电磁学

$\vec{E} = \frac{\vec{F}}{q_0}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

$E_z = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$.

$\vec{p} = q\vec{d}$

$\vec{\tau} = \vec{p}\times\vec{E}$

Gauss' Law $\varepsilon_0\phi = q_{enc}$

$\phi = \oint \vec{E}\cdot d\vec{A}$

uniform sphere of charge $E = \frac{qr}{4\pi\varepsilon_0 R^3}$

infinite sheet: $E = \frac{\sigma}{2\varepsilon_0}$

infinite line of charge $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$

$E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$.

$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$.

$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$.

$U_{tot} = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$

$\nabla\times\vec{E} = 0$, $\vec{E} = -\nabla V$

$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$.

Poisson's Equation. $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

$\oint\vec{E}\cdot d\vec{s} = 0$.

outside a conductor: $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

$i = \frac{dq}{dt}$ $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

$i = \int\vec{J}\cdot d\vec{A}$ $C = \frac{\varepsilon_0 A}{d}$.

$\vec{J} = ne\vec{v}_d$ $C = 2\pi\varepsilon_0\frac{L}{\ln(\frac{b}{a})}$.

$\frac{\partial\rho}{\partial t} = -\nabla\cdot\vec{J}$. capacitance only depends on geometrical factors.

$R = \frac{V}{i}$

$\vec{E} = \rho\vec{J}$

emf $\mathcal{E} = \frac{W}{q}$.

$P = iV$

$P = i^2R = \frac{V^2}{R}$

Energy density $u = \frac{1}{2}\varepsilon_0 E^2$.

$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$.

Gauss' Law with a dielectric $\vec{D} \equiv k\varepsilon_0\vec{E}$

$\oint\vec{D}\cdot d\vec{A} = \varepsilon_0\oint k\vec{E}\cdot d\vec{A} = q$.

$\vec{F}_B = q\vec{v}\times\vec{B}$. $\vec{F}_E = q\vec{E}$

$\vec{F}_B = i\vec{L}\times\vec{B}$

$\vec{\mu} = Ni\vec{A}$, $\vec{p} = q\vec{d}$.

$\vec{\tau}_B = \vec{\mu}\times\vec{B}$, $\tau_E = \vec{p}\times\vec{E}$

$\omega = \frac{|q|B}{m}$. $r = \frac{mv}{qB}$.

Hall effect.

$R_H = \frac{E}{BJ} = \frac{1}{nq}$

Biot-Savart Law

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$, $d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$

$\oint\vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$. $\to$ $\nabla\times\vec{B} = \mu_0\vec{J}$

$\oint\vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$ $\to$ $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

$\nabla\cdot\vec{B} = 0$, $\oint\vec{B}\cdot d\vec{A} = 0$

$B = \frac{\mu_0 i}{2\pi r}$ $B = \mu_0 i\frac{r}{2\pi R^2}$.

$B = \mu_0 i n$ $B = \mu_0 i\frac{N}{2\pi r}$

$\mu = Ni\vec{A}$, $\vec{p} = q\vec{d}$

$\tau_B = \vec{\mu}\times\vec{B}$, $\tau_E = \vec{p}\times\vec{E}$

$U_B = -\vec{\mu}\cdot\vec{B}$, $U_E = -\vec{p}\cdot\vec{E}$

$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$, $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\vec{p}}{z^3}$

$\mathcal{E} = -\frac{d\phi_B}{dt}$

$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$L = \frac{N\phi_B}{i}$

$L = \mu_0 n^2 \ell A$

$U_B = \frac{1}{2}Li^2$

$u_B = \frac{B^2}{2\mu_0}$

$\tau_C = RC,\ \tau_L = \frac{L}{R}$

$M_{12} = M_{21},\ \mathcal{E}_{11} = -L_1\frac{di_1}{dt}$

$\mathcal{E}_{21} = -\frac{d(N_2\phi_{21})}{dt} = -M_{21}\frac{di_1}{dt}$

$\tau_L = \frac{L}{R},\ \mathcal{E} = L\frac{di}{dt} + Ri$

$\tau_C = RC,\ \mathcal{E} = \frac{q}{C} + R\frac{dq}{dt}$

$i = \frac{\mathcal{E}}{R}(1-e^{-\frac{t}{\tau_L}})$

$q = C\mathcal{E}(1-e^{-\frac{t}{\tau_C}})$

$U = U_E + U_B,\ U_E = \frac{q^2}{2C},\ U_B = \frac{1}{2}Li^2$

$\frac{dU}{dt} = -i^2R$

LC oscillations: $L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$

$q = Q\cos(\omega_0 t+\phi),\ \omega_0 = \frac{1}{\sqrt{LC}}$

Damped in RLC: $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$

$q = Qe^{-\frac{t}{\tau}}\cos(\omega t+\phi)$

$q = \tilde{Q}e^{i\hat{\omega}t},\ \omega = \sqrt{\omega_0^2 - (\frac{1}{\tau})^2},\ \frac{1}{\tau} = \frac{R}{2L}$

Forced oscillations: driving angular frequency ω_d.

$\mathcal{E} = \mathcal{E}_m\cos\omega_d t$.

$i = I\cos(\omega_d t+\phi)$

Resonance: equally capacitive and inductive

$\frac{1}{\omega_d C} = \omega_d L$. $(\omega_d = \omega_0)$

$I = I_{max} = \frac{\mathcal{E}_m}{R}.\ \phi = 0$

Maxwell's equations:

$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0},\quad \nabla\cdot\vec{B} = 0$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t},\ \nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

$\nabla^2\vec{E} = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2},\ \nabla^2\vec{B} = \frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$

$\hat{k}\cdot\vec{E} = \hat{k}\cdot\vec{B} = 0$

$\vec{B} = \frac{1}{c}(\hat{k}\times\vec{E})$

Optics

$\frac{1}{p}+\frac{1}{i}=\frac{1}{f}$, $m=-\frac{i}{p}$

plane wave: $f=\infty$

spherical mirror: $f=\frac{r}{2}$

thin lens $\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$ $\left(\frac{n}{n_{medium}}\right)$

$\frac{n_1}{p}+\frac{n_2}{i}=\frac{n_2-n_1}{r}$

Interference.

$\delta=\frac{2\pi}{\lambda}(x_2-x_1)+(\phi_2-\phi_1)$

$E_0^2=E_{01}^2+E_{02}^2+2E_{01}E_{02}\cos(\alpha_2-\alpha_1)$

$\tilde{E}_1=E_{01}e^{i(\alpha_1-\omega t)}$, $\tilde{E}_2=E_{02}e^{i(\alpha_2-\omega t)}$

$\vec{E}_i=(E_{0i}\cos\alpha_i)\hat{x}+(E_{0i}\sin\alpha_i)\hat{y}$

In Young's Double-Slit:

$\delta_2=k\cdot\Delta L=kd\sin\theta\equiv 2\beta.$

$I\propto 2E_0^2[1+\cos(2\beta)]$

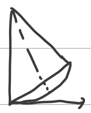
or $I=I_{max}\cos^2\beta.$

$n_1<n_2$, phase shift $\frac{\pi}{2}$

Single-Slit pattern

dark fringes. $a\sin\theta=m\lambda$

$\tilde{E}_\theta=\frac{E_0}{N}e^{-i\omega t}e^{ikr_1}[1+e^{ik(r_2-r_1)}+\cdots+e^{ik(r_N-r_1)}]$



$\frac{I(\theta)}{I_{max}}=\left[\frac{\sin\alpha}{\alpha}\right]^2$

$\alpha=\frac{\pi}{\lambda}\cdot a\sin\theta$

$\tilde{E}_\theta\sim\int_{-\frac{a}{2}}^{\frac{a}{2}}e^{ik_x x}dx$

$E_{sq}=\begin{cases}E_0 & |x|\le\frac{a}{2}\\ 0 & |x|>\frac{a}{2}\end{cases}$

$\tilde{E}_{sq}(k_x)=\int_{-\infty}^{\infty}E_{sq}e^{ik_x x}dx=E_0\int_{-\frac{a}{2}}^{\frac{a}{2}}e^{ik_x x}dx$

$\tilde{E}_{sq}(k_x)=E_0 a\frac{\sin\alpha}{\alpha}$

$I(\theta)=I_{max}\left(\frac{\sin\alpha}{\alpha}\right)^2\cos^2\beta.$

$\beta=\frac{\pi}{\lambda}d\sin\theta$, $\alpha=\frac{\pi}{\lambda}a\sin\theta$

$\delta_2=k_x d$, $\alpha=\frac{a}{2}\cdot k_x$

Fourier Method:

$\tilde{E}_{ds}(k_x)=(e^{-ik_x\frac{d}{2}}+e^{ik_x\frac{d}{2}})\int_{-\frac{a}{2}}^{\frac{a}{2}}E_0e^{ik_x x'}dx'$

right circular: $\vec{E}=E_0[\hat{i}\cos(kz-\omega t)+\hat{j}\sin(kz-\omega t)]$

left circular: $\vec{E}=E_0[\hat{i}\cos(kz-\omega t)-\hat{j}\sin(kz-\omega t)]$

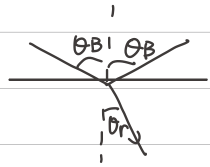
Right-circular $|R\rangle=\frac{1}{\sqrt{2}}(|H\rangle-i|V\rangle)$

monochromatic plane wave → polarized

natural light → unpolarized / randomly polarized

polarized sheet: $\begin{cases}\text{unplorized} & I=\frac{I_0}{2}\\ \text{polarized} & I=I_0\cos^2\theta\end{cases}$

reflected light: partially polarized



Brewster angle θ_B (fully polarized)

$\theta_B+\theta_r=\frac{\pi}{2}$

$n_i\sin\theta_B=n_r\cos\theta_B$

$n_r=\tan\theta_B$. $(n_i=1)$

Rayleigh's criterion

$\theta_R=\sin^{-1}\frac{1.22\lambda}{a}\approx\frac{1.22\lambda}{a}.$

Quantum Mechanics

$p = \frac{h}{\lambda} = \hbar k$

$E = \hbar\omega = K + W$

Compton wavelength $\frac{h}{mc}$

shift $\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$