1-25. $n=2$. $\psi_2(x)=\sqrt{\frac{2}{a}}\sin\left(\frac{2\pi x}{a}\right)$

$$P=\int_0^{\frac{a}{4}}|\psi_2(x)|^2dx=\frac{2}{a}\int_0^{\frac{a}{4}}\sin^2\left(\frac{2\pi x}{a}\right)dx=\frac{2}{a}\cdot\frac{a}{2\pi}\cdot\int_0^{\frac{\pi}{2}}\sin^2x\,dx$$
$$=\frac{1}{\pi}\cdot\left(\frac{\pi}{4}\right)=0.25$$

证明：本征函数是正交的：

$$\int\psi_m^*\psi_n dx=\frac{2}{L}\int_0^L\sin\frac{m\pi x}{L}\sin\frac{n\pi x}{L}dx=\delta_{mn}=\langle\psi_m|\psi_n\rangle=\begin{cases}1, & m=n\\ 0, & m\neq 0\end{cases}$$

$$\psi(r,t)=\sum C_n\psi_n e^{-\frac{iE_nt}{\hbar}}=\sum c_n(t)\psi_n$$

$\psi(r,t)=\sum_{n=1}^{\infty}c_n(t)\psi_n$ 是傅里叶级数

$$c_n(t)=\int_{-\infty}^{\infty}\psi_n^*\psi(r,t)dx$$

$\psi(r,t)$ 是归一化的，那么：

$1=\int_{-\infty}^{\infty}\psi(x)^*\psi(x)dx=\sum_{n=1}^{\infty}|c_n(t)|^2$ 得证

$\frac{1}{2}a^2e$

作业1. $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)\psi_1=E_1\psi_1$ ①

$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)\psi_2=E_2\psi_2$ ②

$$① \Rightarrow -\frac{\hbar^2}{2m}(\alpha^2x^2-1)\alpha^2+V(x)=E_1$$

$$V(x)=E_1+(\alpha^2x^2-1)\frac{\hbar^2}{2m}\cdot\alpha^2$$

$$② \Rightarrow -\frac{\hbar^2}{2m}\left[\frac{(2\alpha^4x^4-11\alpha^2x^2+5)\alpha^2}{2\alpha^2x^2-1}\right]+V(x)=E_2$$

$$\Rightarrow -\frac{\hbar^2}{2m}(\alpha^2x^2-5)\alpha^2+V(x)=E_2,\ V(x)=E_2+(\alpha^2x^2-5)\cdot\frac{\hbar^2}{2m}$$

$$E_2-E_1=\frac{2\hbar^2}{m}\alpha^2=2w\hbar$$

作业2. $|\psi(x)|^2 = [\varphi(x)e^{iEt} + \varphi^*(x)e^{-iEt}][\varphi^*(x)e^{-iEt} + \varphi(x)e^{iEt}]$

$$= \varphi^2(x) + \varphi^2(x) + 2\psi(x)\psi^*(x)\cos(2Et)$$

$$= [2+2\cos(2Et)]\varphi^2(x)$$

，带t，不处于定态

作业3: $J = -\frac{i\hbar}{2m}(\varphi^*\nabla\varphi - \varphi\nabla\varphi^*)$

$$J = -\frac{i\hbar}{2m}\left[\sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x\cdot\sqrt{\frac{2}{L}}\cdot\frac{n\pi}{L}\cdot\cos\frac{n\pi}{L}x - \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x\cdot\sqrt{\frac{2}{L}}\cdot\frac{n\pi}{L}\cdot\cos\frac{n\pi}{L}x\right]$$

$$=0$$

说明定态下，几率流密度在这种情况为0，粒子在任意点的平面的2个方向流动趋势相等。

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