Often, a player owns all the pits on one side of the board.
d) Moves are made by *sowing*, which is a form of counting (see below).
e) After (or during) sowing, counters can be captured. (Hence mancala games are also called *count-and-capture* games.)
f) The goal of the game in general is to capture the majority of the counters.

Mostly two players are involved, but solitaire games are known as well as games for three or more players. Moves in mancala games are made by *sowing*. This means that the player who is to move, selects one of the own pits, takes all of the counters and puts them one-by-one in the consecutive pits in clockwise or anti-clockwise direction. Sometimes, sowing is done in *multiple laps*, which means that if the sowing ends in a non-empty pit, all counters of that pit are taken out and the sowing continues until some stopping condition is encountered. In some games, a player can continue with a new move if the sowing ends in the own store. If the sowing involves many counters, it can reach the pit from which the sowing started. It depends on the rules whether this pit is skipped or not.

After or during sowing, a capture may take place: the player takes all counters out of a pit and puts them in the own store (or keeps them apart if there is no store). In some games the captured stones are reentered into the own row of pits. The condition under which a capture can be made and which pit is to be emptied depends on the rules of the game. Roughly speaking, there are four types of capture:

1) Number capture: a player is allowed to capture counters when the sowing ends in an opponent's pit that, for example, contains 2 or 3 counters after the sowing. In some games also all preceding pits of the opponent that contain 2 or 3 counters can be emptied. (This rule is used in Awale and Awari and will be called the 2-3-*capture rule*.)
2) Place capture: a player is allowed to capture if the sowing ends in a particular pit. For instance, if one ends in an own pit that was empty, the counter in this pit *and* all the counters in the opposite pit of the opponent can be captured. (This rule is used in many Asian games but also in Kalah and will be called the *opposite-capture rule*.) In Indian mancala games the results of a sowing, e.g., capture or the continuation/termination, does not depend on the pit in which the last counter was put, but on the *next* pit.
3) En-passant capture (a special form of number capture): during a players move, the *opponent* can capture the counters in an own pit as soon as it contains, for example, 4 counters.
4) Store capture (a special form of place capture): counters that, if allowed, are put in the own store during a move are captured automatically. (This rule is common in Dakon.)

Next to these general rules, many games have additional rules like the "lending" of counters in some Indian games or the closing of pits in multiple-round games like Dakon. Normally, a game is ended when one of the players has captured the majority of the counters, or when one of the players cannot move anymore.

Mathematics of Mancala

Mancala games seem simple when it is noted that they are deterministic (no chance involved), have perfect information (except for the difficulty of remembering the contents of a crowded pit), and that there are not many choices per move, commonly not more than the number of pits on one row. However, the fact that a single move can have an effect on the contents of all pits on the board, makes it difficult to foresee the consequences of even a few moves ahead, let alone the final outcome of the game.

Mancala positions

A mathematical property that may give insight in the complexity of mancala games is the *number of possible positions*. A position in mancala games consists of a certain distribution of the counters over the pits of the board, but also includes the captured counters either in the stores on the board or kept by the players if there are no stores. Furthermore, a position includes the knowledge which player is to move next. The number of possible positions depends on the number of pits (and stores) and on the number of counters. This number p can be computed by the following formula that is derived from basic combinatorics:

$$p = k \cdot \binom{n + m - 1}{m} \qquad \textit{fig. 1}$$

In this formula, k is the number of players, m is the total number of counters and n is either the total number of pits and stores together or the total number of pits incremented with the number of players if no stores are present. The number p increases very rapidly with increasing numbers of pits and counters. The following table shows the number of positions for a mancala board with two rows of six pits and two stores in the case of two players.

fig. 2

	m	p
	1	28
	2	210
	3	1,120
	4	4,760
	5	17,136
(1 counter per pit)	12	10,400,600
(2 counters per pit)	24	7,124,934,600
(3 counters per pit)	36	5.25194×10^{11}
(4 counters per pit)	48	1.313244×10^{13}
(5 counters per pit)	60	1.725416×10^{14}
(6 counters per pit)	72	1.4776×10^{15}

Of course, the number of possible positions decreases when only the counters still in play are regarded (as is done in for example Retschitzki's analysis (1990)). Only a certain number of all possible positions can actually appear during games. This depends on the starting position of the game and on the exact rules. In the game of Kalah (see below), only about 5% of the possible positions can appear in a real game (Irving *et al*, 2000).

It appears that only a few special positions of mancala games can be understood completely in a mathematical sense. First we will look at a set of special positions in the game of Tchoukaillon.

Tchoukaillon positions

Broline and Loeb (1995) analyse mancala with the use of an artificially created solitaire variant of mancala: Tchoukaillon. This game was developed by Deledicq and Popova and is played on a board with only one row of pits and with one store at the right end of the pits. At the start, every pit except the store contains a given number of counters. The goal is to collect all the counters in the store. Only those moves are allowed that directly end in the store, no sowings beyond the store are allowed. The task of solving Tchoukaillon is to find the correct order of moves that causes all counters to be put in the store. It appears that only a few positions of Tchoukaillon can actually be solved. The following table contains the first ten of these positions.

1	0	0	0	0	1
2	0	0	0	2	0
3	0	0	0	2	1
4	0	0	3	1	0
5	0	0	3	1	1

6	0	0	4	2	0	0
7	0	0	4	2	0	1
8	0	0	4	2	2	0
9	0	0	4	2	2	1
10	0	5	3	1	1	0

fig. 3

For every number of counters still in play, there appears to be only one position that can be solved. To understand this, it is necessary to realise that in any given position only one move is appropriate. If there is more than one pit that contains just enough counters to reach the store, one has to select the rightmost of those pits. If one selects another pit, the contents of this rightmost pit will increase by one which means that the position cannot be solved anymore. Positions that can be solved are called Tchoukaillon positions. These can be derived by a simple algorithm:

1) The first Tchoukaillon position is the position with only a single counter in the rightmost pit.
2) Given a Tchoukaillon position, the next position can be constructed by taking one counter of each pit, starting at the right, until an empty pit is encountered. Put all collected counters plus one extra in this empty pit.

Tchoukaillon positions do not only play a role in this artificial game. In any mancala game that includes the rule that a player can move again if a sowing ends in the own store, these positions are important. These games include Kalah, Dakon, Ruma Tchuka and many others. If a Tchoukaillon position occurs at the player's side, the player is thus able to capture all the counters in this position.

a)

1	0	0	0	0	0
0	0	4	3	1	1

b)

0	0	0	0	0	0
1	0	4	3	1	1

c)

0	0	0	0	0	1
1	0	0	4	2	2

d)

0	0	0	0	1	0
1	0	0	4	2	2

e)

0	0	0	0	0	1
1	0	0	4	2	0

f)

0	0	0	0	1	0
1	0	0	4	2	0

g)

0	0	0	0	0	1
1	0	0	0	3	1

h)

0	0	0	0	1	0
1	0	0	0	3	1

i)

0	0	0	0	0	1
1	0	0	0	0	2

j)

0	0	0	0	1	0
1	0	0	0	0	2

k)

0	0	0	0	0	1
1	0	0	0	0	0

game over

fig. 4

Also mancala games that use the 2-3 capture rule and have no stores (like Wari and Awale) benefit from Tchoukaillon positions. Observe the following endgame of Awale:

0	0	0	0	0	1
7	5	3	1	2	2

fig. 5

The bold numbers indicate the pit to move. In moves c) to k) the shaded areas indicate the Tchoukaillon positions. By following these moves, the South player is able to capture 8 counters. In the best case, it is possible to capture even 20 counters using a sequence of 21 Tchoukaillon positions. The game should have reached the following rare position with North to move:

These "Tchoukaillon-endings" have some similarities with the well-known tactics in Awale called "2-1" and "3-1" (Retschitzki, 1990; N'Guessan, 1992).

Tchuka Ruma

The game Tchoukaillon was derived from the existing solitaire mancala game Tchuka Ruma. The origin of this game and where it is actually played is still subject of scientific dispute (Campbell and Chavey, 1995). Authors refer to India, the Philippines, the Maldives and even to Russia. The name suggests an Indonesian or Malay origin. Probably, the game will have been invented independently in multiple places because the rules of the game are only a small subset of the rules of many two-player mancala games. Tchuka Ruma can be played on any mancala board, with or without stores. The player just selects one pit or store as the main store (the *ruma*) and decides which other pits to include in the game. The goal of Tchuka Ruma is to collect all counters in the ruma. In contrast to Tchoukaillon, sowing appears in multiple laps and can go beyond the ruma. The game is started with an equal number of counters in every pit. Tchuka Ruma can be played on the internet at web-page: http://fanth.cs.unimaas.nl/games/ruma/index.html. It appears to be quite challenging to play Ruma Tchuka. Only with a very small number of pits (up to 4) and counters per pit (1 or 2) can most people solve the game.

Just like in Tchoukaillon, not every starting position of Tchuka Ruma can be solved. Campbell and Chavey (1995) give a detailed analysis of the mathematics in Tchuka Ruma. They show that it is possible to prove that some starting positions cannot be solved. For instance, if there are n pits and k counters per pit, then there is no solution if $k = (n + 1)^i$ with $i \geq 1$, or if $k = n.(n + 1)^i$, with $i \geq 0$. They also show that it is impossible to prove that a given Tchuka-Ruma position can be solved, other than actually *producing* a solution.

	Counters per pit:									
	1	2	3	4	5	6	7	8	9	10
1 pit	1	1	3	3	4	6	7	7	8	9
2 pits	1	2	1	8	10	7	14	16	4	20
3 pits	3	4	2	1	15	17	21	24	27	30
4 pits	3	8	7	2	1	24	28	32	36	40
5 pits	3	7	14	6	2	1	35	40	45	50
6 pits	3	10	15	24	30	2	1	48	54	60
7 pits	5	14	19	27	35	13	2	1	63	70
8 pits	5	12	24	32	40	48	9	2	1	80
9 pits	5	15	27	36	45	54	63	72	2	1
10 pits	5	17	30	40	50	60	70	80	90	2
11 pits	7	19	33	44	55	66	77	88	99	110
12 pits	7	19	36	48	60	72	84	96	108	120

fig. 6

With a little change in the rules, we can define a “solution” as the sequence of moves that captures the *most* counters possible. In the following table, we present the number of counters that can be maximally captured for a number of Tchuka Ruma positions.

The shaded entries in the table indicate the positions in which not all counters on the board can be captured. The cells near the diagonal are positions in which $k = n$ or $k = n+1$, both special cases of the formulae above. The table suggests that solutions are more probable to exist if both the number of pits and the number of counters per pit increase. Campbell and Chavey have checked positions up to 10 pits for very large amount of counters (100,000). Their results confirm this suggestion.

Next to finding whether all counters can be captured, it is also interesting to know what the minimum number of moves is that is needed to capture all possible counters. It is a trivial task to write a computer program that finds these “optimal” solutions. Tchuka Ruma clearly is a game that is challenging for humans, but trivial for computers. This is certainly not the case for other mancala games like Awari and Bao.

Dakon

One of the mancala games that is connected to Tchuka Ruma is the game of Dakon, played under different names on the Maldives, in Indonesia and at the Philippines. This two-person game is played on a board with two rows of pits and two stores. It uses sowing in multiple laps and includes the opposite-capture rule. A user can continue with a move if a sowing ends in the own store. For the exact rules we refer to (Donkers *et al.*, 2000).

A game of Dakon consists of multiple rounds. The first round starts with as many counters in each pit as there are pits in each row. The players move in turn until one player cannot move again. For the second round both players fill the pits with the captured counters. Every pit must contain the original amount of counters. If one player cannot fill all his pits fully, the (partial) empty pits are covered with a leaf and disregarded in this round. A player that cannot fill a single pit loses the game.

It was discovered by hand that for Dakon with 8 pits per side, it is possible for the first player in the first round to collect so many counters in the first turn that the opponent is not able anymore to capture enough counters to start the next round. So this game can be won by the first player in a single move. This means that, mathematically spoken, this Dakon game is not a (2-person) game but a mere (1-person) puzzle that is very similar to Tchuka Ruma. The only difference between solving Tchuka Ruma and finding a winning sequence of moves in Dakon is that in Dakon a player cannot select all pits to move from, but only the pits on the own side of the board. Because one cannot select all pits it is not possible to capture all counters in one turn. If there are n pits per side on the board then it is impossible to capture more than (n / 2˚ – 1 counters. This formula only provides a lower bound; the following table shows the exact number of counters that cannot be captured for different values of n.

n	4	5	6	7	8	9	10
	2	3	2	2	3	3	4

fig. 7

For a computer it is not difficult to find these correct sequences of moves in Dakon, but for human players this is a difficult task. The fact that players were able to find such a sequence for Dakon with n=8 is remarkable, especially because the sequences that were found are very long: one of them counts 93 moves. In (Donkers *et al.*, 2000) we showed that the computer can be used to analyse how people could be able to find these sequences. Instead of just performing the easy task of finding a solution on the computer, we tried to mimic human cognitive behaviour.

The sequences of moves that we found in this way on the computer were similar to the human-found sequences.

As a side step we mention that knowing that there exist winning openings does not prevent the game from being played. With a small adaptation of the rules, Dakon is still very challenging. One odd change that has actually been adopted in the Philippines, is to start the first round *simultaneously* by both players.

Bao

The game of Bao is a popular East-African variant of mancala. This game is also played in an officially organised way in Tanzania. Bao is played by two players on a board with four rows of pits. Bao has no stores (though there are two special pits in the centre of the board). Sowing is done in multiple laps. Further rules of the game can be found in the thesis of De Voogt (1995).

During the play of Bao it can happen that a sowing seems *perpetual*: after some period the player resigns or some special rule is applied to continue the game. So the research question can be posed whether a sowing in Bao can indeed be perpetual. After some time, the original starting position should reappear. It is known that in India (see the paper of dr. Balambal Ramaswamy in these proceedings) such a perpetual sowing exists and is practised as a variant of Pallankhuzi, called Seethi Pandi.

The simplest form of perpetual sowing is on a board with only two pits. (Of course, the trivial situation with one pit must be disregarded.) The first pit has three counters, the second pit has one counter. If the sowing is started with the first pit, a perpetual sequence occurs. It is not difficult to see that in the case of two pits, perpetual sowing occurs if the larger amount of counters is a number that is twice the smaller number plus one ((3,1), (5,2), (7,3), (9,4), and in general ($2k$+1,k)). Sowing should always start with the larger amount. After each lap of sowing, the contents of the pits have switched ((1,3)-(3,1)-(1,3)-…).

If the game starts with the seven counters in one pit and one in the other, again a perpetual sowing occurs, but now a more complicated pattern appears: ($\underline{7}$,1)-(3,$\underline{5}$)-($\underline{6}$,2)-($\underline{3}$,5)-(1,$\underline{7}$)-($\underline{5}$,3)-(2,$\underline{6}$)-(5,$\underline{3}$)-($\underline{7}$,1). The underlined numbers indicate the pit to continue with. After eight sowings the original position reappears. We say that the *period* of this sowing is eight. The following pattern has only a period of three: ($\underline{9}$,1)-(4,$\underline{6}$)-(7,$\underline{3}$)-($\underline{9}$,1). If the game starts with ($\underline{7}$,2), the period of the sequence appears to be 24. A little analysis produced the following table in which k is any positive number:

Starting position	Period
[2k+1,k]	2
[4k+2,k]	4
[6k+3,k]	3
[8k+4,k]	6
[10k+5,k]	10
[12k+6,k]	12

fig. 8

(It can be proven that all positions of the form $(2a.k + a, (2b + 1)k + b)$ where $k \geq 1$, $a \geq 1$ and $b \geq 0$, cause perpetual sowings if the sowing starts with the first pit.) In the case of two pits, it might be possible find a formula for all possible perpetual sowings and their periods. For larger amounts of pits, this task is much more complex, let alone for the case of real mancala game boards with 10, 12 or more pits involved. Bao players suggested that the board situation in which the contents of all the pits alternate between 2 and 3 produces a perpetual sowing if one starts with a pit with 2 counters. With the aid of a small computer program, we checked this and discovered that it was true. The amazing thing is, that the periods of these perpetual sowings is very large. It is not difficult to see why no one was ever able to check the perpetual property of the sowing:

Board size	Period	Rounds
2 x 1 pits	(no perpetual sowing)	
2 x 2 pits	60	55
2 x 3 pits	260	161
2 x 4 pits	6,312	2,943
2 x 5 pits	13,535	5,038
2 x 6 pits	118,824	37,271
2 x 7 pits	905,471	245,520
2 x 8 pits	11,944,136	2,835,141
2 x 9 pits	38,810,079	8,228,045
2 x 10 pits	362,728,700	69,414,849
2 x 11 pits	418,038,808	72,909,333
2 x 12 pits	4,460,747,004	715,501,875
2 x 13 pits	177,651,347,042	26,365,775,535

fig. 9

The column titled *rounds* indicates the number of times that the sowing circles the whole board before the starting position reappears. Suppose that a quick player can do one round in one second. In case of sowing on the 2x8-pits board, which is the common situation in Bao, the player has to continue the sowing for a complete month before the original pattern reappears. The 2x13-pits board would take even more than 800 years!

The question remains open which other perpetual sowings can occur on real Bao boards, but the analysis of the 2-pits board suggests that there should be many positions that will cause perpetual sowing. A more important open question is, whether such positions can occur during an actual game of Bao. Ethnographical research should unveil whether perpetual sowings are actually used like in the game of Seethi Pandi.

Mancala in Artificial Intelligence

Artificial Intelligence (AI) is a branch of computer science which aims to have a computer perform what normally is considered as intelligent (human) behaviour. There are two main motivations why this research is done: the first one is to alleviate human efforts with computer support. The second motivation is to try to understand how human intelligence or mental operations work through computer simulation. Games have played an important role in Artificial Intelligence because game playing is considered as a typically intelligent task, but also because games almost always form a closed environment with limited possibilities and clearly defined rules.

The latter makes it for AI-researchers much easier to treat games than to treat, for instance, the stock market. The most famous result of AI in games is of course the fact that computers nowadays can play chess at grandmaster level and even defeated the former world champion. In the competitive race between computer-chess programs, many techniques have been developed that are now being applied on a regular base in other areas of AI and computer science. One could say that games (and especially chess) are the Formula-1 of computer science.

The family of mancala games has been introduced in Artificial Intelligence relatively early, although most research is restricted to only two games: Kalah and Awari.

Kalah

The game of Kalah is a modern, commercial variant of mancala. It was introduced in the 1950s by a firm called “The Kalah Game Company” (owned by W.J. Champion). In 1960, a first computerized version of the game was created and many others followed. Today, it is even possible to play Kalah (called Bantumi) against your cellular phone (see http://www.nokia.com/games/bantumi.html). In Artificial Intelligence, Kalah has been studied as early as 1964 by Richard Russel. He wrote a program, called KALAH that actually could play the game. In 1968, A.G. Bell wrote another computer program that could learn in some way from the errors that it made. A year later, Slagle and Dixon (1969, 1970) used the game of Kalah to illustrate another algorithm for playing games. After this period, Kalah lost the interest of AI game researchers, that is, until last year. Using some of the advanced techniques that were developed for chess, Irving, who is an undergraduate student at Caltech University, was able to find the winning strategy for Kalah (Irving *et al.*, 2000).

Kalah is played by two persons on a board with two rows of six pits and two additional stores. At the start there are four counters per pit. It uses single-lap sowings and the opposite-capture rule. The own store is included in the sowing, but the opponent’s pit is skipped. A sowing that ends in the own store grants the player another move. In some of the Kalah programs the pit from which a sowing starts is skipped during a large sowing, but in other implementations it is not. The game ends if one of the players cannot move anymore. The other player then captures all counters in the own pits. The player who captured the most counters wins.

It is possible to bend the rules of Kalah a little and to play Kalah with less or more counters per pit, or with another number of pits per row. The following table shows the game-theoretic value of Kalah-instances, i.e., whether the starting player can win the game, will lose it or whether the game is a draw if both players play optimally.

	Counters per pit					
	1	2	3	4	5	6
1 pit	D	L	W	L	W	D
2 pits	W	L	L	L	W	W
3 pits	D	W	W	W	W	L
4 pits	W	W	W	W	W	D
5 pits	D	D	W	W	W	W
6 pits	W	W	W	W	W	?

fig. 10

The smaller instances of Kalah were solved by considering every possible position that actually can arise during a game of Kalah. Databases were created in which every position and its game-theoretic value are stored. The larger instances of Kalah were solved by game-tree search. For these instances, only a winning strategy from the opening position is known explicitly. The program is however able to find an optimal strategy for every position that can occur during a game of Kalah.

This means that Kalah is not of interest anymore for those AI researchers that want the computer to play the game of Kalah as substitutes or superiors of humans. However, for those AI researchers and game psychologists who are interested in the human aspect of game playing, the results that were collected for Kalah remain useful.

Awari

The other mancala game that gained interest from AI game researchers is Awari. This game is played in West Africa and the West Indies. Awari is also known as Wari or Awale. Awari is played on a board with two rows of six pits and no stores. Sowing happens in single laps and captures are of the 2-3 type. If the opponent has no more counters available, a player should select a move that brings new counters to the opponent's pits. If this is not possible, the game is over. In Awari as it was programmed, the pit from which a sowing starts is not excluded during a long sowing (with 12 or more counters), in other variants of this game this pit is skipped. This difference in rules is important for the construction of kroo's. These are pits with so many counters in it that a sowing travels the board one and a halve turn. In this way many counters can be captured in one move. Building and playing a kroo is an important strategem in Awari.

The interest in Awari started in the AI community by the construction of a program called ÔLITHIDION' (Van der Meulen *et al.*, 1990) and has been growing steadily since then. The game of Awari is the only mancala game that is played on the computer olympiad. This is an event in which all kind of computer programs compete in several classical games like Chess, Checkers, and Go, but also in new and artificial games like Hex and Lines Of Action. Last year saw the fifth edition of the computer olympiad and Awari has seen competition all five times.

Although Awari is played with the same amount of pits and the same amount of counters per pit as Kalah, it appears that Awari is more difficult for the computer than Kalah. Since Kalah is originally designed as a children's game and Awari is mostly played by adults, it seems that Awari is the more difficult game for humans too. One way to understand this is the fact that in Kalah counters are automatically captured if a sowing passes the stores. This means that in Kalah a repetition of positions is not possible. Therefore, a game of Kalah will on average last shorter than a game of Awari. The number of choices in Awari and Kalah are the same, so the total game of Kalah must be easier than Awari. The fact that a student has solved Kalah, but several competing teams of AI researchers were still not able to solve Awari despite the serious efforts done in this direction, also illustrates this.

In the competition for solving Awari, one strategy of the participating teams is to build large end-game databases. These databases contain for a huge number of board positions how many counters can be captured and which move is the best to play. The team of Lincke (2000) already has constructed the databases that contain all board positions with 35 or less counters still in play. It is sure that it is impossible to create a complete database with all possible positions. The expectation is that soon the game-tree search from the starting position and the end-game database will meat in the middle.

Conclusions

Mancala games can pose many questions to mathematicians. Only a few questions have yet been put forward and some of them were answered with or without the support of computer science. In a few cases these answers can help in developing better strategies for playing mancala (the Tchoukaillon positions), many others are less relevant for players (the perpetual sowings). We hope that this paper will inspire the reader to pose further questions.

The researchers in Artificial Intelligence have only looked at a few mancala games. The rich family of mancala games provides games that differ much from Awari and Kalah, both in complexity and in strategy. For those AI researchers that want to play or solve new games on their computers, mancala games offer many opportunities. Some of the questions that might be worth to be answered by AI researchers are:

- What is the effect of a rule on the complexity of a mancala game (for different types of complexity)?
- Is it possible to predict the complexity of a mancala game on base of a given set of rules?
- What heuristics can be used for playing mancala games by a computer?
- How should special rules be handled like the lending of counters, covering a pit, or Ôcheating'?

Mancala games also offer opportunities for interdisciplinary research. In our research on Dakon we showed that the computer can be used to assist cognitive-psychological research. Retschitzky and N'Guessan used an intelligent computer program to investigate learning processes in players. The program was only used as a fixed partner to play against. In (Donkers *et al*, 2001) a game algorithm is proposed that uses knowledge on the opponent. This algorithm can in principle also be used to determine the strategy that a (human) player is using against a computer. It could therefore be used in a developmental-psychological research on the learning of mancala games by children.

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