1. $\hat{L}_z = -i\hbar\frac{\partial}{\partial\phi}$，本征值对应为 $m\hbar$

$-i\hbar\frac{\partial\psi}{\partial\phi} = m\hbar\psi \Rightarrow \frac{\partial\psi}{\psi} = im\partial\phi \Rightarrow \ln\psi = im\phi + \ln A$

$\psi(\phi) = Ae^{im\phi}$，由 $\int|\psi(\phi)|^2 d\phi = 1 \Rightarrow A = \frac{1}{\sqrt{2\pi}}$

由 $\psi(\phi) = \psi(\phi+2\pi) \Rightarrow m = 0, \pm1, \pm2, \ldots$

$\therefore$ 本征值：$m\hbar$，本征态：$\frac{1}{\sqrt{2\pi}}e^{i\hbar m}$，$m = 0, \pm1, \pm2, \ldots$

1-26. 证 $\int\psi^*\hat{p}_x\psi dx = \int(\hat{p}_x\psi)^*\psi dx$，$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$

$RHS = \int_{-\infty}^{\infty}(-i\hbar\frac{\partial}{\partial x}\psi)^*\psi dx = \int_{-\infty}^{\infty} i\hbar\frac{\partial\psi^*}{\partial x}\psi dx$

$= \int_{-\infty}^{\infty} i\hbar\left[\frac{\partial}{\partial x}(\psi^*\psi) - \psi^*\frac{\partial\psi}{\partial x}\right]dx$

$= \int_{-\infty}^{\infty} -i\hbar\psi^*\frac{\partial\psi}{\partial x}dx = \int_{-\infty}^{\infty}\psi^*(-i\hbar\frac{\partial}{\partial x})\psi dx = LHS.$

故 $\hat{p}_x$ 是厄密算符

$2\cdot\frac{a}{4}$ $\frac{2n\pi}{a}$

3. $\begin{cases} U(x)=0, & |x|<a \\ U(x)=\infty, & |x|\geq a \end{cases}$

由定态薛定谔方程：$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi$（在势阱内）

$\psi=0$（在势阱外）

对于 $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi \Rightarrow \frac{d^2\psi}{dx^2}+\frac{2mE}{\hbar^2}\psi=0$：

$k^2=\frac{2mE}{\hbar^2}$，一般解：$A\sin kx+B\cos kx$

由边界条件：$\psi_i(a)=\psi_e(0)=0$：$A\sin ka+B\cos ka=0$

$\psi_i(-a)=\psi_e(-a)=0$：$A\sin(-ka)+B\cos(-ka)=0$

$\begin{cases} A\sin ka+B\cos ka=0 \\ -A\sin ka+B\cos ka=0 \end{cases} \Rightarrow \begin{cases} A\sin ka=0 \\ B\cos ka=0 \end{cases}$

由归一化条件及波函数不能全为0可知 $A=0$ 或 $B=0$

当 $A=0$ 时，$\cos ka=0$，$k=\frac{\pi}{2a}\cdot n$（n 为奇数）

此时 $\psi_n=\begin{cases} B\cos\frac{n\pi}{2a}x, & n\text{为奇}, |x|<a \\ 0, & |x|\geq a \end{cases}$

当 $B=0$ 时，$\sin ka=0$，$k=\frac{n\pi}{2a}$，n 为偶数

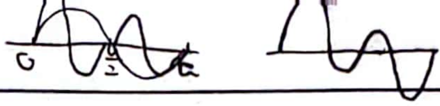
此时 $\psi_n=\begin{cases} A\sin\frac{n\pi}{2a}x, & n\text{为偶}, |x|<a \\ 0, & |x|\geq a \end{cases}$

合并：$\psi_n=\begin{cases} A'\sin\frac{n\pi}{2a}(x+a), & |x|\leq a \\ 0, & |x|\geq a \end{cases}$（相当于 $0\leq x\leq a$）

归一化得到 $A'=\frac{1}{\sqrt{a}}$，$E_n=\frac{k^2\hbar^2}{2m}=\frac{n^2\pi^2\hbar^2}{8ma^2}$，$n=0,1,2,\ldots$

故 $\psi_n(x,t)=\frac{1}{\sqrt{a}}\sin\frac{n\pi}{2a}(x+a)e^{-\frac{i}{\hbar}E_nt}$

$\frac{1}{\frac{a}{4}}\cdot\frac{1}{\sqrt{2}}$ $E_1 = \omega\hbar$, $E_2 = 4\omega\hbar$ $E_1 =$



$t = \frac{}{E_2 - E_1}$

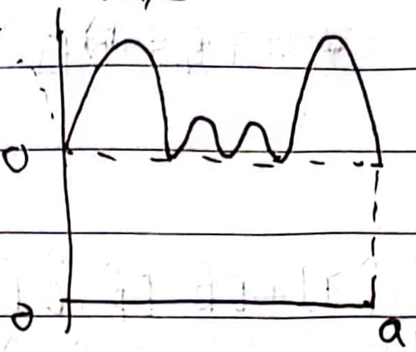
4、由系数 $=\frac{1}{\sqrt{a/4}}\cdot\frac{1}{\sqrt{2}}$，知 $\frac{a}{4}$，故 $\frac{a}{4}$ 就是阱宽度

(1) $t=0$: $\psi(x,t) = \frac{1}{\sqrt{a/2}}\left\{\sin\left(\frac{2\pi x}{a}\right) + \sin\left(\frac{4\pi x}{a}\right)\right\}$

$t = \frac{\pi}{6\omega}$: $\psi(x,t) = \frac{1}{\sqrt{a/2}}\left\{\sin\left(\frac{2\pi x}{a}\right)e^{-i\frac{\pi}{6}} + \sin\left(\frac{4\pi x}{a}\right)e^{-i\frac{2\pi}{3}}\right\}$

$t=1$: $|\psi(x,t)|^2$

在 $(-a, 0)$ 对称



$t = \frac{\pi}{6\omega}$: $|\psi(x,t)|^2$

(2) $\psi_1 : \frac{1}{\sqrt{2}}$，$\psi_2 : \frac{1}{\sqrt{2}}$，$E_n = \frac{n^2\pi^2\hbar^2}{2m}$，$E_1 = \frac{\pi^2\hbar^2}{2m}$，$E_2 = \frac{4\pi^2\hbar^2}{2m}$

$E = c_1^2E_1 + c_2^2E_2 = \frac{\pi^2\hbar^2}{4m} + \frac{\pi^2\hbar^2}{m} = \frac{5\pi^2\hbar^2}{4m}$

平均能量不随时间而变，每次测量随机在 E_1，E_2

(3) 当 V_0 变为0，相当于变为自由粒子，此时无边界条件限制 k，E 可取任意值，$\psi(x,t) = (A\sin kx + B\cos kx)e^{-\frac{i}{\hbar}\frac{k^2\hbar^2}{2m}t}$

$\therefore$ 新：$\psi(x,t) = A'\left\{\sin\left(\frac{2\pi x}{a}\right)e^{-i\frac{\pi}{6}} + \sin\left(\frac{4\pi x}{a}\right)e^{-i\frac{2\pi}{3}}\right\} +$

$B'(A\sin kx + B\cos kx)e^{-\frac{i}{\hbar}\frac{k^2\hbar^2}{2m}t}$

5. $[\hat{F},[\hat{G},\hat{R}]]+[\hat{G},[\hat{R},\hat{F}]]+[\hat{R},[\hat{F},\hat{G}]]=0$

LHS: $\hat{F}[\hat{G},\hat{R}]-[\hat{G},\hat{R}]\hat{F}+\hat{G}[\hat{R},\hat{F}]-[\hat{R},\hat{F}]\hat{G}+\hat{R}[\hat{F},\hat{G}]-[\hat{F},\hat{G}]\hat{R}$

$=\hat{F}(\hat{G}\hat{R}-\hat{R}\hat{G})-(\hat{G}\hat{R}-\hat{R}\hat{G})\hat{F}+\hat{G}(\hat{R}\hat{F}-\hat{F}\hat{R})-(\hat{R}\hat{F}-\hat{F}\hat{R})\hat{G}+\hat{R}(\hat{F}\hat{G}-\hat{G}\hat{F})-(\hat{F}\hat{G}-\hat{G}\hat{F})\hat{R}$

$=\hat{F}\hat{G}\hat{R}-\hat{F}\hat{G}\hat{R}-\hat{F}\hat{R}\hat{G}+\hat{F}\hat{R}\hat{G}-\hat{G}\hat{R}\hat{F}+\hat{G}\hat{R}\hat{F}+\hat{R}\hat{G}\hat{F}-\hat{R}\hat{G}\hat{F}-\hat{G}\hat{F}\hat{R}+\hat{G}\hat{F}\hat{R}-\hat{R}\hat{F}\hat{G}+\hat{R}\hat{F}\hat{G}=0$

$\cancel{5.\ E_1: \frac{1}{4},\ E_2: \frac{3}{4},\ \langle E\rangle=\frac{1}{4}E_1+\frac{3}{4}E_2}$

$\cancel{E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}}$

$\frac{3}{2}+\frac{1}{8}$ $\frac{12}{8}$

5. $E_1: \frac{1}{4},\ E_2: \frac{3}{4}$

$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x,\ 0\le x\le L,\ 0,\ \text{o.w.}$

$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2},\ \langle E\rangle=\frac{1}{4}\cdot\frac{\pi^2\hbar^2}{2mL^2}+\frac{3}{4}\cdot\frac{4\pi^2\hbar^2}{2mL^2}=\frac{13\pi^2\hbar^2}{8mL^2}$