1-21. $\begin{cases} i\hbar\frac{\partial}{\partial t}\psi_1 = [-\frac{\hbar^2}{2m}\nabla^2+U(\vec{r},t)]\psi_1 \\ i\hbar\frac{\partial}{\partial t}\psi_2 = [-\frac{\hbar^2}{2m}\nabla^2+U(\vec{r},t)]\psi_2 \end{cases}$

$\Rightarrow c_1\psi_1+c_2\psi_2$:

$i\hbar\frac{\partial}{\partial t}(c_1\psi_1+c_2\psi_2) = i\hbar c_1\frac{\partial}{\partial t}\psi_1 + i\hbar c_2\frac{\partial}{\partial t}\psi_2$ (微分线性)

$[-\frac{\hbar^2}{2m}\nabla^2+U(\vec{r},t)](c_1\psi_1+c_2\psi_2) = c_1[-\frac{\hbar^2}{2m}\nabla^2+U]\psi_1 + c_2(-\frac{\hbar^2}{2m}\nabla^2+U)\psi_2$

$i\hbar\frac{\partial}{\partial t}(c_1\psi_1+c_2\psi_2) - [-\frac{\hbar^2}{2m}\nabla^2+U(\vec{r},t)](c_1\psi_1+c_2\psi_2)$ (算符线性)

$= c_1[i\hbar\frac{\partial}{\partial t}\psi_1 - (-\frac{\hbar^2}{2m}\nabla^2+U(\vec{r},t))\psi_1] + c_2[i\hbar\frac{\partial}{\partial t}\psi_1 - (-\frac{\hbar^2}{2m}\nabla^2+U)\psi_2]$

$=0 \Rightarrow i\hbar\frac{\partial}{\partial t}(c_1\psi_1+c_2\psi_2) = [-\frac{\hbar^2}{2m}\nabla^2+U(\vec{r},t)](c_1\psi_1+c_2\psi_2)$

是解

1-22. $\psi = Ae^{i(kx-\omega t)}$, $\psi^* = Ae^{i(-kx+\omega t)}$

$\frac{\partial\psi}{\partial x} = ikAe^{i(kx-\omega t)}$ $\quad\frac{\partial\psi^*}{\partial x} = A(-ik)e^{i(-kx+\omega t)}$

$j_x(x,t) = \frac{\hbar}{2mi}(A^2 ik + ikA^2) = \frac{\hbar A^2 k}{m}$

1-23. $\int_{-\infty}^{\infty}|\psi(x)|^2dx = \int_{-\infty}^{\infty}A^2e^{-a^2x^2}dx = \frac{A^2}{a}\int_{-\infty}^{\infty}e^{-a^2x^2}d(ax)$

$= \frac{A^2}{a}\cdot\sqrt{\pi} = 1$, $A = \sqrt{\frac{a}{\sqrt{\pi}}}$

1-24(1) $\int_{-\infty}^{\infty}|\psi(x)|^2dx = 2\int_0^{\infty}A^2e^{-2kx}dx = 2A^2\cdot\frac{1}{2k}\int_0^{\infty}e^{-2kx}d(2kx)$

$= \frac{A^2}{k} = 1 \Rightarrow A = \sqrt{k}$

$\psi = \begin{cases} \sqrt{k}e^{kx}, & x<0 \\ \sqrt{k}e^{-kx}, & x\geq 0 \end{cases}$

(2) $2\int_0^{\frac{1}{k}}ke^{-2kx}dx = 1-e^{-2}$

- $\vec{J} = -\frac{i\hbar}{2m}(\psi^*\nabla\psi - \psi\nabla\psi^*) = \frac{1}{2m}(\psi^*\hat{p}\psi - \psi\hat{p}\psi^*)$

由 $v = \frac{p}{m}$，所以是两个方向流动的结果

而 $\psi^*\psi = |\psi|^2$ 概率密度 → 速×概率密 → $m^{-2}\cdot s^{-1}$

→ 由 $\frac{\partial\rho}{\partial t} + \nabla J_m = 0$ → $m\cdot m^{-3}\cdot s^{-1} = m^{-2}\cdot s^{-1}$

- $\lambda = \frac{h}{p}$，$p = mv = 2.4\times10^{-22}\ kg\cdot m\cdot s^{-1}$

光栅 $d = 100\ nm$，$d\sin\theta = \lambda$，宽度 $\approx L\cdot\sin\theta \approx \frac{L\lambda}{d} \approx 3.45\times 10^{-5}$

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