3-1. $\begin{bmatrix} E_0 & A \\ A & E_0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \lambda \begin{bmatrix} a \\ b \end{bmatrix}$

$\begin{bmatrix} E_0-\lambda & A \\ A & E_0-\lambda \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$

$\Rightarrow (E_0-\lambda)^2 - A^2 = 0$

$\Rightarrow \lambda = E_0 \pm A$

当 $\lambda = E_0 + A$ 时，$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

当 $\lambda = E_0 - A$ 时，$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

幺正矩阵：$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} E_0 & A \\ A & E_0 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} E_0+A & 0 \\ 0 & E_0-A \end{bmatrix}$$

3-2. $\begin{bmatrix} E_1 & A \\ A & E_2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \lambda \begin{bmatrix} a \\ b \end{bmatrix}$

$\Rightarrow (E_1-\lambda)(E_2-\lambda) - A^2 = 0$，$\lambda^2 - (E_1+E_2)\lambda + E_1E_2 - A^2 = 0$

$$\lambda_1 = \frac{E_1+E_2+\sqrt{(E_1+E_2)^2-4(E_1E_2-A^2)}}{2} = E_0+2A$$

$$\lambda_2 = \frac{E_1+E_2-\sqrt{(E_1+E_2)^2-4(E_1E_2-A^2)}}{2} = E_0-2A$$

$\Delta E' = (E_0+2A) - (E_0-2A) = 4A > E_1 - E_2 = 2\sqrt{3}A$

增大

$$\begin{cases} E_1+E_2=2E_0 \\ E_1-E_2=2\sqrt{3}A \end{cases} \Rightarrow \begin{cases} E_1=E_0+\sqrt{3}A \\ E_2=E_0-\sqrt{3}A \end{cases}$$

当$\lambda_1=E_0+2A$时：$\begin{bmatrix} E_0+\sqrt{3}A-(E_0+2A) & A \\ A & E_0-\sqrt{3}A-(E_0+2A) \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}=0$

$\Rightarrow a=\frac{\sqrt{2+\sqrt{3}}}{2}$，$b=\frac{\sqrt{2-\sqrt{3}}}{2}$

当$\lambda_1=E_0-2A$时

$\Rightarrow a=\frac{\sqrt{2-\sqrt{3}}}{2}$，$b=\frac{\sqrt{2+\sqrt{3}}}{2}$

令定态矢记为$|+\rangle$和$|-\rangle$，$|+\rangle=\begin{pmatrix} \frac{\sqrt{2+\sqrt{3}}}{2} \\ \frac{\sqrt{2-\sqrt{3}}}{2} \end{pmatrix}$，$|-\rangle=\begin{pmatrix} \frac{\sqrt{2-\sqrt{3}}}{2} \\ \frac{\sqrt{2+\sqrt{3}}}{2} \end{pmatrix}$

$|1\rangle=\begin{pmatrix} 1 \\ 0 \end{pmatrix}$，$|2\rangle=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|\psi\rangle = C_+|+\rangle + C_-|-\rangle = C_1|1\rangle + C_2|2\rangle$

$$\begin{cases} C_+=\frac{\sqrt{2+\sqrt{3}}}{2}C_1+\frac{\sqrt{2-\sqrt{3}}}{2}C_2 \\ C_-=\frac{\sqrt{2-\sqrt{3}}}{2}C_1+\frac{\sqrt{2+\sqrt{3}}}{2}C_2 \end{cases}$$