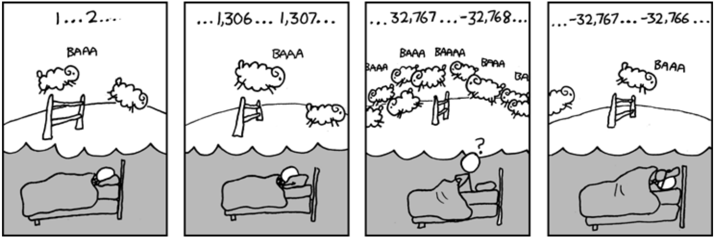

https://xkcd.com/571

- Claim: at least conceptually we could say that

$$123 \equiv \langle 3, 2, 1 \rangle,$$

i.e., the decimal literal 123 is basically just a sequence of digits.

- Question: given
  - a **bit** is a single binary digit, i.e., 0 or 1,
  - a **byte** is an 8-element sequence of bits, and
  - a **word** is a w-element sequence of bits

  and so, e.g.,

  $$01111011 \equiv \langle 1, 1, 0, 1, 1, 1, 1, 0 \rangle,$$

  what do these things *mean* ... what do they *represent*?
- Answer: anything *we* decide they do!

- Concept:

$$\underbrace{\hat{X}}_{\rotatebox{90}{\shortstack{the representation\\of X}}} \quad \underbrace{\mapsto}_{\rotatebox{90}{maps to}} \quad \underbrace{X}_{\rotatebox{90}{\shortstack{the value\\of X}}}$$

  i.e., we need

  1. a concrete representation that we can write down, plus
  2. a mapping that yields the correct value *and* is consistent (in both directions).

- Agenda:
  1. useful properties of bit-sequences,
  2. positional number systems $\rightsquigarrow$ standard integer representations.

Definition

A given literal, say

$$X = 1111011,$$

can be interpreted in *two* ways:

1. A **little-endian** ordering is where we read bits in a literal from right-to-left, i.e.,

   $$X_{LE} = \langle X_0, X_1, X_2, X_3, X_4, X_5, X_6 \rangle = \langle 1, 1, 0, 1, 1, 1, 1 \rangle,$$

   where

   - the Least-Significant Bit (LSB) is the right-most in the literal (i.e., X_0), and
   - the Most-Significant Bit (MSB) is the left-most in the literal (i.e., $X_{n-1} = X_6$).

2. A **big-endian** ordering is where we read bits in a literal from left-to-right, i.e.,

   $$X_{BE} = \langle X_6, X_5, X_4, X_3, X_2, X_1, X_0 \rangle = \langle 1, 1, 1, 1, 0, 1, 1 \rangle,$$

   where

   - the Least-Significant Bit (LSB) is the left-most in the literal (i.e., $X_{n-1} = X_6$), and
   - the Most-Significant Bit (MSB) is the right-most in the literal (i.e., X_0).

Definition

Following the idea of vectorial Boolean function, given an n-element bit-sequence X, and an m-element bit-sequence Y we can clearly

1. overload $\oslash \in \{\neg\}$, i.e., write
$$R = \oslash X,$$
to mean
$$R_i = \oslash X_i$$
for $0 \leq i < n$,
2. overload $\ominus \in \{\wedge, \vee, \oplus\}$, i.e., write
$$R = X \ominus Y,$$
to mean
$$R_i = X_i \ominus Y_i$$
for $0 \leq i < n = m$, where if $n \neq m$, we pad either X or Y with 0 until the $n = m$.

Part 1: useful properties of bit-sequences

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to mean
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for $0 \leq i < n = m$, where if $n \neq m$, we pad either X or Y with 0 until the $n = m$.

- Example: in C, we use the computational (or **bit-wise**) operators `~`, `&`, `|`, and `^` this way: they apply NOT, AND, OR, and XOR to corresponding bits in the operands.

Part 1: useful properties of bit-sequences

Definition

Given two n-bit sequences X and Y, we can define some important properties named after Richard Hamming, a researcher at Bell Labs:

- The **Hamming weight** of X is the number of bits in X that are equal to 1, i.e., the number of times $X_i = 1$. This can be expressed as

$$\mathrm{HW}(X) = \sum_{i=0}^{n-1} X_i.$$

- The **Hamming distance** between X and Y is the number of bits in X that differ from the corresponding bit in Y, i.e., the number of times $X_i \neq Y_i$. This can be expressed as

$$\mathrm{HD}(X, Y) = \sum_{i=0}^{n-1} X_i \oplus Y_i.$$

Note that both quantities naturally generalise to non-binary sequences.

Part 1: useful properties of bit-sequences

Definition

Given two n-bit sequences X and Y, we can define some important properties named after Richard Hamming, a researcher at Bell Labs:

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$$\mathrm{HD}(X, Y) = \sum_{i=0}^{n-1} X_i \oplus Y_i.$$

Note that both quantities naturally generalise to non-binary sequences.

- Example: given $X = \langle 1, 0, 0, 1 \rangle$ and $Y = \langle 0, 1, 1, 1 \rangle$ we find that

$$\begin{array}{lclcl} \mathrm{HW}(X) & = & \sum_{i=0}^{n-1} X_i & & = 1 + 0 + 0 + 1 = 2 \\ \mathrm{HD}(X, Y) & = & \sum_{i=0}^{n-1} X_i \oplus Y_i & = (1 \oplus 0) + (0 \oplus 1) + (0 \oplus 1) + (1 \oplus 1) & = 1 + 1 + 1 + 0 = 3 \end{array}$$

- Concept: a **positional number system** expresses the value of a number x using a base-b (or radix-b) expansion, i.e.,

$$\begin{aligned} \hat{x} &= \langle \hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{n-1} \rangle \\ &\mapsto x \\ &= \pm \sum_{i=0}^{n-1} \hat{x}_i \cdot b^i \end{aligned}$$

  where each $\hat{x}_i$
  - is one of n digits taken from the digit set $X = \{0, 1, \ldots, b-1\}$,
  - is "weighted" by some power of of the base b.

Part 2: positional number systems $\rightsquigarrow$ standard integer representations (1)

- Beware!
  - for $b > 10$ we can't express $\hat{x}_i$ using a single Arabic numeral,
  - for $b = 16$, for example, we use letters instead:

$$
\begin{array}{lcl}
A & \mapsto & 10 \\
B & \mapsto & 11 \\
C & \mapsto & 12 \\
D & \mapsto & 13 \\
E & \mapsto & 14 \\
F & \mapsto & 15
\end{array}
$$

Example

Consider an example where we

1. set $b = 10$, i.e., deal with **decimal** numbers, and
2. have $\hat{x}_i \in X = \{0, 1, \ldots, 10 - 1 = 9\}$.

This means we can write

$$
\begin{array}{lcl}
\hat{x} = 123 & = & \langle 3, 2, 1 \rangle_{(10)} \\
 & \mapsto & x \\
 & = & \sum_{i=0}^{n-1} \hat{x}_i \cdot 10^i \\
 & = & 3 \cdot 10^0 + 2 \cdot 10^1 + 1 \cdot 10^2 \\
 & = & 3 \cdot 1 \quad + 2 \cdot 10 \quad + 1 \cdot 100 \\
 & = & 123_{(10)}
\end{array}
$$

i.e., represent the value "one hundred and twenty three" in a variety of ways using different bases.

Part 2: positional number systems $\rightsquigarrow$ standard integer representations (2)

Example

Consider an example where we

1. set $b = 2$, i.e., deal with **binary** numbers, and
2. have $\hat{x}_i \in X = \{0, 2-1 = 1\}$.

This means we can write

$$
\begin{array}{rcl}
\hat{x} = 1111011 & = & \langle 1,1,0,1,1,1,1 \rangle_{(2)} \\
 & \mapsto & x \\
 & = & \sum_{i=0}^{n-1} \hat{x}_i \cdot 2^i \\
 & = & 1 \cdot 2^0 + 1 \cdot 2^1 + 0 \cdot 2^2 + 1 \cdot 2^3 + 1 \cdot 2^4 + 1 \cdot 2^5 + 1 \cdot 2^6 \\
 & = & 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 4 + 1 \cdot 8 + 1 \cdot 16 + 1 \cdot 32 + 1 \cdot 64 \\
 & = & 123_{(10)}
\end{array}
$$

i.e., represent the value “one hundred and twenty three” in a variety of ways using different bases.

Example

Consider an example where we

1. set $b = 8$, i.e., deal with **octal** numbers, and
2. have $\hat{x}_i \in X = \{0, 1, \ldots, 8 - 1 = 7\}$.

This means we can write

$$
\begin{array}{lcl}
\hat{x} = 173 & = & \langle 3, 7, 1 \rangle_{(8)} \\
 & \mapsto & x \\
 & = & \sum_{i=0}^{n-1} \hat{x}_i \cdot 8^i \\
 & = & 3 \cdot 8^0 + 7 \cdot 8^1 + 1 \cdot 8^2 \\
 & = & 3 \cdot 1 + 7 \cdot 8 + 1 \cdot 64 \\
 & = & 123_{(10)}
\end{array}
$$

i.e., represent the value "one hundred and twenty three" in a variety of ways using different bases.

Example

Consider an example where we

1. set $b = 16$, i.e., deal with **hexadecimal** numbers, and
2. have $\hat{x}_i \in X = \{0, 1, \ldots, 16 - 1 = 15\}$.

This means we can write

$$
\begin{array}{lcl}
\hat{x} = 7B & = & \langle B, 7 \rangle_{(16)} \\
 & \mapsto & x \\
 & = & \sum_{i=0}^{n-1} \hat{x}_i \cdot 16^i \\
 & = & 11 \cdot 16^0 + 7 \cdot 16^1 \\
 & = & 11 \cdot 1 \quad + 7 \cdot 16 \\
 & = & 123_{(10)}
\end{array}
$$

i.e., represent the value "one hundred and twenty three" in a variety of ways using different bases.

Part 2: positional number systems $\rightsquigarrow$ standard integer representations (3)

- **Problem**: we want to represent and perform various operations on elements of $\mathbb{Z}$, *but*
  1. it's an an infinite set, and
  2. so far we've ignored the issue of sign.
- **Solution**: in C, for example, we get

$$\begin{array}{rcrcl} \texttt{unsigned char} & \simeq & \texttt{uint8\_t} & \mapsto & \{ \quad\quad\quad\quad 0, \ldots, +2^8 - 1 \} \\ \texttt{char} & \simeq & \texttt{int8\_t} & \mapsto & \{ -2^7, \ldots, 0, \ldots, +2^7 - 1 \} \end{array}$$

  but why *these*, and how do they work?

Definition

An unsigned integer can be represented in n bits by using the natural binary expansion. That is, we have

$$\begin{array}{rcl} \hat{x} & = & \langle \hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{n-1} \rangle \\ & \mapsto & x \\ & = & \sum_{i=0}^{n-1} \hat{x}_i \cdot 2^i \end{array}$$

for $\hat{x}_i \in \{0, 1\}$, which yields

$$0 \leq x \leq 2^n - 1.$$

Example ($n = 8$)

$$
\begin{array}{lclcr}
11111111 & \mapsto & 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 & = & +255_{(10)} \\
 & \vdots & & \vdots & \\
10000101 & \mapsto & 1 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 & = & +133_{(10)} \\
 & \vdots & & \vdots & \\
10000000 & \mapsto & 1 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 & = & +128_{(10)} \\
01111111 & \mapsto & 0 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 & = & +127_{(10)} \\
 & \vdots & & \vdots & \\
01111011 & \mapsto & 0 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 & = & +123_{(10)} \\
 & \vdots & & \vdots & \\
00000001 & \mapsto & 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 & = & +1_{(10)} \\
00000000 & \mapsto & 0 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 & = & +0_{(10)}
\end{array}
$$

- Fact:
  - each hexadecimal digit $x_i \in \{0, 1, \ldots, 15\}$,
  - four bits gives $2^4 = 16$ possible combinations, so
  - each hexadecimal digit can be thought of as a short-hand for four binary digits.
- Example: we can perform the following translation steps

$$
\begin{array}{lcllll}
8AC & = & \langle & C, & A, & 8, & \rangle_{(16)} \\
 & = & \langle & \langle 0, 0, 1, 1 \rangle_{(2)}, & \langle 0, 1, 0, 1 \rangle_{(2)}, & \langle 0, 0, 0, 1 \rangle_{(2)} & \rangle_{(16)} \\
 & = & \langle & 0, 0, 1, 1, & 0, 1, 0, 1, & 0, 0, 0, 1 & \rangle_{(16)} \\
 & \mapsto & 2220_{(10)} & & & &
\end{array}
$$

such that in C, for example,

$$\texttt{0x8AC} = 2220_{(10)}.$$

- Fact: left-shift (resp. right-shift) of some x by y digits is equivalent to multiplication (resp. division) by b^y.
- Example: taking $b = 2$ we find that

$$\begin{array}{rcl} x \times 2^y & = & (\sum_{i=0}^{n-1} x_i \cdot 2^i) \times 2^y \\ & = & \sum_{i=0}^{n-1} x_i \cdot 2^i \times 2^y \\ & = & \sum_{i=0}^{n-1} x_i \cdot 2^{i+y} \\ & = & x \ll y \end{array}$$

and

$$\begin{array}{rcl} x / 2^y & = & (\sum_{i=0}^{n-1} x_i \cdot 2^i) / 2^y \\ & = & \sum_{i=0}^{n-1} x_i \cdot 2^i / 2^y \\ & = & \sum_{i=0}^{n-1} x_i \cdot 2^{i-y} \\ & = & x \gg y \end{array}$$

such that in C, for example,

$$\begin{array}{lcrcrcl} \texttt{0x8AC << 2} & \mapsto & 2220_{(10)} \times 2^2 & = & 8880_{(10)} & \mapsto & \texttt{0x22B0} \\ \texttt{0x8AC >> 2} & \mapsto & 2220_{(10)} \;/\; 2^2 & = & 555_{(10)} & \mapsto & \texttt{0x22B} \end{array}$$

Definition

A signed integer can be represented in n bits by using the **sign-magnitude** approach; 1 bit is reserved for the sign (0 means positive, 1 means negative) and $n-1$ for the magnitude. That is, we have

$$\begin{array}{rcl} \hat{x} & = & \langle \hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{n-1} \rangle \\ & \mapsto & x \\ & = & (-1)^{\hat{x}_{n-1}} \cdot \sum_{i=0}^{n-2} \hat{x}_i \cdot 2^i \end{array}$$

for $\hat{x}_i \in \{0, 1\}$, which yields

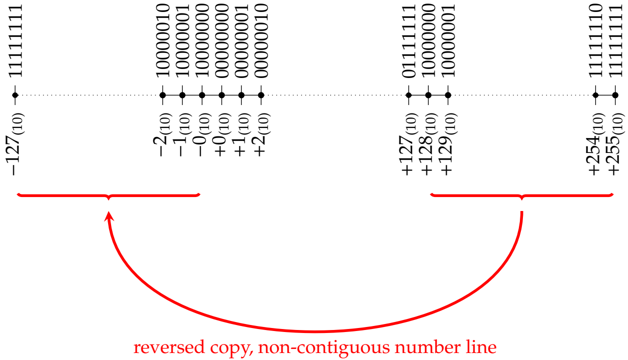
$$-2^{n-1} + 1 \leq x \leq +2^{n-1} - 1.$$

Note that there are two representations of zero (i.e., $+0$ and -0).

Example ($n = 8$)

$$
\begin{array}{lclclcll}
01111111 & \mapsto & (-1)^0 & \cdot & (& 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 &) & = +127_{(10)} \\
 & \vdots & & & & & & \vdots \\
01111011 & \mapsto & (-1)^0 & \cdot & (& 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 &) & = +123_{(10)} \\
 & \vdots & & & & & & \vdots \\
00000001 & \mapsto & (-1)^0 & \cdot & (& 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 &) & = +1_{(10)} \\
00000000 & \mapsto & (-1)^0 & \cdot & (& 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 &) & = +0_{(10)} \\
10000000 & \mapsto & (-1)^1 & \cdot & (& 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 &) & = -0_{(10)} \\
10000001 & \mapsto & (-1)^1 & \cdot & (& 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 &) & = -1_{(10)} \\
 & \vdots & & & & & & \vdots \\
11111011 & \mapsto & (-1)^1 & \cdot & (& 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 &) & = -123_{(10)} \\
 & \vdots & & & & & & \vdots \\
11111111 & \mapsto & (-1)^1 & \cdot & (& 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 &) & = -127_{(10)}
\end{array}
$$

Example ($n = 8$)



11111111
10000010
10000001
10000000
00000000
00000001
00000010
01111111
10000000
10000001
11111110
11111111
$-127_{(10)}$
$-2_{(10)}$
$-1_{(10)}$
$-0_{(10)}$
$+0_{(10)}$
$+1_{(10)}$
$+2_{(10)}$
$+127_{(10)}$
$+128_{(10)}$
$+129_{(10)}$
$+254_{(10)}$
$+255_{(10)}$
reversed copy, non-contiguous number line

Definition

A signed integer can be represented in n bits by using the **two's-complement** approach; the basic idea is to weight the $(n-1)$-th bit using -2^{n-1} rather than $+2^{n-1}$, and all other bits as normal. That is, we have

$$\begin{array}{rcl} \hat{x} & = & \langle \hat{x}_0, \hat{x}_1, \ldots, \hat{x}_{n-1} \rangle \\ & \mapsto & x \\ & = & \hat{x}_{n-1} \cdot -2^{n-1} + \sum_{i=0}^{n-2} \hat{x}_i \cdot 2^i \end{array}$$

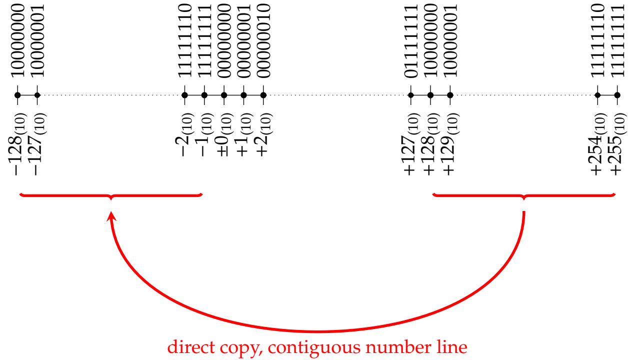
for $\hat{x}_i \in \{0, 1\}$, which yields

$$-2^{n-1} \leq x \leq +2^{n-1} - 1.$$

Example ($n = 8$)

$$
\begin{array}{lclcr}
01111111 & \mapsto & 0 \cdot -2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 & = & +127_{(10)} \\
 & \vdots & & \vdots & \\
01111011 & \mapsto & 0 \cdot -2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 & = & +123_{(10)} \\
 & \vdots & & \vdots & \\
00000001 & \mapsto & 0 \cdot -2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 & = & +1_{(10)} \\
00000000 & \mapsto & 0 \cdot -2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 & = & +0_{(10)} \\
11111111 & \mapsto & 1 \cdot -2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 & = & -1_{(10)} \\
 & \vdots & & \vdots & \\
10000101 & \mapsto & 1 \cdot -2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 & = & -123_{(10)} \\
 & \vdots & & \vdots & \\
10000000 & \mapsto & 1 \cdot -2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 & = & -128_{(10)}
\end{array}
$$

Example ($n = 8$)



10000000
10000001
11111110
11111111
00000000
00000001
00000010
01111111
10000000
10000001
11111110
11111111
$-128_{(10)}$
$-127_{(10)}$
$-2_{(10)}$
$-1_{(10)}$
$\pm 0_{(10)}$
$+1_{(10)}$
$+2_{(10)}$
$+127_{(10)}$
$+128_{(10)}$
$+129_{(10)}$
$+254_{(10)}$
$+255_{(10)}$
direct copy, contiguous number line

Conclusions

- Take away points:
  1. *We* control what bit-sequences mean: we can interpret an instance of the C `char` data-type as
     - a signed 8-bit integer, *or*
     - a generic object which can take one of 2^8 states,

     and, as a result, can represent *anything*, e.g.,
     - a pixel within an image,
     - a character within a document,
     - a number within a matrix,
     - ...
  2. Beyond this, knowing about various standard representations is important and useful in a general sense.

Additional Reading

- *Wikipedia: Numeral system*. URL: https://en.wikipedia.org/wiki/Numeral_system.
- D. Page. "Chapter 1: Mathematical preliminaries". In: *A Practical Introduction to Computer Architecture*. 1st ed. Springer, 2009.
- B. Parhami. "Part 1: Number representation". In: *Computer Arithmetic: Algorithms and Hardware Designs*. 1st ed. Oxford University Press, 2000.
- W. Stallings. "Chapter 9: Number systems". In: *Computer Organisation and Architecture*. 9th ed. Prentice Hall, 2013.
- A.S. Tanenbaum and T. Austin. "Appendix A: Binary numbers". In: *Structured Computer Organisation*. 6th ed. Prentice Hall, 2012.

References

[1] *Wikipedia: Numeral system*. URL: https://en.wikipedia.org/wiki/Numeral_system (see p. 29).

[2] D. Page. "Chapter 1: Mathematical preliminaries". In: *A Practical Introduction to Computer Architecture*. 1st ed. Springer, 2009 (see p. 29).

[3] B. Parhami. "Part 1: Number representation". In: *Computer Arithmetic: Algorithms and Hardware Designs*. 1st ed. Oxford University Press, 2000 (see p. 29).

[4] W. Stallings. "Chapter 9: Number systems". In: *Computer Organisation and Architecture*. 9th ed. Prentice Hall, 2013 (see p. 29).

[5] A.S. Tanenbaum and T. Austin. "Appendix A: Binary numbers". In: *Structured Computer Organisation*. 6th ed. Prentice Hall, 2012 (see p. 29).