Computer Architecture

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Keep in mind there are *two* PDFs available (of which this is the latter):

1. a PDF of examinable material used as lecture slides, and
2. a PDF of non-examinable, extra material:
   - the associated notes page may be pre-populated with extra, written explaination of material covered in lecture(s), plus
   - anything with a "grey'ed out" header/footer represents extra material which is useful and/or interesting but out of scope (and hence not covered).

Notes:

Notes:

- Agenda:
  1. introduce a theoretical computational model that is "*closely related to what happens in [practical] modern digital computers*" [7, Page 199], then
  2. demonstrate how we implement said model in hardware.

Notes:

Part 1: in theory (1)

Definition

A **Register Machine (RM)** is specified by

- a finite number of registers, each of which can store an (infinite) natural number; $\mathsf{R}_i \in \mathbb{N}$ denotes the i-th such register for $0 \leq i < r$, and
- a program, consisting of a finite list of instructions of the form

  $$\text{label} : \text{body}$$

  such that the i-th instruction has label L_i.

Definition

An RM **configuration** is a tuple

$$C = (l, v_0, v_1, \ldots, v_{r-1})$$

where

- l is the current label, and
- v_j is the current value stored in register R_j.

+ve : theoretically attractive: can be proved equivalent to a Turing machine
+ve : practically attractive: has clear analogies with, e.g., a calculator
−ve : some aspects (e.g., infinite sized registers) cannot be realised in practice
−ve : can be very inefficient

Notes:

- Keep in mind item #54 of Perlis' Epigrams on Programming [9]: "*[b]eware of the Turing tar-pit in which everything is possible but nothing of interest is easy*".

Part 1: in theory (2)

Definition

A (finite or infinite) computation by some RM is captured by

$$\langle C_0, C_1, C_2, \ldots \rangle$$

i.e., a sequence of configurations such that

- for $i = 0$,

  $$C_i = (0, v_0, v_1, \ldots, v_{r-1})$$

  is the **initial configuration** where v_j is the initial value stored in register R_j,
- for $i > 0$, C_i results from applying the instruction at label L_l to

  $$C_{i-1} = (l, v_0, v_1, \ldots, v_{r-1}).$$

Definition

A finite computation by some RM is captured by

$$\langle C_0, C_1, C_2, \ldots, C_{h-1} \rangle$$

such that in the **halting configuration**

$$C_{h-1} = (l, v_0, v_1, \ldots, v_{r-1})$$

the instruction labelled L_l either

- explicitly, or intentionally forces computation to halt, i.e., is a halt instruction, or
- implicitly, or unintentionally forces computation to halt, e.g., causes an error condition.

Notes:

Part 1: in theory (3)

- Example: roughly per [7, Chapter 11], consider a **counter machine** such that
  1. $r = 4$, i.e., it has 4 registers,
  2. each $\mathsf{R}_i \in \{0, 1, \ldots, 2^4 - 1 = 15\}$, i.e., the registers store (finite) 4-bit values,
  3. the set of valid instructions is

$$\begin{array}{l}
\mathsf{L}_i : \mathsf{R}_{addr} \leftarrow \mathsf{R}_{addr} + 1 \textbf{ then goto } \mathsf{L}_{i+1} \\
\mathsf{L}_i : \mathsf{R}_{addr} \leftarrow \mathsf{R}_{addr} - 1 \textbf{ then goto } \mathsf{L}_{i+1} \\
\mathsf{L}_i : \textbf{if } \mathsf{R}_{addr} = 0 \textbf{ then goto } \mathsf{L}_{target} \textbf{ else goto } \mathsf{L}_{i+1} \\
\mathsf{L}_i : \text{halt}
\end{array}$$

Notes:

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation.

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_0 & = & (0, 0, 0, 2, 0) \\
\mathsf{L}_0 & \rightsquigarrow & \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
C_1 & = & (1, 0, 0, 2, 0)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_1 & = & (1, 0, 0, 2, 0) \\
\mathsf{L}_1 & \rightsquigarrow & \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
C_2 & = & (2, 0, 0, 1, 0)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_2 & = & (2, 0, 0, 1, 0) \\
\mathsf{L}_2 & \rightsquigarrow & \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
C_3 & = & (3, 0, 0, 1, 1)
\end{array}
$$

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Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_3 & = & (3, 0, 0, 1, 1) \\
\mathsf{L}_3 & \rightsquigarrow & \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
C_4 & = & (4, 0, 1, 1, 1)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_4 & = & (4, 0, 1, 1, 1) \\
\mathsf{L}_4 & \rightsquigarrow & \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
C_5 & = & (0, 0, 1, 1, 1)
\end{array}
$$

Notes:

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_5 & = & (0, 0, 1, 1, 1) \\
\mathsf{L}_0 & \rightsquigarrow & \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
C_6 & = & (1, 0, 1, 1, 1)
\end{array}
$$

Notes:

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_6 & = & (1, 0, 1, 1, 1) \\
\mathsf{L}_1 & \rightsquigarrow & \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
C_7 & = & (2, 0, 1, 0, 1)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_7 & = & (2, 0, 1, 0, 1) \\
\mathsf{L}_2 & \rightsquigarrow & \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
C_8 & = & (3, 0, 1, 0, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_8 & = & (3, 0, 1, 0, 2) \\
\mathsf{L}_3 & \rightsquigarrow & \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
C_9 & = & (4, 0, 2, 0, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_9 & = & (4, 0, 2, 0, 2) \\
\mathsf{L}_4 & \rightsquigarrow & \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
C_{10} & = & (0, 0, 2, 0, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{10} & = & (0, 0, 2, 0, 2) \\
\mathsf{L}_0 & \rightsquigarrow & \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
C_{11} & = & (5, 0, 2, 0, 2)
\end{array}
$$

Notes:

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{11} & = & (5, 0, 2, 0, 2) \\
\mathsf{L}_5 & \rightsquigarrow & \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
C_{12} & = & (6, 0, 2, 0, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{12} & = & (6, 0, 2, 0, 2) \\
\mathsf{L}_6 & \rightsquigarrow & \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
C_{13} & = & (7, 0, 1, 0, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{13} & = & (7, 0, 1, 1, 2) \\
\mathsf{L}_7 & \rightsquigarrow & \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
C_{14} & = & (8, 0, 1, 1, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{14} & = & (8, 0, 1, 1, 2) \\
\mathsf{L}_8 & \rightsquigarrow & \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
C_{15} & = & (5, 0, 1, 1, 2)
\end{array}
$$

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Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{15} & = & (5, 0, 1, 1, 2) \\
\mathsf{L}_5 & \rightsquigarrow & \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
C_{16} & = & (6, 0, 1, 1, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{16} & = & (6, 0, 1, 1, 2) \\
\mathsf{L}_6 & \rightsquigarrow & \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
C_{17} & = & (7, 0, 0, 1, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{17} & = & (7, 0, 0, 1, 2) \\
\mathsf{L}_7 & \rightsquigarrow & \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
C_{18} & = & (8, 0, 0, 2, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{18} & = & (8, 0, 0, 2, 2) \\
\mathsf{L}_8 & \rightsquigarrow & \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
C_{19} & = & (5, 0, 0, 2, 2)
\end{array}
$$

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Notes:

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{19} & = & (5, 0, 0, 2, 2) \\
\mathsf{L}_5 & \rightsquigarrow & \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
C_{20} & = & (9, 0, 0, 2, 2)
\end{array}
$$

Notes:

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{20} & = & (9, 0, 0, 2, 2) \\
\mathsf{L}_9 & \rightsquigarrow & \text{halt} \\
C_{21} & = & (9, 0, 0, 2, 2)
\end{array}
$$

Notes:

Part 1: in theory (4)

- Example: now, given
  1. a program e.g.,

$$
\begin{array}{l}
\mathsf{L}_0 : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_1 \\
\mathsf{L}_1 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 - 1 \textbf{ then goto } \mathsf{L}_2 \\
\mathsf{L}_2 : \mathsf{R}_3 \leftarrow \mathsf{R}_3 + 1 \textbf{ then goto } \mathsf{L}_3 \\
\mathsf{L}_3 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 + 1 \textbf{ then goto } \mathsf{L}_4 \\
\mathsf{L}_4 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_0 \textbf{ else goto } \mathsf{L}_5 \\
\mathsf{L}_5 : \textbf{if } \mathsf{R}_1 = 0 \textbf{ then goto } \mathsf{L}_9 \textbf{ else goto } \mathsf{L}_6 \\
\mathsf{L}_6 : \mathsf{R}_1 \leftarrow \mathsf{R}_1 - 1 \textbf{ then goto } \mathsf{L}_7 \\
\mathsf{L}_7 : \mathsf{R}_2 \leftarrow \mathsf{R}_2 + 1 \textbf{ then goto } \mathsf{L}_8 \\
\mathsf{L}_8 : \textbf{if } \mathsf{R}_0 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_9 \\
\mathsf{L}_9 : \text{halt}
\end{array}
$$

  2. an initial configuration, e.g.,

$$C_0 = (0, 0, 0, 2, 0)$$

  we can produce a **trace** of computation:

$$
\begin{array}{lcl}
C_{20} & = & (9, 0, 0, 2, 2) \\
\mathsf{L}_9 & \rightsquigarrow & \text{halt} \\
C_{21} & = & (9, 0, 0, 2, 2)
\end{array}
$$

  which demonstrates that the program copies R_2 into R_3.

Notes:

Aside: is this the *only* viable example?

- No: (many) alternatives exist, stemming from
  1. different models, e.g.,
     - **counter machine**,
     - **Random-Access Machine (RAM)**,
     - **Random-Access Stored-Program (RASP) machine**,
     - ...

Notes:

Aside: is this the *only* viable example?

- No: (many) alternatives exist, stemming from

2. different instruction set content, e.g., a counter machine with or without

$$\mathsf{L}_i : R_{addr} \leftarrow 0$$

or "clear" instruction.

Notes:

Aside: is this the *only* viable example?

- No: (many) alternatives exist, stemming from

3. different instruction set format, e.g.,
   - **register machine** ≃ 3-operand model:
     - r registers,
     - source and destination operands can be specified independently,
     - (rough) example:

$$\begin{array}{lcl}
 & & C_0 = (\;0,\; 0,\; 0,\; 0,\; 0\;) \\
\mathsf{R}_0 \leftarrow 10 & \rightsquigarrow & C_1 = (\;1,\; 10,\; 0,\; 0,\; 0\;) \\
\mathsf{R}_1 \leftarrow 20 & \rightsquigarrow & C_2 = (\;2,\; 10,\; 20,\; 0,\; 0\;) \\
\mathsf{R}_2 \leftarrow \mathsf{R}_0 + \mathsf{R}_1 & \rightsquigarrow & C_3 = (\;3,\; 10,\; 20,\; 30,\; 0\;)
\end{array}$$

Notes:

Aside: is this the *only* viable example?

- No: (many) alternatives exist, stemming from

3. different instruction set format, e.g.,
   - **register machine** $\simeq$ 2-operand model:
     - r registers,
     - operands may need to be reused as source *and* destination,
     - (rough) example:

$$
\begin{array}{lcl}
 & & C_0 = (\ 0,\ 0,\ 0,\ 0,\ 0\ ) \\
\mathsf{R}_0 \leftarrow 10 & \rightsquigarrow & C_1 = (\ 1,\ 10,\ 0,\ 0,\ 0\ ) \\
\mathsf{R}_1 \leftarrow 20 & \rightsquigarrow & C_2 = (\ 2,\ 10,\ 20,\ 0,\ 0\ ) \\
\mathsf{R}_0 \leftarrow \mathsf{R}_0 + \mathsf{R}_1 & \rightsquigarrow & C_3 = (\ 3,\ 30,\ 20,\ 0,\ 0\ )
\end{array}
$$

Notes:

Aside: is this the *only* viable example?

- No: (many) alternatives exist, stemming from

3. different instruction set format, e.g.,
   - **accumulator machine** $\simeq$ 1-operand model:
     - may have $r > 1$ register, but there is 1 special-purpose case termed the **accumulator**,
     - operations implicitly use accumulator for source and/or destination operands,
     - (rough) example:

$$
\begin{array}{lcl}
 & \rightsquigarrow & C_0 = (\ 0,\ 0,\ 0,\ 0,\ 0\ ) \\
\mathsf{A} \leftarrow 10 & \rightsquigarrow & C_1 = (\ 1,\ 10,\ 0,\ 0,\ 0\ ) \\
\mathsf{A} \leftarrow \mathsf{A} + 20 & \rightsquigarrow & C_2 = (\ 2,\ 30,\ 0,\ 0,\ 0\ )
\end{array}
$$

Notes:

Aside: is this the *only* viable example?

- No: (many) alternatives exist, stemming from

3. different instruction set format, e.g.,
   - **stack machine** $\simeq$ 0-operand model:
     - may have $r > 1$ register, but managed per a **stack** (i.e., FILO-style) policy,
     - operations implicitly use stack for source and/or destination operands,
     - (rough) example:

$$
\begin{array}{lcl}
 & \rightsquigarrow & C_0 = (\ 0,\ 0,\ 0,\ 0,\ 0\ ) \\
\text{push } 20 & \rightsquigarrow & C_1 = (\ 1,\ 20,\ 0,\ 0,\ 0\ ) \\
\text{push } 10 & \rightsquigarrow & C_2 = (\ 2,\ 10,\ 20,\ 0,\ 0\ ) \\
\text{add} & \rightsquigarrow & C_3 = (\ 3,\ 30,\ 0,\ 0,\ 0\ ) \\
\text{pop} & \rightsquigarrow & C_4 = (\ 4,\ 0,\ 0,\ 0,\ 0\ )
\end{array}
$$

Notes:

Part 2: in practice (1)

Design

Definition

Consider a sequence

$$x = \langle x_0, x_1, \ldots, x_{n-1} \rangle$$

where, for each $0 \le i < n$ we have $x_i \in \mathbb{N}$. The associated **Gödel encoding** (or **Gödel numbering**) is

$$\hat{x} \;=\; \prod_{i=0}^{i<n} p_i^{x_i} \;=\; p_0^{x_0} \cdot p_1^{x_1} \cdots p_{n-1}^{x_{n-1}}$$

where p_i is the i-th prime, i.e., $p_0 = 2$, $p_1 = 3$, $p_2 = 5$, and so on. Due to Euclid's unique prime-factorisation theorem, factoring $\hat{x}$ allows recovery of x.

$\therefore$ we can represent *anything* using elements of $\mathbb{N}$, e.g., per [8, Section VII.A],

- let 6 represent "0",
- let 5 represent "=", then
- the logical statement "0 = 0" can be represented as

$$2^6 \cdot 3^5 \cdot 5^6 = 243,000,000.$$

Notes:

- Concept:
  - One can view

$$\text{(human-readable) instruction} \simeq \text{abstraction of (machine-readable) control information,}$$

i.e.,

$$\begin{array}{rcccl} \text{instruction} & = & \text{information} & \mapsto & \text{what to do} \\ \text{data} & = & \text{information} & \mapsto & \text{what to do it on/with} \end{array}$$

Notes:

- Concept:
  - Gödel encoding allows numerical representation of *either* form of information, e.g.,

$$\begin{array}{ccc} 1 & \mapsto & \text{“the integer one” if it represents some data} \\ 1 & \mapsto & \text{“compute an addition” if it represents an instruction} \end{array}$$

are different, valid interpretations of the same number.

Notes:

Part 2: in practice (2)
Design

- Concept:

  - These facts suggest a strategy: an RM is an FSM in disguise, in the sense that
    1. an RM configuration is an FSM state, and
    2. the RM program determines the FSM transition function,

    so we could therefore
    - use, e.g., Gödel encoding or variant thereof, to encode instructions into numerical **machine code**,
    - store the machine code in memory,
    - have our implementation decode machine code into appropriate control signals.

Notes:

Part 2: in practice (3)
Design

- Design: specify an instruction encoding, e.g.,

$\mathsf{L}_i : \mathsf{R}_{addr} \leftarrow \mathsf{R}_{addr} + 1$ **then goto** $\mathsf{L}_{i+1} \mapsto$

8 7 6	5 4	3 2 1 0
000	*addr*	0000

$\mathsf{L}_i : \mathsf{R}_{addr} \leftarrow \mathsf{R}_{addr} - 1$ **then goto** $\mathsf{L}_{i+1} \mapsto$

8 7 6	5 4	3 2 1 0
001	*addr*	0000

$\mathsf{L}_i :$ **if** $\mathsf{R}_{addr} = 0$ **then goto** L_{target} **else goto** $\mathsf{L}_{i+1} \mapsto$

8 7 6	5 4	3 2 1 0
010	*addr*	*target*

$\mathsf{L}_i :$ halt $\mapsto$

8 7 6	5 4	3 2 1 0
011	00	0000

such that

$$\mathsf{L}_i : \textbf{if } \mathsf{R}_2 = 0 \textbf{ then goto } \mathsf{L}_5 \textbf{ else goto } \mathsf{L}_{i+1} \mapsto 010100101_{(2)} = 0A5_{(16)}.$$

Notes:

Part 2: in practice (4)

Design

- **Design**: encode our original program

$$
\begin{array}{lcrcr}
L_0 : \textbf{if } R_2 = 0 \textbf{ then goto } L_5 \textbf{ else goto } L_1 & \mapsto & 010100101_{(2)} & = & 0A5_{(16)} \\
L_1 : R_2 \leftarrow R_2 - 1 \textbf{ then goto } L_2 & \mapsto & 001100000_{(2)} & = & 060_{(16)} \\
L_2 : R_3 \leftarrow R_3 + 1 \textbf{ then goto } L_3 & \mapsto & 000110000_{(2)} & = & 030_{(16)} \\
L_3 : R_1 \leftarrow R_1 + 1 \textbf{ then goto } L_4 & \mapsto & 000010000_{(2)} & = & 010_{(16)} \\
L_4 : \textbf{if } R_0 = 0 \textbf{ then goto } L_0 \textbf{ else goto } L_5 & \mapsto & 010000000_{(2)} & = & 080_{(16)} \\
L_5 : \textbf{if } R_1 = 0 \textbf{ then goto } L_9 \textbf{ else goto } L_6 & \mapsto & 010011001_{(2)} & = & 099_{(16)} \\
L_6 : R_1 \leftarrow R_1 - 1 \textbf{ then goto } L_7 & \mapsto & 001010000_{(2)} & = & 050_{(16)} \\
L_7 : R_2 \leftarrow R_2 + 1 \textbf{ then goto } L_8 & \mapsto & 000100000_{(2)} & = & 020_{(16)} \\
L_8 : \textbf{if } R_0 = 0 \textbf{ then goto } L_5 \textbf{ else goto } L_9 & \mapsto & 010000101_{(2)} & = & 085_{(16)} \\
L_9 : \text{halt} & \mapsto & 011000000_{(2)} & = & 0C0_{(16)}
\end{array}
$$

such that

- we use

$$\mathsf{MEM} = \langle 0A5_{(16)}, 030_{(16)}, \ldots, 0C0_{(16)} \rangle,$$

  a 10-element memory,
- each $\mathsf{MEM}[i]$ is a 9-bit encoding of the instruction labelled L_i.

Notes:

Part 2: in practice (5)

Design

- **Translation**:
  - we're encoding the instructions as 9-element sequence of bits,
  - using a Gödel encoding is too inefficient, so we opt for

$$
\begin{array}{rclcl}
\hat{x} & = & x_0 & \parallel & x_1 & \parallel & x_2 \\
 & \equiv & x_0 \cdot 2^0 & + & x_1 \cdot 2^4 & + & x_2 \cdot 2^6
\end{array}
$$

  so decoding amounts to extraction of contiguous bits from $\hat{x}$, i.e.,

$$
\begin{array}{rclcl}
x_0 & = & \hat{x}_{3\ldots0} & \equiv & (\hat{x} \gg 0) \wedge F_{(16)} \\
x_1 & = & \hat{x}_{5\ldots4} & \equiv & (\hat{x} \gg 4) \wedge 3_{(16)} \\
x_2 & = & \hat{x}_{8\ldots6} & \equiv & (\hat{x} \gg 6) \wedge 7_{(16)}
\end{array}
$$

  - where a field, e.g., *target*, is unused, we just use zero as a placeholder,
  - this approach works, but clearly isn't the *only* one possible.

Notes:

Definition

The **Program Counter (PC)** is a special-purpose register that holds the address of the next instruction to be executed.

Definition

The **Instruction Register (IR)** is a special-purpose register that holds the instruction currently being executed.

Notes:

Definition

The **fetch-decode-execute cycle** (aka. **instruction cycle**) is a 3-stage process

1. fetch stage : $\left\{ \begin{array}{lll} \text{1.a.} & \text{load instruction into } \mathsf{IR} & \mapsto \quad \mathsf{IR} \leftarrow \mathsf{MEM}[\mathsf{PC}] \\ \text{1.b.} & \text{increment } \mathsf{PC} & \mapsto \quad \mathsf{PC} \leftarrow \mathsf{PC} + 1 \end{array} \right.$
2. decode stage : { *decide* what instruction in IR *means*, i.e., translate IR into control signals which reflect instruction semantics
3. execute stage : { *do* whatever instruction in IR *means*, i.e., apply instruction semantics

which describes execution of instructions; in some cases it makes sense to consider a 5-stage process by adding

4. memory access stage : { perform any memory accesses (e.g., loads or stores) required
5. write-back (or commit) stage : { store result(s) stemming from instruction execution (e.g., computation)

i.e., expanding the execute stage to be more precise.

Notes:

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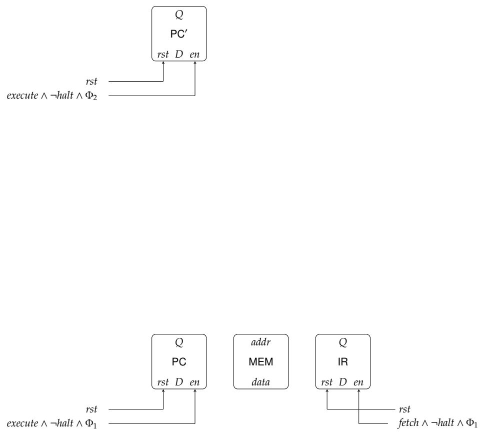
Part 2: in practice (8)
Implementation: control-path

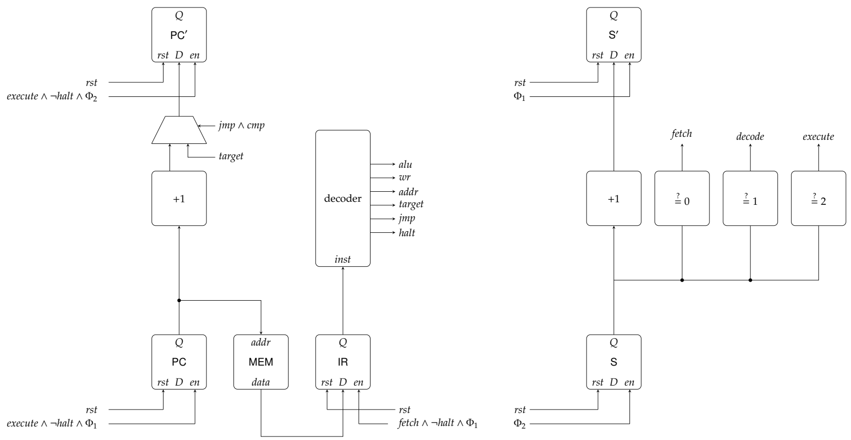
Circuit



Notes:

Part 2: in practice (8)
Implementation: control-path

Circuit

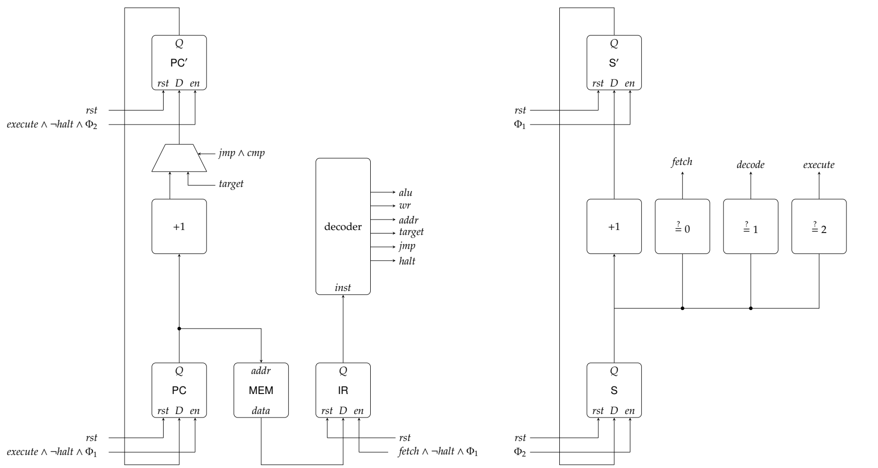


Notes:

Part 2: in practice (8)

Implementation: control-path

Circuit



Notes:

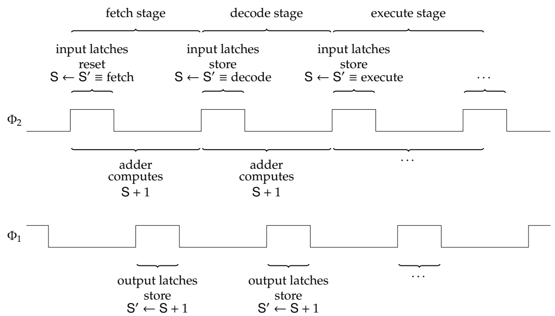
Part 2: in practice (9)

Implementation: control-path

- Caveat(s):
  1. we're using latches (resp. flip-flops) with a dedicated *rst* input (in addition to *en*),
  2. we're assuming a cyclic 2-bit counter, so the state S steps through values

$$0, 1, 2, 3, 0, 1, 2, 3, \ldots,$$

  3. we're not using S = 3, so in a sense this is an idle state,
  4. we've reversed Φ_1 and Φ_2 for the sequencer, so we generate



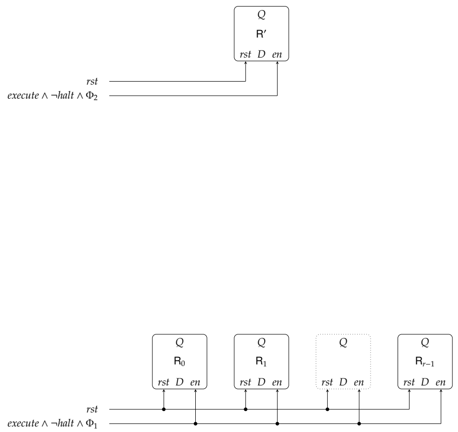
  to correctly drive update to registers the data- and control-path,
  5. we've deviated slightly from the fetch-decode-execute cycle, updating PC in the execute (rather than fetch) stage.

Notes:

Part 2: in practice (10)

Implementation: data-path

Circuit

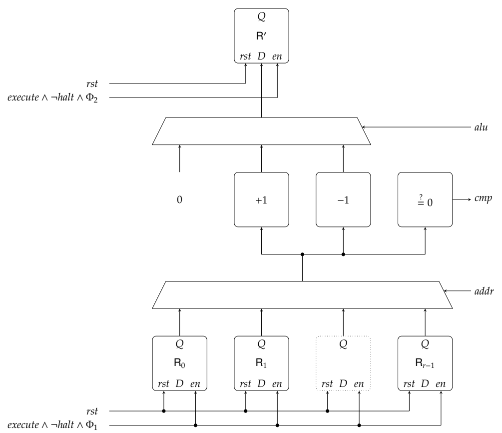


Notes:

Part 2: in practice (10)

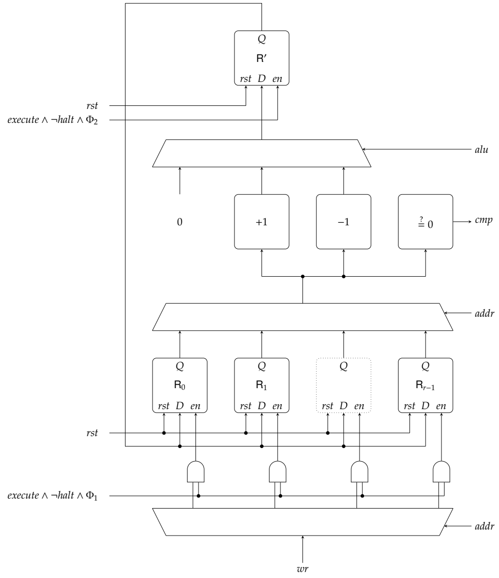
Implementation: data-path

Circuit



Notes:

Circuit



Notes:

Conclusions

- Take away points:
  1. A central aim here is to demonstrate that, per

$$\begin{array}{rcll}
\text{combinatorial logic} & \rightsquigarrow & \text{fixed function,} & \text{not stateful} \\
\text{sequential logic} & \rightsquigarrow & \text{fixed function,} & \text{stateful} \\
\text{FSM} & \rightsquigarrow & \text{fixed function,} & \text{stateful} \\
\text{RM} & \rightsquigarrow & \text{not fixed function,} & \text{stateful} \\
 & \vdots & & \\
\text{micro-processor} & \rightsquigarrow & \text{not fixed function,} & \text{stateful}
\end{array}$$

     and although our counter machine is *still* limited,
     - it has started to exhibit the characteristics of a *real* micro-processor, *and*
     - we can *still* reason end-to-end about the implementation.
  2. In doing so we've encountered some fundamental concepts, e.g.,
     - instruction encoding and decoding,
     - the fetch-decode-execute cycle,
     - the role of PC and IR in supporting it,
     - ...

     which we'll revisit and refine.

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Notes:

Additional Reading

- *Wikipedia: Register machine*. URL: https://en.wikipedia.org/wiki/Register_machine.
- *Wikipedia: Counter machine*. URL: https://en.wikipedia.org/wiki/Counter_machine.
- *Wikipedia: Random-access machine*. URL: https://en.wikipedia.org/wiki/Random-access_machine.
- *Wikipedia: Random-access stored-program machine*. URL: https://en.wikipedia.org/wiki/Random-access_stored-program_machine.
- *Wikipedia: Gödel numbering*. URL: https://en.wikipedia.org/wiki/G%C3%B6del_numbering.
- *Wikipedia: Machine code*. URL: https://en.wikipedia.org/wiki/Machine_code.

Notes:

References

[1] *Wikipedia: Counter machine*. URL: https://en.wikipedia.org/wiki/Counter_machine (see p. 105).

[2] *Wikipedia: Gödel numbering*. URL: https://en.wikipedia.org/wiki/G%C3%B6del_numbering (see p. 105).

[3] *Wikipedia: Machine code*. URL: https://en.wikipedia.org/wiki/Machine_code (see p. 105).

[4] *Wikipedia: Random-access machine*. URL: https://en.wikipedia.org/wiki/Random-access_machine (see p. 105).

[5] *Wikipedia: Random-access stored-program machine*. URL: https://en.wikipedia.org/wiki/Random-access_stored-program_machine (see p. 105).

[6] *Wikipedia: Register machine*. URL: https://en.wikipedia.org/wiki/Register_machine (see p. 105).

[7] M. Minsky. *Computation: Finite and Infinite Machines*. Prentice Hall, 1967 (see pp. 5, 11).

[8] E. Nagel and J.R. Newman. *Gödel's Proof*. Routledge, 1958 (see p. 71).

[9] A.J. Perlis. "Epigrams on programming". In: *ACM SIGPLAN Notices* 17.9 (1982), pp. 7–13 (see p. 8).

Notes: