

$$e^5 \cdot \int_5^{\infty} ye^{-y} dy = -e^{-y} \Big|_5^{\infty} - ye^{-y} \Big|_5^{\infty} + \int_5^{\infty} (e^{-y}) = -e^{-y} \Big|_5^{\infty}$$

$$17. X \sim U(0,1)$$

$$\text{则 } E(X) = \frac{1}{2}, D(X) = \frac{1}{12}$$

$$E(Y) = \int_5^{\infty} ye^{-y+5} dy = 6, D(Y) = E(Y^2) - E^2(Y) = 37 - 36 = 1$$

$$\therefore E(XY) = E(X) \cdot E(Y) = 3$$

$$D(XY) = E(X^2Y^2) - E^2(XY) = E(X^2) \cdot E(Y^2) - E^2(X) \cdot E^2(Y) = \frac{10}{3}$$

$$D(2X - Y) = 4D(X) + D(Y) = \frac{1}{3} + 1 = \frac{4}{3}$$

$$18. E(XY) = \iint xy \cdot 6xy^2 dx dy = \int_0^1 \int_0^1 6x^2y^3 dx dy = \frac{1}{2}$$

$$E(2X^2 + 3Y) = \int_0^1 \int_0^1 (2x^2 + 3Y) 6xy^2 dx dy = \frac{13}{4}$$

$$E(X+Y) = \iint (x+y) 6xy^2 dx dy = \frac{17}{12}$$

$$E(X^2 + 2XY + Y^2) = \iint (x^2 + 2xy + y^2) 6xy^2 dx dy = \frac{21}{10}$$

$$\therefore D(X+Y) = E(X^2 + 2XY + Y^2) - E^2(X+Y) = \frac{67}{120} \approx 0.093$$

$$19. X \sim N(1, 2), Y \sim N(-2, 1)$$

$$E(X) = 1, E(Y) = -2, D(X) = 2, D(Y) = 1$$

$$E(Z) = E(2X - Y + 8) = 2E(X) - E(Y) + 8 = 12$$

$$D(Z) = D(2X - Y + 8) = 4D(X) + D(Y) = 9$$

$$22(1) E(X) = \int_a^b xf(x) dx \leq \int_a^b bf(x) dx = b$$

$$\text{同时 } E(X) = \int_a^b xf(x) dx \geq \int_a^b af(x) dx = a$$

$$\therefore a \leq E(X) \leq b$$

$$(2) D(X) = E(X^2) - (E(X))^2$$

$$\text{由 } a \leq x \leq b \text{ 可得 } (x-a)(x-b) \leq 0 \Rightarrow x^2 - (a+b)x + ab \leq 0$$



$$\int_1^3 x^2 \cdot \frac{1}{2}$$

$$\frac{(a+b)^2}{4} + ab - \frac{1}{2}$$

$$\int_1^3 x^2 \cdot \frac{1}{2} dx = \frac{1}{2} \cdot \frac{1}{3} x^3 \Big|_1^3 = \frac{1}{2} \cdot \frac{1}{3} (27 - 1) = \frac{13}{3}$$

$$\Rightarrow E(X^2 - (a+b)X + ab) \leq 0 \Rightarrow E(X^2) \leq (a+b)E(X) - ab$$

$$\therefore D(X) \leq (a+b)E(X) - ab - (E(X))^2$$

$$\Rightarrow D(X) \leq \min\{(a+b)E(X) - ab - (E(X))^2\}$$

当  $E(X)$  取  $\frac{a+b}{2}$  时, 右式为  $\frac{(b-a)^2}{4}$ , 得证

$$24. E(X) = 2, D(X) = \frac{1}{3}, E(Y) = 0, D(Y) = 1$$

$$\begin{aligned} D(XY) &= E(X^2Y^2) - (E(XY))^2 = E(X^2)E(Y^2) - (E(X))^2(E(Y))^2 \\ &= E(X^2)E(Y^2) = \frac{13}{3} \cdot (1+0) = \frac{13}{3} \end{aligned}$$

$$25. E(X) = \sum X_i P(X_i) = 0.7$$

$$E(Y) = 0.6$$

$$D(X) = E(X^2) - (E(X))^2 = 0.7 - 0.49 = 0.21$$

$$D(Y) = E(Y^2) - (E(Y))^2 = 0.24$$

$$\begin{aligned} \text{Cov}(X, Y) &= E[(X-0.7)(Y-0.6)] = E(XY) + 0.42 - 0.6E(X) - 0.7E(Y) \\ &= E(XY) - 0.42 = 0.4 - 0.42 = -0.02 \end{aligned}$$

$$\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sqrt{D(X)D(Y)}} = \frac{-0.02}{\sqrt{0.21 \times 0.24}} = -0.089$$

补充1.  $(X, Y) \sim N(1, 2; 3, 4; 0.3)$

$$\text{Cov}(X, Y) = \rho \sigma_1 \sigma_2 = 0.3 \times \sqrt{2} \times 2 \approx 0.849$$

补充2.  $E(X) = \frac{5}{3}, E(Y) = \frac{5}{3}$

$$E(XY) = \sum_{i \neq j} P(X_i=1, Y_j=1) = \sum_{i \neq j} \frac{1}{36} = \frac{10 \times 9}{36} = \frac{5}{2}$$

$$\begin{aligned} \therefore \text{Cov}(X, Y) &= E[(X - \frac{5}{3})(Y - \frac{5}{3})] = \frac{5}{2} - \frac{5}{3} \times \frac{5}{3} - \frac{5}{3} \times \frac{5}{3} + \frac{5}{3} \times \frac{5}{3} \\ &= -\frac{5}{18}, \quad \rho_{XY} = \frac{\text{Cov}(X, Y)}{\sqrt{D(X)D(Y)}} = -\frac{1}{5} \quad (D(X)=D(Y)=\frac{50}{36}) \end{aligned}$$