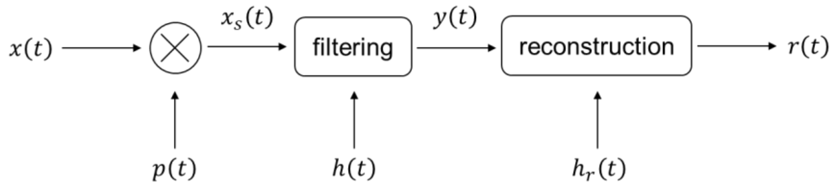
# AI2619 2024Spring Written Assignment #1

**Problem 1**

Suppose we want to do impulse sampling to a continuous-time signal $x(t) = \frac{\sin(\omega_m t)}{\pi t}$. The sampled signal $x_s(t)$ is then filtered and reconstructed, as shown in the figure below.

Here $p(t) = \sum_{-\infty}^{+\infty} \delta(t - nT)$, $H(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & otherwise \end{cases}$ and $H_r(\omega) = \begin{cases} T & |\omega| < \omega_m \\ 0 & otherwise \end{cases}$.

The Fourier transform of $x(t)$ is denoted by $X(\omega)$, and the same for other signals.




(1). Let $T = \frac{2\pi}{3\omega_m}, \omega_c = \frac{1}{2}\omega_m$. Derive the expression for $X_s(\omega), Y(\omega), R(\omega)$ and $r(t)$, then draw $x(t), x_s(t), y(t), r(t)$ and $X(\omega), X_s(\omega), Y(\omega), R(\omega)$. (You can just draw a few periods if they are periodic, but you must clearly mark the necessary values in your figure, such as peak values and cut-off frequencies)

(2). Let $T = \frac{2\pi}{3\omega_m}$. What is the proper range for $\omega_c$ if we want to perfectly reconstruct $x(t)$ in question (1)? Here "perfectly reconstruct" means $r(t) = x(t)$.

(3). Let $T = \frac{4\pi}{3\omega_m}, \omega_c = \frac{1}{2}\omega_m$. Redo what you have done in question (1).

(4). Let $\omega_c = \frac{1}{2}\omega_m$. What is the proper range for $T$ if we want to get the same output $r(t)$ as that in question (1)?

(5). If the sampling method is not impulse sampling, but zero-order hold sampling. Is there still a way to perfectly reconstruct $x(t)$? If so, what should the $h_r(t)$ be then? Please give the procedures. Here, $h(t)$ remains the same, $T$ and $\omega_c$ are the same as in question (1).