26. $E(X)=\int_0^{\infty}\int_0^{\infty} x e^{-(x+y)}dxdy=1$

$E(Y)=\int_0^{\infty}\int_0^{\infty} y e^{-(x+y)}dxdy=1$

$E(X^2)=\int_0^{\infty}\int_0^{\infty} x^2 e^{-(x+y)}dxdy=2$

$E(Y^2)=2$

$D(X)=E(X^2)-[E(X)]^2=1$

$D(Y)=E(Y^2)-[E(Y)]^2=1$

$cov(X,Y)=E(XY)-E(X)E(Y)$

$E(XY)=\int_0^{\infty}\int_0^{\infty} xy e^{-(x+y)}dxdy=1$

$\therefore cov(X,Y)=0$

$\rho_{XY}=\frac{cov(X,Y)}{\sqrt{D(X)D(Y)}}=0$

$C=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

27. $P(X=-1)=\frac{3}{8}$，$P(X=0)=\frac{1}{4}$，$P(X=1)=\frac{3}{8}$

$P(Y=-1)=\frac{3}{8}$，$P(Y=0)=\frac{1}{4}$，$P(X=1)=\frac{3}{8}$

$E(X)=E(Y)=0$，$E(XY)=0\times\frac{1}{2}+1\times\frac{1}{4}+(-1)\times\frac{1}{4}=0$

$\therefore cov(X,Y)=E(XY)-E(X)E(Y)=0$

$E(X^2)=0\times\frac{1}{4}+1\times\frac{3}{4}=\frac{3}{4}=E(Y^2)$

$D(X)=D(Y)=\frac{3}{4}$

$cov(X,Y)=E(XY)-E(X)E(Y)=0$，$\rho_{XY}=\frac{cov(X,Y)}{\sqrt{D(X)D(Y)}}=0$

$\therefore X,Y$ 不相关

$P(X=-1,Y=-1)=\frac{1}{8}\neq P(X=-1)\cdot P(Y=-1)$

$\therefore X,Y$ 不相互独立

28. $E(X)=P(A),\ E(Y)=P(B)$

若X，Y不相关，$cov(X,Y)=E(XY)-E(X)E(Y)=0$

$\Rightarrow E(XY)=E(X)E(Y)=P(A)P(B)$

$\because E(XY)=P(AB)$

$\therefore P(AB)=P(A)P(B)$ 故 X，Y相互独立

29. $E(X)=1,\ E(Y)=0,\ D(X)=9,\ D(Y)=16,\ cov(X,Y)=\rho\sigma_1\sigma_2=-6$

$E(Z)=\frac{1}{3}E(X)+\frac{1}{2}E(Y)=\frac{1}{3}$

$D(Z)=cov(Z,Z)=cov(\frac{1}{3}X+\frac{1}{2}Y,\frac{1}{3}X+\frac{1}{2}Y)=\frac{1}{9}D(X)+\frac{1}{3}cov(X,Y)+\frac{1}{4}D(Y)$

$=1+\frac{1}{3}\times(-6)+4=3$

$cov(X,Z)=\frac{1}{3}D(X)+\frac{1}{2}cov(X,Y)=3-3=0$

$\rho_{XZ}=\frac{cov(X,Z)}{\sqrt{D(X)D(Z)}}=0$

30. $f(x,y)=\begin{cases}\frac{1}{\pi}, & x^2+y^2\le 1\\ 0, & 其他\end{cases}$

$f_X(x)=\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{1}{\pi}dy=\frac{2}{\pi}\sqrt{1-x^2}$

$f_Y(y)=\frac{2}{\pi}\sqrt{1-y^2}$

$E(X)=\int_{-1}^{1}x\frac{2}{\pi}\cdot\sqrt{1-x^2}dx=0$

$E(Y)=0$

$E(XY)=\iint xy\frac{1}{\pi}dxdy=0$

$\therefore cov(X,Y)=E(XY)-E(X)E(Y)=0,\ \rho_{XY}=0$ $\therefore$ X，Y不相关

$f(0,0)=\frac{1}{\pi}\neq f_X(0)\cdot f_Y(0)$ $\therefore$ X，Y不是相互独立

31. $COV(X,X)=1$, $COV(X,Y)=2$, $COV(Y,Y)=5$

$COV(U,V)=2D(X)-5COV(X,Y)+2D(Y)=2$

$D(U)=COV(U,U)=D(X)+4D(Y)-4COV(X,Y)=13$

$D(V)=4D(X)+D(Y)-4COV(X,Y)=1$

$\therefore \rho_{UV}=\frac{COV(U,V)}{\sqrt{D(U)D(V)}}=\frac{2}{\sqrt{13}}=\frac{2\sqrt{13}}{13}$

1. $P(|X-4|\geq 3)\leq \frac{D(X)}{9}=\frac{1}{9}$

$P(1<X<7)=1-P(|X-4|\geq 3)=\frac{8}{9}$

2. $D(X)=490000$

$P(|X-7300|\geq 2100)\leq \frac{D(X)}{2100^2}=\frac{1}{9}$

$P(5200<X<9400)=1-P(|X-7300|\geq 2100)=\frac{8}{9}$