浙江大学 2019－2020 学年春夏 学期

《离散数学及其应用》课程期末考试试卷(答案)

课程号：_____，开课学院：计算机

考试试卷：√A 卷、B 卷（请在选定项上打√）

考试形式：√闭、开卷（请在选定项上打√），允许带_____入场

考试日期：2020 年 8 月 26 日 21:00-23:00 ，考试时间：120 分钟

诚信考试，沉着应考，杜绝违纪。

考生姓名：_____学号：_____ 任课教师：_____所属院系：_____

题序	一	二	三	四	五	六	七	八	总 分
得分									
评卷人									

1、(20 marks) Determine whether the following statements are true or false. If it is true write a √,otherwise a × in the blank before the statement.

1) ( )$n! = \Theta(2^n)$,where n is a nonzero natural number.

2) ( ) Assume that $\forall x \exists y P(x, y)$ is true and that the domain of discourse is nonempty. Then the statement $\exists x \exists y P(x, y)$ must also be true.

3) ( ) If n is a positive integer, then $\left\lceil \frac{n}{2} \right\rceil \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{n^2}{4} \right\rfloor$.

4) ( )Let A, B and C are sets, $\oplus$ is symmetric difference, if $A \oplus C = B \oplus C$, then A= B.

5) ( ) Let f be a function from set A to set B, S and T be subsets of A, then $f(S \cap T) = f(S) \cap f(T)$

6) ( ) Suppose S={1,2,3}. The graphical representation of relation R on S is given in Fig.1. Then R is reflexive, symmetric, anti-symmetric and transitive.

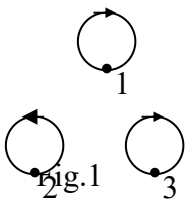


Fig.1

7) ( )Let R and S be relations on nonempty set A， if R and S are transitive, then so is $R \cup S$.

8) ( ) if G = (V,E) is a simple connected non-planar undirected graph, then |V|+|E| ≥ 15.

9) ( )The chromatic number of a simple connected non-bipartite undirected graph is no less than 3.

10) ( ) If both planar graphs G_1 and planar graphs G_2 each have *v* vertices, *e* edges, and *r* regions, these two graphs are isomorphic.

2. (30 marks) Filling in the blanks.

1) Selecting 3 numbers from 1,2,3, …,100, the sum of these 3 numbers must be divided by 4, then the total ways of different selecting is _____

2) Suppose $|A| = 3$ ， there are____binary relations on the set A. Among all binary relations on A, there are __.__.equivalence relations on A.

3) Assume that you have 20 balls and three boxes(labeled A, B, and C).
(a) Assuming that the balls are distinguishable, In how many ways can you put the balls in the boxes, ___
(b) In how many ways can you put the balls in the boxes, assuming that the balls are identical and each box must have at least two balls put into it? ___

4)If $G(x)$ is the generating function for $a_0,a_1,a_2,a_3\ldots$, the generating function for $0,a_1,2a_2,3a_3\ldots$,in terms of G(x) is _____

5) Let *A* be the adjacency matrix for the graph G. The number of triangles (cycles with three edges) that contain v_j is __.___s

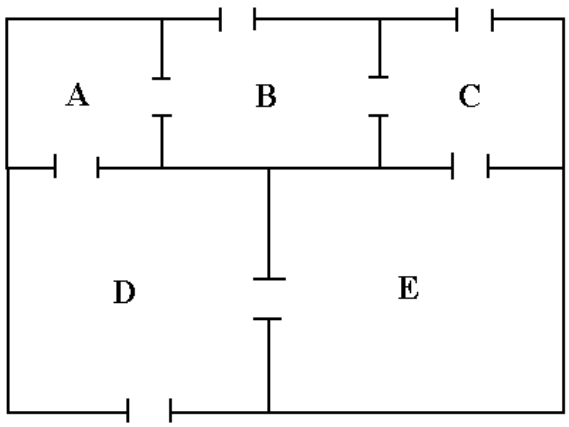
6) The full disjunctive normal form of $\neg r \vee (p \leftrightarrow q)$ is _____.

7) The particular solution of $a_n = 4a_{n-1} - 3a_{n-2} + 4n$ is _____

8） Let *G* be a planar graph with *k* connected components, *v* vertices and *e* edges, then Euler's Formula for this graph is _____.

4. (10 marks) Compute the ways that the digits 0,1,2,3,4,5,6,7,8,9 are arranged so that the first 4 digits are not in their original positions?
Solution:

5. (10 marks) The diagram below represents a floor plan with the doors between the rooms and the outside indicated. The real estate agent would like to be able to tour the house, starting and ending outside, by going through each door exactly once. What is the fewest number of doors that should be added, and where should they be placed in order to make this tour possible? Give reasons for your answer.



Solution:

6. (10 marks) Suppose A is the set of months in the year. That is,

A = {January, February, March, April, May, June, July, August, September, October, November, December }.

We will say that month x is related to month y (written as xRy) if month x and month y begin with the same letter (so, for example, January$R$June).

1) Show that R is an equivalence relation on A.
2) Find its equivalence class.
3) Into how many non-empty sets is the set A partitioned by the equivalence relation R?

   Solution:

7(10 marks) Suppose that Q(*a,b,c,x*) is a quadratic equation in the form of "$ax^2 + bx + c = 0$", where ***a(a≠0)***, *b*, *c* are integers, and *x* is a real number.

1) (1 marks) Use quantifiers and logical connectives to express the fact that any Q(*a,b,c,x*) has at most two real roots.

2) (5 marks) Determine whether the set of real roots of all Q(*a,b,c,x*)s is countable and explain why.

8.(10 marks): Given the following prefix form T:

$$\rightarrow\rightarrow\rightarrow \mathrm{w}vu \rightarrow\rightarrow uw \rightarrow tw$$

(a) Build the corresponding binary expression tree for T.
(b) Give the postfix form for T.
(c) Prove that T is a tautology.