Algorithm Design and Analysis (Fall 2023)

Assignment 6

Deadline: Jan 9, 2023

Choose **two** of the first four questions to submit. Question 5 is the bonus question.

1. Prove that the following problem is NP-complete. Given an undirected graph G and an undirected graph H, decide if H is a subgraph of G.

2. Prove that the following problem is NP-complete. Given an undirected graph G and a positive integer $k \geq 2$, decide if G contains a spanning tree with maximum degree at most k.

3. Given an undirected graph $G = (V, E)$, prove that it is NP-complete to decide if G contains an independent set with size *exactly* $|V|/3$.

4. Consider the decision version of *Knapsack*. Given a set of n items with weights $w_1, \ldots, w_n \in \mathbb{Z}^+$ and values $v_1, \ldots, v_n \in \mathbb{Z}^+$, a capacity constraint $C \in \mathbb{Z}^+$, and a positive integer $V \in \mathbb{Z}^+$, decide if there exists a subset of items with total weight at most C and total value at least V. Prove that this decision version of Knapsack is NP-complete.

5. (**Bonus**) In the class, we have seen that 3SAT is NP-complete. In this question, we investigate the 2SAT problem and its variants. Similar to the 3SAT problem, in the 2SAT problem, we are given a 2-CNF Boolean formula (where each clause contains two literals) and we are to decide if this formula is satisfiable.

   (a) Prove that 2SAT is in P. (Hint: a clause $(a_i \vee a_j)$ with two literals a_i and a_j can be represented as two logical implications: $\neg a_i \implies a_j$ and $\neg a_j \implies a_i$; you may want to construct a directed graph with $2n$ vertices corresponding to $x_1, \neg x_1, x_2, \neg x_2, \ldots, x_n, \neg x_n$.)

   (b) Consider this variant of the 2SAT problem: given a 2-CNF Boolean formula ϕ and a positive integer k, decide if there is a Boolean assignment to the variables such that at least k clauses of ϕ are satisfied. Notice that 2SAT is the special case of this problem with k equals to the number of the clauses. Prove that this problem is NP-complete.

6. How long does it take you to finish the assignment (including thinking and discussion)? Give a score (1,2,3,4,5) to the difficulty. Do you have any collaborators? Please write down their names here.