5. $E(X_n)=\frac{1}{2}(-\sqrt{\ln n})+\frac{1}{2}\sqrt{\ln n}=0$

要证$\{X_n\}$服从大数定律，即证$\frac{1}{n}\sum_{k=1}^{n}(X_k-E(X_k))\xrightarrow{P}0$

$\Rightarrow \lim_{n\to\infty}P(|\frac{1}{n}\sum_{k=1}^{n}(X_k-E(X_k))-0|\geq\varepsilon)=0$

$\Rightarrow \lim_{n\to\infty}P(|\frac{1}{n}\sum_{k=1}^{n}X_k-E(\frac{1}{n}\sum_{k=1}^{n}X_k)-0|\geq\varepsilon)\leq\lim_{n\to\infty}\frac{D(\frac{1}{n}\sum_{k=1}^{n}X_k)}{\varepsilon^2}=\lim_{n\to\infty}\frac{D(\sum_{k=1}^{n}X_k)}{n^2\varepsilon^2}$

$\therefore$ 只需证 $\lim_{n\to\infty}\frac{D(\sum_{k=1}^{n}X_k)}{n^2}=0$

$D(\sum_{k=1}^{n}X_k)=\sum_{k=1}^{n}D(X_k)$ （相互独立）

$D(X_k)=E(X_k^2)-[E(X_k)]^2=\ln k$

$\therefore \sum_{k=1}^{n}D(X_k)=\sum_{k=1}^{n}\ln k\leq n\ln n \quad \Rightarrow \lim_{n\to\infty}\frac{D(\sum_{k=1}^{n}X_k)}{n^2}\leq\lim_{n\to\infty}\frac{\ln n}{n}=0$，得证

6. 要证 $\frac{b-a}{n}\sum_{i=1}^{n}f(x_i) \xrightarrow{P} \int_a^b f(x)dx$

即证 $\lim_{n\to\infty}P(|\frac{b-a}{n}\sum_{i=1}^{n}f(x_i)-\int_a^b f(x)dx|<\varepsilon)=1$ 对于 $\forall\varepsilon>0$

即证 $\lim_{n\to\infty}P(|\frac{1}{n}\sum_{i=1}^{n}(b-a)f(x_i)-\int_a^b f(x)dx|<\varepsilon)=1$

$X_n \sim U(a,b)$ $\therefore h(x)=\begin{cases}\frac{1}{b-a}, & a<x<b \\ 0, & 其他\end{cases}$

则 $E((b-a)f(x_i))=(b-a)E(f(x_i))=(b-a)\int_a^b f(x)\frac{1}{b-a}dx=\int_a^b f(x)dx$

$\because\{X_n\}$ 独立同分布，$\{(b-a)f(X_n)\}$ 独立同分布

且 $E((b-a)f(x_i))=\int_a^b f(x)dx$

$\therefore$ 根据 khintchine 大数定律，有 $\frac{1}{n}\sum_{i=1}^{n}(b-a)f(x_i)\xrightarrow{P}\int_a^b f(x)dx$

即 $\frac{b-a}{n}\sum_{i=1}^{n}f(x_i)\xrightarrow{P}\int_a^b f(x)dx$

7. 要证 $\lim_{n\to\infty}P(|\frac{1}{n}\sum_{i=1}^{n}x_i-\mu|<\varepsilon)=1$ 即证 $\lim_{n\to\infty}P(|\frac{1}{n}\sum_{i=1}^{n}x_i-\mu|\ge\varepsilon)=0$

而上式 $\le\lim_{n\to\infty}\frac{D(\sum_{i=1}^{n}x_i)}{n^2\varepsilon^2}$，而 $D(\sum_{i=1}^{n}x_i)=\sum_{i=1}^{n}D(x_i)+2\sum_{1\le i<j\le n}cov(x_i,x_j)$

$\because D(X_i)=\sigma^2<+\infty$，$cov(x_i,x_j)=0$，$|i-j|\ge2$ 时

$\therefore$ 原式 $=n\sigma^2+2\sum_{i=1}^{n-1}cov(x_i,x_{i+1})\le n\sigma^2+2\cdot\sum_{i=1}^{n-1}\sqrt{D(X_i)D(X_{i+1})}$

$=n\sigma^2+2(n-1)\cdot\sigma^2=(3n-2)\sigma^2$

故 $\lim_{n\to\infty}P(|\frac{1}{n}\sum_{i=1}^{n}x_i-\mu|\ge\varepsilon)\le\lim_{n\to\infty}\frac{(3n-2)\sigma^2}{n^2\varepsilon^2}=0$，得证

8. (1) 记 X_i 为第 i 户的每日用电量，求 $P(\sum_{i=1}^{7500}x_i>76000)$

$E(x_i)=10$，$D(x_i)=33.3$

$\because\{x_i\}$ 独立同分布 $\therefore\sum_{i=1}^{7500}x_i\sim N(75000,250000)$

$F(x)=\phi(\frac{x-\mu}{\sigma})$ $\therefore P(\sum_{i=1}^{7500}x_i>76000)=1-P(\sum_{i=1}^{7500}x_i\le76000)$

$=1-\phi(\frac{76000-75000}{500})=1-\phi(2)=0.0228$

(2) 设供应 a，则有 $P(\sum_{i=1}^{7500}x_i>a)=1-P(\sum_{i=1}^{7500}x_i\le a)=1-\phi(\frac{a-75000}{500})\le0.001$

$\Rightarrow\phi(\frac{a-75000}{500})\ge0.999\Rightarrow\frac{a-75000}{500}\ge3.08\Rightarrow a\ge76540\,kw\cdot h$

9. (1) $E(X)=\sum_{i=1}^{3}P(x_i)x_i=1$

$D(X)=E(X^2)-[E(X)]^2=0.4$

$\therefore P(|\sum_{i=1}^{52}x_i-\sum_{i=1}^{52}E(x)|\geq 10)\leq \frac{D(\sum_{i=1}^{52}x_i)}{100}=0.208$

$\therefore P(42<\sum_{i=1}^{52}x_i<62)=1-P(|\sum_{i=1}^{52}x_i-52|\geq 10)\geq 0.792$

(2) $\sum_{i=1}^{52}x_i\sim N(52,20.8)$, $F(\sum_{i=1}^{52}x_i)=\Phi(\frac{\sum_{i=1}^{52}x_i-52}{\sqrt{20.8}})$

$\therefore P(42<\sum_{i=1}^{52}x_i<62)=\Phi(\frac{10}{\sqrt{20.8}})-\Phi(\frac{-10}{\sqrt{20.8}})=0.9714$

10. $P(|\frac{\xi_n}{n}-p|<\varepsilon)=P(-\varepsilon<\frac{\xi_n-np}{n}<\varepsilon)=P(-\varepsilon\frac{\sqrt{n}}{\sqrt{p(1-p)}}<\frac{\xi_n-np}{\sqrt{np(1-p)}}<\varepsilon\sqrt{\frac{n}{p(1-p)}})$

由 De Moivre-Laplace 中心极限定理得：

上式 $=\int_{-\varepsilon\sqrt{\frac{n}{p(1-p)}}}^{\varepsilon\sqrt{\frac{n}{p(1-p)}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx=\Phi(\varepsilon\sqrt{\frac{n}{p(1-p)}})-\Phi(-\varepsilon\sqrt{\frac{n}{p(1-p)}})$

$\because \Phi(-\varepsilon\sqrt{\frac{n}{p(1-p)}})=1-\Phi(\varepsilon\sqrt{\frac{n}{p(1-p)}})$

$\therefore$ 上式 $=2\Phi(\varepsilon\sqrt{\frac{n}{p(1-p)}})-1$

11. (1) $f(x)=\begin{cases}kx, & x\in[0,30)\\ k(60-x), & x\in[30,60]\\ 0, & \text{其他}\end{cases}$

$\int_0^{30}kx\,dx+\int_{30}^{60}k(60-x)dx=\frac{k}{2}\times 900+1800k-\frac{2700}{2}k=1$

$k=\frac{1}{900}$

(2) $E(X)=\int_0^{60}xf(x)dx=30$

$D(X)=E(X^2)-[E(X)]^2=1050-900=150$

$P(|\sum_{i=1}^{200}x_i-E(\sum_{i=1}^{200}x_i)|\geq 200)\leq\frac{D(\sum_{i=1}^{200}x_i)}{40000}=0.75$

$\therefore P(|\sum_{i=1}^{200}x_i-E(\sum_{i=1}^{200}x_i)|<200)=P(5800<\sum_{i=1}^{200}x_i<6200)\geq 1-0.75=0.25$

(3) $\sum_{i=1}^{200}x_i\sim N(6000,30000)$, $F(\sum_{i=1}^{200}x_i)=\Phi(\frac{\sum_{i=1}^{200}x_i-6000}{\sqrt{30000}})$

$\therefore P(5800<\sum_{i=1}^{200}x_i<6200)=\Phi(\frac{200}{\sqrt{30000}})-\Phi(\frac{-200}{\sqrt{30000}})=2\Phi(1.15)-1=0.7498$

补充题：$X_i \sim G(0.75)$，则 $E(X_i)=\frac{4}{3}$，$D(X_i)=\frac{4}{9}$

$Y_i = 2^{X_i}$，$E(Y_i)=\sum_{k=1}^{\infty} P(X_i=k)\cdot 2^k = \sum_{k=1}^{\infty}(1-\frac{3}{4})^{k-1}\cdot\frac{3}{4}\cdot 2^k$

$=3\sum_{k=1}^{\infty}(\frac{1}{4})^k\cdot 2^k = 3\sum_{k=1}^{\infty}(\frac{1}{2})^k = 3$

$\because Y_1, Y_2, \ldots Y_n$ 独立同分布 且 $E(Y_i)=3$

由 khintchine 大数定律：$\frac{1}{n}\sum_{i=1}^{n} Y_i \xrightarrow{P} E(Y)=3$