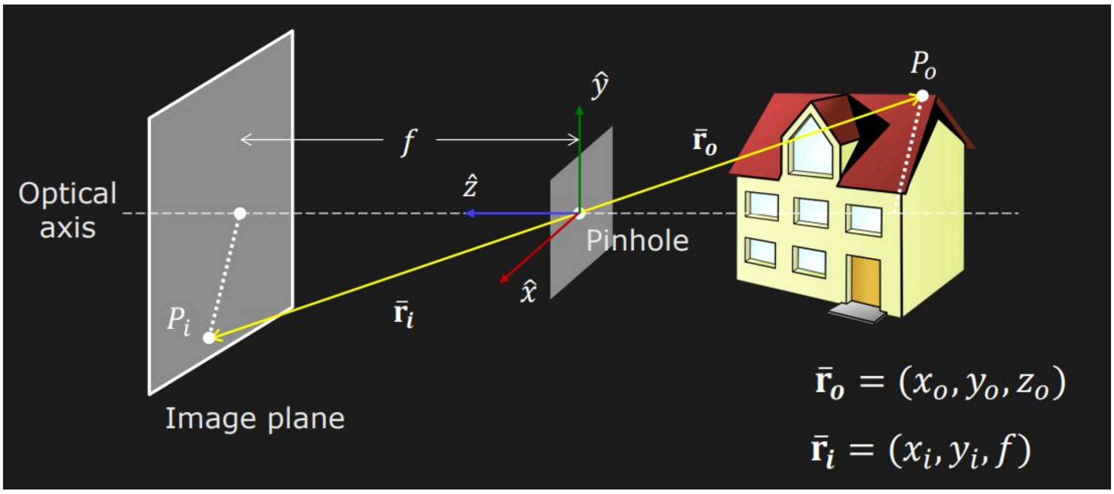
1. (a)



Let the circular disk lies at (x_0, y_0, z_0) with its radius r. The disk can be described as: $(x - x_0)^2 + (y - y_0)^2 \le r^2$, $z = z_0$. The Image plane has its z = f. Using similar triangles, we have: $\frac{x_i}{f} = \frac{x}{z_0}, \frac{y_i}{f} = \frac{y}{z_0}$. Therefore, the image on the image plane satisfies: $\left(\frac{z_0}{f}x_i - x_0\right)^2 + \left(\frac{z_0}{f}y_i - y_0\right)^2 \le r^2$, which can be written as: $\left(x_i - \frac{f}{z_0}x_0\right)^2 + \left(y_i - \frac{f}{z_0}y_0\right)^2 \le \frac{f^2}{z_0^2}r^2$. From the equation above, we can see that the shape of the image of the disk is still a circle with its center at $\left(\frac{f}{z_0}x_0, \frac{f}{z_0}y_0, f\right)$, radius= $\frac{f}{z_0}r$.

(b)

$A = B = D = 0$ and $B = 1$. The plane is y=0.

Suppose the three lines are: $\hat{l_1}(4,0,4), \hat{l_2}(4,0,3), \hat{l_3}(4,0,2)$. The vanishing point of the line $(x_{vp}, y_{vp}) = \left(f\frac{\mathrm{l_x}}{\mathrm{l_z}}, \mathrm{f}\frac{\mathrm{l_y}}{\mathrm{l_z}}\right)$. Therefore, the vanishing points are $(f, 0), \left(\frac{4}{3}f, 0\right), (2f, 0)$.

$B = C = D = 0$ and $A = 1$. The plane is x=0.

Suppose the three lines are: $\hat{l_1}(0,4,4), \hat{l_2}(0,4,3), \hat{l_3}(0,4,2)$. The vanishing point of the line $(x_{vp}, y_{vp}) = \left(f\frac{\mathrm{l_x}}{\mathrm{l_z}}, \mathrm{f}\frac{\mathrm{l_y}}{\mathrm{l_z}}\right)$. Therefore, the vanishing points are $(0,1), \left(0, \frac{4}{3}f\right), (0, 2f)$.

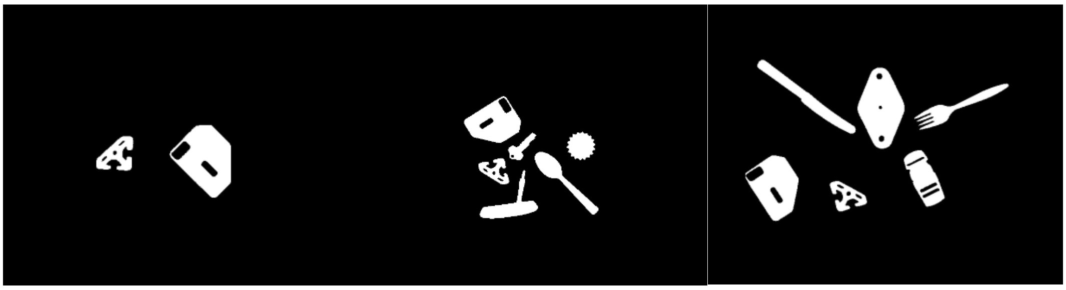
(c)

For any plane $Ax + By + Cz + D = 0$ and any line $\hat{l}(l_x, l_y, l_z)$. The vanishing point of the line $(x_{vp}, y_{vp}) = \left(f\frac{\mathrm{l_x}}{\mathrm{l_z}}, \mathrm{f}\frac{\mathrm{l_y}}{\mathrm{l_z}}\right)$. The vanishing point of the line can further be expressed as $(x_{vp}, y_{vp}, z_{vp}) = \left(f\frac{l_x}{l_z}, f\frac{l_y}{l_z}, f\right) = \left(f\frac{l_x}{l_z}, f\frac{l_y}{l_z}, f\frac{l_z}{l_z}\right)$. Since $\hat{l}$ lies on the plane, we have $Al_x + Bl_y + Cl_z = 0$, which is equal to $A\frac{f}{l_z}l_x + B\frac{f}{l_z}l_y + C\frac{f}{l_z}l_z = 0$. Therefore, we have $Ax_{vp} + By_{vp} + Cz_{vp} = 0$. The vanishing points (x_{vp}, y_{vp}) lie in the line $Ax_{vp} + By_{vp} + Cf = 0$.

Programming Assignment

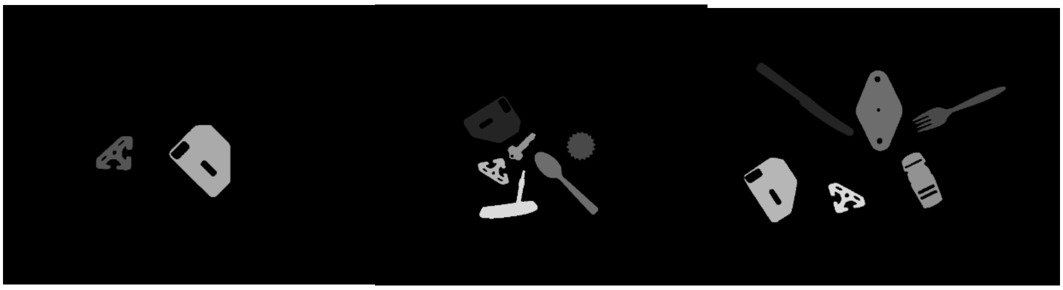
1. (a)

The three binary images with threshold = 128:



(b)

The three labeled images using two-pass algorithm and maintaining an 2d-array equivalence table mentioned in class:



(c)

For each object, calculate its position, orientation and roundness with formula mentioned in class respectively.

Attribute list:

Two objects:


```
[{'position': {'x': 195.3160469667319, 'y': 222.3820939334638}, 'orientation': 0.6875462936637149, 'roundness': 0.479963
64669203495}, {'position': {'x': 349.33298470388286, 'y': 215.45365407242778}, 'orientation': -1.259956523462201, 'round
ness': 0.5336319534756404}]
```


Many objects 1:


```
[{'position': {'x': 265.97616566814276, 'y': 364.13401927585306}, 'orientation': 0.08042727460237048, 'roundness': 0.521
7196889211334}, {'position': {'x': 461.6430812129662, 'y': 312.7504356918787}, 'orientation': 1.2635628997735306, 'round
ness': 0.9902664427338181}, {'position': {'x': 417.71620665251237, 'y': 240.29181410710072}, 'orientation': -0.776023844
3266907, 'roundness': 0.024421609826590758}, {'position': {'x': 326.0154385964912, 'y': 308.29473684210524}, 'orientatio
n': 0.7788385087054038, 'roundness': 0.1331947199392678}, {'position': {'x': 268.30828220858893, 'y': 256.85327198364007
}, 'orientation': -0.5388371734983242, 'roundness': 0.4860732206012445}, {'position': {'x': 303.571394686907, 'y': 177.2
7300759013283}, 'orientation': 0.405201992726549, 'roundness': 0.2702711841586357}]
```

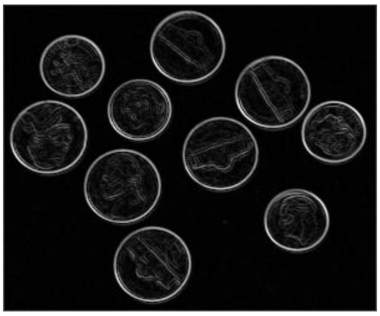

Many objects 2:


```
[{'position': {'x': 188.3515625, 'y': 356.90033143939394}, 'orientation': -0.6431420831724858, 'roundness': 0.0076335289
61638988}, {'position': {'x': 475.3399815894446, 'y': 338.9671678428966}, 'orientation': 0.40324741948779685, 'roundness
': 0.020855451285962178}, {'position': {'x': 331.9617982504706, 'y': 337.21769460746316}, 'orientation': -1.530919572329
0279, 'roundness': 0.3072674402498919}, {'position': {'x': 413.6556685685934, 'y': 203.95137682957082}, 'orientation': -
1.117909417312218, 'roundness': 0.17394416151886236}, {'position': {'x': 130.16157675232074, 'y': 187.1522938248352}, 'o
rientation': -1.4483813438029263, 'roundness': 0.5078766943974373}, {'position': {'x': 265.9671412924425, 'y': 168.64622
12486309}, 'orientation': -0.49296932904138474, 'roundness': 0.4809122478567924}]
```


2. (a)

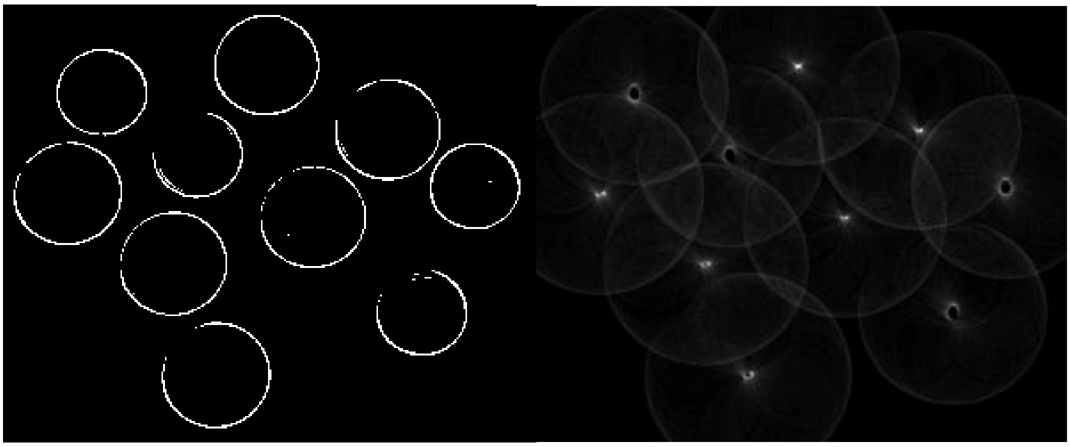
Apply two 3×3 Sobel operators(x, y) to convolve the image separately after padding the image. Then calculate the sum of two processed images with $\sqrt{a^2 + b^2}$.

The results are:



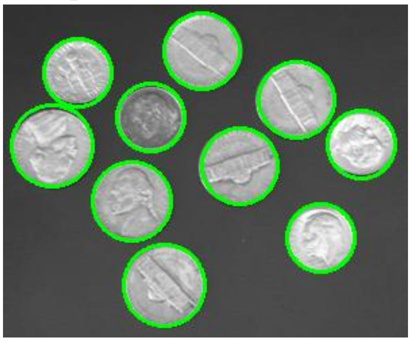
(b)

Set the threshold = 128 so that the edges inside the circle won't be detected. Apply circle detection algorithm in brute force and filter those points outside the image, the results are as follows: (left: binary image; right: accum array with r=28)



(c)

Set the threshold = 60 and filter the accum_array. To filter noisy points (a circle with multiple points), local maximal points are chosen to represent the circle. The results are:



```
(pytorch_dl) E:\Homework\CV_HW1>python p2_hough_circles.py coins 128 60
[(25, 50, 55), (25, 83, 109), (25, 103, 265), (25, 172, 235), (28, 34, 147), (28, 70, 217), (28, 119, 175), (29, 145, 95
), (30, 104, 36), (30, 207, 119)]
```