AI2619 2024Spring Written Assignment #3

Problem 1

Suppose $\tilde{x}[n]$ is a periodic sequence with period N. Then $\tilde{x}[n]$ is also periodic with period $3N$. Let $\tilde{X}[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period N, and let $\tilde{X}_3[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period $3N$.

(a) Express $\tilde{X}_3[k]$ in terms of $\tilde{X}[k]$.

(b) By explicitly calculating $\tilde{X}[k]$ and $\tilde{X}_3[k]$, verify your result in Part (a) when $\tilde{x}[n]$ is as given in Figure 1.

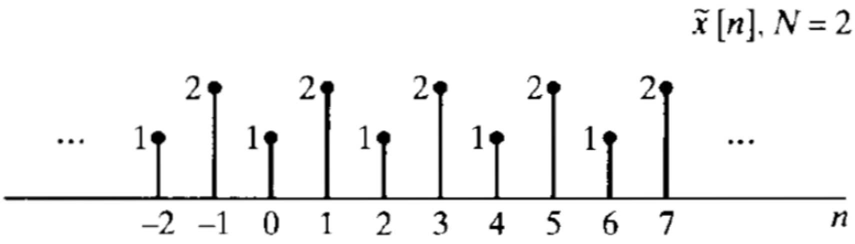

Figure 1: The periodic sequence $\tilde{x}[n]$ in Part (b).

Problem 2

We know some information about a discrete-time signal $x[n]$:

(a). $x[n]$ has non-zero values only when $0 \le n \le 4$;

(b). Its 5-point DFT $X[k] = a + b\cos\left(\frac{2\pi}{5}k\right) + c\cos\left(\frac{4\pi}{5}k\right)$ for $0 \le k \le 4$;

(c). $\sum_{n=0}^{4}(-1)^n x[n] = 1$

(d). $X[0] = 2$

(e). The weighted sum $z[n] = \sum_{m=0}^{6} x[m]y\left[((n-m))_7\right]$ has $z[2] = 3$ when $y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2]$.

(1). Determine $x[n]$ and $X[k]$.

(2). If we change information (b) to "both of $x[n]$ and its 5-point DFT $X[k]$ are real-valued", could we obtain the same results as those in question (1)? Briefly state your reasons.

Problem 3

In this problem, you should give detailed derivation for your conclusion.

We have a N-point discrete-time signal $x[n]$ (whose non-zero values are in $0 \le n \le N-1$), with corresponding z-transform $X(z)$ and N-point DFT $X[k], 0 \le k \le N-1$.

(1). $Y[k] = X(z)|_{z=\frac{1}{2}e^{j\frac{2\pi}{N}k}}$ for $0 \le k \le N-1$. Express $y[n]$ $(0 \le n \le N-1)$ in terms of $x[n]$.

(2). $g[n]$ and $h[n]$ are two $\frac{N}{2}$-point DT signals, with corresponding $\frac{N}{2}$-point DFT $G[k], H[k]$, where $G[k] = X[2k]$ and $H[k] = X[2k+1]$ for $0 \le k \le \frac{N}{2}-1$. Express $g[n]$ and $h[n]$ $(0 \le n \le \frac{N}{2}-1)$ in terms of $x[n]$.