7. 当 $i=j$ 时，$cov(x_i-\bar{x}, x_j-\bar{x}) = D(x_i-\bar{x})$，$\rho=1$

当 $i\neq j$ 时，$cov(x_i-\bar{x}, x_j-\bar{x}) = cov(x_i,x_j) - cov(x_i,\bar{x}) - cov(x_j,\bar{x})$

$+cov(\bar{x},\bar{x}) = 0 + \frac{\sigma^2}{n} - 2cov(x_i, \frac{1}{n}\sum_{i=1}^{n} x_i)$

其中 $cov(x_i, \frac{1}{n}\sum_{i=1}^{n} x_i) = \frac{1}{n}cov(x_i, x_i) = \frac{\sigma^2}{n}$

$\therefore \frac{\sigma^2}{n} - \frac{2\sigma^2}{n} = -\frac{\sigma^2}{n}$

$D(x_i-\bar{x}) = D(x_j-\bar{x}) = \frac{\sigma^2}{n} + \sigma^2 = \sigma^2 \cdot \frac{n+1}{n}$

$\therefore \rho = \frac{cov(x_i-\bar{x}, x_j-\bar{x})}{\sqrt{D(x_i-\bar{x})D(x_j-\bar{x})}} = -\frac{1}{n+1}$

9. (1) $x_i \sim N(0, 0.25)$，$\frac{x_i}{0.5} \sim N(0,1)$

$P(\sum_{i=1}^{10} x_i^2 \geq 4) = P(\sum_{i=1}^{10}(2x_i)^2 \geq 16) = P(\chi^2(10) \geq 16) = 0.1$

(2) $S^2 = \frac{1}{9}\sum_{i=1}^{n}(x_i-\bar{x})^2$，$\frac{9S^2}{0.25} \sim \chi^2(9)$

$\therefore P(\sum_{i=1}^{10}(x_i-\bar{x})^2 \geq 4.23) = P(4\sum_{i=1}^{10}(x_i-\bar{x})^2 \geq 16.92) = P(\chi^2(9) \geq 16.92)$

$= 0.05$

10. (1) $X_i \sim N(0,4)$，$\frac{X_i}{2} \sim N(0,1)$，$a = \frac{1}{4}$

$X_2+X_3+X_4 \sim N(0,12)$，$\frac{X_2+X_3+X_4}{\sqrt{12}} \sim N(0,1)$，$b = \frac{1}{12}$

$S^2 = \frac{1}{4}\sum_{i=5}^{9}(X_i-Y)^2$，$\frac{4S^2}{4} \sim \chi^2(4)$

$\therefore c = \frac{1}{4}$

(2) $X_1+X_2 \sim N(0,8)$，$\frac{X_1+X_2}{2\sqrt{2}} \sim N(0,1)$

$\frac{X_i}{2} \sim N(0,1)$

$d\frac{X_1+X_2}{\sqrt{X_3^2+X_4^2+X_5^2}} = \frac{\frac{X_1+X_2}{2\sqrt{2}}}{\sqrt{\frac{\frac{1}{4}(X_3^2+X_4^2+X_5^2)}{3}}}$，$d = \frac{2\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{2}$，自由度为3

11. $X_{n+1} \sim N(\mu, \sigma^2)$, $X_{n+1}-\bar{X} \sim N(0, \frac{n+1}{n}\sigma^2)$

$$\frac{X_{n+1}-\bar{X}}{\sigma} \cdot \sqrt{\frac{n}{n+1}} \sim N(0,1)$$

由 $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ 得 $\frac{\frac{X_{n+1}-\bar{X}}{\sigma}\sqrt{\frac{n}{n+1}}}{\sqrt{\frac{\frac{(n-1)S^2}{\sigma^2}}{n-1}}} = \frac{X_{n+1}-\bar{X}}{S}\sqrt{\frac{n}{n+1}} \sim t(n-1)$

$$\frac{(X_{n+1}-\bar{X})^2}{S^2} \cdot \frac{n}{n+1} = \left(\frac{X_{n+1}-\bar{X}}{S}\sqrt{\frac{n}{n+1}}\right)^2$$

而 $\frac{X_{n+1}-\bar{X}}{S}\sqrt{\frac{n}{n+1}} \sim t(n-1)$, $\therefore \frac{(X_{n+1}-\bar{X})^2}{S^2}\frac{n}{n+1} \sim F(1, n-1)$

12. $P(X>1) = \frac{1}{2}$

13. (1) $\frac{X_1+X_2}{\sqrt{2}} \sim N(0,1)$, $\frac{X_1-X_2}{\sqrt{2}} \sim N(0,1)$

$$Y = \frac{\left(\frac{X_1+X_2}{\sqrt{2}}\right)^2/1}{\left(\frac{X_1-X_2}{\sqrt{2}}\right)^2/1} \sim F(1,1)$$

(2) $Z = \frac{X_1^2/1}{\sum_{i=2}^{n} X_i^2/(n-1)} \sim F(1, n-1)$