$c_1c_2^{*}+c_2c_1^{*}$ $\quad c_1^{*}+c_2^{*}$

$$[\hat{p}_x, F]\psi = \hat{p}_x \cdot F \cdot \psi - F\hat{p}_x \cdot \psi$$

$$= -i\hbar\frac{\partial(F\psi)}{\partial x} - F(-i\hbar\frac{\partial}{\partial x})\cdot\psi$$

$$= -Fi\hbar\frac{\partial}{\partial x}\cdot\psi - i\hbar\frac{\partial F}{\partial x}\cdot\psi + Fi\hbar\frac{\partial}{\partial x}\psi = -i\hbar\frac{\partial F}{\partial x}\cdot\psi$$

$\therefore [\hat{p}_x, F] = -i\hbar\frac{\partial F}{\partial x}$, （$\psi$ 为任意波函数）

1-30. $\begin{cases}\hat{A}\psi_1(x) = a_1\psi_1(x)\\ \hat{A}\psi_2(x) = a_2\psi_2(x)\end{cases}$ $\Rightarrow \because a_1 \neq a_2 \therefore \psi_1(x)$ 与 $\psi_2(x)$ 正交

$\psi = c_1\psi_1(x) + c_2\psi_2(x)$, $\hat{B}\psi = \hat{B}(c_1\psi_1(x) + c_2\psi_2(x)) =$

$c_2\psi_1(x) + c_1\psi_2(x) = \lambda(c_1\psi_1(x) + c_2\psi_2(x))$

$\because$ 正交 $\therefore c_2 = \lambda c_1$, $c_1 = \lambda c_2$ $\Rightarrow \begin{cases} c_2 = \lambda^2 c_2 \\ c_1 = \lambda^2 c_1\end{cases} \Rightarrow \lambda = \pm 1$

$\therefore |c_1| = |c_2| = \frac{1}{\sqrt{2}}$

$\lambda = 1$ 时，$c_1 = c_2$, $\psi_1 = c_1(\psi_1(x) + \psi_2(x))$, $|c_1| = \frac{1}{\sqrt{2}}$

$\lambda = -1$: $\psi_2 = c_1(\psi_1(x) - \psi_2(x))$, $|c_2| = \frac{1}{\sqrt{2}}$

1-31. 自由粒子：$\frac{\partial V}{\partial x} = 0$

$[\hat{H}, \hat{p}] = [\frac{\hbar^2}{2m}\nabla^2 + V, \hat{p}]$，由于 $\hat{p}$ 是线性

$\therefore [\hat{H}, \hat{p}] = [-\frac{\hbar^2}{2m}\nabla^2, \hat{p}] + [V, \hat{p}] = [V, \hat{p}]$

$[V, \hat{p}] = [V, -i\hbar\frac{\partial}{\partial x}] = i\hbar\frac{\partial V}{\partial x} = 0$ $\therefore$ 可对易

证明1：AB厄密 $\Rightarrow$ AB−BA=0

$\int \psi^*(AB)\psi dx = \int[(AB)\psi]^*\psi dx$

证明1：AB−BA=0 $\Rightarrow$ AB厄密

要证 $\int\psi^*(AB)\psi dx = \int[(AB)\psi]^*\psi dx$

$L.H.S=\int\psi^* A(B\psi)dx$，由A厄密有

$\int\psi^* A(B\psi)dx = \int(A\psi)^* B\psi dx$，由B厄密有

$\int(A\psi)^* B\psi dx = \int(BA\psi)^*\psi dx$

由AB−BA=0，有 $\int[(BA)\psi]^*\psi dx = \int[AB\psi]^*\psi dx$

综上，有 $\int\psi^*(AB)\psi dx = \int[(AB)\psi]^*\psi dx$

AB厄密 $\Rightarrow$ AB−BA=0

$\int\psi^*(AB)\psi dx = \int[(AB)\psi]^*\psi dx$

$\int\psi^*(AB)\psi dx = \int[(BA)\psi]^*\psi dx$（上面已经推过）

$\therefore AB=BA$，AB−BA=0

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证明2：$\frac{d}{dt}\langle\hat{p}_x\rangle=\frac{d}{dt}\int\Phi^*A\Phi\,dx=\int\left(\frac{\partial\Phi^*}{\partial t}\right)\hat{p}_x\Phi\,dx+\left\langle\frac{\partial\hat{p}_x}{\partial t}\right\rangle+\int\Phi^*\hat{p}_x\left(\frac{\partial\Phi}{\partial t}\right)d$

而 $H\Phi=i\hbar\frac{\partial\Phi}{\partial t}$，$(H\Phi)^*=-i\hbar\frac{\partial\Phi^*}{\partial t}$

$\because H^*=H$ $\therefore(H\Phi)^*=\Phi^*H^*=\Phi^*H$

$\therefore\frac{d}{dt}\langle\hat{p}_x\rangle=\frac{1}{i\hbar}\int\Phi^*(\hat{p}_xH-H\hat{p}_x)\Phi\,dx+\left\langle\frac{\partial\hat{p}_x}{\partial t}\right\rangle$

$=\frac{1}{i\hbar}\langle[\hat{p}_x,H]\rangle+\left\langle\frac{\partial\hat{p}_x}{\partial t}\right\rangle$

由 $\frac{\partial\hat{p}_x}{\partial t}=0$，H中前一项为 $\hat{p}$ 故 $\frac{d}{dt}\langle\hat{p}_x\rangle=\frac{1}{i\hbar}\langle[p,V]\rangle$

$=\int\Phi^*V\frac{\partial}{\partial x}\Phi\,dx-\int\Phi^*\frac{\partial}{\partial x}(V\Phi)\,dx=\left\langle-\frac{\partial}{\partial x}V\right\rangle=\langle F\rangle$

$\therefore$ 动量期望变化率 $\frac{d}{dt}\langle p\rangle=-\left\langle\frac{\partial}{\partial x}V\right\rangle$

对比 $F=-\nabla V=\frac{dp}{dt}=ma$，得证