$\frac{E(X)}{2-E(X)}$

1. $P(X=0)=\frac{e^{-\lambda}\lambda^0}{0!}=e^{-\lambda}=\frac{81}{2500}$, $\lambda=\ln\frac{2500}{81}=3.43$

2. $E(X)=\int_1^\theta x\cdot\frac{2\theta^2}{(\theta^2-1)x^3}dx=\frac{2\theta}{\theta+1} \Rightarrow \theta=\frac{2}{2-E(X)}-1=\frac{E(X)}{2-E(X)}$

$\therefore \hat{\theta}=\frac{\bar{X}}{2-\bar{X}}$

3. $E(X)=\lambda$, $\hat{\lambda}=\bar{X}=\frac{2}{5}+\frac{2}{5}+\frac{3}{25}+\frac{2}{25}=1$

$L(\lambda)=\prod_{i=1}^{50}P(x_i;\lambda)=(e^{-\lambda})^{17}\cdot(e^{-\lambda}\cdot\lambda^1)^{20}\cdot(\frac{e^{-\lambda}\cdot\lambda^2}{2})^{10}\cdot(\frac{e^{-\lambda}\cdot\lambda^3}{6})^2\cdot(\frac{e^{-\lambda}\cdot\lambda^4}{24})$

$\ln L(\lambda)=-17\lambda-20\lambda+20\ln\lambda+10(-\lambda+2\ln\lambda-\ln 2)+2(-\lambda+3\ln\lambda-\ln 6)$
$+(-\lambda+4\ln\lambda-\ln 24)$

$\frac{d\ln L(\lambda)}{d\lambda}=-17-20+\frac{20}{\lambda}-10+\frac{20}{\lambda}-2+\frac{6}{\lambda}-1+\frac{4}{\lambda}=-50+\frac{50}{\lambda}=0$

$\hat{\lambda}=1$

4. $E(X)=2\theta(1-\theta)+2\theta^2+3(1-2\theta)=-4\theta+3$

$\theta=-\frac{E(X)-3}{4}$, $\hat{\theta}=-\frac{\bar{X}-3}{4}$, $\bar{X}=\frac{1}{4}+\frac{1}{4}+\frac{3}{2}=2$

$\hat{\theta}=\frac{1}{4}$

$L(\theta)=\prod_{i=1}^{8}P(x_i,\theta)=(1-2\theta)^4\cdot[2\theta(1-\theta)]^2\cdot\theta^2\cdot\theta^2$

$\frac{d\ln L}{d\theta}=-\frac{8}{1-2\theta}+\frac{4}{2\theta}-\frac{2}{1-\theta}+\frac{4}{\theta}=0$, $\theta=\frac{7-\sqrt{13}}{12}$

5. $L(\lambda)=\prod_{i=1}^{10}P(x_i,\lambda)=\lambda^{10}e^{-\lambda(1050+\dots+1150)}=\lambda^{10}\cdot e^{-\lambda\cdot 11680}$

$\frac{d\ln L}{d\lambda}=10\frac{1}{\lambda}-11680=0 \Rightarrow \lambda=0.000856$

$4\ln(1-2\theta)$ $4\theta^2$. $(2\theta)^2$.

$\theta = \frac{}{1-\bar{x}}$ θ x $1+\frac{}{\sqrt{\theta}} = E(X) \ldots$

6. (1) $E(X) = kp$, $\hat{p} = \frac{\bar{X}}{k}$

$L(\lambda) = \prod_{i=1}^{n} p(x_i, p) = C_k^{x_1} p^{x_1} (1-p)^{(k-x_1)} \cdot C_k^{x_2} p^{x_2} (1-p)^{(k-x_2)} \cdot \ldots \cdot C_k^{x_n} p^{x_n} (1-p)^{k-x_n}$

$\frac{d\ln L}{dp} = \sum_{i=1}^{n} x_i \cdot \frac{1}{p} - \frac{nk - \sum x_i}{1-p} = 0$, $\hat{p} = \frac{\sum_{i=1}^{n} x_i}{nk} = \frac{\bar{x}}{k}$

(2) $E(X) = \int_0^{\infty} x \cdot \frac{x}{\theta^2} e^{-\frac{x}{\theta}} dx = 2\theta$, $\hat{\theta} = \frac{\bar{X}}{2}$

$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta) = \frac{\prod x_i}{\theta^{2n}} e^{-\frac{\sum x_i}{\theta}}$

$\frac{d\ln L}{d\theta} = 0 \Rightarrow -2n \frac{1}{\theta} + \frac{\sum x_i}{\theta^2} = 0 \Rightarrow \hat{\theta} = \frac{\bar{X}}{2}$

(3) $E(X) = \int_0^1 x \sqrt{\theta} x^{\sqrt{\theta}-1} dx = \int_0^1 \sqrt{\theta} x^{\sqrt{\theta}} dx = \frac{\sqrt{\theta}}{\sqrt{\theta}+1}$, $\hat{\theta} = \left(\frac{\bar{X}}{1-\bar{X}}\right)^2$

$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta) = (\sqrt{\theta})^n \cdot (\prod x_i)^{\sqrt{\theta}-1}$

$\frac{d\ln L(\theta)}{d\theta} = \frac{n}{2} \cdot \frac{1}{\theta} + \frac{1}{2\sqrt{\theta}} \cdot \sum_{i=1}^{n} \ln x_i = 0 \Rightarrow \theta = \left(\frac{n}{\sum_{i=1}^{n} \ln x_i}\right)^2$

(4) $E(X) = \int_{\theta}^{\infty} x \frac{2\theta^2}{x^3} dx = 2\theta$, $\hat{\theta} = \frac{\bar{X}}{2}$

$L(\theta) = \prod_{i=1}^{n} f(x_i, \theta) = \frac{(2\theta^2)^n}{(\prod x_i)^3}$, if $\theta \leq \min\{x_1, \ldots, x_n\}$

$\frac{d\ln L}{d\theta} = n \frac{4\theta}{2\theta^2} = \frac{2n}{\theta}$ $\therefore \hat{\theta} = \min\{x_1, \ldots, x_n\}$

(5) $E(x) = \int_{\mu}^{\infty} x \frac{1}{\theta} e^{-\frac{x-\mu}{\theta}} dx$

$E(X^2) = \int_{\mu}^{\infty} x^2 \frac{1}{\theta} e^{-\frac{x-\mu}{\theta}} dx$

$\Rightarrow \begin{cases} \hat{\theta} = \sqrt{M_2 - M_1^2} \\ \hat{\mu} = M_1 - \sqrt{M_2 - M_1^2} \end{cases}$

$L = \prod_{i=1}^{n} f(x_i; \theta, \mu) = \frac{1}{\theta^n} \cdot e^{-\frac{\sum x_i - n\mu}{\theta}}$

$\begin{cases} \frac{d\ln L}{d\theta} = 0 \\ \frac{d\ln L}{d\mu} = 0 \end{cases} \Rightarrow \begin{cases} \hat{\theta} = \bar{x} - x_{(1)} \\ \hat{\mu} = x_{(1)} \end{cases}$

7. $X = e^{\ln X}$，令 $\ln X$ 为 Y

$E(X) = E(e^{Y}) = \int_{-\infty}^{\infty} e^{Y} \cdot f_Y(y)\,dy = e^{\mu + \frac{1}{2}\sigma^2}$

$E(X^2) = E(e^{2Y}) = \int_{-\infty}^{\infty} e^{2Y} f_Y(y)\,dy = e^{2\mu + 2\sigma^2}$

$D(X) = E(X^2) - [E(X)]^2 = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2} = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$

$= E^2(X)(e^{\sigma^2} - 1)$

由 $Y \sim N(\mu, \sigma^2) \Rightarrow \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \ln X_i$

$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(\ln X_i - \hat{\mu})^2$

$\Downarrow$

$E(X) = e^{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2}$

$D(X) = e^{2\hat{\mu} + \hat{\sigma}^2}(e^{\hat{\sigma}^2} - 1)$