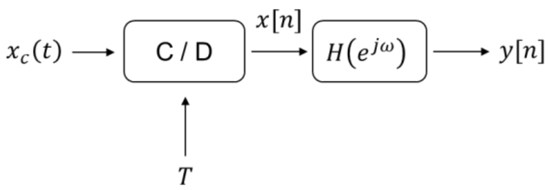
AI2619 2023Spring Written Assignment #3

due: 2023/4/4 23:59 (17 days)

In this assignment, we denote the FT of continuous-time signal $x(t)$ *by* $X(j\Omega)$*, and the DTFT of discrete-time signal* $x[n]$ *by* $X(e^{j\omega})$*.*

Problem 1

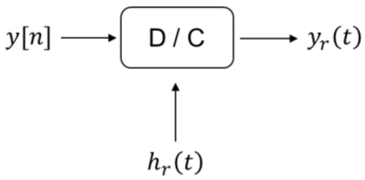
A continuous-time signal $x_c(t)=\frac{\sin(\Omega_m t)}{\pi t}$ is sampled by impulse series with period T and converted to discrete-time signal $x[n]$. $x[n]$ is then filtered by $H(e^{j\omega})=\begin{cases}A & |\omega|<\omega_c \\ 0 & \omega_c<|\omega|\le\pi\end{cases}$ to create $y[n]$, as shown in the figure below. In this problem, we will experiment on $y[n]$ with different post-processing systems.



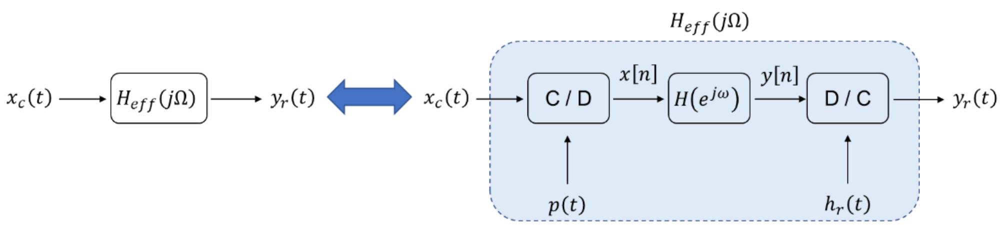
(1). Suppose $y[n]$ is reconstructed by a D/C converter to get $y_r(t)$. Here $A=1, \omega_c=\frac{4\pi}{5}$ and the recovery filter $H_r(j\Omega)=\begin{cases}T & |\Omega|<\frac{\pi}{T} \\ 0 & \text{otherwise}\end{cases}$. Sketch $X_c(j\Omega), X(e^{j\omega}), Y(e^{j\omega})$ and $Y_r(j\Omega)$ when

i. $T=\frac{\pi}{\Omega_m}$

ii. $T=\frac{4\pi}{3\Omega_m}$



(2). For cases in question (1), sketch the equivalent continuous-time system $H_{eff}(j\Omega)$, and compare it with $H(e^{j\omega})$. Does the formula $H_{eff}(j\Omega)=\begin{cases}H(e^{j\Omega T}) & |\Omega|<\frac{\pi}{T} \\ 0 & \text{otherwise}\end{cases}$ always hold?



(3). Suppose $T=\frac{2\pi}{3\Omega_m}, \omega_c=\pi$, $y[n]$ is sampled in the frequency domain with period $\frac{2\pi}{N} (N\in\mathbb{N}^+)$ to get $\hat{y}[n]$. Is it possible to have $\hat{y}[n]=x[n], n=0,1,2,\dots,N-1$ for some specific A and N? If possible, give such values (or ranges); if not, briefly state the reason.

(4). Suppose $T = \frac{2\pi}{3\Omega_m}, \omega_c = \pi$, $y[n]$ is truncated by a rectangular window $w[n] = \begin{cases} 1 & n = 0,1,\dots,L-1 \\ 0 & \text{otherwise} \end{cases}$ to get $y_t[n] = y[n]w[n] = \begin{cases} y[n] & n = 0,1,\dots,L-1 \\ 0 & \text{otherwise} \end{cases}$, and then sampled in the frequency domain as question (3) does. Is it possible to have $\hat{y}[n] = x[n], n = 0,1,2,\dots,L-1$ for some specific A, L and N? If possible, give such values (or ranges); if not, briefly state the reason.