

1. (1) E 的可能取值: $E_1 = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{8a}$, 概率率: $\frac{3}{4}$

$$E_2 = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{2a}, \text{ 概率率: } \frac{1}{4}$$

$$\bar{E} = -\frac{1}{4\pi\epsilon_0} \frac{7e^2}{32a}$$

L^2 可能取值: $L_1^2 = 2\hbar^2$, 概率率: $\frac{3}{4}$

$$L_2^2 = 0, \text{ 概率率: } \frac{1}{4}$$

$$\bar{L}^2 = \frac{3}{2}\hbar^2$$

L_z 可能取值: $L_{z1} = 0$, 概率率: $\frac{1}{2}$

$$L_{z2} = -\hbar, \text{ 概率率: } \frac{1}{2}$$

$$\bar{L}_z = -\frac{1}{2}\hbar$$

$$(2) \psi(r, t) = \frac{1}{2} R_{10} Y_{10} \cdot e^{-i\frac{E_1}{\hbar}t} - \frac{1}{2} R_{10} Y_{00} e^{-i\frac{E_1}{\hbar}t} + \frac{1}{\sqrt{2}} R_{21} Y_{1-1} e^{-i\frac{E_2}{\hbar}t}$$

E, L^2, L_z : 同第(1)问

2-10. $|\psi_{nlm}(r, \theta, \varphi)|^2$ 代表在球坐标下 (r, θ, φ) 处的概率密度
 在 $r \rightarrow r+dr$ 的概率: $r^2 dr \int d\Omega |\psi_{nlm}(r, \theta, \varphi)|^2 = |R_{nl}(r)|^2 r^2 dr$
 $\int_{r_1}^{r_2} |R_{nl}(r)|^2 r^2 dr$: 在 $r_1 \rightarrow r_2$ 的概率

电子出现在 (θ, φ) 的立体角 $d\Omega$ 中的概率为

$$d\Omega \int |\psi_{nlm}(r, \theta, \varphi)|^2 r^2 dr = |Y_{lm}(\theta, \varphi)|^2 d\Omega$$

$$= |Y_{lm}(\theta, \varphi)|^2 \sin\theta d\theta d\varphi = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} |Y_{lm}(\theta, \varphi)|^2 \sin\theta d\theta d\varphi$$

2-11. $L = \sqrt{l(l+1)} \hbar$, $n=3$ 时, $l=0, 1, 2$
 L 可取 $0, \sqrt{2} \hbar, \sqrt{6} \hbar$

2-12. $n=3, l=2, m=0, \pm 1, \pm 2$

$$L = \sqrt{6} \hbar, \quad L_z = 0, \pm \hbar, \pm 2 \hbar$$

$$\theta = \frac{\pi}{2} \text{ 或 } \cos^{-1}\left(\frac{1}{\sqrt{6}}\right) \text{ 或 } \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) \text{ 或 } \cos^{-1}\left(\frac{-1}{\sqrt{6}}\right)$$

$$\text{或 } \cos^{-1}\left(\frac{-2}{\sqrt{6}}\right)$$

2-13. $n=4, l=0, 1, 2, 3$

$$L = 0, \sqrt{2} \hbar, \sqrt{6} \hbar, 2\sqrt{3} \hbar$$

2-14. (1) 电子偶素: $M = \frac{me^2}{2me} = \frac{me}{2}$
 $\therefore E_1 = - \frac{mee^4}{4(4\pi\epsilon_0)^2 \hbar^2}$