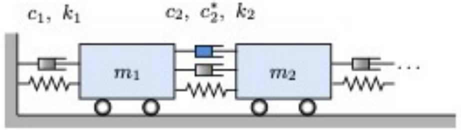
CVD Formulation

CVD: A simultaneous decomposition that provides common modal structures, given two data matrices.

System and Observer Model

$$\dot{\mathbf{x}} = \boldsymbol{f}(\mathbf{x}, \mathbf{p}, t)$$
$$\mathbf{y} = \mathbf{h}(\mathbf{x})$$



Matrix Pair: $\{\mathbf{Y}_1, \mathbf{Y}_2\}$ → CVD: $\mathbf{Y}_1 = \mathbf{U}\boldsymbol{\Sigma}_1\mathbf{V}^H$, $\mathbf{Y}_2 = \mathbf{U}\boldsymbol{\Sigma}_2\mathbf{V}^H$ → Common Modal Structure: $\{\mathbf{U}, \boldsymbol{\Sigma}_2\boldsymbol{\Sigma}_1^{-1}, \mathbf{V}^H\}$

Generalization (GCVD): identify balanced modal structures that are dynamic-direction invariant. It also allows ensemble decomposition with more than two matrices

$$\mathbf{RV} = \mathbf{V}\overline{\mathbf{S}} \quad \text{with}$$

$$\mathbf{R} = \frac{1}{4}\sum_{i=1}^{2}\sum_{\substack{j=1\\ j\neq i}}^{2}\sum_{k=1}^{2}\left(\mathbf{K}_{ik}\mathbf{K}_{jk}^{-1}\right)$$

CVD Algorithm and Generalization

CVD Algorithm

Given

$\{\mathbf{Y}_1, \mathbf{Y}_2\}$

Compute Quotient

$\mathbf{K}_{ij} \leftarrow \mathbf{Y}_i^H\mathbf{Y}_j$

$\mathbf{R}' \leftarrow \mathbf{K}_{21}\mathbf{K}_{11}^{-1}$

EVP

$\mathbf{R}'\mathbf{V} = \mathbf{V}\mathbf{S}'$

Transformation & Normalization

$\mathbf{W} \leftarrow \mathbf{V}^{-H}$

$\mathbf{P}_1 = [\mathbf{p}_1, \ldots, \mathbf{p}_{2n}] \triangleq \mathbf{U}\mathbf{S}_1 \leftarrow \mathbf{Y}_1\mathbf{W}$

$\mathbf{P}_2 \leftarrow \mathbf{Y}_2\mathbf{W}$

$\mathbf{S}_1 = \mathrm{diag}[s_1, \ldots, s_2],\ s_{1,i} \leftarrow \|\mathbf{p}_i\|,\ \forall i$

$\mathbf{U} \leftarrow \mathbf{P}_1\mathbf{S}_1^{-1}$

$\mathbf{S}_2 \leftarrow \mathbf{S}_1\mathbf{P}_1$

Return

$\{\mathbf{U}, \mathbf{S}_2\mathbf{S}_1^{-1}, \mathbf{V}\}$

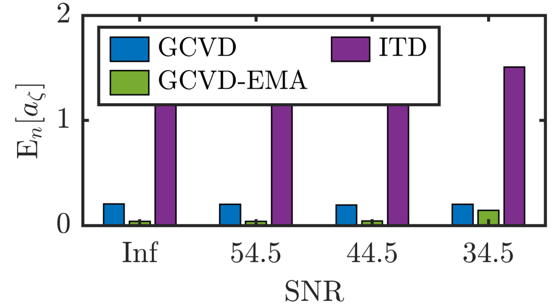
A Unifying Paradigm and Accurate Modal Analysis Method

CVD unifies mainstream time-domain data-driven modal analysis methods

CVD: ERA, DMD, ITD, S-VD

CVD identifies modal parameters with superior accuracy under additive noise

EMA (forcing known)



OMA (output only)