21. (1) $P(A)=\sum_{i=0}^{3}P(B_i)P(A|B_i)$ ，B_i为击中i次，A为摧毁

$=C_3^1\times0.4\times0.6^2\times0.2+C_3^2\times0.4^2\times0.6\times0.5+0.4^3\times0.8$

$=0.2816$

(2) $P(B_3|A)=\dfrac{P(B_3A)}{P(A)}=\dfrac{0.4^3\times0.8}{0.2816}\approx0.1818$

25. (1) $P=C_5^2\times0.46^2\times0.54^3\approx0.33$

(2) $P=C_5^3\times0.46^3\times0.4^2\approx0.16$

(3) $P=0.97^5\approx0.86$

26. (1) $P=0.5\times(1-0.7)\times(1-0.6)+(1-0.5)\times0.7\times(1-0.6)+(1-0.5)\times$

$(1-0.7)\times0.6=0.29$

(2) $P=(1-0.5)\times0.7\times0.6+0.5\times(1-0.7)\times0.6+0.5\times0.7\times(1-0.6)$

$=0.44$

(3) $P=1-(1-0.5)(1-0.7)(1-0.6)=0.94$

1. $\lim\limits_{x\to+\infty}\arctan x=\dfrac{\pi}{2}$ $\therefore a+\dfrac{\pi}{2}\cdot b=1$

又有 $a-\dfrac{\pi}{2}b=0$， $\begin{cases}a=\dfrac{1}{2}\\ b=\dfrac{1}{\pi}\end{cases}$

2. $\lim\limits_{x\to+\infty}F(x) = a\lim\limits_{x\to+\infty}G(x) + b\lim\limits_{x\to+\infty}H(x)$

∵ $G(x)$，$H(x)$ 都是随机变量分布函数

∴ $\lim\limits_{x\to+\infty}G(x) = \lim\limits_{x\to+\infty}H(x) = 1$，$\lim\limits_{x\to-\infty}G(x) = \lim\limits_{x\to-\infty}H(x) = 0$

∴ $\lim\limits_{x\to+\infty}F(x) = a+b = 1$，$\lim\limits_{x\to-\infty}F(x) = a\cdot 0 + b\cdot 0 = 0$

∵ $G(x)$，$H(x)$ 单调不减

∴ $F(x+\Delta x) = aG(x+\Delta x) + bH(x+\Delta x) \geq aG(x) + bH(x) = F(x)$

∴ $F(x)$ 单调不减

$G(x)$，$H(x)$ 右连续，$G(x) = G(x+0)$，$H(x) = H(x+0)$

∴ $F(x+0) = aG(x+0) + bH(x+0) = aG(x) + bH(x) = F(x)$

∴ $F(x)$ 右连续

∴ 也是分布函数

4. 设二等有 $\overset{\frac{6}{6}}{x}$ 个，则一等有 $\overset{\frac{18}{6}}{3x}$ 个，三等有 $\frac{1}{6}x$ 个

分布律：
$$\begin{cases} P(X=1) = \frac{18}{25} \\ P(X=2) = \frac{6}{25} \\ P(X=3) = \frac{1}{25} \end{cases}$$

分布函数：
$$F(x) = \begin{cases} 0, & x<1 \\ \frac{18}{25}, & 1\leq x<2 \\ \frac{24}{25}, & 2\leq x<3 \\ 1, & 3\leq x \end{cases}$$

① $P(1<X\leq 3) = P(X=2) + P(X=3) = \frac{7}{25}$

② $P(1<X\leq 3) = P(1<X\leq 2) + P(2<X\leq 3) = F(2)-F(1)+F(3)-F(2)$
$= F(3)-F(1) = \frac{7}{25}$

5. $$F(x)=\begin{cases} 0, & x<7 \\ \frac{1}{20}, & 7\le x<9 \\ \frac{1}{5}, & 9\le x<13 \\ \frac{1}{2}, & 13\le x<18 \\ 1, & 18\le x \end{cases}$$

$P(X=7)=F(7)-F(7-0)=\frac{1}{20}$

$P(2<X<7)=F(7-0)-F(2)=0$

$P(7\le X<13)=F(13-0)-F(7-0)=\frac{1}{5}$

8. 第1名：令$X=n$为投了n次

则 $P(X=n)=(1-0.4)^{n-1}(1-0.6)^{n-1}\cdot 0.4+(1-0.4)^{n}\cdot(1-0.6)^{n-1}\cdot 0.6$

$=0.24^{n-1}\times 0.4+0.24^{n-1}\times 0.36=0.76\times 0.24^{n-1}$

第2名：$P(X=n)=(1-0.4)^{n}(1-0.6)^{n}\cdot 0.4+(1-0.4)^{n}\cdot(1-0.6)^{n-1}\cdot 0.6$

$=0.24^{n}\times 0.4+0.24^{n-1}\times 0.36=0.24^{n-1}\times 0.456$

11. 二项分布 $X\sim B(15,0.2)$

$P(X=k)=C_{15}^{k}\cdot 0.2^{k}\cdot 0.8^{15-k}$

(1) $C_{15}^{3}\times 0.2^{3}\times 0.8^{12}=0.2501$

(2) $1-0.8^{15}-C_{15}^{1}\times 0.2\times 0.8^{14}\approx 0.8329$

(3) $P(X=1)+P(X=2)+P(X=3)=0.6130$

(4) $1-P(X=0)-P(X=1)-P(X=2)-P(X=3)-P(X=4)-P(X=5)$

$=0.061$

13. 设A为带菌，B为阳，C为所求事件

(1) $P(C)=P(C|A)\cdot P(A)+P(C|\bar{A})P(\bar{A})$

$=C_3^2\times 0.95^2\times 0.05\times 0.1+C_3^2\times 0.01^2\times 0.99\times 0.9$

$=0.0138$

(2) $P(A|C)=\dfrac{P(AC)}{P(C)}=\dfrac{P(A)\cdot P(C|A)}{P(C)}=\dfrac{0.1\times C_3^2\times 0.95^2\times 0.05}{0.0138}$

$=0.98$