9. $X_{i+1}-X_i \sim N(0, 2\sigma^2)$

$E[(X_{i+1}-X_i)^2] = D(X_{i+1}-X_i) = 2\sigma^2$

$E\left(c\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right) = c\cdot(n-1)\cdot 2\sigma^2 = \sigma^2$

$\therefore c = \frac{1}{2(n-1)}$

10. $E(\bar{X}) = \lambda$, $D(\bar{X}) = \frac{\lambda}{n}$, $E(\bar{X}^2) = \lambda^2 + \frac{\lambda}{n}$

$\therefore \lambda^2 = E(\bar{X}^2) - D(\bar{X})$, $E(\bar{X}^2) - \frac{E(\bar{X})}{n} = \lambda^2$

$\therefore$ 无偏估计量: $\bar{X}^2 - \frac{\bar{X}}{n}$

14. $D(S_1^2) = D\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2\right)$

而 $\frac{X_i-\mu}{\sigma} \sim N(0,1)$ $\therefore \sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma^2} \sim \chi^2(n)$

$\therefore D\left(\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma^2}\right) = 2n$, $D\left(\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2\right) = \frac{2\sigma^4}{n}$

$D(S_2^2) = D\left(\frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2\right)$

又 $\sum_{i=1}^{n}\frac{(X_i-\bar{X})^2}{\sigma^2} \sim \chi^2(n-1)$

$\therefore D\left(\frac{(n-1)S_2^2}{\sigma^2}\right) = 2(n-1)$

$\therefore D(S_2^2) = \frac{2}{n-1}\cdot\sigma^4 > D(S_1^2)$

$\therefore S_1^2$ 有效

x^2 $n \cdot \frac{x^{2n-1}}{\theta^{2n-2}}$ $\frac{2}{3}\frac{x^3}{\theta^2}$ $\frac{2x^2 n}{2n+1}$ θ^3 D 1 x^{2n-1} $\frac{n!}{2n} \cdot \frac{x^{2n}}{\theta^{2n-2}}$

15. $a+b=1$, a^2+b^2 最小, $a=b=\frac{1}{2}$

16. $E(\bar{X}) = E(X) = \mu$, 无偏

$g(x_i) = \frac{\partial \ln p(x_i;\mu)}{\partial \mu} = \frac{x-\mu}{\sigma^2}$,

$E[g(x)^2] = \frac{1}{\sigma^2} \cdot E\left[\left(\frac{x-\mu}{\sigma}\right)^2\right] = \frac{1}{\sigma^2}$

$\therefore D(\mu|\sigma^2) \geq \frac{\sigma^2}{n}$

而 $D(\bar{X}) = \frac{\sigma^2}{n}$ $\therefore$ ~~书~~ $\bar{X}$ 是 μ 的有效估计值

x^2

补：$F(x) = \int_0^x \frac{2x}{\theta^2} dx = \frac{x^2}{\theta^2}$ $(0<x<\theta)$

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^2}{\theta^2}, & 0<x<\theta \\ 1, & \theta \leq x \end{cases}$$

$$\cancel{F(x)}\ F_{X(n)}(x) = F^n(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^{2n}}{\theta^{2n}}, & 0<x<\theta \\ 1, & \theta \leq x \end{cases}$$

$f_{X(n)}(x) = n \cdot \frac{x^{2n-2}}{\theta^{2n-2}}$, $0<x<\theta$

$E(X_{(n)}(x)) = \int_0^\theta x \cdot n \cdot \frac{x^{2n-2}}{\theta^{2n-2}} dx = \frac{\theta^2}{2}$

$\tilde{\theta} = 2X_{(10)}$ 为 θ^2 的无偏估计量

由 $\lim\limits_{n\to\infty} D(2X_{(10)}(x)) = 4 \cdot \lim\limits_{n\to\infty} D(X_{(10)}(x)) = 4\lim\limits_{n\to\infty}\left(\frac{n}{2n+1}\theta^3 - \frac{\theta^4}{4}\right) = 0$

$\therefore P(|\tilde{\theta} - \theta^2| \geq \varepsilon) \leq \frac{D(\tilde{\theta})}{\varepsilon^2} = 0$ $\therefore$ 一致