1. Rank the following functions at ascending order; that is, find an arrangement f1, f2, ⋯, f8 of the functions satisfying f1 = O(f2), f2 = O(f3), · · · , f7 = O(f8). Briefly show your work for this problem.

The order satisfying f1 = O(f2), f2 = O(f3), · · · , f7 = O(f8) is:

f1: 1 $=\Theta(n^{\frac{1}{\lg n}})$

f2: $n^{\frac{1}{\lg n}}$ $=O(n)$

f3: n $=\Theta(2^{\lg n})$

f4: $2^{\lg n}$ $=O(n^2)$

f5: n^2 $=\Theta(n^2+n)$

f6: n^2+n $=O(n^{\lg\lg n})$

f7: $n^{\lg\lg n}$ $=\Theta((\lg n)^{\lg n})$

f8: $(\lg n)^{\lg n}$

or write it as:

$$1=\Theta(n^{\frac{1}{\lg n}})<n=\Theta(2^{\lg n})<n^2=\Theta(n^2+n)<n^{\lg\lg n}=\Theta((\lg n)^{\lg n})$$

satisfying

$$1=O(n^{\frac{1}{\lg n}}),n^{\frac{1}{\lg n}}=O(n),n=O(2^{\lg n}),2^{\lg n}=O(n^2),n^2=O(n^2+n),n^2+n=O(n^{\lg\lg n}),n^{\lg\lg n}=O((\lg n)^{\lg n})$$

proof f1(n)=1 and f2(n)=$n^{\frac{1}{\lg n}}$:

because $\log_b a=\dfrac{1}{\log_a b}$ so $n^{\frac{1}{\lg n}}=n^{\log_n 2}$

because a=$b^{\log_b a}$ so $n^{\frac{1}{\lg n}}=n^{\log_n 2}=2$

we have f1(n)=1 and f2(n)=2

we have $c_1=1, c_2=1, n_0=1$ satisfying 1=<f1(n)<=f2(n)

so $1=\Theta(n^{\frac{1}{\lg n}})$

or

because $\lim\limits_{x\to\infty}\dfrac{f1(n)}{f2(n)}=\dfrac{1}{2}$

so $1=\Theta(n^{\frac{1}{\lg n}})$

proof f2(n)= $n^{\frac{1}{\lg n}}$ and f3(n)=n :

because $\log_b a = \dfrac{1}{\log_a b}$ so $n^{\frac{1}{\lg n}} = n^{\log_n 2}$

because a=$b^{\log_b a}$ so $n^{\frac{1}{\lg n}} = n^{\log_n 2}$ =2

because $\lim\limits_{x\to\infty} \dfrac{f2(n)}{f3(n)} = \lim\limits_{x\to\infty} \dfrac{2}{n} = 0$

so $n^{\frac{1}{\lg n}}$ =O(n)

proof f3(n)= n and f4(n)=$2^{\lg n}$:

because a=$b^{\log_b a}$ so $2^{\lg n}$=n

so f3(n)=f4(n) , n=Θ($2^{\lg n}$)

proof f4(n)=$2^{\lg n}$ and f5(n)=n^2 :

because a=$b^{\log_b a}$ so $2^{\lg n} = n$

because $\lim\limits_{x\to\infty} \dfrac{f4(n)}{f5(n)} = \lim\limits_{x\to\infty} \dfrac{n}{n^2} = \lim\limits_{x\to\infty} \dfrac{1}{n} = 0$

so $2^{\lg n}$ =O(n^2)

proof f5(n)=n^2 and f6(n)=$n^2 + n$:

because we have $c_1 = \dfrac{1}{3}, c_2 = 1, n_0 = 1$, n>=n_0

n>=$n_0 = 1$ ==> $n^2 + n >= n^2$

n>=$n_0 = 1$ ==> $\dfrac{2}{3} n >= \dfrac{1}{3}$ ==> $\dfrac{2}{3} n^2 >= \dfrac{1}{3} n$ ==> $n^2 >= \dfrac{1}{3} n^2 + \dfrac{1}{3} n$ ==>$n^2 >= \dfrac{1}{3}(n^2 + n)$

we have while $n_0 = 1$, $c_1 (n^2 + n)$<= n^2 <=$c_2 (n^2 + n)$

so n^2 =Θ($n^2 + n$)

or

because $\lim\limits_{x\to\infty} \dfrac{f6(n)}{f5(n)} = \lim\limits_{x\to\infty} \dfrac{n^2 + n}{n^2} = \lim\limits_{x\to\infty} (\dfrac{2n+1}{2n}) = \lim\limits_{x\to\infty} (\dfrac{2}{2}) = 1$

so n^2 =Θ($n^2 + n$)

proof f5(n)=n^2 and f7(n)=$n^{\lg\lg n}$:

because we have $c_1=4, n_0=4$, n>=n_0

n>=n_0 =4 ==> $\lg n >= 2$ ==> $\lg n >= \lg 4$ ==> $2*\frac{\lg 4}{\lg n} <= 2$ ==> $\frac{\lg 16}{\lg n} <= 2$

==> $\lg(\frac{\lg 16}{\lg n}) <= 1$ ==> $\lg\lg 16 - \lg\lg n <= 1$ ==> $2 - \lg\lg n <= 1$

==> $n^{2-\lg\lg n} <= n$ ==> $\frac{n^2}{n^{\lg\lg n}} <= n$ ==> $n^2 <= c_1 n^{\lg\lg n}$

we have while n_0 =4,c1=4 , 0<= $n^2 <= c_1 n^{\lg\lg n}$

so n^2 =O($n^{\lg\lg n}$)

proof f7(n)=$n^{\lg\lg n}$ and f8(n)=$(\lg n)^{\lg n}$:

because $a^{\log_b c} = c^{\log_b a}$ so $(\lg n)^{\lg n} = (\log_2 n)^{\log_2 n} = n^{\log_2 \log_2 n} = n^{\lg\lg n}$

so f7(n)=f8(n) , $n^{\lg\lg n}$ =Θ($(\lg n)^{\lg n}$)

2. Partition your list into equivalence classes such that f(n) and g(n) are in the same class if and only if f(n) = Θ(g(n)).

class: a	f1,f2	$1, n^{\frac{1}{\lg n}}$
class: n	f3,f4	$n, 2^{\lg n}$
class: p(n)	f5,f6	n^2, n^2+n
class: $(\lg n)^{\lg n}$	f7,f8	$n^{\lg\lg n}, (\lg n)^{\lg n}$