$2xy^2$ 2 $2x-8x^3$ x^2-2x^4 $\frac{1}{4}-2x\frac{1}{16}$

30.(1) $f(x,y)=f_x(x)\cdot f_Y(y)=4xy,\ 0<x<1,\ 0<y<1$

$$f(x,y)=\begin{cases}4xy, & 0<x<1,\ 0<y<1\\ 0, & \text{其他}\end{cases}$$

(2) $P(Y>2X)=\int_0^{\frac{1}{2}}\int_{2x}^{1}4xy\cdot dy\,dx=\frac{1}{8}$ $\frac{2}{3}-\frac{2}{3}(z-1)^3$

(3) $f_Z(z)=\int_{-\infty}^{\infty}f(x,z-x)dx$ $-\frac{4}{3}x^3+2zx^2$

$-4x^2+4zx$

当 $z<0$ 时，$f_Z(z)=0$

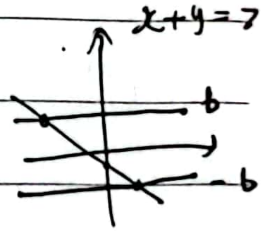
当 $0\le z\le 1$ 时，$0\le x\le z$：$f_Z(z)=\int_0^z 4x(z-x)dx=\frac{2}{3}z^3$

当 $1<z\le 2$ 时，$z-1\le x\le 1$：$f_Z(z)=\int_{z-1}^{1}4x(z-x)dx=\frac{2}{3}-\frac{2}{3}(z-1)^3$

当 $z>2$ 时，$f_Z(z)=0$ $x=z-y$ $y=-x+z$

$$\therefore f_Z(z)=\begin{cases}\frac{2}{3}z^3, & 0\le z\le 1\\ \frac{2}{3}[1-(z-1)^3], & 1<z\le 2\\ 0, & \text{其他}\end{cases}$$

$x+y=z$



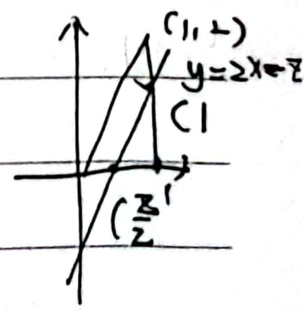
31. $f_Z(z)=\int_{-\infty}^{\infty}f_X(x)\cdot f_Y(z-x)dx$

$$\because f_X(x)\cdot f_Y(z-x)=\begin{cases}\frac{1}{2b\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, & -b<z-x<b\\ 0, & \text{其他}\end{cases}$$

$y=2x-z$

$$f_Z(z)=\int_{z-b}^{z+b}\frac{1}{2b\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$$

$$=\frac{1}{2b}\cdot\left[\Phi\left(\frac{z+b-\mu}{\sigma}\right)-\Phi\left(\frac{z-b-\mu}{\sigma}\right)\right]$$



32. $z\le 0$ 时，$f_Z(z)=0$，$z\ge 2$ 时，$f_Z(z)=0$

当 $0<z<2$ 时，$f_Z(z)=\int_{\frac{z}{2}}^{1}dx=1-\frac{z}{2}$

36. (1) $f_X(x)=\begin{cases}\frac{x}{4}e^{-\frac{x^2}{8}}, & x\ge 0\\ 0, & x<0\end{cases}$

$F_X(x)=\int_0^x \frac{x}{4}e^{-\frac{x^2}{8}}dx = 1-e^{-\frac{x^2}{8}}$

$\therefore F_X(x)=\begin{cases}1-e^{-\frac{x^2}{8}}, & x\ge 0\\ 0, & x<0\end{cases}$

$\therefore F_Z(z)=[F_X(x)]^5=(1-e^{-\frac{x^2}{8}})^5$

(2) $P(Z>4)=1-F_Z(4)=1-(1-e^{-2})^5=0.517$

1. $E(X)=P(X=1)+2\cdot P(X=2)+3\cdot P(X=3)+4\cdot P(X=4)=1.5625$

2. $E(X)=5000\times 0.99=4950$

3. $P(X=3)=2\times(\frac{1}{2})^3=\frac{1}{4}$

$P(X=4)=2\times\frac{1}{2}\times C_3^2\times(\frac{1}{2})^2\times(\frac{1}{2})=\frac{3}{8}$

$P(X=5)=2\times\frac{1}{2}\times C_4^2\times(\frac{1}{2})^2\times(\frac{1}{2})^2=\frac{3}{8}$

$\therefore E(X)=\sum_{i=3}^{5} i P(X=i)=4.125$

4. $P(X=0)=\frac{3}{4}$, $P(X=1)=\frac{3}{12}\times\frac{9}{11}$, $P(X=2)=\frac{3}{12}\times\frac{2}{11}\times\frac{9}{10}$

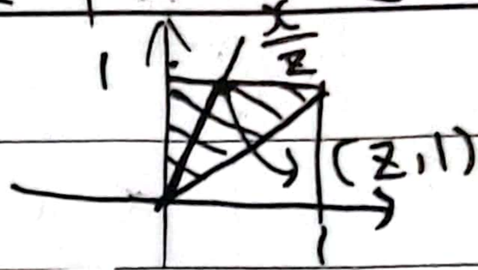
$P(X=3)=\frac{3}{12}\times\frac{2}{11}\times\frac{1}{10}$

$\therefore E(X)=\sum_{i=0}^{3} i P(X=i)=0.3$

$x = z1$

$z = \frac{x}{y}$ $\quad y = \frac{x}{z}$ $\quad 0 < z < 1$

补：$f(x,y) = \begin{cases} \frac{1}{1-x}, & 0<x<y<1 \\ 0, & 其他 \end{cases}$



$$\begin{cases} f_Z(z) = \int_{-\infty}^{\infty} f(x, \frac{x}{z}) \cdot \left|\frac{x}{z^2}\right| dx = \int_0^z \frac{x}{1-x} dx \cdot \frac{1}{z^2} \\ \quad = \frac{1}{z^2} \cdot [-z - \ln|z-1|] = -\frac{1}{z} - \frac{\ln(1-z)}{z^2}, \quad 0<z<1 \\ f_Z(z) = 0, \quad 其他 \end{cases}$$