2-1. $\hat{H}\Psi(\vec{r}) = E\Psi(\vec{r})$

当 $E > V_0$ 时，有：

$$\begin{cases} \dfrac{d^2\psi}{dx^2} + \dfrac{2mE}{\hbar^2}\psi = 0, & x<0 \\ \dfrac{d^2\psi}{dx^2} + \dfrac{2m}{\hbar^2}(E-V_0)\psi = 0, & x\geq 0 \end{cases}$$

解为：
$$\begin{cases} x<0 & \psi_1 = Ae^{ikx} + A'e^{-ikx}, \quad k = \dfrac{\sqrt{2mE}}{\hbar} \\ x\geq 0 & \psi_2 = Be^{ik'x} + B'e^{-ik'x}, \quad k' = \dfrac{\sqrt{2m(E-V_0)}}{\hbar} \end{cases}$$

由于透射后的波在 $x\geq 0$ 处无反射，故 $B'=0$

由 0 处的连续性条件可得：$(\psi_1(0) = \psi_2(0),\ \psi_1'(0) = \psi_2'(0))$

$$\begin{cases} A + A' = B \\ ik(A-A') = ik'B \end{cases} \Rightarrow \begin{cases} R = \dfrac{|A'|^2}{|A|^2} = \dfrac{\left|1-\frac{k'}{k}\right|^2}{\left|1+\frac{k'}{k}\right|^2} = \dfrac{V_0^2}{(\sqrt{E}+\sqrt{E-V_0})^4} \\ T = 1-R = 1-\dfrac{V_0^2}{(\sqrt{E}+\sqrt{E-V_0})^4} \end{cases}$$

当 $V_0>E>0$ 时，有：

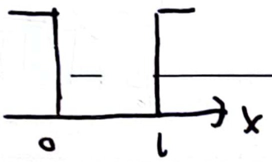
$$\begin{cases} \dfrac{d^2\psi}{dx^2}+\dfrac{2mE}{\hbar^2}\psi=0, & x<0 \\ \dfrac{d^2\psi}{dx^2}+\dfrac{2m}{\hbar^2}(E-V_0)\psi=0, & x\geq 0 \end{cases}$$

解为
$$\begin{cases} x<0 & \psi_1=Ae^{ikx}+A'e^{-ikx}, \quad k=\dfrac{\sqrt{2mE}}{\hbar} \\ x\geq 0 & \psi_2=Be^{-k'x}+B'e^{-k'x}, \quad k'=\dfrac{\sqrt{2m(V_0-E)}}{\hbar} \end{cases}$$

同样有 $B'=0$，连续性：

$$\begin{cases} A+A'=B \\ ik(A-A')=-k'B \end{cases} \Rightarrow \begin{cases} R=\dfrac{|A'|^2}{|A|^2}=\dfrac{|1+i\frac{k'}{k}|^2}{|1-i\frac{k'}{k}|^2}=1 \\ T=1-R=0 \end{cases}$$

2-4.



(1) $\int_0^l |\psi_n(x)|^2 dx = \int_0^l A^2\sin^2\frac{n\pi x}{l}dx = \frac{l}{n\pi}\cdot A^2\int_0^{n\pi}\sin^2 t\, dt$

$= \frac{lA^2}{n\pi}\cdot\left(\frac{n\pi}{2}-\frac{1}{4}\sin 2n\pi\right) = \frac{lA^2}{2}=1, \quad A=\sqrt{\frac{2}{l}}$

(2) $\frac{d^2\psi}{dx^2}+\frac{2mE}{\hbar^2}\psi=0, \quad 0\leq x\leq l$ 势阱中

此处 $k=\frac{n\pi}{l}$ 已知，$E_n=\frac{p^2}{2m}=\frac{k^2\hbar^2}{2m}=n^2\cdot\frac{\pi^2\hbar^2}{2ml^2}$

(3) $n=3\longrightarrow n=1$，$\Delta E=E_3-E_1=\frac{4\pi^2\hbar^2}{ml^2}=h\frac{c}{\lambda}$

$\lambda=\frac{hc\cdot ml^2}{4\pi^2\hbar^2}=\frac{cml^2}{2\pi\hbar}$