If $a\%5 = \alpha$ and $b\%5 = \beta$ then $(a+b)\%5 = (\alpha+\beta)\%5$.

$3\%5 = 3$

$6\%5 = 1$

$(3+6)\%5 = 9\%5 = 4$

$(3+1)\%5 = 4\%5 = 4$

If a current string has a numerical value of x ,
=> placing a 0 on the right hand side results in a string having a value of x+x

Example: 1010 = 10
10100 = 20

=> placing a 1 results in a string having a value x+x+1

Example: 1010 = 10
10101 = 21

Possible remainder: 0, 1, 2, 3, or 4

Five possible states

Each state number indicates the remainder

Start state is 000 and final state is 000 because remainder of 0 means that number is divisible by 5

input output

true = 1 on logism

false = 0 on logism

=> state (000) Remainder = 0
-> if next input is 00, (0), 0 mod 5 = 0 — 0
-> if next input is 01, (1), 1 mod 5 =1 — 1

=> state (001) Remainder = 1
-> if next input is 10, (2), 2 mod 5 = 2 — 0
-> if next input is 11, (3), 3 mod 5 = 3 — 1

=> state (010) Remainder = 2
-> if next input is 100, (4), 4 mod 5 = 4 — 0
-> if next input is 101, (5), 5 mod 5 = 0 — 1

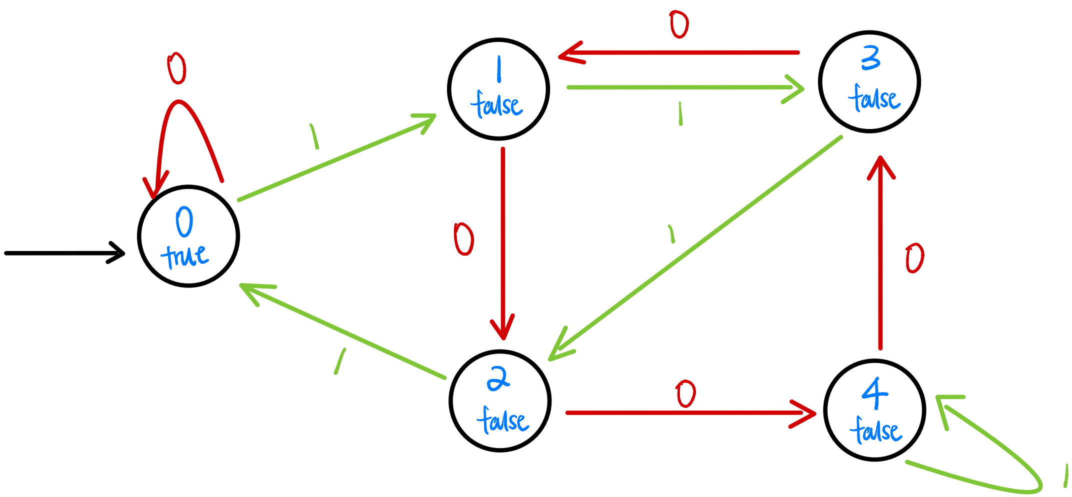
=> state (011) Remainder = 3
-> if next input is 110, (6), 6 mod 5 = 1 — 0
-> if next input is 111, (7), 7 mod 5 = 2 — 1

=> state (100) Remainder = 4
-> if next input is 1000, (8), 8 mod 5 = 3 — 0
-> if next input is 1001, (9), 9 mod 5 = 4 — 1

Symbolic state transition table

State	Input	next state	output
Remainder = 0	0	Remainder = 0	True
	1	Remainder = 1	True
Remainder = 1	0	Remainder = 2	False
	1	Remainder = 3	False
Remainder = 2	0	Remainder = 4	False
	1	Remainder = 0	False
Remainder = 3	0	Remainder = 1	False
	1	Remainder = 2	False
Remainder = 4	0	Remainder = 3	False
	1	Remainder = 4	False

Finite state machine diagram



True Table

Encoded state transition table

D flip-flop

	S_2	S_1	S_0	i	S_2'	S_1'	S_0'	output	D_2	D_1	D_0
0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	1	0	0	1	1	0	0	1
1	0	0	1	0	0	1	0	0	0	1	0
	0	0	1	1	0	1	1	0	0	1	1
2	0	1	0	0	1	0	0	0	1	0	0
	0	1	0	1	0	0	0	0	0	0	0
3	0	1	1	0	0	0	1	0	0	0	1
	0	1	1	1	0	1	0	0	0	1	0
4	1	0	0	0	0	1	1	0	0	1	1
	1	0	0	1	1	0	0	0	1	0	0

Expressions

$$\text{output} = \bar{S_2} \cdot \bar{S_1} \cdot \bar{S_0}$$

$$D_2 = S_2' = \bar{S_2} \cdot S_1 \cdot \bar{S_0} \cdot \bar{i} + S_2 \cdot \bar{S_1} \cdot \bar{S_0} \cdot i$$

$$\begin{aligned} D_1 = S_1' &= \bar{S_2} \cdot \bar{S_1} \cdot S_0 \cdot i + \bar{S_2} \cdot \bar{S_1} \cdot S_0 \cdot \bar{i} + \bar{S_2} \cdot S_1 \cdot S_0 \cdot i + S_2 \cdot \bar{S_1} \cdot \bar{S_0} \cdot \bar{i} \\ &= \bar{S_2} \cdot S_0 \cdot i (\bar{S_1} + S_1) + \bar{S_1} \cdot \bar{i} (\bar{S_2} \cdot S_0 + S_2 \cdot \bar{S_0}) \\ &= \bar{S_2} \cdot S_0 \cdot i + \bar{S_1} \cdot \bar{i} (\bar{S_2} \cdot S_0 + S_2 \cdot \bar{S_0}) \end{aligned}$$

$$\begin{aligned} D_0 = S_0' &= \bar{S_2} \cdot \bar{S_1} \cdot \bar{S_0} \cdot i + \bar{S_2} \cdot \bar{S_1} \cdot S_0 \cdot i + \bar{S_2} \cdot S_1 \cdot S_0 \cdot \bar{i} + S_2 \cdot \bar{S_1} \cdot \bar{S_0} \cdot \bar{i} \\ &= \bar{S_2} \cdot \bar{S_1} \cdot i (\bar{S_0} + S_0) + \bar{i} (\bar{S_2} \cdot S_1 \cdot S_0 + S_2 \cdot \bar{S_1} \cdot \bar{S_0}) \\ &= \bar{S_2} \cdot \bar{S_1} \cdot i + \bar{i} (\bar{S_2} \cdot S_1 \cdot S_0 + S_2 \cdot \bar{S_1} \cdot \bar{S_0}) \end{aligned}$$