5. 令 $X_i=1$ 表示第 i 层有人下，$X_i=0$ 表示第 i 层无人下

每层下的概率为 $\frac{1}{n-1}$，$X=\sum_{i=2}^{n} X_i$

$\therefore P(X_i=0)=(1-\frac{1}{n-1})^m$，$P(X_i=1)=1-(1-\frac{1}{n-1})^m$

$\therefore E(X)=\sum_{i=2}^{n} E(X_i)=\sum_{i=2}^{n}[1-(1-\frac{1}{n-1})^m]=(n-1)[1-(1-\frac{1}{n-1})^m]$

9. $\begin{cases} -P(X=-1)+P(X=1)=0.1 \\ P(X=-1)+P(X=1)=0.9 \end{cases} \Rightarrow \begin{cases} P(X=-1)=0.4 \\ P(X=1)=0.5 \end{cases}$

$\therefore P(X=0)=0.1$

X	-1	0	1
$P(X)$	0.4	0.1	0.5

$250000 \quad \frac{4}{3} \quad 2-\frac{2}{3} \quad \frac{x}{\ }-\frac{x^3}{3} \quad x' \quad k\cdot \quad \frac{x^2}{2}-\frac{x^4}{4}\Big|_{-1}^{1}$

10. $E(X)=\int_{-1}^{1}(500+x)^2\cdot k(1-x^2)\,dx$

$k\int_{-1}^{1}(1-x^2)\,dx=1 \Rightarrow k=\frac{3}{4}$

$\therefore E(X)=250000.2$

11. $E(X)=\int_0^1 x^2dx+\int_1^2 x(2-x)\,dx=1$

$E(2X+1)=\int_0^1(2x+1)x\,dx+\int_1^2(2x+1)(2-x)\,dx=3$

$E(e^{-X})=\int_0^1 e^{-x}\cdot x\,dx+\int_1^2 e^{-x}(2-x)\,dx=0.4$

12. $X\sim N(0,1)$

$E(|X|)=-\int_{-\infty}^{0}x\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx+\int_0^{\infty}x\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx=\sqrt{\frac{2}{\pi}}$

$E(X^4)=\int_{-\infty}^{\infty}x^4\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx=3$

13. $E(T)=-\int_{-\infty}^{10}\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-M)^2}{2}}dx+20\int_{10}^{12}\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-M)^2}{2}}dx-5\int_{12}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-M)^2}{2}}dx$

$E'(T)=\frac{1}{\sqrt{2\pi}}\left[e^{-\frac{(10-M)^2}{2}}+20e^{-\frac{(10-M)^2}{2}}-20e^{-\frac{(12-M)^2}{2}}-5e^{-\frac{(12-M)^2}{2}}\right]=0$

$\therefore 21e^{-\frac{(10-M)^2}{2}}=25e^{-\frac{(12-M)^2}{2}}$

$\therefore -\frac{(10-M)^2}{2}+\frac{(12-M)^2}{2}=\ln\frac{25}{21}$, $M=11-\frac{1}{2}\cdot\ln\frac{25}{21}\approx 10.91$

15. $P(X_i)=\frac{1}{n}$, $E(X)=\sum_{i=1}^{n}i\cdot\frac{1}{n}=\frac{1}{n}\cdot\frac{(1+n)\cdot n}{2}=\frac{1+n}{2}$

$D(X)=E[(X-E(X))^2]=\sum_{i=1}^{n}\left[i-\left(\frac{1+n}{2}\right)\right]^2\cdot\frac{1}{n}=\frac{n^2-1}{12}$

补. $R\sim U(9,11)$, $H\sim U(9,11)$, $V=\pi R^2\cdot H$

$E(\pi R^2H)=\pi\cdot E(R)\cdot E(R)\cdot E(H)=1000\pi$