1. $W_N \triangleq e^{-j(2\pi/N)}$

$$\tilde{X}[k]=\sum_{n=0}^{N-1}\tilde{x}[n]W_N^{kn}, \quad \tilde{X}_3[k]=\sum_{n=0}^{3N-1}\tilde{x}[n]W_{3N}^{kn}$$

$\because \tilde{x}[n]=\tilde{x}[n+N]$

$$\therefore \tilde{X}_3[k]=\sum_{n=0}^{N-1}\tilde{x}[n]W_{3N}^{kn}+\sum_{n=N}^{2N-1}\tilde{x}[n]W_{3N}^{kn}+\sum_{n=2N}^{3N-1}\tilde{x}[n]W_{3N}^{kn}$$

while $\sum_{n=N}^{2N-1}\tilde{x}[n]W_{3N}^{kn}=W_{3N}^{kN}\sum_{n=0}^{N-1}\tilde{x}[n]W_{3N}^{kn}$

we have $\sum_{n=2N}^{3N-1}\tilde{x}[n]W_{3N}^{kn}=W_{3N}^{2kN}\sum_{n=2N}^{3N-1}\tilde{x}[n]W_{3N}^{kn}$

$$\therefore \tilde{X}_3[k]=\sum_{n=0}^{N-1}\tilde{x}[n]W_{3N}^{kn}+W_{3N}^{kN}\sum_{n=0}^{N-1}\tilde{x}[n]W_{3N}^{kn}+W_{3N}^{2kN}\sum_{n=0}^{N-1}\tilde{x}[n]W_{3N}^{kn}$$

$$=\sum_{n=0}^{N-1}\tilde{x}[n]W_N^{\frac{kn}{3}}+W_N^{\frac{kN}{3}}\sum_{n=0}^{N-1}\tilde{x}[n]W_N^{\frac{kn}{3}}+W_N^{\frac{2kN}{3}}\sum_{n=0}^{N-1}\tilde{x}[n]W_N^{\frac{kn}{3}}$$

$$=\left(1+W_N^{\frac{kN}{3}}+W_N^{\frac{2kN}{3}}\right)\sum_{n=0}^{N-1}\tilde{x}[n]W_N^{\frac{kn}{3}}$$

$$=\left(1+e^{-j2\pi\cdot\frac{k}{3}}+e^{-j2\pi\cdot\left(\frac{2k}{3}\right)}\right)\tilde{X}[k]$$

$$=3\tilde{X}\left[\frac{k}{3}\right]\ (k=3\ell)\ ;\ 0,\ \text{o.w.}$$

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] W_N^{kn} = \sum_{n=0}^{1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn} = 1 + 2e^{-j\pi k} = 1+2(-1)^k$$

$$= \begin{cases} 3, & k = 2Z \\ -1, & k = 2Z+1 \end{cases}$$

$3N/kn$

$$\tilde{X}_3[k] = \sum_{n=0}^{3N-1} \tilde{x}[n] W_{3N}^{kn} = \sum_{n=0}^{5} \tilde{x}[n] e^{-j\frac{\pi}{3}kn} = (1 + e^{-j\cdot 2\pi\cdot\frac{k}{3}} + e^{-j2\pi\cdot(\frac{2k}{3})}) \cdot (1+2(-1)^{\frac{k}{3}})$$

$$= \begin{cases} 9 & k = 6Z \\ -3, & k = 3+6Z \\ 0, & \text{o.w. in a period} \end{cases}$$

$\therefore$ Satisfied

P2.

(1) $x[0] - x[1] + x[2] - x[3] + x[4] = 1$

$a + b + c = 2$

$z[2] = x[0] \cdot 3 + x[1] \times 2 + x[2] = 3$

$2\times 1 + 2 = 3$

$N=5$

$1+2+3+4 = 1$

$2\times 1 + 2\times 2 = 1$ $\quad z = -2$

$X[0] = x[0] + x[1] + x[2] + x[3] + x[4] = 2$

$$X[k] = \sum_{n=0}^{4} x[n] \cdot e^{-j\frac{2\pi}{5}\cdot kn} = x[0] + x[1]\cdot e^{-j\frac{2\pi}{5}\cdot k} + x[2]\cdot e^{-j\frac{2\pi}{5}\cdot 2k} + x[3]\cdot e^{-j\frac{2\pi}{5}\cdot 3k} + x[4] e^{-j\frac{2\pi}{5}\cdot 4k}$$

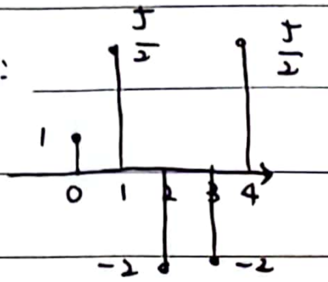
$$= x[0] + x[1]\cdot\left[\cos(-\tfrac{2\pi}{5}k) + j\sin(-\tfrac{2\pi}{5}k)\right] + x[2]\cdot\left[\cos(-\tfrac{4\pi}{5}k) + j\sin(-\tfrac{4\pi}{5}k)\right] + x[3]\left[\cos(-\tfrac{6\pi}{5}k) + j\sin(-\tfrac{6\pi}{5}k)\right] + x[4]\left[\cos(-\tfrac{8\pi}{5}k) + j\sin(-\tfrac{8\pi}{5}k)\right]$$

$$= x[0] + (x[1]+x[4])\cos\left(\tfrac{2\pi}{5}k\right) + (x[2]+x[3])\cos\left(\tfrac{4\pi}{5}k\right)$$

$x[1] = x[4]$, $x[2] = x[3]$

$$\begin{cases} x[0]=1 \\ x[1]=\frac{5}{2} \\ x[2]=-2 \\ x[3]=-2 \\ x[4]=\frac{5}{2} \end{cases} \qquad \begin{cases} a=1 \\ b=5 \\ c=-4 \end{cases}$$

$x[n]$:



$$X[k]=1+5\cos\left(\frac{2\pi}{5}k\right)-4\cos\left(\frac{4\pi}{5}k\right)$$

(2) Yes. Since $X[k]=x[0]+(x[1]+x[4])\cos\left(\frac{2\pi}{5}k\right)+(x[2]+x[3])\cos\left(\frac{4\pi}{5}k\right)+j(x[4]-x[1])\sin\left(\frac{2\pi}{5}k\right)+j(x[3]-x[2])\sin\left(\frac{2\pi}{5}k\right)$

If it's real-valued, $x[4]=x[1]$, $x[3]=x[2]$.

It's exactly what we get from the original (b)

P3.

(1) $X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}=\sum_{n=0}^{N-1}x[n]z^{-n}$

$$Y[k]=\sum_{n=0}^{N-1}x[n]\left(\frac{1}{2}e^{j\frac{2\pi}{N}k}\right)^{-n}=\sum_{n=0}^{N-1}x[n]\cdot 2^{n}e^{-j\frac{2\pi}{N}kn}$$

$$X[k]=\sum_{n=0}^{N-1}x[n]e^{-j\frac{2\pi}{N}kn}$$

Therefore, $y[n]=x[n]\cdot 2^{n}$, $0\le n\le N-1$

(2) $G[k]=X[2k]=\sum_{n=0}^{N-1}x[n]e^{-j\frac{2\pi}{N}(2k)\cdot n}$

$$=\sum_{n=0}^{N-1}x[n]e^{-j\frac{2\pi}{N}k(2n)}$$

上式 $=\sum_{2n=0}^{2N-2}x\left[\frac{2n}{2}\right]e^{-j\frac{2\pi}{N}k\cdot(2n)}$, $2n=0,2,4,\ldots$

$\therefore g[n]=\begin{cases} x\left[\frac{n}{2}\right], & n=0,2,\ldots,2N-2 \\ 0, & \text{o.w.} \end{cases}$, similarly, $h[n]=\begin{cases} x\left[\frac{n-1}{2}\right], & n=1,3,\ldots,2N-1 \\ 0, & \text{o.w.} \end{cases}$