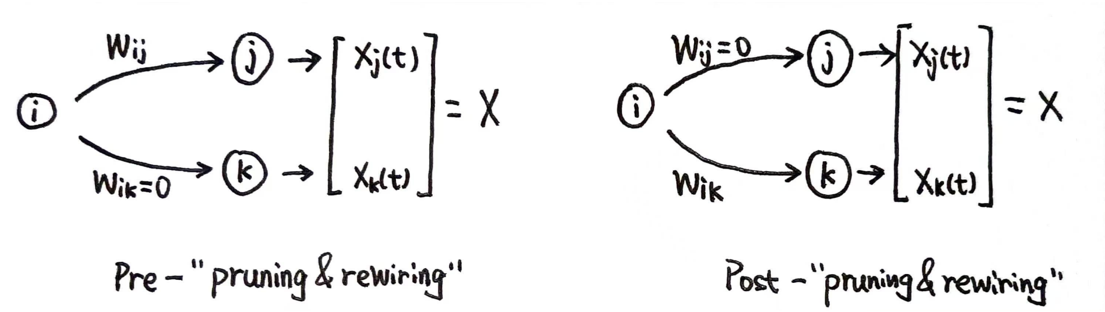
Proof1：The Analysis of Lyapunov Function Stability

Given the NCP connection subgraph as shown in the figure, in order to simplify the problem for analysis, only one disconnection and its corresponding reconnection process are provided. The state of neurons j and k is directly represented as output (the output matrix $\boldsymbol{X}$ is a 2×1 matrix). The connection weights between neurons are as shown in the figure ($W_{ik} \gg W_{ij}$).



Due to the nonlinear time-dependent behavior of the NCP neurons' LTC (Liquid Time Constant):

$$\frac{d\boldsymbol{X}}{dt} = \begin{bmatrix} \frac{dx_j(t)}{dt} \\ \frac{dx_k(t)}{dt} \end{bmatrix} = \begin{bmatrix} -(\frac{1}{\tau} + \frac{W_{ij}}{C_m}\sigma(x_i(t)))x_j(t) + \frac{W_{ij}}{C_m}\sigma(x_i(t))E + \frac{x_{leak}}{\tau}) \\ -(\frac{1}{\tau} + \frac{W_{ik}}{C_m}\sigma(x_i(t)))x_k(t) + \frac{W_{ik}}{C_m}\sigma(x_i(t))E + \frac{x_{leak}}{\tau}) \end{bmatrix} + \begin{bmatrix} R_j \\ R_k \end{bmatrix}$$

Further matrix decomposition is performed as follows:

$$\frac{d\boldsymbol{X}}{dt} = \begin{bmatrix} -(\frac{1}{\tau} + \frac{W_{ij}}{C_m}\sigma(x_i(t))) & 0 \\ 0 & -(\frac{1}{\tau} + \frac{W_{ik}}{C_m}\sigma(x_i(t))) \end{bmatrix} \begin{bmatrix} x_j(t) \\ x_k(t) \end{bmatrix} + \frac{\sigma(x_i(t))E}{C_m} \begin{bmatrix} W_{ij} \\ W_{ik} \end{bmatrix} + \begin{bmatrix} \frac{x_{leak}}{\tau} \\ \frac{x_{leak}}{\tau} \end{bmatrix} + \begin{bmatrix} R_j \\ R_k \end{bmatrix}$$

$$= \boldsymbol{diag}\{-(\frac{1}{\tau} + \frac{W_{ij}}{C_m}\sigma(x_i(t))),\ -(\frac{1}{\tau} + \frac{W_{ik}}{C_m}\sigma(x_i(t)))\} \begin{bmatrix} x_j(t) \\ x_k(t) \end{bmatrix} + \frac{\sigma(x_i(t))E}{C_m} \begin{bmatrix} W_{ij} \\ W_{ik} \end{bmatrix} + \boldsymbol{L} + \boldsymbol{R}$$

Further define the Lyapunov function (where P is a positive definite matrix, and the function itself is a positive definite function) as follows:

$$V(\boldsymbol{X}) = \boldsymbol{X}^T \boldsymbol{P} \boldsymbol{X}$$

Then we can calculate its derivative:

$$\frac{dV(\boldsymbol{X})}{dt} = \frac{d(\boldsymbol{X}^T\boldsymbol{P}\boldsymbol{X})}{dt} = \frac{d(\boldsymbol{X}^T)\boldsymbol{P}\boldsymbol{X}}{dt} + \frac{\boldsymbol{X}^T d(\boldsymbol{P}\boldsymbol{X})}{dt} = 2\frac{\boldsymbol{X}^T\boldsymbol{P}d(\boldsymbol{X})}{dt}$$

Substitute the derivative of the state matrix $\boldsymbol{X}$ obtained earlier:

$$\frac{dV(\boldsymbol{X})}{dt} = 2\boldsymbol{X}^T\boldsymbol{P}(\boldsymbol{diag}\{-(\frac{1}{\tau} + \frac{W_{ij}}{C_m}\sigma(x_i(t))),\ -(\frac{1}{\tau} + \frac{W_{ik}}{C_m}\sigma(x_i(t)))\} \begin{bmatrix} x_j(t) \\ x_k(t) \end{bmatrix} + \frac{\sigma(x_i(t))E}{C_m} \begin{bmatrix} W_{ij} \\ W_{ik} \end{bmatrix} + \boldsymbol{L} + \boldsymbol{R})$$

The most significant negative term is obtained by multiplying the diagonal matrix with the state matrix. From this, we can deduce the following insight: T<u>he more significant the state values of the newly connected neurons, the more stable the newly connected system becomes.</u>