1980 IAU THEORY OF NUTATION: THE FINAL REPORT OF THE IAU WORKING GROUP ON NUTATION

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Abstract. In 1979 the Seventeenth General Assembly of the International Astronomical Union (IAU) in Montreal, Canada, adopted the 1979 IAU Theory of Nutation upon the recommendation of this Working Group. Subsequently the International Union of Geodesy and Geophysics (IUGG) passed a resolution requesting that this action be reconsidered in favor of a theory based on a different Earth model. As a consequence of that reconsideration the 1980 IAU Theory of Nutation was adopted. The details of that theory and the history of its adoption are described here in the Final Report of the IAU Working Group on Nutation. A summary of these events and the essence of our recommendations is provided first while the body of the report discusses these matters in greater detail. The theory itself is contained in Table I.

Introduction

(1) The President of IAU Commission 4, Dr V. K. Abalakin, established the Working Group on Nutation at the request of IAU Symposium No. 78 on Nutation and the Earth's Rotation, held at Kiev in May 1977. The final membership of the Working Group comprise the authors of this report.

(2) The theory of nutation currently in use is due to Woolard (1953) and has the following characteristics:

(a) It is based on a rigid model of the Earth with dynamical axisymmetry ($A = B$).

(b) The 'constant of nutation' is an empirical value and is not consistent with other adopted astronomical constants.

(c) Eulerian motion and forced nearly-diurnal polar motion are not included in the current theory of nutation, but are assumed to be part of polar motion.

(d) The pole of reference is the instantaneous celestial rotation pole.

(3) Modern theoretical and observational developments of various types have revealed the following problems with the current theory of nutation:

(a) The Earth is not a rigid body and the effects of non-rigidity can be observationally significant.

(b) Determinations of UT1 and polar motion using optical observations of stars, Doppler and laser range tracking of satellites, laser ranges to the Moon, and radio interferometric measurements are sufficiently accurate that their usefulness can be degraded by use of the present theory of nutation in the data reduction process.

(c) As Jeffreys and Atkinson pointed out, the instantaneous axis of rotation as

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defined by Woolard rotates relative to an Earth-fixed coordinate system with a quasi-diurnal period. For accurate observation reduction, this rotation cannot be ignored and a resolution was passed at the Sixteenth General Assembly of the IAU in 1976 in Grenoble to adopt a different pole of reference.

(d) Observational data indicate that with the current (Woolard, 1953) theory of nutation and a redefined pole of reference, a body-fixed coordinate system would still rotate with respect to the reference pole; therefore, the theory of nutation should be revised.

(4) The goal of this report is the adoption of a set of nutation coefficients which will provide an adequate working standard for determination of UT1 and polar motion, the reduction of optical observations of stars, Doppler or laser range tracking of satellites, laser ranges to the Moon, radio interferometric measurements and other high precision requirements.

(5) Therefore, the proposed solution incorporates the following changes:

(a) A non-rigid model of the Earth without axial symmetry is used. This model of the Earth is subject to tidal distortions, has a solid inner core, fluid outer core, but no oceans.

(b) The constants are consistent with the 1976 IAU System of Astronomical Constants and are in agreement with available observational data of various types.

(c) The reference pole is selected so that there are no diurnal or quasi-diurnal motions of this pole with respect to either a space-fixed or Earth-fixed coordinate system. The phenomenon of dynamical variation of latitude, otherwise known as forced diurnal polar motion, is included implicitly in the new nutation theory. The new nutation theory thus includes externally-forced motions of the Earth's rotation axis, but does not include free motions or such complex phenomena as ocean tides, atmospheric winds, and currents in the oceans or core. The new reference pole shall be referred to as the 'Celestial Ephemeris Pole' (CEP).

(6) The new nutation theory, incorporating the above changes, shall be referred to as the '1980 IAU Theory of Nutation'. This nutation theory was developed by Wahr (1979, 1981) based on previous work by Kinoshita (1977) and Gilbert and Dziewonski (1975).

1. Resolution

We request that the following draft resolution be submitted to Commissions 4, 7, 8, 19, 24, and 31, with the view of its being adopted by mail:

The IAU adopts the 1980 IAU Theory of Nutation in place of the 1979 IAU Theory of Nutation, endorses the recommendations given in the Report of the Working Group on Nutation and recommends that they shall be used in the national and international ephemerides from the year 1984 onwards, and in all other relevant astronomical work.

2. Recommendations

Whereas, the complete theory of the general nutational motion of the Earth about its center of mass may be described by the sum of two components, *astronomical nutation*, commonly referred to as nutation, which is motion with respect to a space-fixed coordinate system, and *polar motion*, which is motion with respect to a body-fixed coordinate system, it is recommended that:

(a) astronomical nutation be computed for the 'Celestial Ephemeris Pole' using a non-rigid model of the Earth in such fashion that there are no nearly-diurnal motions of this celestial pole with respect to either space-fixed or body-fixed (crust-fixed) coordinates which can be calculated from torques external to the Earth and its atmosphere.

(b) the numerical values given in Table I of the complete report be used for computing astronomical nutation of the 'Celestial Ephemeris Pole'.

TABLE I

Nutation in Longitude and Obliquity referred to mean ecliptic of date.

Epoch J2000.0 (JD 2451 545.0 TDB) T in Julian Centuries

	Argument					Period	Longitude		Obliquity	
	l	l'	F	D	Ω	(days)	(0.0001″)		(0.0001″)	
1	0	0	0	0	1	6798.4	− 171 996	− 174.2T	92025	8.9T
2	0	0	0	0	2	3399.2	2062	0.2T	− 895	0.5T
3	− 2	0	2	0	1	1305.5	46	0.0T	− 24	0.0T
4	2	0	− 2	0	0	1095.2	11	0.0T	0	0.0T
5	− 2	0	2	0	2	1615.7	− 3	0.0T	1	0.0T
6	1	− 1	0	− 1	0	3232.9	− 3	0.0T	0	0.0T
7	0	− 2	2	− 2	1	6786.3	− 2	0.0T	1	0.0T
8	2	0	− 2	0	1	943.2	1	0.0T	0	0.0T
9	0	0	2	− 2	2	182.6	− 13187	− 1.6T	5736	− 3.1T
10	0	1	0	0	0	365.3	1426	− 3.4T	54	− 0.1T
11	0	1	2	− 2	2	121.7	− 517	1.2T	224	− 0.6T
12	0	− 1	2	− 2	2	365.2	217	− 0.5T	− 95	0.3T
13	0	0	2	− 2	1	177.8	129	0.1T	− 70	0.0T
14	2	0	0	− 2	0	205.9	48	0.0T	1	0.0T
15	0	0	2	− 2	0	173.3	− 22	0.0T	0	0.0T
16	0	2	0	0	0	182.6	17	− 0.1T	0	0.0T
17	0	1	0	0	1	386.0	− 15	0.0T	9	0.0T
18	0	2	2	− 2	2	91.3	− 16	0.1T	7	0.0T
19	0	− 1	0	0	1	346.6	− 12	0.0T	6	0.0T
20	− 2	0	0	2	1	199.8	− 6	0.0T	3	0.0T
21	0	− 1	2	− 2	1	346.6	− 5	0.0T	3	0.0T
22	2	0	0	− 2	1	212.3	4	0.0T	− 2	0.0T
23	0	1	2	− 2	1	119.6	4	0.0T	− 2	0.0T
24	1	0	0	− 1	0	411.8	− 4	0.0T	0	0.0T
25	2	1	0	− 2	0	131.7	1	0.0T	0	0.0T
26	0	0	− 2	2	1	169.0	1	0.0T	0	0.0T
27	0	1	− 2	2	0	329.8	− 1	0.0T	0	0.0T
28	0	1	0	0	2	409.2	1	0.0T	0	0.0T

Table I (*continued*)

	Argument					Period	Longitude		Obliquity	
	l	l'	F	D	Ω	(days)	(0.0001″)		(0.0001″)	
29	−1	0	0	1	1	388.3	1	0.0T	0	0.0T
30	0	1	2	−2	0	117.5	−1	0.0T	0	0.0T
31	0	0	2	0	2	13.7	−2274	−0.2T	977	−0.5T
32	1	0	0	0	0	27.6	712	0.1T	−7	0.0T
33	0	0	2	0	1	13.6	−386	−0.4T	200	0.0T
34	1	0	2	0	2	9.1	−301	0.0T	129	−0.1T
35	1	0	0	−2	0	31.8	−158	0.0T	−1	0.0T
36	−1	0	2	0	2	27.1	123	0.0T	−53	0.0T
37	0	0	0	2	0	14.8	63	0.0T	−2	0.0T
38	1	0	0	0	1	27.7	63	0.1T	−33	0.0T
39	−1	0	0	0	1	27.4	−58	−0.1T	32	0.0T
40	−1	0	2	2	2	9.6	−59	0.0T	26	0.0T
41	1	0	2	0	1	9.1	−51	0.0T	27	0.0T
42	0	0	2	2	2	7.1	−38	0.0T	16	0.0T
43	2	0	0	0	0	13.8	29	0.0T	−1	0.0T
44	1	0	2	−2	2	23.9	29	0.0T	−12	0.0T
45	2	0	2	0	2	6.9	−31	0.0T	13	0.0T
46	0	0	2	0	0	13.6	26	0.0T	−1	0.0T
47	−1	0	2	0	1	27.0	21	0.0T	−10	0.0T
48	−1	0	0	2	1	32.0	16	0.0T	−8	0.0T
49	1	0	0	−2	1	31.7	−13	0.0T	7	0.0T
50	−1	0	2	2	1	9.5	−10	0.0T	5	0.0T
51	1	1	0	−2	0	34.8	−7	0.0T	0	0.0T
52	0	1	2	0	2	13.2	7	0.0T	−3	0.0T
53	0	−1	2	0	2	14.2	−7	0.0T	3	0.0T
54	1	0	2	2	2	5.6	−8	0.0T	3	0.0T
55	1	0	0	2	0	9.6	6	0.0T	0	0.0T
56	2	0	2	−2	2	12.8	6	0.0T	−3	0.0T
57	0	0	0	2	1	14.8	−6	0.0T	3	0.0T
58	0	0	2	2	1	7.1	−7	0.0T	3	0.0T
59	1	0	2	−2	1	23.9	6	0.0T	−3	0.0T
60	0	0	0	−2	1	14.7	−5	0.0T	3	0.0T
61	1	−1	0	0	0	29.8	5	0.0T	0	0.0T
62	2	0	2	0	1	6.9	−5	0.0T	3	0.0T
63	0	1	0	−2	0	15.4	−4	0.0T	0	0.0T
64	1	0	−2	0	0	26.9	4	0.0T	0	0.0T
65	0	0	0	1	0	29.5	−4	0.0T	0	0.0T
66	1	1	0	0	0	25.6	−3	0.0T	0	0.0T
67	1	0	2	0	0	9.1	3	0.0T	0	0.0T
68	1	−1	2	0	2	9.4	−3	0.0T	1	0.0T
69	−1	−1	2	2	2	9.8	−3	0.0T	1	0.0T
70	−2	0	0	0	1	13.7	−2	0.0T	1	0.0T
71	3	0	2	0	2	5.5	−3	0.0T	1	0.0T
72	0	−1	2	2	2	7.2	−3	0.0T	1	0.0T
73	1	1	2	0	2	8.9	2	0.0T	−1	0.0T
74	−1	0	2	−2	1	32.6	−2	0.0T	1	0.0T
75	2	0	0	0	1	13.8	2	0.0T	−1	0.0T
76	1	0	0	0	2	27.8	−2	0.0T	1	0.0T
77	3	0	0	0	0	9.2	2	0.0T	0	0.0T
78	0	0	2	1	2	9.3	2	0.0T	−1	0.0T
79	−1	0	0	0	2	27.3	1	0.0T	−1	0.0T

Table I (*continued*)

	Argument l	l'	F	D	Ω	Period (days)	Longitude (0.0001″)		Obliquity (0.0001″)	
80	1	0	0	−4	0	10.1	−1	0.0T	0	0.0T
81	−2	0	2	2	2	14.6	1	0.0T	−1	0.0T
82	−1	0	2	4	2	5.8	−2	0.0T	1	0.0T
83	2	0	0	−4	0	15.9	−1	0.0T	0	0.0T
84	1	1	2	−2	2	22.5	1	0.0T	−1	0.0T
85	1	0	2	2	1	5.6	−1	0.0T	1	0.0T
86	−2	0	2	4	2	7.3	−1	0.0T	1	0.0T
87	−1	0	4	0	2	9.1	1	0.0T	0	0.0T
88	1	−1	0	−2	0	29.3	1	0.0T	0	0.0T
89	2	0	2	−2	1	12.8	1	0.0T	−1	0.0T
90	2	0	2	2	2	4.7	−1	0.0T	0	0.0T
91	1	0	0	2	1	9.6	−1	0.0T	0	0.0T
92	0	0	4	−2	2	12.7	1	0.0T	0	0.0T
93	3	0	2	−2	2	8.7	1	0.0T	0	0.0T
94	1	0	2	−2	0	23.8	−1	0.0T	0	0.0T
95	0	1	2	0	1	13.1	1	0.0T	0	0.0T
96	−1	−1	0	2	1	35.0	1	0.0T	0	0.0T
97	0	0	−2	0	1	13.6	−1	0.0T	0	0.0T
98	0	0	2	−1	2	25.4	−1	0.0T	0	0.0T
99	0	1	0	2	0	14.2	−1	0.0T	0	0.0T
100	1	0	−2	−2	0	9.5	−1	0.0T	0	0.0T
101	0	−1	2	0	1	14.2	−1	0.0T	0	0.0T
102	1	1	0	−2	1	34.7	−1	0.0T	0	0.0T
103	1	0	−2	2	0	32.8	−1	0.0T	0	0.0T
104	2	0	0	2	0	7.1	1	0.0T	0	0.0T
105	0	0	2	4	2	4.8	−1	0.0T	0	0.0T
106	0	1	0	1	0	27.3	1	0.0T	0	0.0T

$\varepsilon_{J2000} = 23°26'21''.448$
$\sin \varepsilon_{J2000} = 0.397\,777\,16$

3. The Working Group

At IAU Symposium No. 78 on 'Nutation and the Earth's Rotation' held in Kiev in May 1977 the following resolution was adopted without objection:

IAU Symposium No. 78 requests that the President of IAU Commission 4 sets up a small working group of experts to prepare a fully-documented proposal for the adoption of a new series for nutation at the IAU General Assembly in 1979 and recommends that the group shall take into account the desirability of basing this proposal on resolution No. 2 of this symposium.

Resolution No. 2 was as follows:

IAU Symposium No. 78 recommends that the following set of coefficients be substituted for the corresponding coefficients in Woolard's series for the nutation in order to provide a more accurate representation of the forced nutation of the axis of rotation of the Earth due to the lunisolar perturbing forces and that this amended series be referred to as the 'IAU (1977) nutation series':

Period	in $\Delta\varepsilon$	in $\Delta\psi \sin \varepsilon$
18.6 yr	$+9''.206$	$-6''.843$
9 yr	-0.091	$+0.083$
1 yr	$+0.006$	$+0.058$
0.5 yr	$+0.569$	-0.520
122 days	$+0.022$	-0.020
27 days	0.000	$+0.028$
13.7 days	$+0.091$	-0.083

Subsequently, the President of Commission 4, Dr V. K. Abalakin, established the Working Group on Nutation and its final membership comprise the authors of this report.

In proposing a theory of nutation to the Montreal IAU General Assembly in 1979, the Working Group sought to present a numerical expression that represented nutation with an accuracy better than existing astronomical observations. Two series were considered; both based upon Kinoshita's rigid-body calculations but differing slightly in how the effects of the Earth's deformability were accommodated. The first series was prepared by Kinoshita and co-workers and used the well-known results given by Molodensky in 1961. The other series, due to Wahr, had become available to the Working Group only a few months before the IAU meeting; it had not yet been published and, hence, was not widely known. Furthermore, it was clear that for any known astronomical application, the differences between the series were not currently detectable. Thus, both series satisfied the Working Group's requirements for a theory of nutation for astronomical observations.

The General Assembly, at the recommendation of the Working Group, adopted the 1979 IAU Theory of Nutation, which was based on the rigid-body theory of Kinoshita and the deformable theory for Earth model No. 2 by Molodensky. At that time it was emphasized that this action was the adoption of a set of nutation coefficients and not the endorsement of a particular Earth model.

In December 1979 the International Union of Geodesy and Geophysics (IUGG) adopted a resolution requesting that the IAU reconsider its choice of a nutation series. The IUGG objection to the IAU resolution was not based on a criticism of the numerical values of the coefficients of the nutation theory, but rather on their interpretation that the IAU had implicitly endorsed the Molodensky Earth model 2, which was no longer judged adequate on geophysical grounds. Wahr's results, on the other hand, were obtained using a representative model available in 1979.

The IAU has avoided this misunderstanding by accepting the IUGG suggestion. The IUGG believes that the model on which Wahr's computations are based (the model 1066A of Gilbert and Dziewonski) is the best Earth model presently available and that observed geophysical constraints are such that any modern Earth model would have to be very similar. This implies that the observational residuals that result from the use of the Wahr nutation series will be more meaningful, geophysically.

On the other hand, since the IUGG objections were predicated on the question of the choice of Earth model, changing to the Wahr series implies at least an indirect endorsement by the IAU of the IUGG's preferred Earth model. This is not necessarily bad, especially in view of the belief that future models will not differ appreciably so far as nutation is concerned, but it must be clear that the nutation series will not be changed in the near future in response to shifts in the Earth model preferred by the IUGG.

After considerable correspondence and a discussion at IAU Colloquium 56 in September 1980 in Warsaw, Poland, the process of changing to the 1980 IAU Theory of Nutation was initiated. Clearly the situation has changed since the General Assembly in Montreal and it is advisable to make any adjustments before the 1979 theory has been introduced into wide spread use. Therefore, the IAU Commissions involved were requested to take a vote by mail to adopt the 1980 Theory of Nutation.

4. Nutation and Polar Motion Concepts

The variations of an Earth-fixed coordinate system with respect to a space-fixed coordinate system are primarily due to the torques produced by the gravitational attraction of the Moon, and to a lesser extent that of the Sun and planets, on the equatorial bulge of the Earth, and to deformations of the Earth. Poorly understood mechanisms, such as ocean tides, are not included in the model used here. The long-period motion of the Earth's rotation axis with respect to the axis through the ecliptic pole caused by lunisolar torques is called *lunisolar precession.* The planets affect the orientation of the mean orbital plane of the Earth, causing a slow rotation of the ecliptic about a slowly moving axis of rotation, and this is called *planetary precession.* This results in a motion of the equinox and, at present, a decrease in the obliquity of the ecliptic. The combination of lunisolar precession and planetary precession is referred to as *general precession.*

The short period motion of the Earth's rotation axis with respect to the space-fixed coordinate system is referred to as *nutation.* It includes the forced nutations that account for the motion due to all external torques as well as any free nutations which may be excited by internal processes which can only be determined from observation. Precession and nutation change the observed celestial coordinates (right ascension and declination).

Polar motion is movement of the rotation axis with respect to the crust of the Earth. This motion causes changes in the astronomical coordinates of observatories (longitude and latitude). Also, tidal motions and crustal displacements introduce variations of the astronomical coordinates of observatories.

Although the altitude of a celestial body at transit is affected by both nutation and polar motion, the two effects are observationally separable, except for the components with periods shorter than a few days. As observed from a given location on the Earth's surface, nutation affects the location of the celestial pole with respect to the stars while polar motion affects the direction of the celestial pole with respect to a local reference direction (for example the zenith or a radio interferometer baseline). As viewed from the observatory, the change in nutation will result in a variation of the radius of a diurnal quasi-circular path traced out in the sky, while the change in polar motion will result in a variation in the zenith distance of the celestial pole. Observations of the object at various hour angles serve to determine both the radius of the object's diurnal quasi-circular path and the zenith distance of the celestial pole.

5. Definitions of Terms

One of the principal barriers to the effective communication of ideas about nutation and polar motion is the lack of widely-accepted definitions of the terms involved, especially in discussions of axes and poles. In this report the following definitions and conventions will be used.

An *axis* is a straight line which is parallel to some associated vector and which passes through the center of mass of the Earth. The point at which an axis intersects the surface of the Earth or the celestial sphere is a *pole*; in the former case a *terrestrial pole*, in the latter case a *celestial pole*. If not explicitly specified, terrestrial pole will be understood. It is assumed that the celestial sphere serves to define a space-fixed coordinate system.

The rigid-body rotational velocity, ω, describes the motion of the particle with instantaneous position $\mathbf{P}(t)$ by

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) = \omega \times \mathbf{P}(t).$$

Following Munk and MacDonald (1960) and Jeffreys (1970, p. 276), for a non-rigid body we can determine an instantaneous angular velocity ω which minimizes

$$\int_V \left| \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) - \omega \times \mathbf{P}(t) \right|^2 \mathrm{d}v$$

where V represents the volume of the Earth or, more generally, that portion of the Earth whose instantaneous angular velocity we wish to know.

The *instantaneous rotation axis* $\mathbf{I}$ is a line parallel to the instantaneous angular velocity ω of the Earth and which passes through its center of mass. The intersection of $\mathbf{I}$ with the surface of the Earth is the *instantaneous terrestrial rotation pole* $\mathbf{PI}_T$ (or simply the *instantaneous rotation pole* $\mathbf{PI}$). The intersection of $\mathbf{I}$ with the celestial sphere is the *instantaneous celestial rotation pole* $\mathbf{PI}_C$.

The *instantaneous angular momentum axis* $\mathbf{H}$ is a line parallel to the instantaneous angular momentum vector of the Earth and which passes through the center of mass of the Earth. The intersection of $\mathbf{H}$ with the surface of the Earth is the *instantaneous terrestrial angular momentum pole* $\mathbf{PH}_T$ (or simply the *instantaneous angular momentum pole* $\mathbf{PH}$). The intersection of $\mathbf{H}$ with the celestial sphere is the *instantaneous celestial angular momentum pole* $\mathbf{PH}_C$.

The *principal axes* of a body are the orthogonal directions which define a cartesian coordinate system in which the moment of inertia tensor is a diagonal matrix, i.e. the principal axes are parallel to the eigenvectors of the inertia tensor. For a rigid ellipsoid of revolution the principal moments of inertia are equal about two of the principal axes and larger about the third. This latter axis is the *axis of figure*. The Earth imperfectly resembles an ellipsoid of revolution, and, in addition, is elastic. Thus, its inertia tensor, and the directions of the principal axes, are functions of time. Nevertheless, the *instantaneous axis of figure* **F** can be defined as the line passing through the center of mass of the Earth which is parallel to the primary eigenvector of the instantaneous inertia tensor of the Earth. The intersection of **F** with the surface of the Earth is the *instantaneous pole of figure* $\mathbf{PF}_T$ (or **PF**). The intersection of **F** with the celestial sphere will be denoted $\mathbf{PF}_C$.

The instantaneous axis of figure of a deformable Earth is subject to substantial motions due to distortions of the Earth, such as those caused by the body tides. This effect makes it difficult to interpret the motions of the axis **F**, above, in terms of intuitive, rigid-body concepts. It then becomes useful to define the *mean surface geographic axis*, **B** as an axis attached in a least-square sense to the Earth's outer surface. Consider a network of observatories on a rigid Earth and an axis fixed with respect to the positions of the observatories. The only possible motion of the observatory network is a, possibly time-dependent, rigid rotation; that portion of the rigid rotation which is not parallel to the axis will cause motion of the axis as the latter follows motions of the observatories defining it. On a deformable planet we must allow for the possibility of other motions of the observatories and we generalize the definition of the axis so that it is defined in a least-square sense by the position of the observatories. The axis **B** is exactly this axis in the limit of an infinite number of uniformly distributed observatories. Thus if we decompose the motion of the Earth's surface into a mean rigid rotation plus a residual deformation, **B** moves with the sense prescribed by the mean rotation. The axis **B** does not respond to body tides. The intersection of **B** with the surface of the Earth is denoted $\mathbf{PB}_T$, and the intersection with the celestial sphere is denoted $\mathbf{PB}_C$. Note that for a rigid Earth **B** coincides with **F** and **PB** coincides with **PF** at all times. There is no *non-tidal* periodic motion of observatories on the surface of the Earth with respect to $\mathbf{PB}_T$. There may be motion of observatories with respect to $\mathbf{PB}_T$ caused by crustal motions which have not yet been adequately modelled and, of course, by tidal forces.

The Conventional International Origin, the CIO, is defined by the adopted latitudes of the five International Latitude Service (ILS) observatories contributing to the International Polar Motion Service (IPMS) (Markowitz and Guinot, 1968). This definition was adopted to assure computational uniqueness and certainty in using results. It was not meant to define a fixed point on the Earth's surface or the position of the mean ILS pole for a specific epoch. It is not known whether the CIO is fixed with respect to $\mathbf{PB}_T$, but it is assumed that the CIO roughly coincides with the mean pole over the period from 1900 to 1905.

The terms *nutation* and *polar motion* have been used quite loosely. Having accounted for precessional motion, nutation has commonly referred to the motion of **PI** in a space-fixed reference frame, while polar motion has referred to the motion of **PI** with respect to **PB**, as viewed from an Earth-fixed frame, but this usage is far from universal. In particular, nutation is frequently used to refer, in a general sense, to the relative motions of any or all of the axes. In this report reference will be made to the motions of specific axes (as defined above) as viewed from specific reference frames. The orientations of all of the axes are, in general, constantly changing. If we wish to compute the motion of one of these axes we must find the complete solution to the differential equations, i.e. the forced and free solutions. The forced solution is the solution of the Earth's equations of rotational motion that accounts for all external forces (specifically the gravitational forces due to the Sun, Moon, and planets). The free solution is that solution which results from setting the external forces to zero (the particular solution to the differential equations). For a rigid Earth this results in the Eulerian free motion, a component of polar motion. For a non-rigid Earth the free solution describes the Chandler component of polar motion. Some authors, however, include in the free solution any motion of any of the axes which is not due to external gravitational forces. Thus, some authors include the forced annual polar motion in the free solution along with motions resulting from geophysical strains and meteorological effects.

For an Earth with a fluid core there exists a second free solution, the so-called *nearly-diurnal free polar motion* (also called the *nearly-diurnal free wobble* or the *free core nutation*), which has not yet been clearly observed. Like the polar motion it must be added to the calculable astronomical nutations in transforming from space-fixed to body-fixed coordinates (Yatskiv, 1980).

Sometimes the terms *predictable* and *unpredictable* are used to refer to different types of these motions. In this sense predictable nutations are motions determined using a specific Earth model and known external torques, i.e. the *forced* solution, or *astronomical nutation*. The unpredictable nutations are those determined *ex post facto* from observations and caused by presently unmodelled mechanisms related to the internal dynamics of the Earth and its atmosphere, i.e. the free solution or polar motion. However, these terms are not rigorously distinct. In particular, the annual forced polar motion can be approximately predicted based on meteorological data, and the existence (but not the amplitude or phase) of a nearly diurnal free polar motion has been predicted theoretically.

6. Current Nutational Coefficients

The rotation of the Earth about its center of mass is described by combining a dynamical theory of its motion with numerical values for parameters needed by that theory. These parameters may be determined directly from observations or from combinations of other well-known astronomical constants. Among the latter are the ratio of the masses of the Moon and the Earth, and the Earth's dynamical ellipticity, $(C - A)/C$, where C and A are the moments of inertia about the polar axis and any equatorial axis. For a rigid-Earth theory of nutation, by both definition and tradition, the amplitude of the principal term in obliquity is called the *constant of nutation*. The constant of nutation serves as a scaling factor for the other terms in the nutation theory. For a non-rigid Earth theory of nutation a single constant of nutation is not adequate and the meaning of the corresponding term is not the same. In 1952 the IAU adopted the rigid-Earth theory of Woolard (1953) along with the coefficients listed in his classical work. These coefficients are related by the rigid-Earth theory to a value for the coefficient of the principal term in obliquity. The value used by Woolard was based on observational results and not on the theoretical value determined from other astronomical constants. The discrepancy between the theoretical and observational values of this coefficient inspired investigations of the theory of nutation for a non-rigid Earth (Jeffreys and Vicente, 1957a, b; Molodensky, 1961; Pederson, 1967; Kakuta, 1970; Shen and Mansinha, 1976; Sasao *et al.*, 1979; Wahr, 1979). Another source of the nutation coefficients is the study of solid Earth tides (Melchior, 1978).

Recent astronomical observations indicate that the Woolard rigid-Earth theory of nutation is no longer adequate for precise investigations. The available choices of action appear to be the following:

(1) The present theory and constant of nutation could be retained. This has the advantage of maintaining continuity with the past. It has the disadvantage that the observational data indicate that significant corrections are required. In practice, the determinations of UT1 using optical observations of stars, Doppler or laser range tracking of satellites, laser ranges to the Moon and radio interferometric measurements are sufficiently accurate that their usefulness can be degraded with the present theory of nutation. If this first option were followed, it might result in degradation of other efforts as well. Also it is certain that numerous different nutation coefficients would be used in critical applications precisely because of the absence of a suitable standard theory. In view of the fact that corrections to the constant of precession and other astronomical constants were adopted by the IAU in 1976 for use after 1983, corrections to the nutation theory should be introduced at the same time. Retention of the current theory and constant of nutation would be ill-advised.

(2) A complete new theory of nutation based on a non-rigid Earth model could be adopted. Such a theory should be based on the newly-adopted IAU constants for the Earth, should be compatible with the observational data to the accuracy of the observations, should be as internally theoretically consistent as possible, and if feasible should be in good agreement with geophysical observations.

(3) An empirically-obtained set of nutation coefficients could be adopted which appear to be reasonably consistent with astronomical observational data. This approach, in fact, is already being used in some cases where greater accuracy is required.

At a time when we are adopting new astronomical constants, including a new precession constant, a new dynamical time scale, and new methods for reduction to apparent place, it would be a mistake not to correct the theory and constant of nutation which are known to be at variance with the observational data. It should be recognized that while a single constant of nutation is sufficient for a rigid-Earth theory of nutation, it is not satisfactory for a non-rigid Earth model. The adoption of a theory and coefficients of nutation based on the non-rigid Earth model, and satisfying the observational data, is desirable. Therefore, the goal of this report is the adoption of a set of nutation coefficients to provide a working standard for determination of UT1 and polar motion, the reduction of optical observations of stars, Doppler or laser range tracking of satellites, laser ranges to the Moon, radio interferometric measurements, and other high-precision requirements.

7. Background on Reference Pole

Oppolzer (1880) defined nutation to be the motion of the instantaneous axis of rotation and referred to the additional displacement of what he called the axis of figure from the axis of rotation as the 'diurnal variation of latitude'. Thus, the observable phenomenon was arbitrarily divided into two parts which were represented in different ways. Morgan (1952) was able to detect Oppolzer's diurnal variation of latitude in photographic zenith tube observations. Woolard's (1953) nutation theory (which was adopted by the IAU for introduction in 1960) is based on a rigid Earth model and is given in terms of the instantaneous axis of rotation. He gives expressions for polar motion based on the lunisolar component and the theoretical Eulerian motion for his rigid model.

Jeffreys (1963) and Atkinson (1973, 1975) have questioned the choice of the axis of rotation and presented arguments for choosing a different reference pole. Based on scientific discussions of this matter, the Joint Report of the Working Groups of IAU Commission 4 on Precession, Planetary Ephemerides, Units and Time-scales, adopted in 1976 by the IAU General Assembly in Grenoble, included as Recommendation 4(b):

"it is recommended that the tabular nutation shall include the forced periodic terms listed by Woolard for the axis of figure in place of those given by the instantaneous axis of rotation, and the two calibrations performed by him shall be revised accordingly, taking account of the change in the adopted precession."

The Notes on the Recommendation were as follows:

"Nutation is taken into account in the current procedure for computing true places by a reduction from the mean celestial pole of date to a celestial pole which approximates to the direction of the instantaneous axis of rotation of the Earth. It has, however, been demonstrated that observations give the place of a pole whose position with respect to the mean pole of date can be obtained by the procedure described in Recommendation 4(b). This pole may continue to be called the true celestial pole of date. The prescribed procedure may be achieved by removing the seven small forced periodic-terms in Woolard's Equation 55 (*Astron. Papers American Ephemeris* **15**, 133, 1953), by substituting the corresponding terms in Equations 54 (p. 132), and by scaling."

At Symposium No. 78 on 'Nutation and the Earth's Rotation' in Kiev in May 1977, the following resolution was adopted:

IAU Symposium No. 78 recommends that the decision of the 16th General Assembly of the IAU that 'the tabular nutation shall include the forced periodic terms listed by Woolard for the axis of figure...' shall be annulled and that the nutation of true pole of date with respect to the mean pole of date should be computed for the motion of the instantaneous axis of rotation of the mantle.

Since then some of the supporters of this resolution have indicated that they have reconsidered their opinion. Therefore, the Grenoble resolution is considered to represent the preferred opinion.

8. On the Choice of Pole

Accepting the Earth in all its complexity and the various motions that are present, let us consider the situation in light of what is actually observable.

8.1. Determinations of Astronomical Latitude and Longitude

For determining the astronomical latitude, φ, declinations, δ, of stars are assumed and the motion of a plumb line direction, or zenith of the observatory, with respect to these star's positions is observed. We have the relation

$$\varphi(\tau) = z(\tau) + \delta(\tau)$$

where $z(\tau)$ is the zenith distance of a star at the moment of observation τ, and $\varphi(\tau)$ is the instantaneous latitude of the observatory. $\varphi(\tau)$ contains the effects of precession and nutation of the crust of the Earth, plus any geophysical effects, both local and global. If we want to combine the observations of $\varphi(\tau)$ in some way to represent the orientation of the crust in space with respect to a space-fixed reference system, the following actions should be taken:

(a) theoretically, a precession/nutation model should be constructed to be applied to the declinations;

(b) theoretically or observationally, a polar motion model should be constructed to be applied to the latitudes;

(c) theoretically or observationally, a plumb line variation model should be constructed to be applied to the zenith distances.

After these corrections, $\varphi(\tau)$ should be constant. Due to the errors of the models and the unmodelled local effects, this is not the case. Thus, to rigorously and accurately reduce observations they must be corrected for both astronomical nutation and polar motion.

For determining the astronomical longitude, λ, the right ascension, α, of stars and Greenwich sidereal time, T, are assumed in a space fixed coordinate system. From the observation of a star transit one can find

$$\lambda(\tau) = \alpha - h - T + \Delta\lambda p - \Delta\lambda g$$

where h is an hour angle measured in a body fixed system, $\Delta\lambda p$ and $\Delta\lambda g$ are corrections to the astronomical longitude of the site and the reference meridian which accounts for α and T being determined in a space-fixed coordinate system.

This $\lambda(\tau)$ contains the geophysical effects and depends on the choice of the reference system. If the instantaneous rotational axis is chosen, $\lambda(\tau)$ will exhibit the dynamical variation in longitude, which is nearly diurnal.

8.2. Absolute observation of declination and fundamental latitude

Atkinson (1975) has pointed out that observations of an individual star made above and below the pole do not yield the place on the celestial sphere of the instantaneous pole of rotation, but do yield (for a rigid Earth) the mean position, over the period between culminations, of a pole which is displaced from the pole of figure by the Eulerian motion.

By considering various types of observations, made at all hour angles, Kinoshita *et al.* (1979) concluded that a *forced nutation of the pole of figure* should be given in ephemerides. If nutations of celestial bodies are observed with respect to a reference frame in which stars are moved with precession and nutation, a pole of figure coincides with a pole of rotation in absence of free nutation. Wahr (1979) has also considered the various choices for reference axes and has come to essentially the same conclusion as Atkinson and Kinoshita. Wahr's axis B, which his nutation series describes, is essentially the axis of figure for the Tisserand mean outer surface of the Earth, as discussed earlier.

8.3. Choices

We must recognize that a complete theory for both astronomical nutation and polar motion cannot currently be specified, although such a theory would certainly be desirable. For a non-rigid Earth, we can construct theories of nutation for the motion of the angular momentum axis (H), instantaneous axis of rotation (I), instantaneous axis of figure (F), or the mean surface geographic figure axis (B). Any of these could be chosen to define the reference pole. (We should distinguish between our concept of the reference pole and our ability to calculate the position of the pole. The difference between the concept and realization will be due to the limitations of the theory. Thus, our concept of the pole will remain the same, but our ability to calculate the position of the pole will improve with increased knowledge of the Earth's behavior.)

Which axis is most meaningful for the observer who requires knowledge of the orientation of the Earth in our space-fixed coordinate system? In reality no axis or pole is 'observed'. Only the motion of the observer's reference direction can be observed. This is related to a motion of the pole only by some model.

The orientation of the Earth would ideally be described by the orientation of some 'body-fixed axis'. However, for a non-rigid Earth a body-fixed axis must be carefully defined.

To rigorously predict the orientation of a reference line it is necessary to know both astronomical nutation and polar motion. This can be done either by knowledge of the nutation for the instantaneous axis of rotation (I), mean figure axis (B), instantaneous axis of figure (F), or angular momentum axis (H) – just as long as the polar motion is defined accordingly and possible apparent local geophysical motion is taken into account.

In a sense, then, the choice of the pole for the nutation series is arbitrary, although some choices will clearly be more convenient than others.

8.4. Conclusion

Therefore, it would be best to refer the nutation theory to the pole that has no nearly-diurnal motion with respect to a space-fixed coordinate system or an Earth-fixed coordinate system. We therefore propose the adoption, as the reference pole for astronomical nutation, the axis of figure for the mean surface of a model Earth in which the free motion has zero amplitude. This is the axis B defined in Section 5. To clarify the change from previous practice and to avoid confusion we suggest that this reference pole be known as the *Celestial Ephemeris Pole* (CEP). The CEP is the center of the quasi circular diurnal paths of the stars in the sky.

One of the conclusions of this report is that the adopted nutation theory include the effect of dynamical variation of latitude. It is sometimes stated that including the dynamical variation of latitude in the nutation theory amounts to referring the nutation theory to the 'body-fixed axis', or the axis of figure, but such terminology is misleading. In fact, only the predictable, *forced* components of the motion of the mean axis of figure with respect to the space-fixed coordinate system due to luni-solar forces are included. The pole to which the proposed theory of nutation refers differs from the instantaneous celestial rotation pole described by the current IAU nutation series in that a more complete model of the motions of the mantle of the Earth is included in the new theory, so that separate corrections for the 'dynamical variation of latitude', or 'diurnal polar motion' are not necessary. As stated in the notes with the Grenoble resolution for 1976, the Celestial Ephemeris Pole may be called the true celestial pole of date.

9. Nutation Theory

Recognizing the need for an improved nutation theory, an outline of the theory and the numerical constants involved are given in this section. The theoretical basis for the nutation theory, in terms of the Celestial Ephemeris Pole, for the mean shape of the outer surface of the elastic mantle to that for a rigid Earth is also given.

9.1. Theory for a Rigid Earth

Define

A, B, C = principal moment of inertia of the Earth,
$H = (2C - A - B)/2C$ = dynamical ellipticity of the Earth,
$l_{☾}$ = mean anomaly of the Moon,
$l_{\odot}$ = mean anomaly of the Sun,
$L_{☾}$ = mean longitude of the Moon,
$L_{\odot}$ = mean longitude of the Sun,
$\Omega_{☾}$ = mean longitude of the Moon's node,
$F = L_{☾} - \Omega_{☾}$,
$D = L_{☾} - L_{\odot}$,
$\nu = (i_1, i_2, i_3, i_4, i_5)$,
$\Theta_\nu = i_1 l_{☾} + i_2 l_{\odot} + i_3 F + i_4 D + i_5 \Omega_{☾}$,
$N_\nu = (\mathrm{d}/\mathrm{d}t)\Theta_\nu$,
ε = mean obliquity of the ecliptic of date,
$M_{☾}, M_{\oplus}, M_{\odot}$ = masses of the Moon, the Earth, and the Sun,
$\mu = M_{☾}/M_{\oplus}$,
$n_{☾}$ = sidereal mean motion of the Moon,
$n_{\odot}$ = sidereal mean motion of the Sun,
F_2 = factor for the mean distance of the Moon,
ω = Earth's angular speed,
$N_g = (C/A)\omega$,
N_f = nutation constant for axis of figure of a rigid Earth,
N_A = nutation constant for axis of angular momentum.

Nutations in longitude and obliquity of a rigid Earth have the following forms (Kinoshita, 1977):

$$\Delta\psi = k\sum_\nu \left[\frac{E_\nu}{N_\nu} - \frac{1}{\sin\varepsilon}\left(\frac{C_\nu(+)}{N_g - N_\nu} - \frac{C_\nu(-)}{N_g + N_\nu}\right)\right]\sin\Theta_\nu$$

$$\Delta\varepsilon = k\sum_\nu \left[\frac{i_5 B_\nu}{\sin\varepsilon} - \left(\frac{C_\nu(+)}{N_g - N_\nu} + \frac{C_\nu(-)}{N_g + N_\nu}\right)\right]\cos\Theta_\nu \tag{1}$$

whose common factors, $k_{☾}$ and $k_{\odot}$ are expressed as

$$k_{☾} = 3H\frac{M_{☾}}{M_{☾} + M_{\oplus}}\frac{1}{F_2^3}\frac{n_{☾}^2}{\omega}$$

$$k_{\odot} = 3H\frac{M_{\odot}}{M_{\odot} + M_{☾} + M_{\oplus}}\frac{n_{\odot}^2}{\omega}. \tag{2}$$

The quantities $E_\nu, C_\nu(+), C_\nu(-)$, and B_ν depend on the mean obliquity and the orbital elements of the Moon and the Sun, and their explicit expressions and numerical values are obtained from Brown's theory of the Moon as improved by Eckert *et al.* (1966) and Newcomb's theory of the Sun. The second terms in (1) are the so-called Oppolzer terms.

In order to get nutational coefficients, we have to know the values of common factors in (1), $k_{☾}$ and $k_{\odot}$ which are functions of the dynamical ellipticity and the ratios of the masses of the Moon, the Earth, and the Sun. Because of the large uncertainty of $\mu = M_{☾}/M_{\oplus}$ at the time, Woolard (1953) adopted 9″21 as the coefficient of $\cos \Omega_{☾}$, which was obtained by Newcomb (1895). Using 9″21, Woolard obtained $k_{☾}$, and then derived $k_{\odot}$ with 5037″08 per tropical century at 1900 of the lunisolar precession in longitude. The value 9″21 determined from observations includes an effect due to the non-rigidity of the Earth.

The lunisolar precession in longitude is of the form (Kinoshita, 1975 and 1977):

$$f_{2000} = 3H\left\{\frac{\mu}{1+\mu}\frac{1}{F_2^3}\frac{n_{☾}^2}{\omega}\left[\left(M_0 - \frac{M_2}{2}\right)\cos\varepsilon + M_1\frac{\cos 2\varepsilon}{\sin\varepsilon} + \right.\right.$$
$$\left. + M_3\frac{\mu}{1+\mu}\frac{n_{☾}^2}{\omega\Omega_{☾}}H(6\cos^2\varepsilon - 1)\right] +$$
$$\left. + \frac{M_{\odot}}{M_{\odot}+M_{☾}+M_{\oplus}}\frac{n_{\odot}^2}{\omega}S_0\cos\varepsilon\right\}_{2000} - P_g \tag{3}$$

with

$$\begin{aligned} M_0 &= 496\,303.3 \times 10^{-6} \\ M_1 &= -20.7, \\ M_2 &= 0.1, \\ M_3 &= 3020.2, \\ S_0 &= 500\,209.1, \end{aligned}$$

which are values at J2000.0 and depend only on the orbital elements of the Moon and the Sun. The lunisolar precession does not depend on the non-rigidity of the Earth. The terms having M_1, M_2, and M_3 as factors are not included in Newcomb's (1906) precessional theory; M_1 and M_2 come from the long-periodic terms in the motion of the Moon, and M_3 arises from the second-order secular perturbation. P_g is geodesic precession. As seen from (3), if we know the ratio, μ, of the masses of the Moon and the Earth, we can determine the dynamical ellipticity, H, and then $k_{☾}$ and $k_{\odot}$. The ratio μ is well-determined from recent data obtained by lunar and planetary spacecraft. The General Assembly of the IAU at Grenoble in 1976 adopted the value 0.012 300 02 for μ as one of the primary constants. To obtain the dynamical ellipticity, we adopt the following values from the IAU (1976) System of Astronomical Constants:

$$\begin{aligned} \varepsilon_{2000} &= 23°26'21''.448, \\ f_{2000} &= 5038''.7784 \text{ per Julian century}, \\ P_g &= 1''.92 \text{ per Julian century}. \end{aligned}$$

These values were derived by Lieske *et al.* (1977), applying Fricke's (1971) correction to luni-solar precession of $\Delta\varphi = 1''.1$ per century at 1900.0, and the correction to planetary precession at 1900.0, $\Delta\chi = -0''.029$ per century. Using these values, we have from (3) and (2)

$$H = 0.003\,273\,993\,5 = 1/305.437\,38,$$
$$k_{☾} = 7567.8292 \text{ per Julian century},$$
$$k_{\odot} = 3475.4416 \text{ per Julian century},$$

$$N_{f,2000} = N_{A,2000} + \text{Oppolzer Terms} = 9''.228\,78 - 0''.001\,01 = 9''.227\,77.$$

The effects of these values of the inaccuracies in f_{2000} and μ are

$$\Delta H = 6.5 \times 10^{-7}\,\Delta f_{2000} - 1.8 \times 10^{-1}\,\Delta\mu,$$
$$\Delta k_{☾} = 1.5\,\Delta f_{2000} + 2.0 \times 10^{5}\,\Delta\mu,$$
$$\Delta k_{\odot} = 0.7\,\Delta f_{2000} - 1.9 \times 10^{5}\,\Delta\mu,$$

$$\Delta N_{f,2000} = 1.8 \times 10^{-3}\,\Delta f + 2.4 \times 10^{2}\,\Delta\mu.$$

The order of the accuracy we now have is about $0''.15$ per century for f_{2000} and 10^{-8} for μ. The accuracy of f_{2000} has more of an effect on the determination of these values than does that of μ. However, the present accuracies of f_{2000} and μ are sufficient to determine nutational coefficients for a rigid Earth within $0''.0001$ other than the term with argument $\Omega_{☾}$.

Numerical values foı quantities used are listed below, where the fundamental epoch is J2000.0 (JD 2451 545.0 Barycentric Dynamical Time).

$\mu = 0.012\,300\,02$ (from IAU (1976) System of Astronomical Constants.

$M_{\odot}/(M_{☾} + M_{\oplus}) = 328\,900.5$ (from IAU (1976) System of Astronomical Constants),

$\varepsilon_{2000} = 23°26'21''.448$ (from IAU (1976) System of Astronomical Constants),

$p_{2000} = 5029''.0966$ per Julian century (from IAU (1976) System of Astronomical Constants),

$P_g = 1''.92$ per Julian century (from Lieske *et al.*, 1977),

$f_{2000} = 5038''.7784$ per Julian century (from Lieske *et al.*, 1977),

$$\dot{L}_{\odot\,2000} = \dot{L}_{\odot\,1900} + 1''.089 \times 2,$$
$$= 129\,602\,770''.308 \text{ per Julian century},$$
$$\dot{L}_{☾\,2000} = \dot{L}_{☾\,1900} - 4''.08 \times 2 + 0''.0068 \times 3,$$
$$= 1\,732\,564\,371''.17 \text{ per Julian century},$$
$$n_{☾\,2000} = L_{☾\,2000} - p_{2000}$$
$$= 1\,732\,559\,342''.07 \text{ per Julian century},$$
$$n_{\odot} = L_{\odot\,2000} - p_{2000}$$
$$= 129\,597\,741''.2114 \text{ per Julian century},$$
$$\omega = 360°.985\,612\,3 \text{ day}^{-1},$$
$$F_2 = 0.999\,093\,142,$$
$$\dot{l}_{☾\,2000} = 8328.6914 \text{ radians per Julian century},$$
$$\dot{l}_{\odot,2000} = 628.3019 \text{ radians per Julian century},$$

$$\dot{D}_{2000} = 7771.3771 \text{ radians per Julian century,}$$
$$\dot{F}_{2000} = 8433.4661 \text{ radians per Julian century,}$$
$$\dot{\Omega}_{☾\,2000} = -33.7570 \text{ radians per Julian century.}$$

9.2. Notes

(1) The value newly adopted for ε_{2000} corresponds to the previously adopted value $\varepsilon_{1900} = 23°27'8''.26$ evaluated at J2000.0 using the revised rate of change, and the value for ε_{1900} was derived by using Newcomb's precessional formula (1906) and $\varepsilon_{1850} = 23°27'31''.68 \pm 0''.15$ (Newcomb, 1895), which was determined from observations of the Sun and planets. The value for ε_{2000}, therefore, is based on the value for ε_{1850}. The Equation (1) for nutation in obliquity gives a constant term, $-0''.0087$. ε_{2000} is merely an integration constant which can be determined only from observations of the Sun and planets. Considering the above situations, the value for ε_{2000} must remain unchanged, since diminishing the value for ε_{2000} by $0''.0087$ introduces an unnecessary confusion.

(2) ω is derived from

$$\omega = 360\left(1 + \frac{\dot{L}_\odot}{360} - \frac{m\dot{L}_\odot}{360^2}\right)$$

where m is the general precession in right ascension in degree per tropical year and $\dot{L}_\odot$ is expressed in units of degree per day.

9.3. Modifications to rigid-Earth Theory

For each circular motion component of nutation with frequency $N_☾ = (\mathrm{d}/\mathrm{d}t)\Theta$, Θ being the argument of nutation, a theoretical ratio, a/a_0, of nutation amplitude for the non-rigid Earth to that for a rigid Earth model is given by the following equation (Wahr, 1979)

$$\frac{a}{a_0} = 1 + \frac{N_☾}{\omega}\left(1.003\,28 - \frac{N_☾}{\omega}\right)\left(0.416 + \left(0.073 - \frac{N_☾}{\omega}\right) \times \right.$$
$$\left. \times \left(\frac{1.06}{1.003\,28 - \frac{N_☾}{\omega}} - \frac{0.810}{1.002\,48 - \frac{N_☾}{\omega}} + \frac{0.665}{0.002\,171\,4 + \frac{N_☾}{\omega}}\right)\right) \tag{1}$$

where ω is the mean rotation rate of the Earth. Amplitudes of longitude and obliquity components, $\Delta\psi$ and $\Delta\varepsilon$, of the nutation of the CEP are expressed in terms of those for the rigid Earth $(\Delta\psi)_R$ and $(\Delta\varepsilon)_R$. by the following,

$$-\Delta\psi \sin\varepsilon = \frac{1}{2}\left(-\left(\left(\frac{a}{a_0}\right)_n + \left(\frac{a}{a_0}\right)_{-n}\right)(\Delta\psi)_R \sin\varepsilon + \right.$$
$$\left. + \left(\left(\frac{a}{a_0}\right)_n - \left(\frac{a}{a_0}\right)_{-n}\right)(\Delta\varepsilon)_R.\right) \tag{2}$$

$$\Delta\varepsilon = \frac{1}{2}\left(-\left(\left(\frac{a}{a_0}\right)_n - \left(\frac{a}{a_0}\right)_{-n}\right)(\Delta\psi)_R \sin\varepsilon + \right.$$

$$\left. + \left(\left(\frac{a}{a_0}\right)_n + \left(\frac{a}{a_0}\right)_{-n}\right)(\Delta\varepsilon)_R \cdot \right) \tag{3}$$

where ε is the obliquity and $(a/a_0)_n$ is the ratio given by expression (1) above. Expression (1) represents the correction for the nutational motion of the Earth model 1066A (Gilbert and Dziewonski, 1975). This model accounts for a solid inner and liquid outer core in the Earth and a distribution of elastic parameters inferred from inversion of a large set of seismological data.

10. Nutation Coefficients

The fundamental arguments in the FK5 reference system are (Van Flandern, 1981):

$$l = 134°57'46''.733 + (1325^r + 198°52'02''.633)T + 31''.310T^2 + 0''.064T^3,$$

$$l' = 357°31'39''.804 + (99^r + 359°03'01''.224)T - 0''.577T^2 - 0''.012T^3,$$

$$F = 93°16'18''.877 + (1342^r + 82°01'03''.137)T - 13''.257T^2 + 0''.011T^3,$$

$$D = 297°51'01''.307 + (1236^r + 307°06'41''.328)T - 6''.891T^2 + 0''.019T^3,$$

$$\Omega = 125°02'40''.280 - (5^r + 134°08'10''.539)T + 7''.455T^2 + 0''.008T^3,$$

where the fundamental epoch is J2000.0 = 2000 January $1^d.5$ TDB = JD 2451 545.0 TDB, and

$1^r = 360° = 1\,296\,000''.0$,

T is measured in Julian centuries of 36 525 days of 86 400 s of dynamical time each,

l = mean longitude of the Moon minus the mean longitude of the Moon's perigee,

l' = mean longitude of the Sun minus the mean longitude of the Sun's perigee,

F = mean longitude of the Moon minus the mean longitude of the Moon's node,

D = mean elongation of the Moon from the Sun,

Ω = longitude of the mean ascending node of the lunar orbit on the ecliptic, measured from the mean equinox of date.

Note that the fundamental arguments are the best values currently available for the FK5 reference system and the 1976 IAU constants. These values may change slightly based on an improved lunar ephemeris, but the changes will not significantly affect the nutation theory. It is possible that the different expressions should be used for work which depends in a very critical way on the precise solar or lunar theories; however, nutation theory is not in this class of work. Therefore, *the above expressions, while provisional, are of sufficient accuracy for the evaluation of the nutation theory to 0''.0001.*

The new expression for the obliquity of the ecliptic is (Lieske *et al.*, 1977):

$$\varepsilon = 23°26'21''.448 - 46''.8150T - 0''.000\,59T^2 + 0''.001\,813T^3.$$

A complete list of the nutation theory coefficients, including the period of the terms and a consecutive term number, is given in Table I.

11. Differences between Theories

The differences between the astronomical nutation theory proposed in this report and Woolard's theory are twofold:

(1) The proposed astronomical nutation series is computed for the Celestial Ephemeris Pole in accordance with the Grenoble resolution rather than the instantaneous pole of rotation as has been the practice in the past. Therefore, the proposed series will represent the motion of the Celestial Ephemeris Pole with respect to the mean celestial pole of date. The 'dynamical variation of latitude' terms are included in the proposed series.

(2) The proposed series is based upon the theory of Wahr (1979, 1981) and model 1066A of Gilbert and Dziewonski (1975) to account for the elasticity and the solid inner and liquid outer core of the Earth. The agreement with solid Earth tidal observations is adequate and this theory appears to be a fairly good representation of the Earth for this purpose (for further observational evidence of the effects of non-rigidity, see also Federov, 1963; Wako, 1970; Melchior, 1971; Yokoyama, 1974; McCarthy *et al.*, 1977; Sasao *et al.*, 1977; Kinoshita *et al.*, 1979).

The complete theory of nutation by Woolard is given in Tables 24 and 25 (Woolard, 1953). The differences in the sense 'proposed series minus Woolard' are given in Table IIa of this report. These differences are referred to the ecliptic of date, epoch J2000.0 (JD 2 451 545.0 TDB), by applying the secular terms given by Woolard to the nutation coefficients listed in Tables 24 and 25.

An abbreviated version of Woolard's theory containing 69 terms with magnitude greater than or equal to $0''.0002$ is given in Table 26 (Woolard, 1953) and in the Explanatory Supplement, pages 44 and 45. Since this shorter table is commonly used and is the basis for the nutation data in the Astronomical Almanac the differences in the sense "proposed theory minus abbreviated Woolard theory" are given in Table IIb. Again, these differences refer to epoch J2000.0. Since terms smaller than $0''.0002$ were neglected previously, differences of $0''.0001$ might be neglected.

12. Polar Motion

If an accuracy of $\pm 0''.5$ is adequate for a description of an observer's reference direction in a space-fixed coordinate system, the use of the proposed nutation series along with precession is sufficient. However, if greater accuracy is required, the effects of polar motion must be considered. This phenomenon is the motion of the Celestial Ephemeris Pole with respect to some adopted terrestrial reference pole such as the Conventional International Origin. This orientation may be described by the coordinates of the Celestial Ephemeris Pole with respect to the terrestrial reference axes, x and y. These

TABLE IIA

Proposed series minus complete Woolard series (Woolard's Tables 24 and 25)

Epoch J2000.0 (JD 2 451 545.0 TDB), T in Julian Centuries

	Argument					Period	Longitude		Obliquity	
	l	l'	F	D	Ω	(days)	(0.0001″)		(0.0001″)	
1	0	0	0	0	1	6798.4	504.6	− 0.47T	− 84.1	− 0.2T
2	0	0	0	0	2	3399.2	− 26.2	0.01T	8.6	0.1T
3	− 2	0	2	0	1	1305.5	1			
4	2	0	− 2	0	0	1095.2	1			
5	− 2	0	2	0	2	1615.7			− 1	
6	1	− 1	0	− 1	0	3232.9	− 1			
7	0	− 2	2	− 2	1	6786.3	2		− 1	
8	2	0	− 2	0	2	943.2	1			
9	0	0	2	− 2	2	182.6	− 456.2	− 0.30T	216.4	− 0.2T
10	0	1	0	0	0	365.3	167.9	− 0.33T	54	− 0.1T
11	0	1	2	− 2	2	121.7	− 20.8	0.01T	8.8	
12	0	− 1	2	− 2	2	365.2	3.9	0.02T	− 2.7	
13	0	0	2	− 2	1	177.8	5.1		− 4.1	
14	2	0	0	− 2	0	205.9	3		1	
15	0	0	2	− 2	0	173.3	− 1			
16	0	2	0	0	0	182.6	0.8			
17	0	1	0	0	1	386.0	− 0.1		1	
18	0	2	2	− 2	2	91.3	− 1			
19	0	− 1	0	0	1	346.6	− 2		1	
20	− 2	0	0	2	1	199.8	− 1			
21	0	− 1	2	− 2	1	346.6				
22	2	0	0	− 2	1	212.3				
23	0	1	2	− 2	1	119.6	1			
24	1	0	0	− 1	0	411.8	− 1			
25	2	1	0	− 2	0	131.7				
26	0	0	− 2	2	1	169.0				
27	0	1	− 2	2	0	329.8				
28	0	1	0	0	2	409.2				
29	− 1	0	0	1	1	388.3				
30	0	1	2	− 2	0	117.5				
31	0	0	2	0	2	13.7	− 237.1		93.9	
32	1	0	0	0	0	27.6	36.9	0.04T	− 7	
33	0	0	2	0	1	13.6	− 43.9	− 0.05T	17	
34	1	0	2	0	2	9.1	− 40.3	− 0.03T	16.1	
35	1	0	0	− 2	0	31.8	− 9	− 0.02T	− 1	
36	− 1	0	2	0	2	27.1	9	− 0.01T	− 3	
37	0	0	0	2	0	14.8	3		− 2	
38	1	0	0	0	1	27.7	5	0.10T	− 2	
39	− 1	0	0	0	1	27.4	− 1	− 0.10T	2	
40	− 1	0	2	2	2	9.6	− 7		4	
41	1	0	2	0	1	9.1	− 7		4	
42	0	0	2	2	2	7.1	− 6		2	
43	2	0	0	0	0	13.8	1		− 1	
44	1	0	2	− 2	2	23.9	3		− 1	
45	2	0	2	0	2	6.9	− 5		2	
46	0	0	2	0	0	13.6	1		− 1	
47	− 1	0	2	0	1	27.0	2			

Table IIA (*continued*)

	Argument					Period	Longitude	Obliquity
	l	l'	F	D	Ω	(days)	(0.0001″)	(0.0001″)
48	−1	0	0	2	1	32.0	2	−1
49	1	0	0	−2	1	31.7		
50	−1	0	2	2	1	9.5	−1	
51	1	1	0	−2	0	34.8		
52	0	1	2	0	2	13.2		
53	0	−1	2	0	2	14.2	−1	
54	1	0	2	2	2	5.6	−2	
55	1	0	0	2	0	9.6		
56	2	0	2	−2	2	12.8		−1
57	0	0	0	2	1	14.8		
58	0	0	2	2	1	7.1	−2	
59	1	0	2	−2	1	23.9	1	
60	0	0	0	−2	1	14.7		
61	1	−1	0	0	0	29.8	1	
62	2	0	2	0	1	6.9	−1	1
63	0	1	0	−2	0	15.4		
64	1	0	−2	0	0	26.9		
65	0	0	0	1	0	29.5		
66	1	1	0	0	0	25.6		
67	1	0	2	0	0	9.1		
68	1	−1	2	0	2	9.4		
69	−1	−1	2	2	2	9.8	−1	
70	−2	0	0	0	1	13.7		
71	3	0	2	0	2	5.5	−1	
72	0	−1	2	2	2	7.2	−1	
73	1	1	2	0	2	8.9		
74	−1	0	2	−2	1	32.6		
75	2	0	0	0	1	13.8		
76	1	0	0	0	2	27.8		
77	3	0	0	0	0	9.2	1	
78	0	0	2	1	2	9.3	1	
79	−1	0	0	0	2	27.3		
80	1	0	0	−4	0	10.1		
81	−2	0	2	2	2	14.6		
82	−1	0	2	4	2	5.8	−1	
83	2	0	0	−4	0	15.9		
84	1	1	2	−2	2	22.5		
85	1	0	2	2	1	5.6		
86	−2	0	2	4	2	7.3		1
87	−1	0	4	0	2	9.1		
88	1	−1	0	−2	0	29.3		
89	2	0	2	−2	1	12.8		−1
90	2	0	2	2	2	4.7		
91	1	0	0	2	1	9.6		
92	0	0	4	−2	2	12.7		
93	3	0	2	−2	2	8.7		
94	1	0	2	−2	0	23.8		
95	0	1	2	0	1	13.1		
96	−1	−1	0	2	1	35.0		
97	0	0	−2	0	1	13.6		

Table IIA (*continued*)

	Argument					Period	Longitude	Obliquity
	l	l'	F	D	Ω	(days)	(0.0001″)	(0.0001″)
98	0	0	2	−1	2	25.4		
99	0	1	0	2	0	14.2		
100	1	0	−2	−2	0	9.5		
101	0	−1	2	0	1	14.2		
102	1	1	0	−2	1	34.7	−1	
103	1	0	−2	2	0	32.8		
104	2	0	0	2	0	7.1		
105	0	0	2	4	2	4.8		
106	0	1	0	1	0	27.3		
	−1	−1	2	−1	2	11113.6	1	
	0	0	2	−2	3	161.0	−1	

$\sin \varepsilon_{J2000} = 0.397\,777\,16.$

The last two terms above are listed in Woolard's Table 24, but not in this paper. Terms 8 and 102 listed above are not included in Woolard's Table 24.

TABLE IIB

Proposed series minus Woolard series (Woolard's Table 26 and Explanatory Supplement pp. 44–45)

Epoch J2000.0 (JD 2 451 545.0 TDB), T in Julian Centuries

	Argument					Period	Longitude		Obliquity	
	l	l'	F	D	Ω	(days)	(0.0001″)		(0.0001″)	
1	0	0	0	0	1	6798.4	504.7	$-0.50T$	−84.1	$-0.2T$
2	0	0	0	3	2	3399.2	−26.2		8.6	$0.1T$
3	−2	0	2	0	1	1305.5	1			
4	2	0	−2	0	0	1095.2	1			
5	−2	0	2	0	2	1615.7			−1	
6	1	−1	0	−1	0	3232.9	−1			
7	0	−2	2	−2	1	6786.3	2		−1	
8	2	0	−2	0	1	943.2	1			
9	0	0	2	−2	2	182.6	−456.7	$-0.30T$	216.9	$-0.2T$
10	0	1	0	0	0	365.3	168.1	$-0.30T$	54	$-0.1T$
11	0	1	2	−2	2	121.7	−21.2		8.6	
12	0	−1	2	−2	2	365.2	3.5		−2.3	
13	0	0	2	−2	1	177.8	4.9		−4	
14	2	0	0	−2	0	205.9	3		1	
15	0	0	2	−2	0	173.3	−1			
16	0	2	0	0	0	182.6	1.1			
17	0	1	0	0	1	386.0			1	
18	0	2	2	−2	2	91.3	−1.1			
19	0	−1	0	0	1	346.6	−2		1	
20	−2	0	0	2	1	199.8	−1			
21	0	−1	2	−2	1	346.6				
22	2	0	0	−2	1	212.3				
23	0	1	2	−2	1	119.6	1			
24	1	0	0	−1	0	411.8	−1			

Table IIB (*continued*)

	Argument					Period	Longitude		Obliquity
	l	l'	F	D	Ω	(days)	(0.0001″)		(0.0001″)
25	2	1	0	−2	0	131.7	1		
26	0	0	−2	2	1	169.0	1		
27	0	1	−2	2	0	329.8	−1		
28	0	1	0	0	2	409.2	1		
29	−1	0	0	1	1	388.3	1		
30	0	1	2	−2	0	117.5	−1		
31	0	0	2	0	2	13.7	−236.8		93.5
32	1	0	0	0	0	27.6	36.9		−7
33	0	0	2	0	1	13.6	−43.6		17
34	1	0	2	0	2	9.1	−40		16.1
35	1	0	0	−2	0	31.8	−9		−1
36	−1	0	2	0	2	27.1	9		−3
37	0	0	0	2	0	14.8	3		−2
38	1	0	0	0	1	27.7	5	$0.10T$	−2
39	−1	0	0	0	1	27.4	−1	$-0.10T$	2
40	−1	0	2	2	2	9.6	−7		4
41	1	0	2	0	1	9.1	−7		4
42	0	0	2	2	2	7.1	−6		2
43	2	0	0	0	0	13.8	1		−1
44	1	0	2	−2	2	23.9	3		−1
45	2	0	2	0	2	6.9	−5		2
46	0	0	2	0	0	13.6	1		−1
47	−1	0	2	0	1	27.0	2		
48	−1	0	0	2	1	32.0	2		−1
49	1	0	0	−2	1	31.7			
50	−1	0	2	2	1	9.5	−1		
51	1	1	0	−2	0	34.8			
52	0	1	2	0	2	13.2			
53	0	−1	2	0	2	14.2	−1		
54	1	0	2	2	2	5.6	−2		
55	1	0	0	2	0	9.6			
56	2	0	2	−2	2	12.8			−1
57	0	0	0	2	1	14.8			
58	0	0	2	2	1	7.1	−2		
59	1	0	2	−2	1	23.9	1		
60	0	0	0	−2	1	14.7			
61	1	−1	0	0	0	29.8	1		
62	2	0	2	0	1	6.9	−1		1
63	0	1	0	−2	0	15.4			
64	1	0	−2	0	0	26.9			
65	0	0	0	1	0	29.5			
66	1	1	0	0	0	25.6			
67	1	0	2	0	0	9.1			
68	1	−1	2	0	2	9.4			1
69	−1	−1	2	2	2	9.8	−1		1
70	−2	0	0	0	1	13.7			1
71	3	0	2	0	2	5.5	−1		1
72	0	−1	2	2	2	7.2	−1		1
73	1	1	2	0	2	8.9			−1
74	−1	0	2	−2	1	32.6			1

Table IIB (*continued*)

	Argument					Period	Longitude	Obliquity
	l	l'	F	D	Ω	(days)	(0.001″)	(0.0001″)
75	2	0	0	0	1	13.8		−1
76	1	0	0	0	2	27.8		1
77	3	0	0	0	0	9.2	2	
78	0	0	2	1	2	9.3	2	−1
79	−1	0	0	0	2	27.3	1	−1
80	1	0	0	−4	0	10.1	−1	
81	−2	0	2	2	2	14.6	1	−1
82	−1	0	2	4	2	5.8	−2	1
83	2	0	0	−4	0	15.9	−1	
84	1	1	2	−2	2	22.5	1	−1
85	1	0	2	2	1	5.6	−1	1
86	−2	0	2	4	2	7.3	−1	1
87	−1	0	4	0	2	9.1	1	
88	1	−1	0	−2	0	29.3	1	
89	2	0	2	−2	1	12.8	1	−1
90	2	0	2	2	2	4.7	−1	
91	1	0	0	2	1	9.6	−1	
92	0	0	4	−2	2	12.7	1	
93	3	0	2	−2	2	8.7	1	
94	1	0	2	−2	0	23.8	−1	
95	0	1	2	0	1	13.1	1	
96	−1	−1	0	2	1	35.0	1	
97	0	0	−2	0	1	13.6	−1	
98	0	0	2	−1	2	25.4	−1	
99	0	1	0	2	0	14.2	−1	
100	1	0	−2	−2	0	9.5	−1	
101	0	−1	2	0	1	14.2	−1	
102	1	1	0	−2	1	34.7	−1	
103	1	0	−2	2	0	32.8	−1	
104	2	0	0	2	0	7.1	1	
105	0	0	2	4	2	4.8	−1	
106	0	1	0	1	0	27.3	1	

$\sin \varepsilon_{J2000} = 0.397\,777\,16.$
Woolard's Table does not give coefficients of magnitude 0.0001″, so differences of that magnitude in this table might be neglected.

coordinates can only be obtained by observations, and numerical values are available from the International Polar Motion Service, the Bureau International de l'Heure and the Defense Mapping Agency polar motion service. It must also be realized that local geophysical effects may be significant at the 0″.03 level in determining the orientation of a local body-fixed reference direction.

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