Modelo Liu

$M\dot{\nu} + C(\nu)\nu + D\nu + B\nu|\nu| = \tau$

$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix}$ (massa + massa adicional)

$C = \begin{bmatrix} 0 & 0 & -m_{22}v \\ 0 & 0 & m_{11}u \\ m_{22}v & -m_{11}u & 0 \end{bmatrix}$ (centrípeta e coriolis)

$D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}$ (linear + potencial)

$B = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$ (força hidrodinâmica não-potencial e força do vento)

$b_{11} = \frac{1}{2}\rho LTC_{c1}(180^o) + \frac{1}{2}\rho_{ar}B(h-T)C_{w1}(180^o)$

$b_{22} = \frac{1}{2}\rho LTC_{c2}(90^o) + \frac{1}{2}\rho_{ar}L(h-T)C_{w2}(90^o)$

$b_{33} = \frac{1}{32}\rho L^4TC_{c2}(90^o) + \frac{1}{2}\rho_{ar}L^2(h-T)C_{w3}(90^o)$

$\tau = \begin{Bmatrix} \tau_u \\ \tau_v \\ \tau_r \end{Bmatrix}, \ \tau_v = \tau_r = 0$

$J = \frac{V_A}{nD} = \frac{u}{nD}$

$K_T = \frac{T}{pn^2D^4}$

$\tau_u = 2\eta T = 2\eta K_t pn^2 D^4$

Simplificação:

$\tau_v = \tau_r = v = r = 0$

$$m_{11}\dot{u} + d_{11}u + b_{11}u|u| = 2\eta K_t p D^4 n^2$$

$\dot{u} = -du - bu|u| + gn^2$, onde $d = \dfrac{d_{11}}{m_{11}}$, $b = \dfrac{b_{11}}{m_{11}}$ e $g = \dfrac{2\eta K_t p D^4}{m_{11}}$

Lei de controle:

$$\dot{u}_m = -d_m u_m + g_m s$$

$$n = \sqrt{\widehat{a}_r s + \widehat{a}_u u + \widehat{a}_{u|u|} u|u|}$$

$$a_s^* = \frac{g_m}{g}$$

$$a_u^* = \frac{d - d_m}{g}$$

$$a_{u|u|}^* = \frac{b}{g}$$

Erro

$$z = u - u_m$$

$$\dot{z} = \dot{u} - \dot{u_m} = -du - bu|u| + gn^2 + d_m u_m - g_m s$$

$$\dot{z} = -du - bu|u| + g(\widehat{a}_s s + \widehat{a}_u u + \widehat{a}_{u|u|} u|u|) + d_m u_m - g_m s$$

$$(g\widehat{a}_s - g_m)s = gs(\widehat{a}_s - a_s^*) = gs\widetilde{a}_s$$

$$(g\widehat{a}_{u|u|} - b)u|u| = (\widehat{a}_{u|u|} - a_{u|u|}^*)gu|u| = gu|u|\widetilde{a}_{u|u|}$$

$$-du + g\widehat{a}_u u + d_m u_m = -(d_m + ga_u^* g)u + g\widehat{a}_u u + d_m u_m = -d_m(u - u_m) + gu(\widehat{a}_u - a_u^*) = -d_m z + gu\widetilde{a}_u$$

$$\dot{z} = -d_m z + gu\widetilde{a}_u + gu|u|\widetilde{a}_{u|u|} + gs\widetilde{a}_s = -d_m z + g(u\widetilde{a}_u + u|u|\widetilde{a}_{u|u|} + s\widetilde{a}_s)$$

$$\dot{z} = -d_m z + g(u\widetilde{a}_u + u|u|\widetilde{a}_{u|u|} + s\widetilde{a}_s)$$

$\widetilde{a}_s = \widehat{a}_s - a_s^*$, $\widetilde{a}_u = \widehat{a}_u - a_u^*$ e $\widetilde{a}_{u|u|} = \widehat{a}_{u|u|} - a_{u|u|}^*$

Lei de adaptação:

$$\begin{cases} \dot{\widehat{a}}_s = -sgn(g)\gamma zs \\ \dot{\widehat{a}}_u = -sgn(g)\gamma zu \\ \dot{\widehat{a}}_{u|u|} = -sgn(g)\gamma zu|u| \end{cases}$$

Função de Lyapunov

$V = \frac{1}{2}z^2 + \frac{1}{2\gamma}|g|(\widetilde{a}_s^2 + \widetilde{a}_u^2 + \widetilde{a}_{u|u|}^2)$ P.D

Assumindo g, a_s^*, a_u^* e $a_{u|u|}^*$ constantes

$$\dot{V} = z\dot{z} + \frac{|g|}{\gamma}(\widetilde{a}_s\hat{\dot{a}}_s + \widetilde{a}_u\hat{\dot{a}}_u + \widetilde{a}_{u|u|}\hat{\dot{a}}_{u|u|})$$

$$\dot{V} = z(-d_m z + g(u\widetilde{a}_u + u|u|\widetilde{a}_{u|u|} + s\widetilde{a}_s)) - |g|(\widetilde{a}_s sgn(g)zs + \widetilde{a}_u sgn(g)zu + \widetilde{a}_{u|u|}sgn(g)zu|u|)$$

$$\dot{V} = z(-d_m z + g(u\widetilde{a}_u + u|u|\widetilde{a}_{u|u|} + s\widetilde{a}_s)) - gz(\widetilde{a}_s s + \widetilde{a}_u u + \widetilde{a}_{u|u|}u|u|)$$

$\dot{V} = -d_m z^2$ S.N.D.

$z \rightarrow 0$ e $\widetilde{a}_s$, $\widetilde{a}_u$ e $\widetilde{a}_{u|u|}$ são limitados.

Filtro de Kalman Estendido (Discreto)

Modelo:

$$u_{k+1} = u_k + (-d_0 0 u_k - b_0 u_k|u_k| + g_0 n_k^2)\Delta t + w_k$$

$$F(u_k, n_k) = u_k + (-d0u_k - b_0 u_k|u_k| + g_0 n_k^2)\Delta t$$

$$u_{k+1} = F_k(u, n) + w_k$$

$Q = 1$ constante

$$A_k = 1 + (-d_0 - 2b_0 u_k)\Delta t$$

Sensores:

$$z_k = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} u_k + \begin{Bmatrix} v_{gps} \\ v_{dvl} \\ v_{imu} \end{Bmatrix}$$

$$G(u_k) = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} u_k$$

$$z_k = G(u_k) + v_k$$

$$C_k = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$R = \begin{bmatrix} R_{gps} & 0 & 0 \\ 0 & R_{dvl} & 0 \\ 0 & 0 & R_{imu} \end{bmatrix}$$

Predição:

$$\bar{\hat{u}}_{k+1} = F(u_k, n_k) = \hat{u}_k + (-d_0\hat{u}_k - b_0\hat{u}_k|\hat{u}_k| + g_0 n_k^2)\Delta t$$

$$\bar{P}_{k+1} = A_k P_k A_k^T + Q$$

Atualização:

$$K_k = \bar{P}_{k+1} C_k^T (C_k \bar{P}_{k+1} C_k^T + R)^{-1}$$

$$\hat{u}_{k+1} = \bar{\hat{u}}_{k+1} + K_k(z_k + G(\bar{\hat{u}}_k))$$

$$P_{k+1} = (I - K_k C_k)\bar{P}_{k+1}$$

Filtro de Kalman Estendido (Contínuo)

Modelo:

$$\dot{u} = -d_0 u - b_0 u|u| + g_0 n^2 + w(t) = f(u, n) + w(t)$$

$$z = \begin{Bmatrix} z_{gps} \\ z_{dvl} \\ z_{imu} \end{Bmatrix} = \begin{Bmatrix} u_{gps} \\ u_{dvl} \\ u_{imu} \end{Bmatrix} + \begin{Bmatrix} v_{gps} \\ v_{dvl} \\ v_{imu} \end{Bmatrix} = h(u) + v(t)$$

$Q = 10^{-7}$ e $R = \begin{bmatrix} R_{gps} & 0 & 0 \\ 0 & R_{dvl} & 0 \\ 0 & 0 & R_{imu} \times 10^4 \end{bmatrix}$

Inicialização:

$\widehat{u}_0 = E[u_0]$ e $P_0 = Var[u_0]$

Predição-atualização:

$$F(t) = \frac{\partial f}{\partial x}|_{\widehat{u},n} = -d_0 - 2b_0\widehat{u}$$

$$H(t) = \frac{\partial h}{\partial x}|_{\widehat{u}} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\dot{P}(t) = F(t)P(t) + P(t)F(t)^T - K(t)H(t)P(t) + Q(t)$$

$$K(t) = P(t)H(t)^T R(t)^{-1}$$

$$\dot{\widehat{u}} = f(\widehat{u}, n) + K(z - h) = -d_0\widehat{u} - b_0\widehat{u}|\widehat{u}| + g_0 n^2 + \begin{Bmatrix} K_{gps} & K_{dvl} & K_{imu} \end{Bmatrix}\left(\begin{Bmatrix} z_{gps} \\ z_{dvl} \\ z_{imu} \end{Bmatrix} - \begin{Bmatrix} \widehat{u} \\ \widehat{u} \\ \widehat{u} \end{Bmatrix}\right)$$