

| 1. $P_{ij}$              |   | X              |                 |                 |                | $P_{.j} = \sum_i P_{ij}$ |
|--------------------------|---|----------------|-----------------|-----------------|----------------|--------------------------|
|                          |   | 0              | 1               | 2               | 3              |                          |
| Y                        | 0 | 0              | 0               | $\frac{2}{35}$  | $\frac{2}{35}$ | $\frac{1}{7}$            |
|                          | 1 | 0              | $\frac{6}{35}$  | $\frac{12}{35}$ | $\frac{2}{35}$ | $\frac{4}{7}$            |
|                          | 2 | $\frac{1}{35}$ | $\frac{6}{35}$  | $\frac{3}{35}$  | 0              | $\frac{2}{7}$            |
| $P_{i.} = \sum_j P_{ij}$ |   | $\frac{1}{35}$ | $\frac{12}{35}$ | $\frac{18}{35}$ | $\frac{4}{35}$ | 1                        |

| (1) 3. $P_{ij}$ |       | X                  |                    |     |                    |                    | $P_{.j}$      |
|-----------------|-------|--------------------|--------------------|-----|--------------------|--------------------|---------------|
|                 |       | 1                  | 2                  | ... | $n-1$              | $n$                |               |
| Y               | 1     | 0                  | $\frac{1}{n(n-1)}$ | ... | $\frac{1}{n(n-1)}$ | $\frac{1}{n(n-1)}$ | $\frac{1}{n}$ |
|                 | 2     | $\frac{1}{n(n-1)}$ | 0                  |     | $\frac{1}{n(n-1)}$ | $\frac{1}{n(n-1)}$ | $\frac{1}{n}$ |
|                 | ...   | ...                | ...                |     | ...                | ...                | ...           |
|                 | $n-1$ | $\frac{1}{n(n-1)}$ | $\frac{1}{n(n-1)}$ |     | 0                  | $\frac{1}{n(n-1)}$ | $\frac{1}{n}$ |
|                 | $n$   | $\frac{1}{n(n-1)}$ | $\frac{1}{n(n-1)}$ | ... | $\frac{1}{n(n-1)}$ | 0                  | $\frac{1}{n}$ |
| $P_{i.}$        |       | $\frac{1}{n}$      | $\frac{1}{n}$      | ... | $\frac{1}{n}$      | $\frac{1}{n}$      | 1             |

| (2) $P_{ij}$ |   | X             |               |               | $P_{.j}$      |
|--------------|---|---------------|---------------|---------------|---------------|
|              |   | 1             | 2             | 3             |               |
| Y            | 1 | 0             | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{3}$ |
|              | 2 | $\frac{1}{6}$ | 0             | $\frac{1}{6}$ | $\frac{1}{3}$ |
|              | 3 | $\frac{1}{6}$ | $\frac{1}{6}$ | 0             | $\frac{1}{3}$ |
| $P_{i.}$     |   | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 1             |

|       |          |               |               |               |               |  |  |  |
|-------|----------|---------------|---------------|---------------|---------------|--|--|--|
| 4.    | $P_{ij}$ | $Y_1$         |               |               | $P_{.j}$      |  |  |  |
|       |          | 0             | 1             | 2             |               |  |  |  |
|       | $-1$     | 0             | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{2}{3}$ |  |  |  |
| $Y_2$ | 1        | $\frac{1}{3}$ | 0             | 0             | $\frac{1}{3}$ |  |  |  |
|       | $P_{i.}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 1             |  |  |  |

  

|       |               |               |               |
|-------|---------------|---------------|---------------|
| $Y_1$ | 0             | 1             | 2             |
| $P$   | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

  

|       |               |               |
|-------|---------------|---------------|
| $Y_2$ | -1            | 1             |
| $P$   | $\frac{2}{3}$ | $\frac{1}{3}$ |

5. (1)  $\int_0^{+\infty} \int_0^{+\infty} k e^{-2x-4y} dx dy = 1 \Rightarrow k=8$

(2)  $P(0 \leq X \leq 2, 0 < Y \leq 1) = \int_0^2 \int_0^1 8e^{-2x-4y} dx dy$   
 $= (e^{-4}-1)^2 = 0.9637$

(3)  $P(X+Y < 1) = \int_0^1 dx \int_0^{1-x} 8e^{-2x-4y} dy = 1 - 2e^{-2} + e^{-4}$

(4) 当  $x$  或  $y$  不全大于 0 时  $F(x, y) = 0$

当  $x > 0, y > 0$  时  $F(x, y) = \int_0^y \int_0^x 8e^{-2x-4y} dy dx =$   
 $1 + e^{-2x-4y} - e^{-4y} - e^{-2x} = (1 - e^{-4y})(1 - e^{-2x})$

6.  $P(X > 100, Y > 100) = 1 - P(X \leq 100) - P(Y \leq 100) + P(X < 100, Y < 100)$

$F_X(x) = F(x, +\infty) = 1 - e^{-0.01x}, x \geq 0$

$F_Y(y) = F(+\infty, y) = 1 - e^{-0.01y}, y \geq 0$

$\therefore$  上式  $= 1 - (1 - e^{-1}) - (1 - e^{-1}) + (1 - e^{-1} \times 2 + e^{-2})$

7. (1)  $\int_0^2 dx \int_0^{\frac{x}{2}} kx dy = \frac{4}{3}k = 1, k = \frac{3}{4}$

(2)  $P(X+Y \leq 2) = \int_0^{\frac{2}{3}} dy \int_{2-y}^2 \frac{3}{4}x dx = \frac{5}{9}$

(3)  $f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$

当  $x$  不在  $[0, 2]$  时,  $f_X(x) = 0$ , 当  $x \in [0, 2]$  时

$f_X(x) = \int_0^{\frac{x}{2}} \frac{3}{4}x dy = \frac{3}{8}x^2$

$$\frac{21}{8}x^2y^2 \Big|_{x^2}$$

$$\frac{3}{8}x^2 \quad \frac{3}{4}$$

$$\frac{21}{12} \quad \frac{7}{4}x^3y \Big|_{-1}^{1}$$

$$\frac{21}{8}x^2y^2 \Big|$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx, \text{ 当 } y \notin [0, 1] \text{ 时, } f_Y(y) = 0$$

$$\text{当 } y \in [0, 1] \text{ 时, } f_Y(y) = \int_{-y}^y \frac{3}{4}x dx = \frac{3}{2} - \frac{3}{2}y^2$$

$$8. (1) f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$\text{当 } x \notin [-1, 1] \text{ 时 } f_X(x) = 0$$

$$\text{当 } x \in [-1, 1] \text{ 时, } f_X(x) = \int_{x^2}^1 \frac{21}{4}x^2y dy = \frac{21}{8}x^2 - \frac{21}{8}x^4$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$\text{当 } y \notin [0, 1] \text{ 时, } f_Y(y) = 0$$

$$\text{当 } y \in [0, 1] \text{ 时, } f_Y(y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{21}{4}x^2y dx = \frac{7}{2}y^{\frac{5}{2}}$$

$$(2) P(X \geq Y) = \int_0^1 dx \int_{x^2}^x \frac{21}{4}x^2y dy = \frac{3}{20}$$