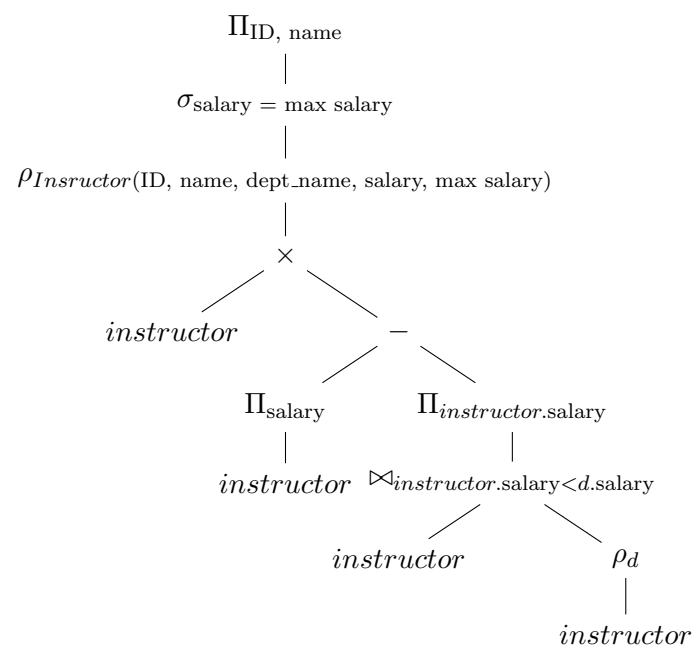


# AI3613 Database Principles: Homework 1

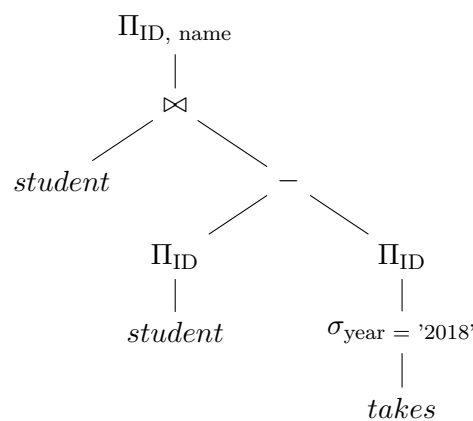
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## Problem 1

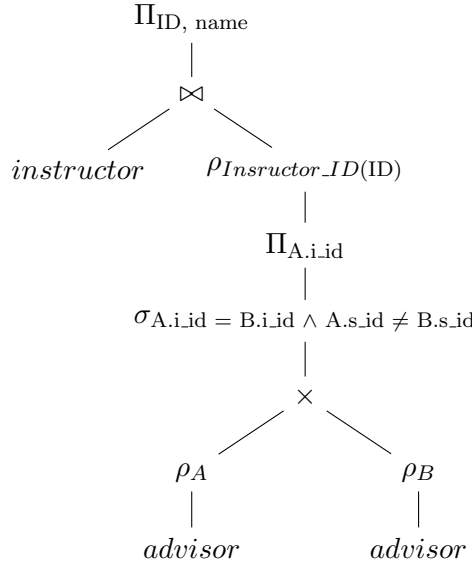
1. Below is the relational algebra expression tree that achieves to retrieve every instructor whose salary is the highest.



2. Below is the relational algebra expression tree that achieves to retrieve every student who hasn't taken any course in 2018.



3. Below is the relational algebra expression tree that achieves to retrieve every instructor who has advised more than one student.



## Problem 2

The final RA expression is:

$$\Pi_{A_1, \dots, A_n}(R) - \Pi_{A_1, \dots, A_n}(\Pi_{A_1, \dots, A_n}(R) \times S - R)$$

The RA expression first finds the combinations of all possible (A, B) that are missing in R. Then, project the missing combinations onto  $A_1, \dots, A_n$ , which is finally used to find the combinations that share  $B_1, \dots, B_n$  in both R and S.

Meanwhile, the RA expression could also be formulated as:

$$\Pi_{A_1, \dots, A_n}(R - (\Pi_{A_1, \dots, A_n}(R) \times S - R))$$

However, the efficiency decreases when  $m$  is large enough for  $B_1, B_2, \dots, B_m$  as the operation Difference checks every  $B$  before the final projection.

## Problem 3

1. First, find all lines that share a vertex:

$$E^2 = \rho_{E^2(u,v,w)}(\Pi_{E.src\_id, E.dst\_id, E'.dst\_id}(E \bowtie_{E.dst\_id=E'.src\_id} \rho_{E'}(E)))$$

Then, find the triangles in  $E^2$ :

$$Triangles = \rho_{Triangles(u,v,w)}(\Pi_{E^2.u, E^2.v, E^2.w}(\sigma_{E^2.u=E.dst\_id}(E^2 \bowtie_{E^2.w=E.src\_id} E)))$$

2. First, find all  $(u, v)$  where  $v$  is reachable from  $u$  within 1 hop to avoid self-loop.

$$\sigma_{src\_id \neq dst\_id}(E)$$

Then, find all  $(u, v)$  where  $v$  is reachable from  $u$  with 2 hops.

$$E^2 = \rho_{E^2(src\_id, dst\_id)}(\Pi_{E.src\_id, E'.dst\_id}(E \bowtie_{E.dst\_id=E'.src\_id \wedge E.src\_id \neq E'.dst\_id} \rho_{E'}(E)))$$

Next, find all  $(u, v)$  where  $v$  is reachable from  $u$  with 3 hops.

$$E^3 = \rho_{E^3(src\_id, dst\_id)}(\Pi_{E^2.src\_id, E.dst\_id}(E^2 \bowtie_{E^2.dst\_id=E.src\_id \wedge E^2.src\_id \neq E.dst\_id} E))$$

Finally, find all  $(u, v)$  where  $v$  is reachable from  $u$  within 3 hops using the above three relations.

$$R1 = \rho_{R1(u, v)}(\sigma_{src\_id \neq dst\_id}(E) \cup E^2 \cup E^3)$$

3. **No.** Rational Algebra does not include recursion operation, which limits RA expression to only find  $(u, v)$  where  $v$  is reachable from  $u$  within **finite** hops. However, find all  $(u, v)$  in a graph with unbounded path lengths could be an infinite process. That explains why there is no RA expression to find all  $(u, v)$ .

## Question 4

It takes me half a day to finish the assignment.

Question 1: 4/5

Question 2: 3/5

Question 3: 4/5