Assignment 2A - Hill Climbing

In this assignment, you will implement the necessary data structures and subroutines to run hill climbing on traveling salesman problems.

You are given four files

- 1. This Notebook You will be running the cells in this file.
- 2. sa_utils.py. You should not change anything in this file.
- 3. tsp_utils.py. You will be writing most of the code in this file.

from tsp_utils import plot_cities, plot_path, compare_sols

4. berlin52.tsp. A TSP file, downloaded from http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/

TODO

Enter your information below.

Name: HaoLiu **CWID:** A20473685

```
import numpy as np
import pandas as pd

from matplotlib import pylab
import matplotlib.pyplot as plt
pylab.rcParams['figure.figsize'] = (10.0, 8.0)
from sa_utils import Node
from sa_utils import hill_climbing
In [2]: from tsp_utils import City, TSPNode, read_cities, subsample_cities, create_initial_node
```

Reading the file

TODO: Implement the read_cities function in tsp_utils.py. This function should read a given TSP file from http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/ Check out the documentation file. Your function should support only the EUC_2D types of files. See berlin52.tsp as an example.

read_cities should accept a string (filename) and return a dictionary of the City objects, where the key is the City name and the objects are City objects with the correct coordinates.

```
In [3]: # Run. It should show the dictionary.
         all_cities = read_cities('berlin52.tsp')
        TSPNode._cities = all_cities
        all_cities
Out[3]: {'1': City: 1 (565.00 575.00),
          '2': City: 2 (25.00 185.00),
         '3': City: 3 (345.00 750.00),
         '4': City: 4 (945.00 685.00),
         '5': City: 5 (845.00 655.00),
         '6': City: 6 (880.00 660.00),
         '7': City: 7 (25.00 230.00),
         '8': City: 8 (525.00 1000.00),
         '9': City: 9 (580.00 1175.00),
         '10': City: 10 (650.00 1130.00),
         '11': City: 11 (1605.00 620.00),
         '12': City: 12 (1220.00 580.00),
         '13': City: 13 (1465.00 200.00),
         '14': City: 14 (1530.00 5.00),
         '15': City: 15 (845.00 680.00),
         '16': City: 16 (725.00 370.00),
         '17': City: 17 (145.00 665.00),
         '18': City: 18 (415.00 635.00),
         '19': City: 19 (510.00 875.00),
          '20': City: 20 (560.00 365.00),
          '21': City: 21 (300.00 465.00),
         '22': City: 22 (520.00 585.00),
         '23': City: 23 (480.00 415.00),
         '24': City: 24 (835.00 625.00),
         '25': City: 25 (975.00 580.00),
         '26': City: 26 (1215.00 245.00),
         '27': City: 27 (1320.00 315.00),
         '28': City: 28 (1250.00 400.00),
         '29': City: 29 (660.00 180.00),
          '30': City: 30 (410.00 250.00),
          '31': City: 31 (420.00 555.00),
          '32': City: 32 (575.00 665.00),
          '33': City: 33 (1150.00 1160.00),
         '34': City: 34 (700.00 580.00),
         '35': City: 35 (685.00 595.00),
         '36': City: 36 (685.00 610.00),
         '37': City: 37 (770.00 610.00),
         '38': City: 38 (795.00 645.00),
         '39': City: 39 (720.00 635.00),
         '40': City: 40 (760.00 650.00),
         '41': City: 41 (475.00 960.00),
         '42': City: 42 (95.00 260.00),
         '43': City: 43 (875.00 920.00),
         '44': City: 44 (700.00 500.00),
         '45': City: 45 (555.00 815.00),
         '46': City: 46 (830.00 485.00),
         '47': City: 47 (1170.00 65.00),
         '48': City: 48 (830.00 610.00),
         '49': City: 49 (605.00 625.00),
         '50': City: 50 (595.00 360.00),
         '51': City: 51 (1340.00 725.00),
         '52': City: 52 (1740.00 245.00)}
```

Subsample Cities

TODO: Complete the implementation of the subsample_cities function in tsp_utils.py . The arguments are

- cities: the dictionary of the cities
- number_of_cities : the number of cities in the subsample
- random_seed : the random seed used to create the subsample

It should return a new dictionary of cities.

```
In [4]: # Run
subsample_size = 10
subsample_seed = 2
cities = subsample_cities(all_cities, number_of_cities=subsample_size, random_seed=subsample_seed)
cities
```

Implement TSPNode

TODO: Complete the implementation of the TSPNode class. You need to implement

- expand This should create all possible children of this node, where a child is a swap of two neighbor cities. A state is an ordered list of city names to visit. Remember that the last city travels back to the start city and hence they are also neighbors.
- value This is the negative of the cost of the state. The cost of the state is the sum of the distances between the neighbor cities. The distance between two neighbors is the Eucledian distance (square root of the sum of the squares of the differences).

```
In [5]: # Run
        tsp_node = TSPNode(sorted(list(cities.keys())))
        tsp_node
        TSPNode: 14-15-16-26-4-40-45-46-48-6
Out[5]:
In [6]: # Run
        children_nodes = tsp_node.expand()
        len(children_nodes)
Out[6]:
In [7]: # Run
        children_nodes
Out[7]: [TSPNode: 15-14-16-26-4-40-45-46-48-6,
         TSPNode: 14-16-15-26-4-40-45-46-48-6,
         TSPNode: 14-15-26-16-4-40-45-46-48-6,
         TSPNode: 14-15-16-4-26-40-45-46-48-6,
         TSPNode: 14-15-16-26-40-4-45-46-48-6,
         TSPNode: 14-15-16-26-4-45-40-46-48-6,
         TSPNode: 14-15-16-26-4-40-46-45-48-6,
         TSPNode: 14-15-16-26-4-40-45-48-46-6,
         TSPNode: 14-15-16-26-4-40-45-46-6-48,
         TSPNode: 6-15-16-26-4-40-45-46-48-14]
In [8]: tsp_node.value()
        -4315.526779689034
```

Create a random start state

```
In [9]: # Run
initial_seed = 5
initial_node = create_initial_node(cities, random_seed=initial_seed)
initial_node

Out[9]: TSFNode: 4-48-26-46-40-16-15-6-45-14

In [10]: # Run
initial_node.value()

Out[10]: -4480.278437200555
```

Implement Plot Functions

TODO: Implement

In [11]: # Run

• plot_cities Given an matplotlib axes, a dictionary of cities, and a state, it should plot the cities in the state. The cities should be plotted at their coordinates.

TODO: Implement

600

200

100

0

• plot_path Given an matplotlib axes, a dictionary of cities, and a state, it should plot the edges between the cities.

1200

1400

1000

```
In [12]: # Run
fig, ax = plt.subplots()
plot_cities(ax, all_cities, initial_node.state)
plot_path(ax, cities, initial_node.state)
```

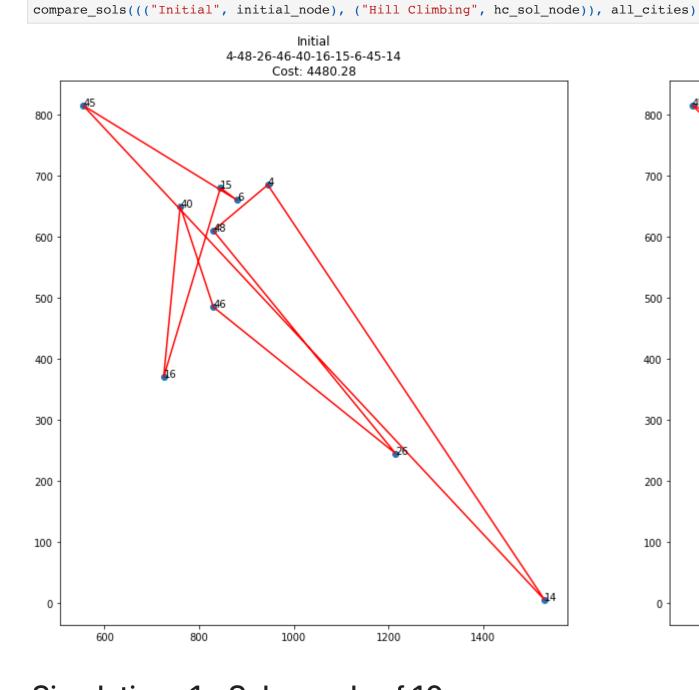
Run Hill Climbing

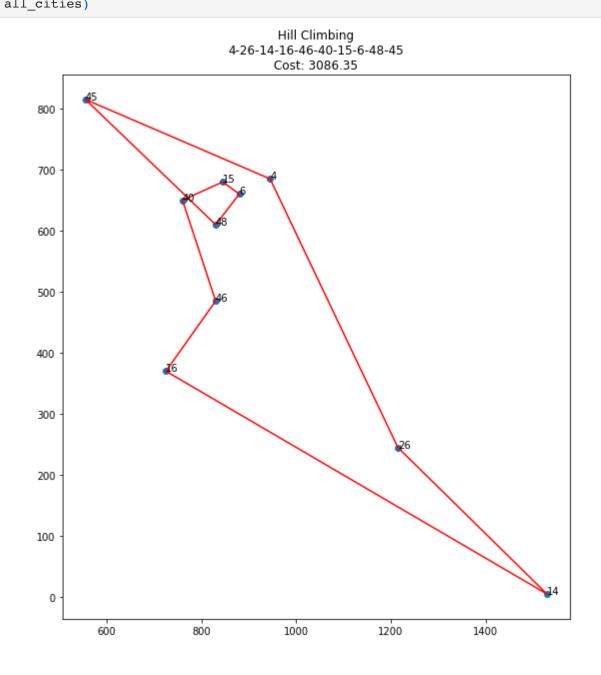
print("Initial Node")

In [13]: # Run

In [16]: # Run

```
print(type(initial_node))
          hc_sol_node = hill_climbing(initial_node)
         hc_sol_node
         Initial Node
         <class 'tsp_utils.TSPNode'>
         TSPNode: 4-26-14-16-46-40-15-6-48-45
Out[13]:
In [14]: hc_sol_node.value()
         -3086.348120526929
Out[14]:
In [15]: # Plot the solution
          fig, ax = plt.subplots()
         plot_cities(ax, all_cities, hc_sol_node.state)
         plot_path(ax, all_cities, hc_sol_node.state)
          800
          700
          600
          500
          400
          300
          100
                                             1000
                                                          1200
                                                                        1400
```



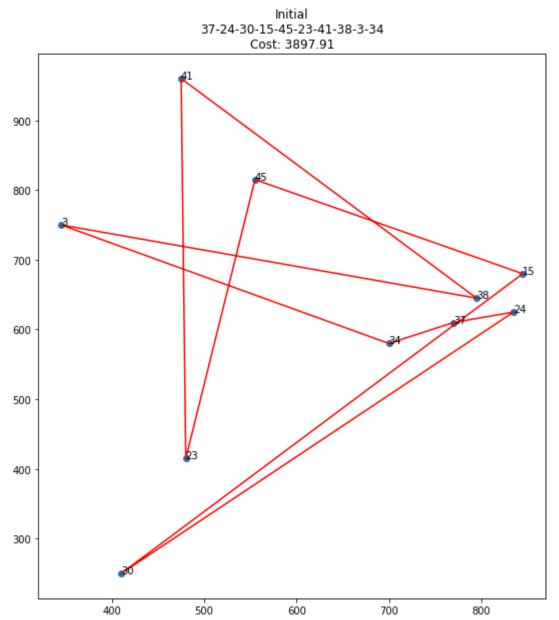


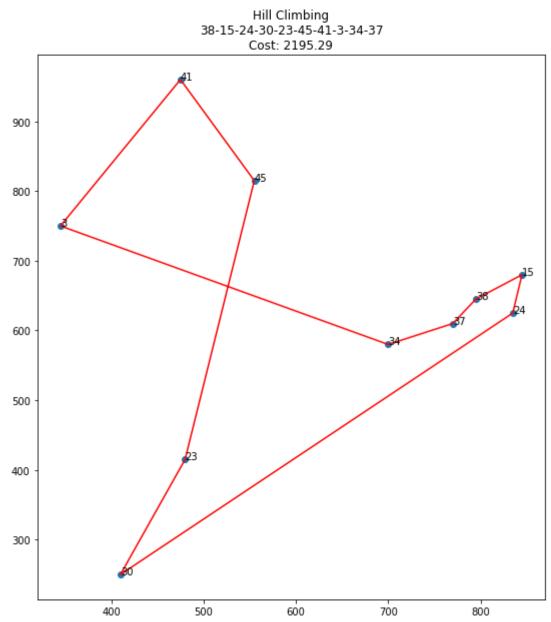
Simulations 1 - Subsample of 10

- 1. Create multiple subsamples of cities, of size 10 each
- 2. Create multiple initializations for each.
- 3. Run HC for each.
- 4. Present the initial and the HC results as a table.

```
• subsample_size = 10
           • subsample_seeds of [0, 1, 2, 3, 4]
           • initial_seeds = [11, 12, 13, 14, int(last_three_digits_of_your_CWID)]
In [17]: # TODO - Write code and run the simulation
          def run_simulations(subsample_size, subsample_seeds, initial_seeds):
              Run the simulations for the given parameters.
             initial_states = {}
              hc states = {}
              for subsample_seed in subsample_seeds:
                  cities = subsample_cities(all_cities, number_of_cities=subsample_size, random_seed=subsample_seed)
                  initial_states[subsample_seed] = {}
                 hc_states[subsample_seed] = {}
                  for initial_seed in initial_seeds:
                      initial_node = create_initial_node(cities, random_seed=initial_seed)
                      initial_states[subsample_seed][initial_seed] = initial_node
                      hc_states[subsample_seed][initial_seed] = hill_climbing(initial_node)
              return initial_states, hc_states
In [18]: CWID='A20473685'
          subsample_size = 10
          subsample\_seeds = range(0, 5)
         initial_seeds = [11, 12, 13, 14, 15, int(CWID[6:])]
          # TODO - Complete the code. It finally should display a table.
          initial_states, hc_states = run_simulations(subsample_size, subsample_seeds, initial_seeds)
          import pandas as pd
          df = pd.DataFrame(columns=['Subsample Seed', 'Initial Seed', 'Initial Value', 'HC Value'])
          df['Subsample Seed'] = [ j for j in subsample_seeds for i in initial_seeds]
         df['Initial Seed'] = [ i for j in subsample_seeds for i in initial_seeds]
         df['Initial Value'] = [initial_states[j][i].value() for j in subsample_seeds for i in initial_seeds]
         df['HC Value'] = [hc_states[j][i].value() for j in subsample_seeds for i in initial_seeds]
         display(df)
             Subsample Seed Initial Seed
                                        Initial Value
                                                       HC Value
                         0
                                   11 -6910.854964 -4845.283431
          0
                         0
                                   12 -4805.420205 -4578.810278
          2
                         0
                                   13 -6281.931261 -6230.336304
          3
                         0
                                   14 -6547.397559 -4943.598730
                         0
                                      -6567.276372 -3994.034490
          4
          5
                         0
                                  685 -4089.327327 -3939.512675
          6
                         1
                                   11 -3897.908590 -2195.289900
                                      -3851.562313 -3140.493238
          8
                         1
                                   13 -3343.303102 -2746.034130
          9
                                   14 -2431.764939 -2159.997566
                         1
          10
                                   15 -2541.940049 -2390.651920
                                  685 -3797.873196 -2297.324830
          11
                         2
          12
                                   11 -4544.428908 -4089.549443
          13
                         2
                                   12 -4312.788187 -3211.747659
```

In [19]: # Pick subsample seed and initial seed, and visualize the initialization and the HC.
compare_sols((("Initial", initial_states[1][11]), ("Hill Climbing", hc_states[1][11])), all_cities)





14

15

16

17 18

19

20

21

22

23

24

2526

27

28 29 2

2

2

2

3

3

13 -4733.314832 -3118.205023

14 -3813.323874 -3334.947124

15 -4393.977833 -2850.467881

685 -4094.846276 -3139.174043

11 -4799.996881 -3990.085683

12 -5107.349706 -3764.067382

13 -4792.187279 -3898.438303

14 -4842.838365 -4342.710220 15 -4777.704743 -4060.666300

685 -5640.635557 -4341.438565

12 -7169.922143 -5886.325861

14 -4637.973404 -3997.536911

15 -6147.884236 -6078.827882

685 -7015.783332 -5089.433264

-6146.594024 -4957.369556

-4993.819117

11 -6450.980927

Repeat the above simulation for

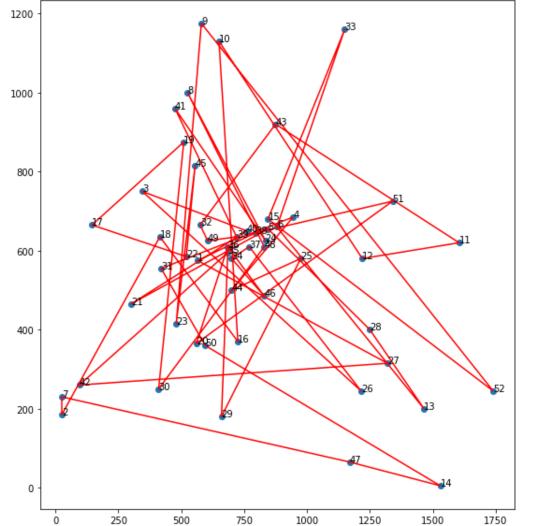
- subsample of 20
- subsample of 30
- subsample of 40

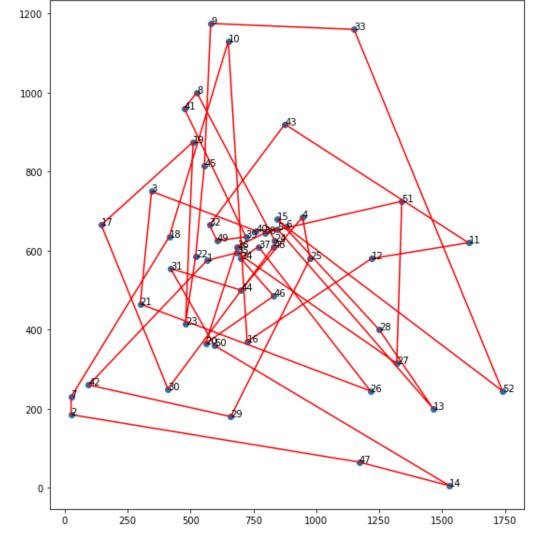
Finally, repeat it for the full set of cities (i.e., no subsampling, only random initialization).

```
In [20]:
    subsample_size_list=[20, 30, 40, len(all_cities)]
    subsample_seeds = range(0, 5)
    initial_seeds = [11, 12, 13, 14, 15, int(CWID[6:])]
    for subsample_size in subsample_size_list:
        initial_states, ho_states = run_simulations(subsample_size, subsample_seeds, initial_seeds)
        df = pd.DataFrame(columns=['Subsample Seed', 'Initial Seed', 'Initial Value', 'HC Value'])
        df['Subsample Seed'] = [ j for j in subsample_seeds for i in initial_seeds]
        df['Initial Seed'] = [ i for j in subsample_seeds for i in initial_seeds]
        df['Initial Value'] = [initial_states[j][i].value() for j in subsample_seeds for i in initial_seeds]
        df['HC Value'] = [hc_states[j][i].value() for j in subsample_seeds for i in initial_seeds]
        display(df)
        compare_sols((("Initial", initial_states[1][11]), ("Hill Climbing", hc_states[1][11])), all_cities)
```

	display(df)			
	compare_sols((("Initial		ates[1][11]),
	Subsample Seed	Initial Seed	Initial Value	HC Value
0	0	11	-14528.512545	-9409.188607
1	0	12	-12492.176845	-9553.188395
2	0	13	-12938.237671	-11515.194157
3	0	14	-12529.839281	-10628.089376
4	0	15	-11961.551958	-10324.493947
5	0	685	-12764.576346	-7868.568848
6	1	11	-8315.996528	-5971.753474
7	1	12	-8106.832988	-5710.267338
8	1	13	-7919.323807	-6222.817913
9	1	14	-7841.527843	-5757.361661
10	1	15	-7579.257232	-5198.359541
11	1	685	-8142.940583	-7096.208365
12	2	11	-8682.089796	-7607.631794
13	2	12	-10238.872421	-7579.814402
14	2	13	-9250.016401	-7040.697803
15	2	14	-10050.787735	-7726.043985
16	2	15	-9214.227042	-7682.478157
17 18	3	685	-9768.657953 -10567.370527	-7975.684458 -9742.518415
19	3	12	-10567.370527	-9742.518415 -8856.363487
20	3	13	-11736.349148	-10009.973969
21	3	14	-11730.549148	-9536.974683
22	3	15	-11551.965621	-8076.005732
23	3	685	-11500.495571	-8592.875125
24	4	11	-13671.503005	-11329.118532
25	4	12	-11587.825714	-8361.201935
26	4	13	-12308.573258	-9682.017457
27	4	14	-11053.665807	-7622.474497
28	4	15	-12269.771205	-10862.073814
20	4	685	12104 05 4000	0047.075500
29	•	000	-13194.954000	-8917.875528
29				
	Subsample Seed	Initial Seed	Initial Value	HC Value
0	Subsample Seed 0	Initial Seed	Initial Value -18452.333046	HC Value
0	Subsample Seed 0 0	Initial Seed 11 12	Initial Value -18452.333046 -16392.240025	HC Value -14751.060989 -13710.665979
0 1 2	Subsample Seed 0 0 0	Initial Seed 11 12 13	Initial Value -18452.333046 -16392.240025 -19073.027869	HC Value -14751.060989 -13710.665979 -13446.357383
0 1 2 3	Subsample Seed 0 0 0 0	11 12 13 14	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203
0 1 2 3	Subsample Seed 0 0 0 0 0	11 12 13 14 15	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435
0 1 2 3	Subsample Seed 0 0 0 0 0 0 0	11 12 13 14 15 685	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203
0 1 2 3 4 5	Subsample Seed 0 0 0 0 0	11 12 13 14 15	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682
0 1 2 3 4 5	Subsample Seed 0 0 0 0 0 1	11 12 13 14 15 685 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737
0 1 2 3 4 5 6	Subsample Seed	11 12 13 14 15 685 11 12	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251
0 1 2 3 4 5 6 7 8	Subsample Seed	11 12 13 685 11 12 13	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115
0 1 2 3 4 5 6 7 8	Subsample Seed 0 0 0 0 0 1 1 1 1	Initial Seed 11 12 13 14 15 685 11 12 13	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060
0 1 2 3 4 5 6 7 8 9	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 15 11 15 15 11 11 11 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421
0 1 2 3 4 5 6 7 8 9 10	Subsample Seed	11 12 13 685 11 12 13 14 15 685 685	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744
0 1 2 3 4 5 6 7 8 9 10 11	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 11 11 11 11 11 11 11 11 11 11 11 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274
0 1 2 3 4 5 6 7 8 9 10 11 12 13	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 15	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 15 11 12 13 14 15 15 11 11 12 13 11 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 11 12 13 14 15 11 11 12 13 14 15 11 11 12	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 12 13 14 15 15 11 12 13 14 15 15 11 11 12	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 14 15 685 11 12 13 14 15 685 11 12 13 685	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 15 685 11 12 13 14 15 685 11 15 685 11 11 11 11 11 11 11 11 11 11 11 11 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	Subsample Seed	Initial Seed 11 12 13 14 15 685 11 15 685 11 12 13 14 15 685 11 15 685 11 11 12 13 14 15 15 11 11 12 11 11 12	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	Subsample Seed 0 0 0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 15 11 11 12 13 13 14 15 15 11 11 12 13 13	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	Subsample Seed 0 0 0 0 0 0 0 1 1 1 1 1 1 1 2 2 2 2 2	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 11 12 13 14 15 15 11 11 12 13 14 15 16 11 11 12 13 14 14 15 15 11 14 15 16 11 11 12 13 14 14 15 14 15 16 11 11 12 13 14 14 15 14 15 16 11 14 15 16 16 11 11 12 13 14 14 15 14 14 15 16 16 17 18 18 18 18 18 18 18 18 18 18 18 18 18	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217 -17842.382778	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287 -14800.082032
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22	Subsample Seed 0 0 0 0 0 0 0 1 1 1 1 1 1 1 2 2 2 2 2	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15 15 18 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 11 12 13 14 15 15 11 11 11 12 13 14 15 15 11 11 11 11 11 11 11 11 11 11 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217 -17842.382778 -16098.062045	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287 -14800.082032 -12958.085754
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23	Subsample Seed 0 0 0 0 0 1 1 1 1 1 2 2 2 2 2 2 3 4 5 6 7 8 9 10	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 15 685	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217 -17842.382778 -16098.062045 -16440.250739	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287 -14800.082032 -12958.085754 -11984.257858
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24	Subsample Seed 0 0 0 0 0 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 4	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15 685 11 11 11 12 13 14 15 15 16 11 11 11 11 11 11 11 11 11 11 11 11	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217 -17842.382778 -16098.062045 -16440.250739 -17191.618369	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287 -14800.082032 -12958.085754 -11984.257858 -12626.906252
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27	Subsample Seed 0 0 0 0 0 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 4 4 4 4 4 4 4 4	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15 18 14 15 18 11 11 12 13 14 15 16 11 11 12 13 14 15 16 11 11 12 13 14 15 16 11 11 12 13 14 14 15 15 11 14 15 16 11 11 12 13 14 14 15 16 11 11 12 13 14 14 15 15 16 11 11 12 13 14 14 15 16 18 18 18 18 18 18 18 18 18 18 18 18 18	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217 -17842.382778 -16098.062045 -16440.250739 -17191.618369 -16373.436395 -18476.092408 -17528.624119	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287 -14800.082032 -12958.085754 -11984.257858 -12626.906252 -13487.906800 -16402.522209 -12792.169598
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	Subsample Seed 0 0 0 0 0 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 4 4 4 4 4 4 4	Initial Seed 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15 13 14 15 15 11 12 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 13 14 15 15 11 11 12 13 13 14 15 15 16 11 11 12 13 13 14 15 15 16 11 11 12 13 13 14 15 15 16 11 11 12 13 13 14 15 15 16 11 11 12 13 13	Initial Value -18452.333046 -16392.240025 -19073.027869 -18543.235765 -18072.423051 -19757.068889 -14617.254235 -12924.087636 -13329.522781 -13921.360320 -14768.205467 -14854.343435 -12965.237731 -14217.105152 -14082.051073 -16210.674008 -14568.237519 -16189.480659 -16658.990034 -16427.212697 -16750.003217 -17842.382778 -16098.062045 -16440.250739 -17191.618369 -16373.436395 -18476.092408	HC Value -14751.060989 -13710.665979 -13446.357383 -14528.437203 -12982.142435 -12084.093737 -11752.905682 -11395.282251 -11252.846115 -10533.700060 -12661.765421 -12545.604744 -9998.038274 -12282.945719 -12146.101451 -12451.308075 -12123.613370 -14084.468451 -14731.745296 -13249.806835 -12192.455287 -14800.082032 -12958.085754 -11984.257858 -12626.906252 -13487.906800 -16402.522209

	Subsample Seed	Initial Seed	Initial Value	HC Value
0	0	11	-22422.827965	-18173.154615
1	0	12	-24004.194769	-18844.451113
2	0	13	-24314.444478	-19406.699556
3	0	14	-23950.875799	-18255.824281
4	0	15	-25058.430067	-18536.135701
5	0	685	-22871.791725	-19439.177595
6 7	1	11	-21039.012888 -22582.888615	-15656.884408 -17234.947213
8	1	13	-20314.188732	-15903.115885
9	1	14	-21356.947180	-16087.632942
10	1	15	-20338.593483	-16509.662754
11	1	685	-22128.838784	-15499.226735
12	2	11	-24160.336086	-16567.714593
13	2	12	-24050.568443	-17847.127587
14	2	13	-20854.844012	-18709.864400
15	2	14	-24817.518102	-19279.484009
16	2	15	-21517.972362	-17015.181136
17	2	685	-24082.028716	-18842.906614
18 19	3	11	-20334.250556 -20291.047321	-18257.623656 -17871.852822
19	3	13		-17871.852822 -18508.736349
21	3	14		-18506.239308
22	3	15	-21698.490496	-16922.600429
23	3	685	-22193.636820	-18498.234506
24	4	11	-21728.882616	-18831.781799
25	4	12	-21626.604248	-16252.476551
26	4	13	-25140.093334	-18509.374608
27	4	14	-23924.830059	-19249.368027
28	4	15	-25000.958085	-18799.549114
29	4	685	-23818.456296	-19737.394912
	Subsample Seed	Initial Seed	Initial Value	HC Value
0	0	11	-31341.440592	-24386.679891
1	0	11 12	-31341.440592 -28501.327303	-24386.679891 -22336.651571
1 2	0 0	11 12 13	-31341.440592 -28501.327303 -29191.886193	-24386.679891 -22336.651571 -22960.907423
1 2 3	0 0 0	11 12 13 14	-31341.440592 -28501.327303 -29191.886193 -33497.821978	-24386.679891 -22336.651571 -22960.907423 -24220.819770
1 2	0 0	11 12 13	-31341.440592 -28501.327303 -29191.886193	-24386.679891 -22336.651571 -22960.907423
1 2 3 4	0 0 0 0	11 12 13 14 15	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512
1 2 3 4 5	0 0 0 0 0	11 12 13 14 15 685	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536
1 2 3 4 5	0 0 0 0 0 0	11 12 13 14 15 685	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925
1 2 3 4 5 6	0 0 0 0 0 0 1 1	11 12 13 14 15 685 11	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339
1 2 3 4 5 6 7 8	0 0 0 0 0 0 1 1	11 12 13 14 15 685 11 12	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339
1 2 3 4 5 6 7 8 9 10	0 0 0 0 0 0 1 1 1 1 1	11 12 13 14 15 685 11 12 13	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234
1 2 3 4 5 6 7 8 9 10 11	0 0 0 0 0 0 1 1 1 1 1	11 12 13 14 15 685 11 12 13 14 15 685 11	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838
1 2 3 4 5 6 7 8 9 10 11 12	0 0 0 0 0 0 1 1 1 1 1 1 2	11 12 13 14 15 685 11 12 13 14 15 685 11	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813
1 2 3 4 5 6 7 8 9 10 11 12 13	0 0 0 0 0 1 1 1 1 1 1 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769
1 2 3 4 5 6 7 8 9 10 11 12 13 14	0 0 0 0 0 1 1 1 1 1 1 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853
1 2 3 4 5 6 7 8 9 10 11 12 13 14	0 0 0 0 0 1 1 1 1 1 1 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 3	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 11 12 13 14 15 15 11 11 11 12 13 14 15 15 11 11 11 12 13	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175 -31006.810601	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615 -22004.844266
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22	0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 3 3	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 11 12 13 14 15	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175 -31006.810601 -30719.495773	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615 -22004.844266 -22759.561740
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	0 0 0 0 0 0 1 1 1 1 1 1 2 2 2 2 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 18 18 18 18 18 18 18 18 18 18 18 18 18	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175 -31006.810601 -30719.495773 -28248.855505 -30332.390858 -29153.398684	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615 -22004.844266 -22759.561740 -23771.499303 -23359.367994 -21496.265113
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	0 0 0 0 0 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 11 12 13 14 15 11 12 13 14 15 15 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 16 11 11 12 13 14 15 15 16 11 11 12 13 14 15 15 16 11 11 12 13 14 15 15 16 16 17 17 18 18 18 18 18 18 18 18 18 18 18 18 18	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175 -31006.810601 -30719.495773 -28248.855505 -30332.390858 -29153.398684 -31219.015970	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615 -22004.844266 -22759.561740 -23771.499303 -23359.367994 -21496.265113 -24257.962331
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27	0 0 0 0 0 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 12 13 14 15 12 13 14 15 11 12 13 14 15 15 11 11 12 13 14 15 14 15 15 11 11 12 13 14 15 14 15 15 11 11 12 13 14 15 14 15 15 11 11 12 13 14 15 14 15 15 16 11 11 12 13 14 15 14 15 16 17 18 18 18 18 18 18 18 18 18 18 18 18 18	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175 -31006.810601 -30719.495773 -28248.855505 -30332.390858 -29153.398684 -31219.015970 -28269.002791	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615 -22004.844266 -22759.561740 -23771.499303 -23359.367994 -21496.265113 -24257.962331 -23651.328536
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26	0 0 0 0 0 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2	11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 685 11 12 13 14 15 12 13 14 15 12 13 14 15 12 13 14 15 11 12 13 14 15 11 12 13 14 15 15 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 11 11 12 13 14 15 15 16 11 11 12 13 14 15 15 16 11 11 12 13 14 15 15 16 11 11 12 13 14 15 15 16 16 17 17 18 18 18 18 18 18 18 18 18 18 18 18 18	-31341.440592 -28501.327303 -29191.886193 -33497.821978 -28864.122676 -29767.168904 -27733.692461 -33503.439105 -33253.988918 -28666.006138 -28543.404999 -26764.807334 -31283.939751 -27940.575589 -28888.282236 -27127.133115 -28939.621876 -30799.715848 -32396.582311 -29835.971857 -31618.508175 -31006.810601 -30719.495773 -28248.855505 -30332.390858 -29153.398684 -31219.015970	-24386.679891 -22336.651571 -22960.907423 -24220.819770 -23114.078512 -23499.217536 -22213.206925 -26638.790908 -25542.650339 -24951.430933 -21941.293088 -20185.195234 -23705.428838 -21245.902813 -23766.845769 -24352.542707 -21644.230853 -23423.217027 -26930.542723 -23660.091484 -26419.659615 -22004.844266 -22759.561740 -23771.499303 -23359.367994 -21496.265113 -24257.962331 -23651.328536 -21970.458833





Optional (for fun only - no extra credit)

Given a Hill Climbing solution, trace the path to the initial state (using the path() function) and visualize the differences between each successsor state.

Here is a simple one. I'm sure you can come up with fancier ones.

```
In [21]: def plot_path_diff(cities, state, path):
             Plot the path differences.
             path=[i for i in path]
             xtmp=[]
             ytmp=[]
             for i in range(len(path) - 1):
                 fig, ax = plt.subplots()
                 state1 = path[i].state
                 state2 = path[i + 1].state
                 state=state1
                 x = [cities[i].x for i in state]
                 y = [cities[i].y for i in state]
                 ax.scatter(x, y)
                 for j in range(len(state)):
                     ax.annotate(state[j], (x[j], y[j]))
                 x+=x[0:1]
                 y+=y[0:1]
                 ax.plot(x, y, color='black')
                 if i==0:
                     xtmp=x
                     ytmp=y
                 else:
                     xtmp_2=[xtmp[j] if x[j]!=xtmp[j] else 0 for j in range(len(x))]
                     ytmp_2=[ytmp[j] if y[j]!=ytmp[j] else 0 for j in range(len(y))]
                     xtmp=x
                     ytmp=y
                     xtmp_2=[xtmp_2[j] for j in range(len(xtmp_2)) if xtmp_2[j]!=0]
                     ytmp_2=[ytmp_2[j] for j in range(len(ytmp_2)) if ytmp_2[j]!=0]
                     #print(xtmp_2,ytmp_2)
                     ax.plot(xtmp_2, ytmp_2, color='red')
```

In [22]: plot_path_diff(all_cities, hc_sol_node.state, hc_sol_node.path())

