magnetic resonance integral equation

Marie

Body modeling:
Numerical overview
File formats
Data structures

Beta Version, Spring 2015

http://thanospol.github.io/MARIE/

MARIE - magnetic resonance integral equation suite.

A prototype MATLAB open source software for the fast electromagnetic analysis of MRI systems

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- **This document introduces**
  - the information related to the body models in MARIE
  - file formats for body definition
    - step-by-step of how to generate valid MARIE geometries
  - structures with body information

contents

- **Body modeling**
  - brief numerical overview
- **Voxel based body models**
  - how to generate a voxel body model (.vmm)
  - how to use a voxel body model
- **Body data structures**
  - main fields and use inside MARIE

- **Main reference**

[1] Polimeridis et al., VIE paper

brief numerical overview

- **Why integral equation methods for body analysis?**

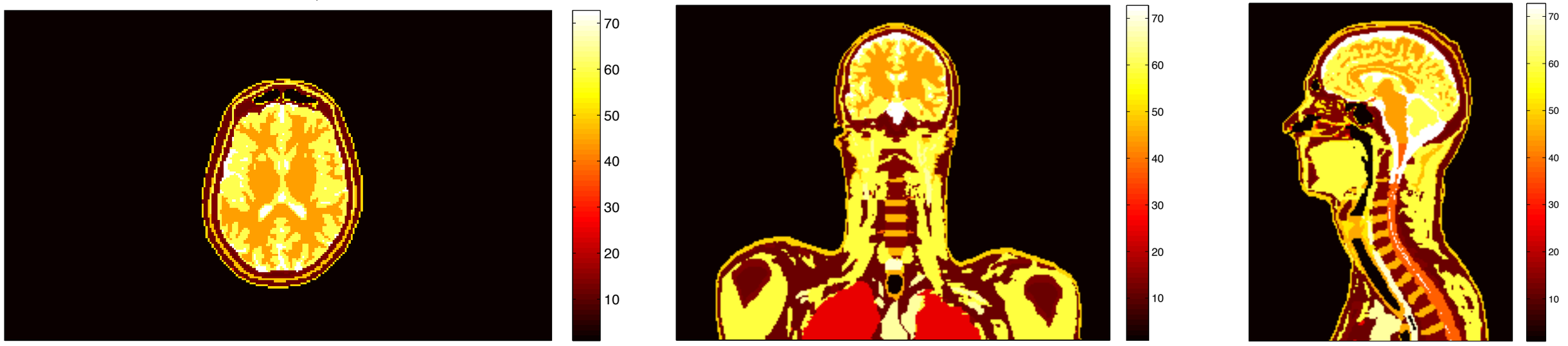
  - are based on applying the Green function
    - gives the fundamental solution at any given point in space
    - satisfies (by definition) the radiation conditions

  - consequently
    - only need to discretize the volume of the material
      - smaller (but dense) systems, no need for air voxels
    - do not require to define a bounded domain or boundary conditions
      - simplifies things for the user

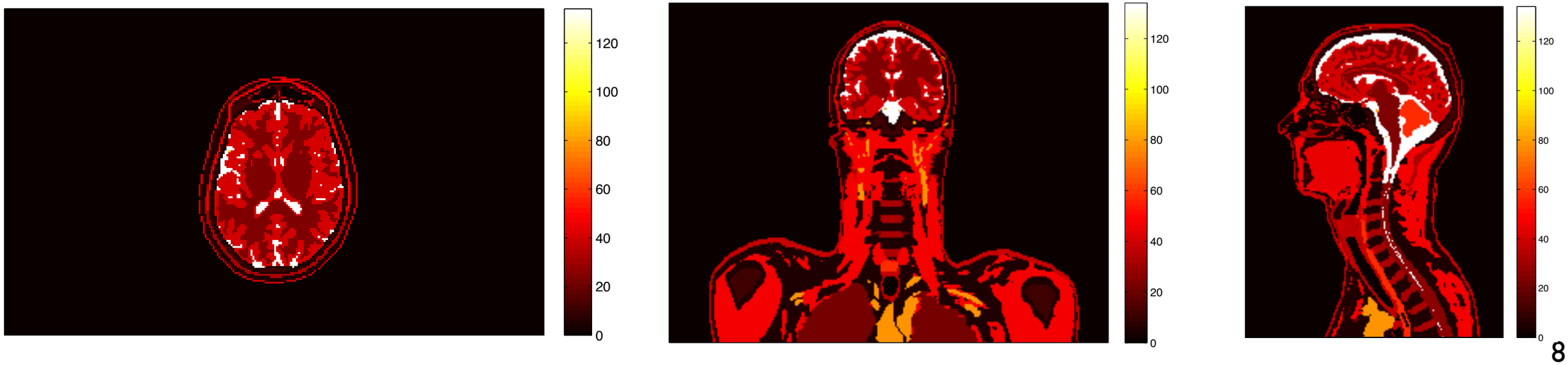
  - efficient combination with other models (coils) and fast solvers
    - superposition and translational invariance properties

- **Body models properties given voxel-by-voxel in MRI**
  - high contrats and highly inhomogeneous

$$\mathrm{Re}\{\epsilon_r\}$$



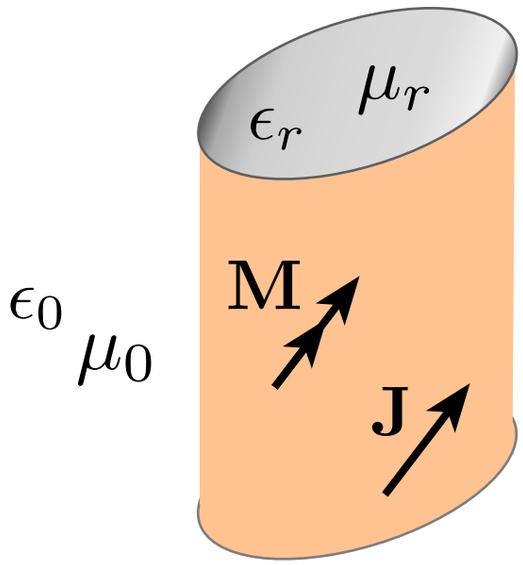
$$\mathrm{Im}\{\epsilon_r\}$$



integral equation methods

- **MARIE full wave solvers are Volume Integral Equation (VIE) based**
  - frequency domain (MRI is single frequency analysis)
  - reduces dimensionality and satisfy radiation conditions
  - easy to “couple” with other solvers
- **New current-based formulation (JVIE)**
  - natural formulation for MRI applications
- **Machine precision integration**
  - DEMCEM and DIRECTFN packages (http://web.mit.edu/thanos_p/www/)
- **FFT-based fast solver**
  - exploit voxel based data from MRI
- **Well conditioned system - fast convergence**
  - even for high contrast

- **Basic formulation**



$$\mathbf{e} = \mathbf{e}^{\text{inc}} + \mathbf{e}^{\text{sca}} = \mathbf{e}^{\text{inc}} + \frac{1}{c_\epsilon}\mathcal{L}\mathbf{j} - \mathcal{K}\mathbf{m}$$

$$\mathbf{h} = \mathbf{h}^{\text{inc}} + \mathbf{h}^{\text{sca}} = \mathbf{h}^{\text{inc}} + \frac{1}{c_\mu}\mathcal{L}\mathbf{m} + \mathcal{K}\mathbf{j}$$

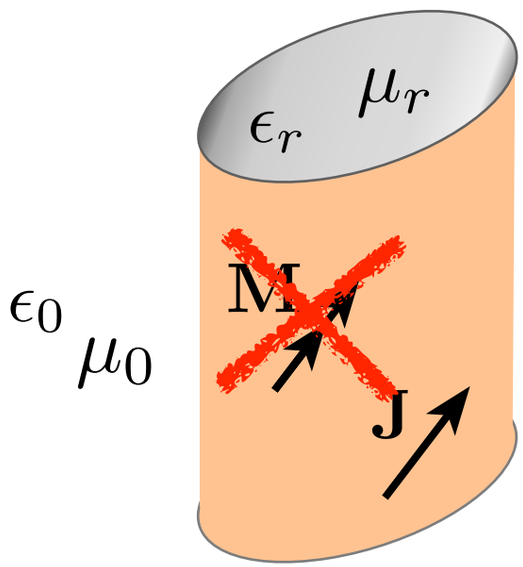
$$\mathbf{j}(\mathbf{r}) \triangleq c_\epsilon \chi_\epsilon(\mathbf{r})\,\mathbf{e}(\mathbf{r})$$

$$c_\epsilon = j\omega\epsilon_0$$
$$\chi_\epsilon = \epsilon_r(\mathbf{r}) - 1$$

$$\mathcal{L}\mathbf{u} \triangleq (k_0^2 + \nabla\nabla\cdot)\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r})$$
$$\mathcal{K}\mathbf{u} \triangleq \nabla \times \mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r})$$

$$\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r}) \triangleq \int_\Omega G(\mathbf{R})\mathbf{u}(\mathbf{r}')d\mathbf{r}'$$

- **Basic formulation**
  - for non-magnetic material



$$\mathbf{e} = \mathbf{e}^{\text{inc}} + \mathbf{e}^{\text{sca}} = \mathbf{e}^{\text{inc}} + \frac{1}{c_\epsilon}\mathcal{L}\mathbf{j} - \mathcal{K}\mathbf{m}$$

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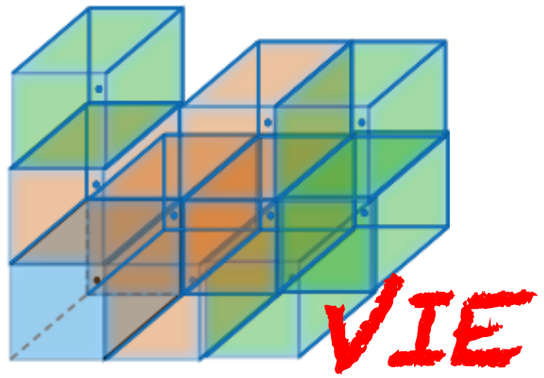
- **Voxel as support**
  - natural discretization of MRI applications
  - transform volume integrals into surface
  - allows to apply FFT based approaches

Toeplitz → *Circulant*

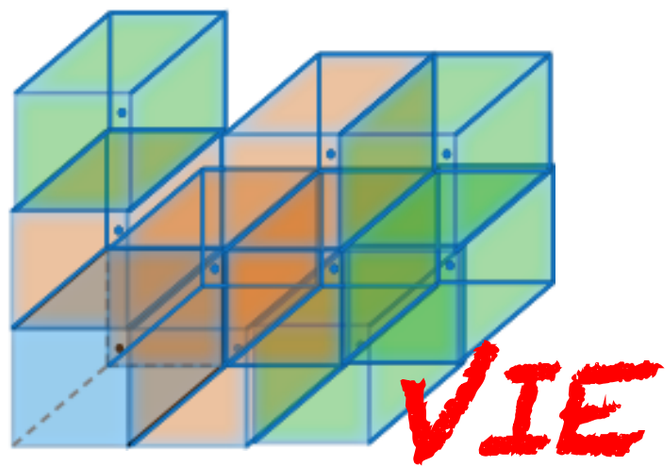
→

→



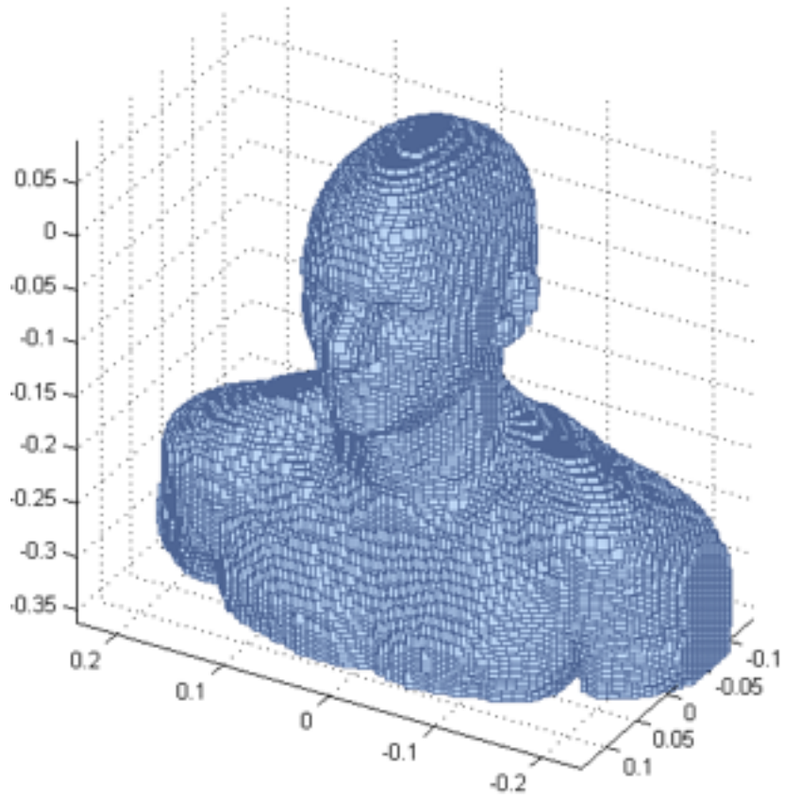
$\mathbf{Aj} = \mathbf{e}^{\mathrm{inc}}$ → *Iterative Solver* → *M-v product* → *FFT* $N \log(N)$

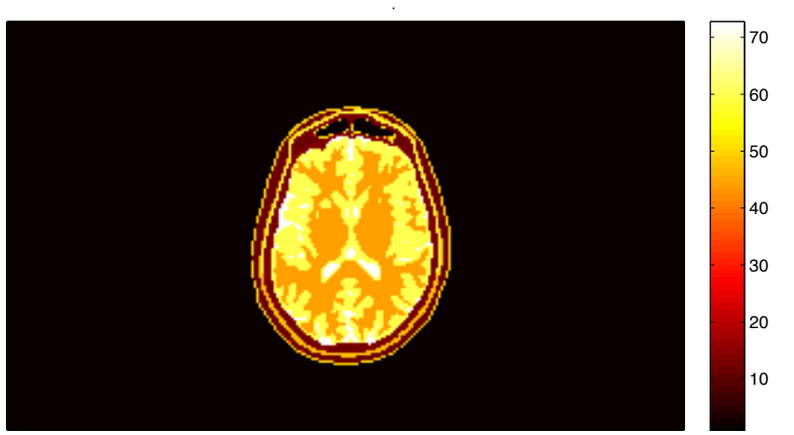
- **Solve the Volume Integral Equation (VIE) system to obtain**
  - Electric currents inside the body model
    - Due to external incident EM field
    - These currents directly related to E field and SAR inside body
  - Electric and Magnetic fields
    - Scattered inside and outside the body
    - Total fields by combination of incident plus scattered

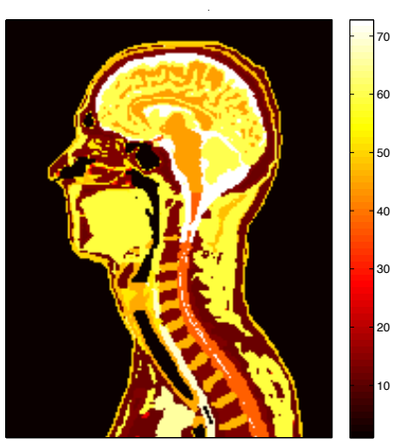


voxel body models

- **What is a voxel body model in MARIE?**
  - a 3D domain that is discretized into regular voxels
  - each voxel has constant properties (related to tissue)







- **Discretization and modeling methodology**
  - Regular grid, with constant properties at each voxel
    - natural MRI format
  - Single frequency modeling
  - Pulse basis functions with voxels as support
    - integration boils down to surface integrals
  - Machine precision numerical integration
    - DEMCEM

- **MARIE uses .vmm (volume marie model) file defining body**
  - these are just text files a few lines

RHBM1_HT_5mm.vmm

```
1  My RHBM1 Head & Shoulders, 5mm, 7T
2  RHBM1_HT_5mm.mat
```

  - first line is the model name and/or description
  - rest of lines can be
    - a name of a .mat file (Matlab data), which stores the RHBM structure

- **MARIE uses .vmm (volume marie model) file defining body**
  - these are just text files a few lines

RHBM1_HT_5mm.vmm

```
1  My RHBM1 Head & Shoulders, 5mm, 7T
2  rhmb1_voxels.dat
3  rhbm1_x_axis.dat
4  rhbm1_y_axis.dat
5  rhbm1_z_axis.dat
```

  - first line is the model name and/or description
  - rest of lines can be
    - 4 lines, with .dat files: voxel properties, x, y and z coordinates
    - this is the typical output format of some EM simulators

- **.vmm and .dat files**
  - these are just text files a few lines

RHBM1_HT_5mm.vmm

```
1  My RHBM1 Head & Shoulders, 5mm, 7T
2  rhmb1_voxels.dat
3  rhbm1_x_axis.dat
4  rhbm1_y_axis.dat
5  rhbm1_z_axis.dat
```

  - first .dat file is the tissue information

RHBM1_HT_5mm.vmm | rhbm1_voxels.dat

```
1  % Voxel information for simulation 'Simulation 1': voxel information
2  % x-index   y-index z-index material-index  epsilon_r   sigma_E mu_r    sigma_H rho
3  0   0   0   0   1.0 0.0 1.0 0.0 1e-12
4  0   0   1   0   1.0 0.0 1.0 0.0 1e-12
5
6  ...
7
8  47  76  42  3   58.21   0.77    1.0 0.0 1047.0
9  47  76  43  3   58.21   0.77    1.0 0.0 1047.0
10 47  76  44  3   58.21   0.77    1.0 0.0 1047.0
11
12 ...
13
```

- **.vmm and .dat files**
  - these are just text files a few lines

RHBM1_HT_5mm.vmm

```
1  My RHBM1 Head & Shoulders, 5mm, 7T
2  rhmb1_voxels.dat
3  rhbm1_x_axis.dat
4  rhbm1_y_axis.dat
5  rhbm1_z_axis.dat
```

  - second .dat file is the x positions

RHBM1_HT_5mm.vmm | rhbm1_voxels.dat | rhbm1_x_axis.dat

```
1  % Voxel information for simulation 'Simulation 1': x axis
2  % [meter]
3  -0.449827048333
4  -0.406126600822
5  -0.369680401637
6  -0.339279409392
7  -0.313916089595
8  -0.292750830173
9  -0.275083954396
10 -0.260332398323
11 -0.248010275337
12 -0.237712679924
```

- **.vmm and .dat files**
  - these are just text files a few lines

RHBM1_HT_5mm.vmm

```
1  My RHBM1 Head & Shoulders, 5mm, 7T
2  rhmb1_voxels.dat
3  rhbm1_x_axis.dat
4  rhbm1_y_axis.dat
5  rhbm1_z_axis.dat
```

  - third .dat file is the y positions

RHBM1_HT_5mm.vmm | rhbm1_voxels.dat | rhbm1_x_axis.dat | rhbm1_y_axis.dat

```
1  % Voxel information for simulation 'Simulation 1': y axis
2  % [meter]
3  -0.449827048333
4  -0.406126600822
5  -0.369680401637
6  -0.339279409392
7  -0.313916089595
8  -0.292750830173
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- **.vmm and .dat files**
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RHBM1_HT_5mm.vmm

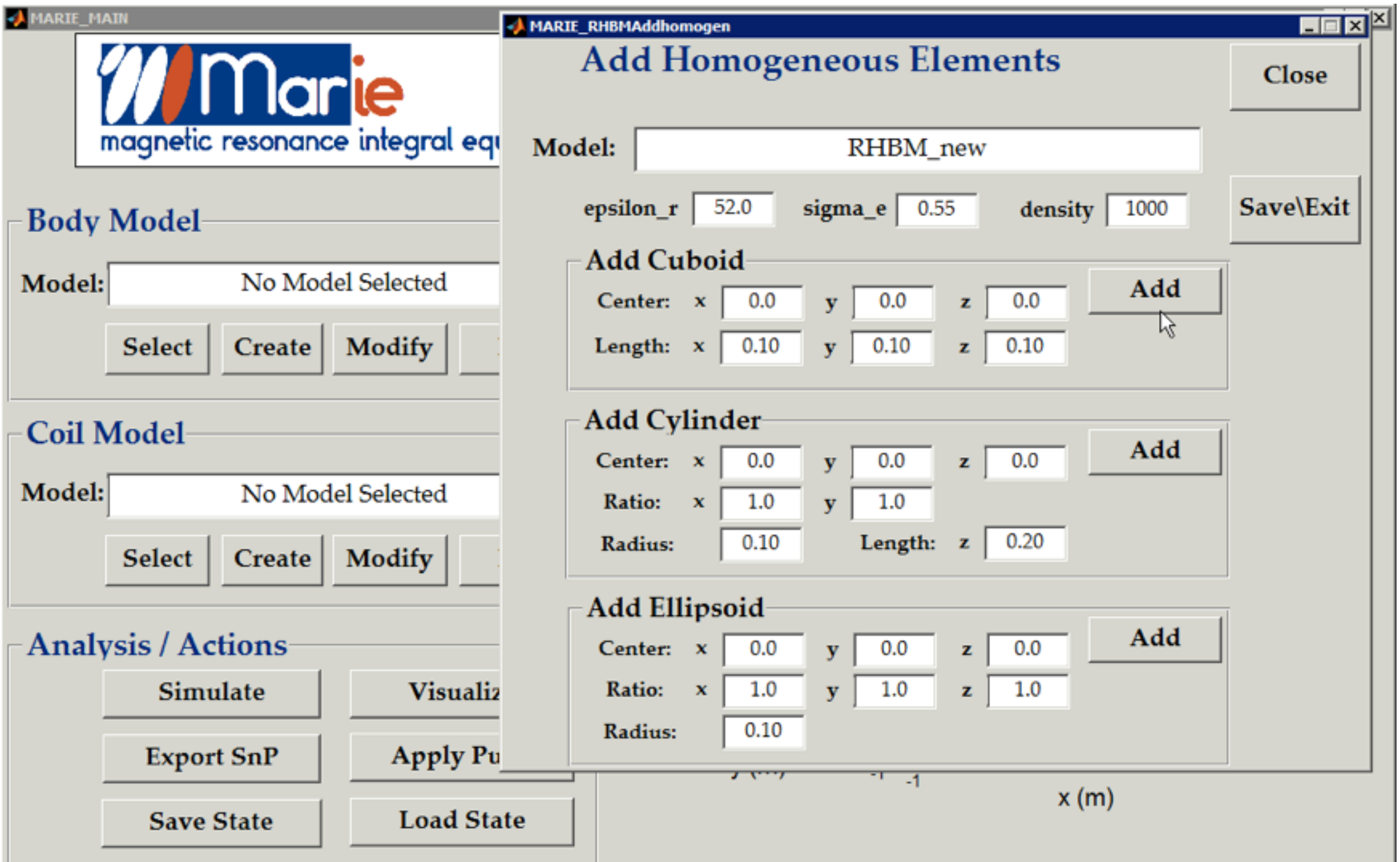
```
1  My RHBM1 Head & Shoulders, 5mm, 7T
2  rhmb1_voxels.dat
3  rhbm1_x_axis.dat
4  rhbm1_y_axis.dat
5  rhbm1_z_axis.dat
```

  - fourth .dat file is the z positions

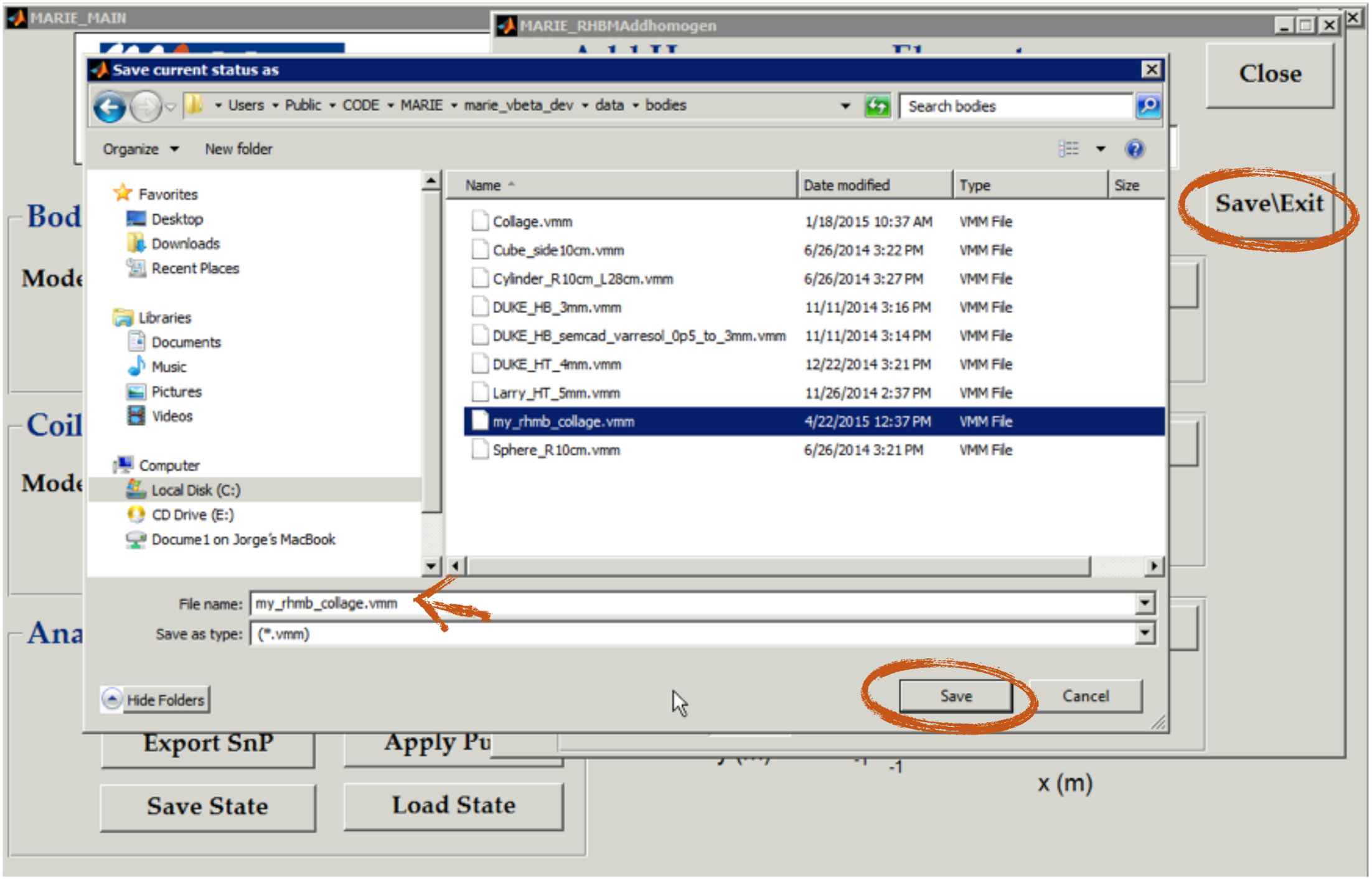
RHBM1_HT_5mm.vmm | rhbm1_voxels.dat | rhbm1_z_axis.dat | rhbm1_x_axis.dat | rhbm1_y_axis.dat

```
1  % Voxel information for simulation 'Simulation 1': z axis
2  % [meter]
3  -0.449827048333
4  -0.406126600822
5  -0.369680401637
6  -0.339279409392
7  -0.313916089595
8  -0.292750830173
9  -0.275083954396
10 -0.260332398323
11 -0.248010275337
12 -0.237712679924
```

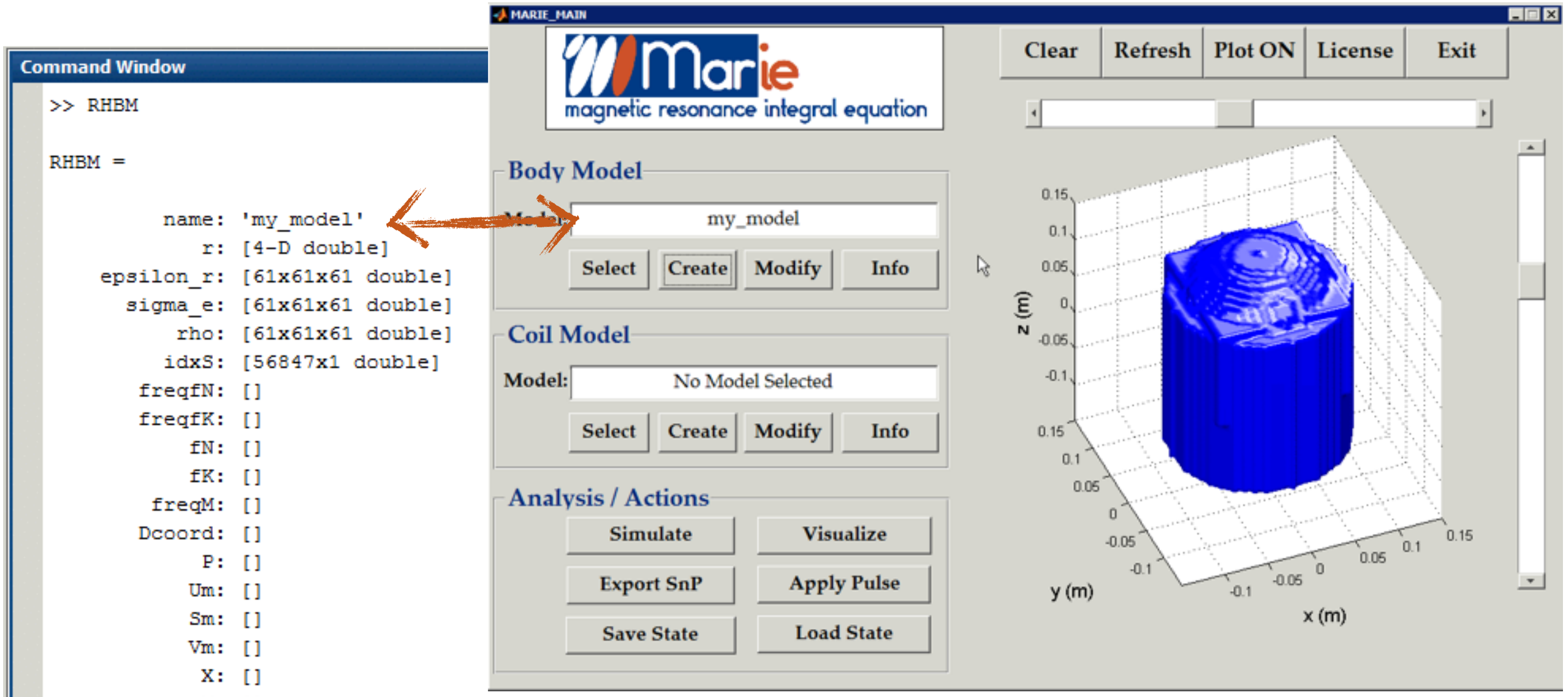
- **MARIE GUI incorporate an automatic RHBM generator**
  - For piecewise homogeneous geometries
  - A new homogeneous element is added at a time



- **MARIE GUI incorporate an automatic RHBM generator**
  - the model can be saved as a .vmm with .mat info



- **Load the RHBM**
  - MARIE will load the data given by the .vmm
    - either load the .mat file or parse the .dat files (the former can be slow)



body data structures

- **Body data structure**
  - **RHBM** (realistic human body model)
  - has geometric information of the scatterer
    - can be an inhomogeneous realistic human body model
    - or a homogeneous phantom or element (e.g. sphere)
  - the information is given as voxel based material properties
    - used by the Volume Integral Equation modeling
- may include some pre-computed numeric information
  - VIE-related data (circulants from last solve)
  - Magnetic Resonance Green Functions (MRGFs)
  - to speed-up analysis and simulations

- **RHBM (body data structure)**

Command Window

```
  FIGIDX      1x1                      8  double    global
  PULSE       1x1                   1760  struct    global
  RHBM        1x1             1312290508  struct    global
  SOL         1x1              157354104  struct    global
  p           1x6341               12682  char

>> RHBM

RHBM =

          name: 'DUDE Head & Shoulders, 5mm, 7T'
             r: [4-D double]
       epsilon_r: [103x103x103 double]
       sigma_e: [103x103x103 double]
           rho: [103x103x103 double]
          idxS: [126858x1 double]
        freqfN: 300000000
        freqfK: 300000000
            fN: [4-D double]
            fK: [4-D double]
         freqM: []
        Dcoord: []
             P: []
            Um: []
            Sm: []
            Vm: []
             X: []
             M: []

fx >>
```

body data structure

- **RHBM (body data structure, standard info)**

  .name - name or description of the model

  .r - coordinates of the 3D grid defining the domain

  .epsilon_r - permittivity of each voxel of the grid

  .sigma_e - conductivity of each voxel of the grid

  .rho - densitity of each voxel of the grid

  .idxS - index of the elements of the grid that are not air (body voxels)

- this information is used by the Volume Integral Equation modeling

body data structure

- **RHBM (body data structure, numeric data for VIE)**

  .fN / .freqfN- N operator and frequency at which was generated

  .fK / .freqfK - K operator and frequency at which was generated

- if the new computations are at the same frequency
  - fN and fK do not need to be re-generated

body data structure

- **RHBM (body data structure, numeric MRGF data)**

  .freqM - frequency at which the MRGF was generated

  .Dcoord - coordinates of the DEIM interpolation points

  .P - incidence matrix of the DEIM interpolation points

  .Um - coil perturbation MRGF factor

  .Sm - coil perturbation MRGF factor

  .Vm - coil perturbation MRGF factor

  .X - MRGF weighting matrix

  .M - MRGF pre-computed solution

- this information is used by the accelerated Integral Equation solvers

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