Leading order loop calculation

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This is a document detailing the calculation of the full leading order vertices in the interactions of a BSM pseudoscalar a with gauge bosons. The gluon gluon fusion cross section, which occurs through a quark loop, is calculated by hand, and the amplitude is used to write down expressions for the branching ratios.

1 The ggf cross section

The pseudoscalar, a colour singlet, cannot couple directly to gluons. As a result, gluon gluon fusion occurs through a loop of quarks. This process is represented by the triangle diagrams in Figure 1. These two diagrams give the same result. [1].

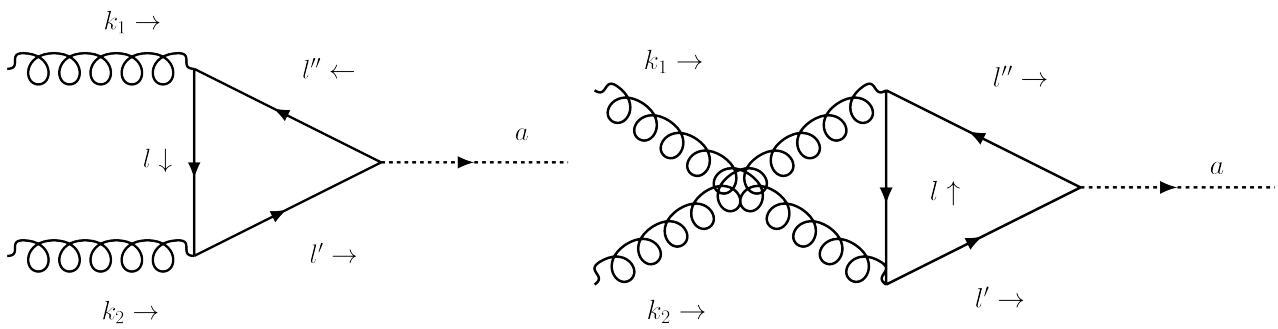


Figure 1: Feynman diagrams

We have chosen this direction for the momenta so that the propagators are written the same way in the integral. We expect that these diagrams will produce a finite result as there are no LO tree level processes which could absorb any infinities arising from this one loop process. For now, let's consider the top quark loop as the dominant contribution, and denote the mass of the quark in the loop as m_f; given that the bottom quark is ~ 35 times lighter than the top, it experiences a relative suppression of around 35^2. However, we may include the bottom loop too, for completeness.

We can express a generic cross section as [2]

$$\hat{\sigma}(gg \rightarrow a) = \frac{1}{2\hat{s}} \int dPS \overline{\sum}_{spins,colour} |\mathcal{M}|^2 \tag{1}$$

1.1 Amplitude $\mathcal{M}$

In order to calculate the amplitude in Equation 1, we need to consider the vertex factors and propagators in the diagram. We will use that

i, j label quark colours
a, b label gluon colours
α, β label spinor indices
μ, ν label Lorentz group indices for gluons

To each ffa vertex we assign a factor of

$$\frac{-im_f}{v}\gamma^5 g_{Af} \tag{2}$$

where g_{Af} is a dimensionless coupling constant.
To each $gf_\alpha f_\beta$ vertex we assign

$$-ig_s\gamma^\mu_{\alpha\beta}[t^a]_{ij}. \tag{3}$$

The propagator is assigned

$$\frac{i(\not{p}+m)_{\beta\alpha}\delta_{ij}}{p^2-m_f^2+i\epsilon} \tag{4}$$

An external boson leg gets a factor of 1. The polarisation of external gluons is $\epsilon_\mu(\lambda, k_i)$ for a gluon in spin state λ with momentum k. Notably, gluons may only have momentum in a direction transverse to their polarisation, so $\epsilon_\mu(\lambda, k_i)\cdot k_i = 0$. The gluons are, of course, massless, and we will use that $k_1^2 = 0 = k_2^2$, and $(k_1+k_2)^2 = \hat{s} = M_a^2$. This can also be done off mass-shall for the gluons (see [3]).

1.1.1 Loop integral

The momentum in the loop is l, and we regularise the loop integral by using $d = 4-2\varepsilon$. The amplitude is then

$$\begin{aligned}\mathcal{M}_1 = \int \frac{d^dl}{(2\pi)^d}\frac{1}{D_1D_2D_3}Tr\Bigg[&\left(-i\frac{y_t}{\sqrt{2}}\gamma^5\right) i\left(\not{l}+\not{k}_2+m\right)\left(-ig_s\gamma^\mu\left[t^a\right]\right)\epsilon_\mu(\lambda,k_2)i\left(\not{l}+m\right)\left(-ig_s\gamma^\nu\left[t^b\right]\right)\epsilon_\nu(\lambda,k_1)i\left(\not{l}-\not{k}_2+m\right)\\ &+\left(-i\frac{y_t}{\sqrt{2}}\gamma^5\right) i\left(-\not{l}+\not{k}_1+m\right)\left(-ig_s\gamma^\mu\left[t^b\right]\right)\epsilon_\nu(\lambda,k_1)i\left(-\not{l}+m\right)\left(-ig_s\gamma^\nu\left[t^a\right]\right)\epsilon_\mu(\lambda,k_2)i\left(-\not{l}-\not{k}_2+m\right)\Bigg],\end{aligned} \tag{5}$$

where

$$\begin{aligned}D_1 &= l^2-m_f^2,\\ D_2 &= (l-k_1)^2-m_f^2\\ D_3 &= (l+k_2)^2-m_f^2.\end{aligned} \tag{6}$$

m is the mass of the propagator, which we take here to be the top quark (as it is the dominant contribution).

1.1.2 The numerator

Let's look at the numerator first. We have a factor of -1 emerging from the fermion loop

$$\begin{aligned}\mathcal{M}_{num} = \mathcal{M}_{1,num}+\mathcal{M}_{2,num} = 2\mathcal{M}_{1,num} = \frac{y_t}{\sqrt{2}}g_s^2(-i)(-1)(i^3)\mathrm{Tr}\left[t^at^b\right]\epsilon_\mu(\lambda,k_2)\epsilon_\nu(\lambda,k_2)\int\frac{d^dl}{(2\pi)^d}\\ \mathrm{Tr}\Bigg[\left(\gamma^5\not{l}\gamma^\mu\not{l}+m\gamma^5\not{l}\gamma^\mu+\gamma^5\not{k}_2\gamma^m u\not{l}+m\gamma^5\not{k}_2\gamma^m u+m\gamma^5\gamma^\mu\not{l}+m_f^2\gamma^5\gamma^\mu\right)\left(\gamma^\nu\not{l}-\gamma^\nu\not{k}_2+m\gamma^\nu\right)\\ +\left(\gamma^5\not{l}\gamma^\nu\not{l}-m\gamma^5\not{l}\gamma^\nu-\gamma^5\not{k}_1\gamma^\nu\not{l}+m\gamma^5\not{k}_1\gamma^\nu-m\gamma^5\gamma^\nu\not{l}+m_f^2\gamma^5\gamma^\nu\right)\left(-\gamma^\mu\not{l}-\gamma^\mu\not{k}_2+m\gamma^\mu\right)\Bigg].\end{aligned} \tag{7}$$

We recall the useful properties of γ matrices;

$$\begin{aligned}&\mathrm{Tr}(\text{odd \# }\gamma\text{ matrices}) = 0\\ &\mathrm{Tr}(\gamma^5) = 0\\ &\mathrm{Tr}(\gamma^5\ \times\ \text{odd \# }\gamma\text{ matrices}) = 0\\ &\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu) = 0\\ &\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\varepsilon^{\mu\nu\rho\sigma}.\end{aligned} \tag{8}$$

We can then expand Equation 7 using these properties, and using $\not{l} = l_\sigma\gamma^\sigma$,

$$\mathcal{M}_{1,num} = 4m\epsilon^{\sigma\rho\mu\nu}k_{1,\sigma}k_{2,\rho}\epsilon_\mu(\lambda,k_2)\epsilon_\nu(\lambda,k_2). \tag{9}$$

The remaining integral is an integral over the loop momentum l;

$$\mathcal{M}_{1,den} \int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2 - m_f^2)\left((l-k_1)^2 - m_f^2\right)\left((l+k_2)^2 - m_f^2\right)}. \tag{10}$$

1.1.3 The denominator

This integral can be performed using conventional techniques (see handwritten notes, with reference to [2, 4]) and evaluates to

$$C_0 = \frac{-i}{16\pi^2}\frac{1}{M_a^2}\begin{cases} \frac{1}{2}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right)\right]^2 & \text{if } \tau < 1 \\ -2\arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) & \text{if } \tau \geq 1. \end{cases} \tag{11}$$

We are left with

$$\mathcal{M}_1 = \frac{4m_f^2 g_{Af}}{v} g_s^2 \mathrm{Tr}\left[t^a t^b\right] C_0 \epsilon^{\sigma\rho\mu\nu} k_{1,\sigma} k_{2,\rho} \epsilon_\mu(\lambda, k_2)\epsilon_\nu(\lambda, k_2). \tag{12}$$

We'll use

$$\epsilon(k_1, k_2, \mu, \nu) = \epsilon^{\sigma\rho\mu\nu} k_{1,\sigma} k_{2,\rho} \epsilon_\mu(\lambda, k_2)\epsilon_\nu(\lambda, k_2) \tag{13}$$

for brevity.

1.2 Phase Space

We should consider all kinematically allowed final states; the 2 to 1 one particle phase space is [2, 5]

$$\begin{aligned} \int dPS &= \int \frac{d^4 q}{(2\pi)^4}(2\pi)\delta(q^2 - M_a^2)(2\pi)^4\delta^4(k_1 + k_2 - q) \\ &= 2\pi\delta(\hat{s} - M_a^2) \end{aligned} \tag{14}$$

where $\hat{s}$ is the gg invariant energy squared [6].

The cross section then becomes

$$\hat{\sigma}(gg \to a) = \frac{\pi}{\hat{s}}\delta(\hat{s} - M_a^2)\overline{\sum}_{spins,colour}|\mathcal{M}|^2 \tag{15}$$

1.3 Putting it all together

The factors in front of the cross section are

$$g_{Af}\frac{m_f}{v} g_s^2 \mathrm{Tr}\left[t^a t^b\right] \tag{16}$$

We'll use that

$$\begin{aligned} \frac{m_f^2}{v^2} &= \sqrt{2} G_F m_f^2, \\ g_s^2 &= \sqrt{16\pi^2\alpha_s^2} \end{aligned} \tag{17}$$

and also [2] that $\mathrm{Tr}[t_a t_b]^2 = \frac{1}{4}\delta_{ab}\delta^{ab} = 2$, to write

$$S = 32\sqrt{2} g_{Af} G_F \pi^2 \alpha_s^2 m_f^2 \overline{\sum}|\mathcal{M}_1|^2, \tag{18}$$

with

$$|\mathcal{M}_1|^2 = |4m\,\epsilon(k_1, k_2, \mu, \nu)C_0|^2 \tag{19}$$

Next, let's average over the relevant initial states of $\mathcal{M}^2$. In averaging over the initial states we get a factor of $\left(\frac{1}{2}\right)^2$ for the spins and $\left(\frac{1}{8}\right)^2$ for the colour of the gluons [2].

such that

$$
\begin{aligned}
S &= \frac{32\sqrt{2}g_{Af}G_F\pi^2\alpha_s^2 m_f^2|\mathcal{M}_1|^2}{8^2 2^2} \\
S &= \frac{\sqrt{2}g_{Af}G_F\pi^2\alpha_s^2 m_f^2|\mathcal{M}_1|^2}{8}.
\end{aligned}
\tag{20}
$$

Using Equation 15, we then have

$$
\begin{aligned}
si\hat{g}ma =& \frac{\sqrt{2}}{8M_a^2} g_{Af}^2 G_F \pi^3 \alpha_s^2 m_f^2 |\mathcal{M}_1|^2 \\
=& \frac{\sqrt{2}}{8M_a^2} g_{Af}^2 G_F \pi^3 \alpha_s^2 m_f^2 16 m_f^2 |\epsilon(k_1, k_2, \mu, \nu)|^2 |C_0|^2
\end{aligned}
\tag{21}
$$

Let's write Equation 11 instead as

$$
f(\tau) = \begin{cases} \frac{1}{2}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right)\right]^2 & \text{if } \tau < 1 \\ -2\arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) & \text{if } \tau \geq 1, \end{cases}
\tag{22}
$$

and move the factor $|\left(\frac{i}{16\pi^2}\frac{1}{M_a^2}\right)|^2$ into the main amplitude. Then Equation 21 becomes

$$
\begin{aligned}
\hat{\sigma} &= \frac{\sqrt{2}}{8M_a^2} g_{Af}^2 G_F \pi^3 \alpha_s^2 m_f^2 16 m_f^2 \left(\frac{1}{16\pi^2}\frac{1}{M_a^2}\right)^2 |\epsilon(k_1, k_2, \mu, \nu)|^2 |f|^2 \\
&= \frac{\sqrt{2} g_{Af}^2 G_F \alpha_s^2}{2048\pi M_a^2} \frac{16 m_f^4}{M_a^4} |\epsilon(k_1, k_2, \mu, \nu)|^2 |f|^2 \\
&= \frac{\sqrt{2} g_{Af}^2 G_F \alpha_s^2}{2048\pi M_a^2} |\epsilon(k_1, k_2, \mu, \nu)|^2 |\tau f(\tau)|^2
\end{aligned}
\tag{23}
$$

We need to multiply by 4 to get the result of both diagrams ($|\mathcal{M}|^2 = |M_1 + M_2|^2 = |2M_1|^2 = 4|M_1|^2$), and we can redefine f such that

$$
f(\tau) = \begin{cases} \frac{1}{4}\left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right)\right]^2 & \text{if } \tau < 1 \\ -\arcsin^2\left(\frac{1}{\sqrt{\tau}}\right) & \text{if } \tau \geq 1, \end{cases}
\tag{24}
$$

We then write Equation 23 as

$$
\begin{aligned}
\hat{\sigma} &= \frac{\sqrt{2} g_{Af}^2 G_F \alpha_s^2}{128\pi M_a^2} |\tau f(\tau)|^2 |\varepsilon^{\mu\nu\sigma\rho} \epsilon_\mu(\lambda, k_2) \epsilon_\nu(\lambda, k_2) k_{1,\sigma} k_{2,\rho}|^2 \\
&= \frac{\sqrt{2} g_{Af}^2 G_F \alpha_s^2}{128\pi M_a^2} |\tau f(\tau)|^2 |\epsilon(k_1, k_2, \mu, \nu)|^2 \\
&= \frac{\sqrt{2} g_{Af}^2 G_F \alpha_s^2}{64\pi M_a^2} |\tau f(\tau)|^2 (k_1 \cdot k_2)^2,
\end{aligned}
\tag{25}
$$

where $|\epsilon(k_1, k_2, \mu, \nu)|^2 = |-2(k_1 \cdot k_2)^2|$. For massless particles, we have that the mandelstam variable s reduces to [5]

$$
s = (k_1 + k_2)^2 = 2k_1 \cdot k_2 = M_a^2 \Longrightarrow (k_1 \cdot k_2)^2 = \frac{M_a^4}{4}
\tag{26}
$$

such that 25 becomes

$$
\hat{\sigma} = \frac{\sqrt{2} g_{Af}^2 G_F \alpha_s^2}{256\pi} |\tau f(\tau)|^2 M_a^2
\tag{27}
$$

Now that we have the full leading order cross section $\hat{\sigma}$, which is written [2]

$$
\hat{\sigma}_{LO} = \sigma_0 M_a^2 \delta(\hat{s} - M_a^2).
\tag{28}
$$

The cross section for finite fermion masses reduces to

$$\sigma_0 = \frac{\sqrt{2}G_F}{256\pi} g_{Af}^2 \alpha_s^2 \, |\tau f(\tau)|^2, \tag{29}$$

agreeing with the literature [7, 3] Absorbing the dimensionless coefficient g_{Af} into the function $A(\tau)$, we have

$$\sigma_0 = \frac{\sqrt{2}G_F}{256\pi} \alpha_s^2 \, |A(\tau)|^2 \tag{30}$$

which agrees with the SM Higgs cross section for this process up to the function $A(\tau)$ (see for instance [8].

This is generalisable to include other SM fermions, by

$$\sigma_0 = \frac{\sqrt{2}G_F}{256\pi} \alpha_s^2 \, |\sum_f A(\tau_f)|^2, \tag{31}$$

summing over contributions from each quark species. Note that for bottom quarks, $\tau_b < 1$.

2 Branching ratio calculations

A general branching ratio is written [9]

$$\Gamma(a \to XX') = \frac{1}{2M_a} \int dQ_2 \sum |\mathcal{M}_{a\to XX'}|^2, \tag{32}$$

where [9]

$$\int dQ_2 = \int \frac{1}{(2\pi)^2} \frac{p}{4\sqrt{s}} d\Omega. \tag{33}$$

For gluons and photons, which are massless and identical final state particles, the two particle phase space is

$$\begin{aligned} \int dQ_2 &= \frac{1}{8\pi} S \\ &= \frac{1}{16\pi} \end{aligned} \tag{34}$$

where we have used that $d\Omega = 4\pi$, and $S = 1/n!$ is the symmetry factor for n identical final state particles. We then need to include the amplitude $|\mathcal{M}|^2$, including a sum over final state colours and spins.

The amplitude $|\mathcal{M}|$ for the general aXX process including the fermion triangle diagram is, from the previous section (without summing over colours or spin states yet),

$$|\mathcal{M}(gga)|^2 = \frac{\sqrt{2}}{2\pi^2} G_F \alpha_s^2 M_a^4 |A(\tau)|^2. \tag{35}$$

To calculate the photon amplitude, we need only replace the strong coupling constant by the fine structure constant, and multiply by a colour factor;

$$|\mathcal{M}(\gamma\gamma a)|^2 = \frac{\sqrt{2}}{2\pi^2} G_F \alpha^2 M_a^4 |N_c Q_f^2 A(\tau)|^2, \tag{36}$$

where N_c is 3 for quarks and 1 for leptons, and Q_f^2 is the charge of the fermion.

For final state gluons we'll have a factor of 8 from summing over the colours

$$\sum_{a,b} \delta_{ab}\delta_{ab} = \sum_a \delta_{aa} = 8, \tag{37}$$

so that, given

$$\Gamma(a \to XX') = \frac{1}{32\pi M_a} \int \sum |\mathcal{M}_{a\to XX'}|^2, \tag{38}$$

we have

$$\begin{aligned}\Gamma(a \to gg) &= \frac{8\sqrt{2}G_F\alpha_s^2 M_a^3}{64\pi^3}|A(\tau)|^2 \\ &= \frac{\sqrt{2}G_F\alpha_s^2 M_a^3}{8\pi^3}|A(\tau)|^2,\end{aligned} \tag{39}$$

and for photons

$$\Gamma(a \to \gamma\gamma) = \frac{\sqrt{2}G_F\alpha^2 M_a^3}{64\pi^3}|N_c Q_f^2 A(\tau)|^2 \tag{40}$$

The full photon branching ratio should also include a W loop?

3 Application to SM + a

The interaction Lagrangian for our pseudoscalar a is

$$\begin{aligned}\mathcal{L}_{int} = &-\frac{1}{2}m_a^2 a^2 - \Sigma_f \frac{iC_f m_f}{f_a} a\bar{\Psi}_f\gamma^5\Psi_f \\ &+ \frac{g_s^2\kappa_g}{16\pi^2 f_a} aG^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \frac{g^2\kappa_W}{16\pi^2 f_a} aW^i_{\mu\nu}\tilde{W}^{i\mu\nu} + \frac{g'^2\kappa_B}{16\pi^2 f_a} aB_{\mu\nu}\tilde{B}^{\mu\nu}\end{aligned} \tag{41}$$

where the coefficients $K_{g,W,B}$ denote contributions in addition to those from SM loops.

For agg vertices, the factor $K_g$has been modified in [10] in order to include loops of SM particles by

$$K_{g,eff} \equiv Kg + A(\tau) = K_g + g_{Af}\tau f(\tau). \tag{42}$$

The cross section that I have calculated is just with sm fermions in the loop. Should add this cross section to the one MG spits out with pure Kg?

The branching ratio for gluons and photons due to contributions in addition to those from SM loops are given by [11]

$$\begin{aligned}\Gamma_{BSM}(a \to gg) &= \frac{\alpha_s^2\kappa_g^2 M_a^3}{8\pi^3 f_a^2} \\ \Gamma_{BSM}(a \to \gamma\gamma) &= \frac{\alpha^2(\kappa_W + \kappa_B)^2 M_a^3}{64\pi^3 f_a^2}\end{aligned} \tag{43}$$

Using $Gf = 1/\sqrt{2}vev^2$, the total branching ratios including those with SM loops are

$$\Gamma_{BSM}(a \to gg) = \frac{\alpha_s^2 M_a^3}{8\pi^3 f_a^2}|\kappa_g + A(\tau)|^2, \tag{44}$$

$$\Gamma_{BSM}(a \to \gamma\gamma) = \frac{\alpha^2 M_a^3}{64\pi^3 f_a^2}|(\kappa_W + \kappa_B) + N_c Q_f^2 A(\tau)|^2 \tag{45}$$

which becomes, for top loops,

$$\Gamma_{BSM}(a \to \gamma\gamma) = \frac{\alpha^2 M_a^3}{64\pi^3 f_a^2}|(\kappa_W + \kappa_B) + \frac{4}{3}A(\tau)|^2 \tag{46}$$

Plotted here are the branching ratios for the pseudo-scalar to dominant modes. They look very similar to Gabriele's. I have used the FeynRules calculation for the fermion branching modes, and have included all possible decay modes (where Gabriele only included the most dominant ones), as well as using (46) for the photon BR.

I have used (44) for the branching ratio to gluons, with a modification to measure the branching ratio to hadrons (see 7a [12]).

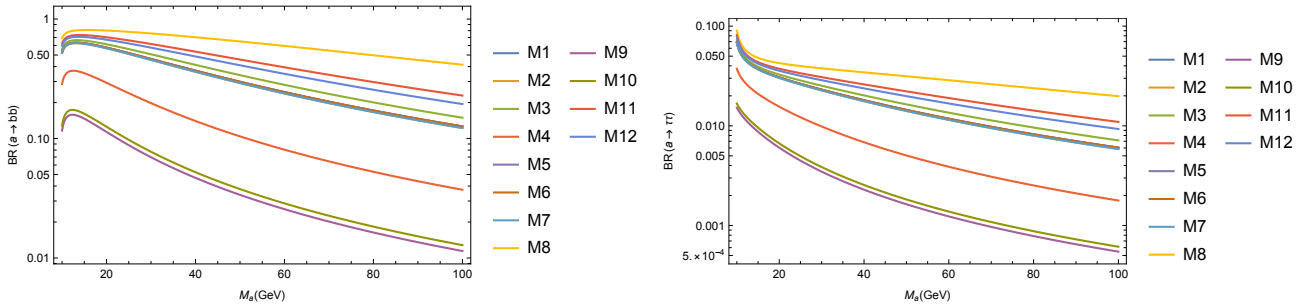


Figure 2: Branching ratios of a to b quarks (left) and τ leptons (right) for each model.

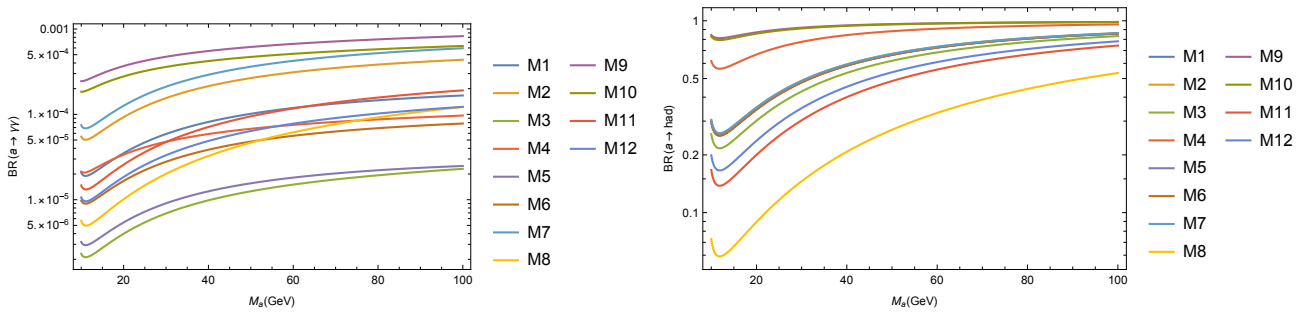


Figure 3: Branching ratios of a to $\gamma\gamma$ (left) and hadrons (right) for each model.

4 Integrated cross section

In the narrow width approximation we can assume that the peudo-scalar is always on mass-shell. If not, we should replace the delta function with the Breit-Wigner function, and instead write

$$\hat{\sigma}_{LO} = \sigma_0 M_a^2 BW(\hat{s}), \tag{47}$$

where

$$BW(\hat{s}) = \frac{M_a \Gamma_a/\pi}{(s - M_a^2) + M_a^2 \Gamma_a^2} \tag{48}$$

The total integrated cross section is given by [9]

$$\sigma = \int dx_1 \int dx_2 g(x_1, \mu) g(x_2, \mu) \hat{\sigma} \delta(s - M_a^2), \tag{49}$$

where protons carry momentum P_i, gluons carry momentum $p_{g_i} = x_i P_i$ with momentum fraction $0 \leq x_i \leq 1$, and $g(x_i, \mu$ are the parton distribution functions for the gluons, representing the probability density of a given gluon within a proton at energy scale μ.

We will define a quantity S which is related to the momenta by [9]

$$\begin{aligned} S &= (P_1 + P_2)^2 = 2P_1 P_2 = 4p^2, \\ P_1^\mu &= (p, -\mathbf{p}); \quad P_2^\mu = (p, \mathbf{p}). \end{aligned} \tag{50}$$

This allows us to relate S with the traditional s, where [9]

$$s = (p_{g_1} + p_{g_2})^2 = (x_1 P_1 + x_2 P_2)^2 = x_1 x_2 S. \tag{51}$$

We'll also define $\tau_0 \equiv M_a^2/S$, to write 49 as [9]

$$\sigma = \int dx_1 \int dx_2 g(x_1, \mu) g(x_2, \mu) \hat{\sigma} \frac{\tau_0}{x_1} \delta(x_2 - \frac{\tau_0}{x_1}). \tag{52}$$

4.1 Appendix

Handwritten notes detailing the calculation of the integral of the fermion propagators.

1.

$$\frac{1}{ABC} = \int_0^1 dx\,dy\,dz \frac{2!\,\delta(1-x-y-z)}{[xA + yB + zC]^3}$$

$$D = (\ell^2 - m^2 + i\varepsilon)\,((\ell - k_1)^2 - m^2 + i\varepsilon)\,((\ell + k_2)^2 - m^2 + i\varepsilon)$$

$$\longrightarrow \left[x(\ell^2 - m^2 + i\varepsilon) + y(\ell^2 - 2\ell\cdot k_1 - m^2 + i\varepsilon) + z(\ell^2 + 2k_2\cdot\ell - m^2 + i\varepsilon)\right]$$

$$= \ell^2(x+y+z) - m^2(x+y+z) + i\varepsilon(x+y+z) - 2y\,\ell\cdot k_1 + 2z\,k_2\cdot\ell$$

$$= \ell^2 - m^2 + i\varepsilon + 2\ell(zk_2 - yk_1)$$

$p = \ell + (zk_2 - yk_1)$ shift integration params

$$\rightarrow p^2 = \ell^2 + 2\ell(zk_2 - yk_1) + (zk_2 - yk_1)^2$$

$$= p^2 - m^2 + i\varepsilon - \underbrace{(zk_2 - yk_1)^2}_{\cancel{z^2k_2^2} + \cancel{y^2k_1^2} - 2zy\,k_2\cdot k_1} \qquad k_1^2 = k_2^2 = 0.$$

$$D = p^2 - m^2 + i\varepsilon + 2zy\underbrace{k_2\cdot k_1}_{\frac{M_a^2}{2}}$$

$x + y + z = 1$ $\quad z = 1 - y$

$$D = p^2 - \underbrace{[m^2 - zy\,M_a^2]}_{\text{effective mass fermion loop?}} + i\varepsilon$$

$$\frac{1}{D} = \int_0^1 dy \int_0^{1-y} dz \frac{2}{(p^2 - \Delta + i\varepsilon)^3}$$

(ignoring factors out the front for a second.)

we have

$$I = 2\int_0^1 dy \int_0^{1-y} dz \underbrace{\int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - \Delta + i\varepsilon)^3}}_{= \frac{-i}{32\pi^2\Delta}\ \text{FINITE}}$$

$$\frac{1}{\Delta} = \frac{1}{m^2 - zyM_a^2} = \frac{4}{4M_a^2\left(\frac{m^2}{M_a^2} - zy\right)} = \frac{4}{M_a^2}\frac{1}{\tau - 4zy} = \frac{\tau}{m^2}\frac{1}{\tau - 4zy}$$

(2)

$$I = 2\int_0^1 dy \int_0^{1-y} dz \left(\frac{-i}{32\pi^2}\right) \frac{\tau}{m^2} \frac{1}{\tau - 4zy}$$ sign ✓

$$= \frac{\tau}{m^2}\left(\frac{-i}{16\pi^2}\right)\int_0^1 dy \left.\frac{\ln(\tau - 4zy)}{-4y}\right|_0^{1-y}$$

$$= \frac{\tau}{m^2}\left(\frac{-i}{16\pi^2}\right)\int_0^1 dy \frac{1}{-4y}\Big(\underbrace{\ln(\tau - 4y(1-y))}_{\ln\left(\tau\left(1-\frac{4y(1-y)}{\tau}\right)\right)} - \ln(\tau)\Big)$$

$$= \frac{\tau}{4m^2}\left(\frac{i}{16\pi^2}\right)\int_0^1 dy \frac{1}{y}\left(\ln\left(1 - \frac{4(1-y)y}{\tau}\right)\right).$$

consider $J(\tau) = \int_0^1 dy \, \frac{\ln\overbrace{\left(1 - \frac{4(1-y)y}{\tau}\right)}^{*}}{y}$

when is $*$ positive? $\quad 1 - 4(1-y)y \cdot \frac{Ma^2}{4m^2}$

$1 - \frac{4(1-y)y}{\tau} > 0$

if $1 > \frac{4(1-y)y}{\tau}$

IBP: $\quad J(\tau) = \ln\left(1 - \frac{4(1-y)y}{\tau}\right) \cdot \ln(y)\Big|_0^1 - \int \frac{\ln y \left(-\frac{4}{\tau}(1-2y)\right)}{\left(1 - \frac{4(1-y)y}{\tau}\right)}$

$$= \int_0^1 \frac{\ln y \left(\frac{4}{\tau}(1-2y)\right)}{1 - \frac{4(1-y)y}{\tau}}.$$

denominator vanishes at two places $\left(\delta\left(1 - \frac{4}{\tau}y + \frac{4}{\tau}y^2\right)\right)$. ⊛

Split integral using $\int \frac{dx'}{x' - x \mp i\varepsilon} = P\int \frac{dx'}{x'-x} \pm i\pi\,\delta(x'-x)$

$m^2 - i\varepsilon$ prescription

$\beta = \sqrt{1-\tau}$

⊛ $\quad \delta\left(1 - \frac{4}{\tau}y + \frac{4}{\tau}y^2\right) = \frac{\tau}{4}\frac{1}{\beta}\left\{\delta\left(y - \frac{1}{2}(1+\beta)\right) + \delta\left(y - \frac{1}{2}(1-\beta)\right)\right\}$

(5)

$$\mathrm{Im}\, J(\xi) = -\pi \ln\left(\frac{1+\beta}{1-\beta}\right) \quad ; \quad \frac{4}{\tau} > 4 \quad \rightarrow \quad \tau < 1$$

imag. part.

Real part:

$$\frac{dJ}{d\left(\frac{4}{\tau}\right)} = \frac{d}{d\left(\frac{4}{\tau}\right)} \int_0^1 dy \, \frac{\ln\left(1 - \frac{4}{\tau} y(1-y)\right)}{y}$$

$$= \int_0^1 dy \, \frac{\cancel{y}(1-y)}{\cancel{y}\left(1 - \frac{4}{\tau} y(1-y)\right)}$$

sub $y = \frac{1}{2}(1+u)$

$$\Rightarrow \quad dy = \frac{1}{2} du \quad , \quad 1-y = \frac{1}{2}(1-u)$$

$$\frac{dJ}{d\left(\frac{4}{\tau}\right)} = \int_{-1}^{1} \frac{1}{2} du \, \frac{\frac{1}{2}(1-u)}{1 - \frac{1}{4}(1+u)(1-u)\frac{4}{\tau}}$$

$$= \int_{-1}^{1} \frac{\frac{1}{4} du \, (1-u)}{1 - \left(\frac{1}{4} - \frac{u^2}{4}\right)\left(\frac{4}{\tau}\right)} = \int_{-1}^{1} \frac{\frac{1}{4}}{\frac{1}{4}} \frac{du \, (1-u)}{4 - \frac{4}{\tau} + u^2 \frac{4}{\tau}}$$

For ~~[illegible]~~ $\tau \leq 1$:

$$\frac{dJ}{d\left(\frac{4}{\tau}\right)} = \frac{1}{2\beta\left(\frac{4}{\tau}\right)} \int_{-1}^{1} du \left(\frac{1}{1+\beta} - \frac{1}{1-\beta}\right) = \frac{1}{\beta \frac{4}{\tau}} \ln\left|\frac{1+\beta}{1-\beta}\right|$$

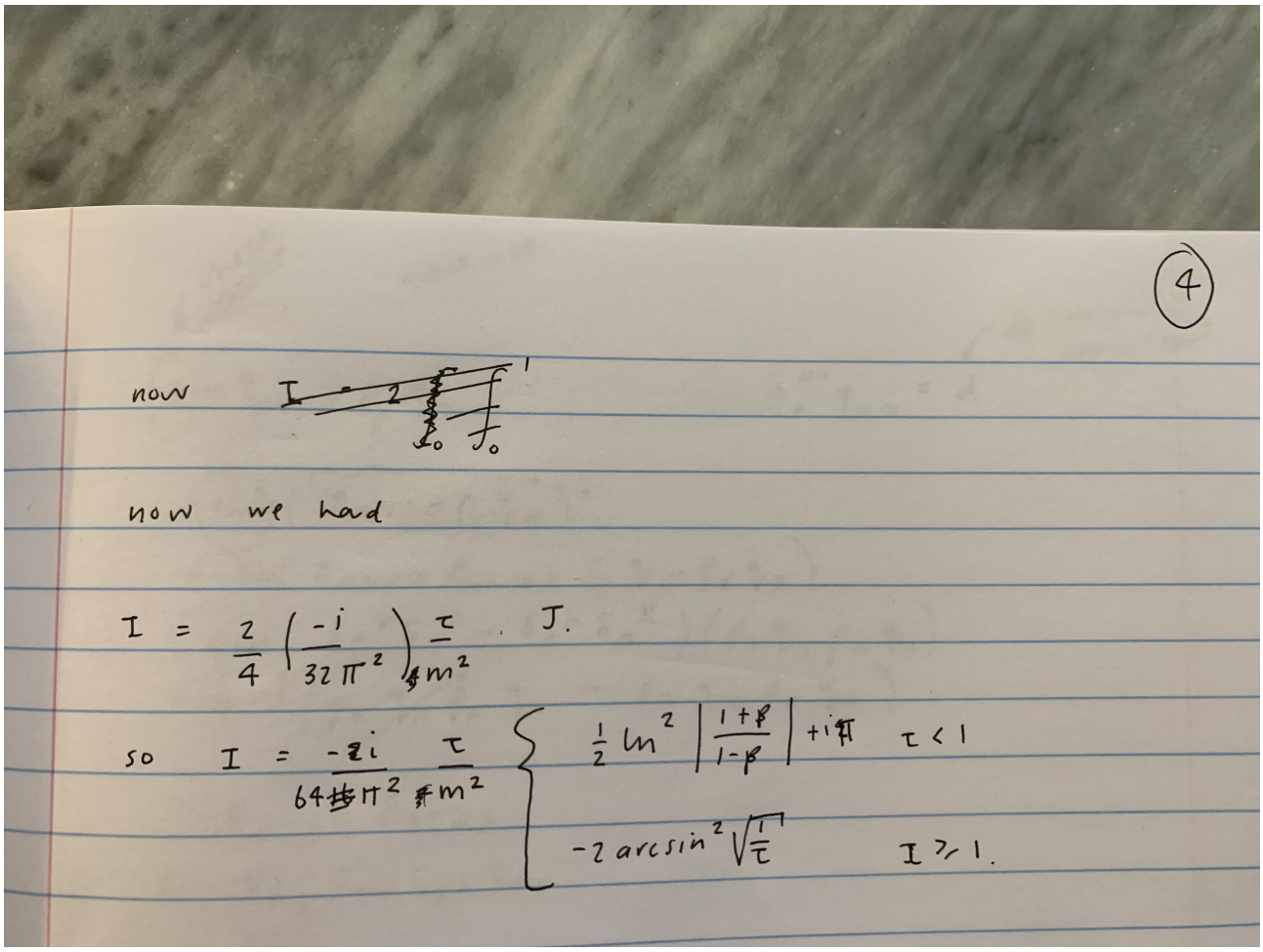
$\tau \geq 1$:

$$\frac{dJ}{d\left(\frac{4}{\tau}\right)} = \frac{-1}{\sqrt{\frac{4}{\tau}\left(4 - \frac{4}{\tau}\right)}} \arctan\left(u \sqrt{\frac{4/\tau}{4 - 4/\tau}}\right)\Bigg|_{-1}^{1} = \frac{-2}{\sqrt{\frac{4}{\tau}\left(4 - \frac{4}{\tau}\right)}} \arcsin\sqrt{\frac{1}{4}\frac{4}{\tau}}$$

integrate over $\frac{4}{\tau}$

$$\rightarrow \quad \tau < 1 : \quad \mathrm{Re}(J) = \frac{1}{2} \ln^2\left|\frac{1+\beta}{1-\beta}\right| + \text{const}$$

$$\tau \geq 1 : \quad J = -2 \arcsin^2\sqrt{\frac{1}{\tau}} + \text{const.}$$



4

now

now we had

$$I = \frac{2}{4}\left(\frac{-i}{32\pi^2}\right)\frac{\tau}{m^2} \cdot J.$$

so $$I = \frac{-2i}{64\pi^2}\frac{\tau}{m^2}\begin{cases} \frac{1}{2}\ln^2\left|\frac{1+\beta}{1-\beta}\right| + i\pi & \tau < 1 \\ -2\arcsin^2\sqrt{\frac{1}{\tau}} & \tau \geq 1. \end{cases}$$

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