Name_____ Student Number_____ Class_____ Tutor_____

装 订 线

Discrete Mathematics Final Exam
Zhejiang University 2019

Number	1	2	3	4	5	6	7	Total
Records								

1. (20%) Determine whether the following statements are true or false.
   (1) ( )There is at least a maximal in any nonempty poset.
   (2) ( )Let R be a relation on the nonempty set A, if $R \subseteq R^2$ then R is transitive.
   (3) ( )The graph Q_3 is a Euler graph.
   (4) ( )Mathematical induction and strong induction are equivalent.
   (5) ( )The Hasse diagram for the partial ordering $(\{1,2,3,4,5,6,7,8,9\}, |)$ is a tree.
   (6) ( )$((p \to (q \to r)) \to ((p \to q) \to (p \to r)))$ is a tautology.
   (7) ( )Let R and S be relations on a nonempty set A, if R and S are transitive, then so $R \circ S$.
   (8) ( )There is a tree with degrees 3, 2, 2, 2, 1, 1, 1, 1, 1.
   (9) ( )Suppose $A = \{a, \{a\}\}$, then we have $a \subseteq P(A)$.
   (10)( )The negation of "Alan and Bill are absent" is "Alan and Bill are present".

2. (18%) Filling in the blanks.
   (1) If T is a full binary tree with 99 vertices, then its minimum height is _____.
   (2) If G is a planar connected graph with 20 vertices, each of degree 4, then G has _____ regions.
   (3) How many spanning trees does $K_{2,4}$ have? _____.
   (4) There are_____ non-isomorphic rooted trees with 6 vertices.
   (5) There is a binary tree. Its preorder traversal is $ABDECF$ and its inorder traversal is $DBEACF$. Then its postorder traversal is _____.
   (6) Determine the number of 10-combinations from three apples, four oranges, and five pears._____.
   (7) Suppose $|A| = 3$. Among all binary relations on A there are _____ anti-symmetric relations and _____ equivalence relations.
   (8) The diagram at the Figure.1 is the Hasse diagram for a partially ordered set. Referring to this diagram, list the minimal elements:_____.

3. (12%) Let $A = \{1,3,5,7\}$. Let R be the relation on $A \times A$, where $((a,b),(c,d)) \in R$ if $a + b = c + d$.
   (1) Show that R is an equivalence relation.
   (2) Find $[(3,5)]$.
   (3) Find the partition of $A \times A$ from the equivalence relation R.

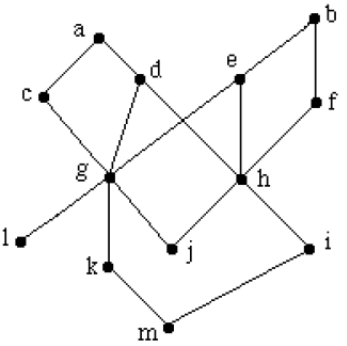

Figure 1: The Hasse diagram

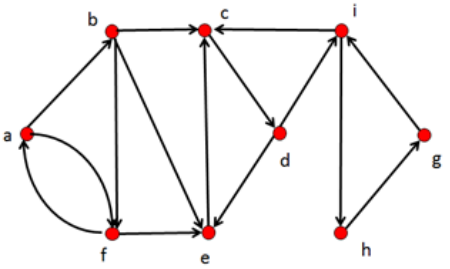

Figure 2: Directed graph G.

4. (8%) Let A be a set, and let R and S be relations on A. Let T be a relation on A defined by xTy if and only if xRy and xSy. Prove or disprove: If R and S are equivalence relations, then T is also an equivalence relation.

5. (8%) A directed graph G is shown in Figure.2.
   (1) Find the strongly connected components of the graph G.
   (2) Determine if G has Hamilton circuit/path. If yes, give a path or circuit; otherwise, give the reason.
   (3) Determine if the underlying undirected graph of the directed graph G has Euler circuit/path. If yes, give a path or circuit; otherwise, give the reason.

6. (12%) G is a directed graph(See Figure.3).
   (1) Find the number of different paths of length 3.
   (2) Find the strongly connected components of the graph G.
   (3) Determine if G has Euler circuit/path or Hamilton circuit/path. If yes, give a path or circuit; otherwise, give the reason.
   (4) Find the chromatic number of the underlying undirected graph of the directed graph G.
   (5) Find the spanning tree for the underlying undirected graph of the directed graph G. Choose b as the root of the spanning tree.

7. (10%) Determine whether the following statements are true and prove them.
   (1) If G is a simple connected planar graph with 6 vertices and 12 edges, each face of G has three edges.

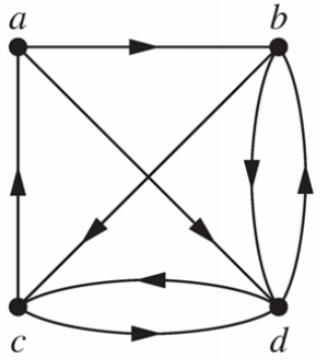

Figure 3: Directed graph G.

(2) We do not have a simple connected planar graph with 7 edges if each vertex' degree is greater than or equal to 3.

8. (12%) Use Huffman coding to encode these symbols with given frequencies: $a : 0.15, b : 0.35, c : 0.23, d : 0.22, e : 0.04, f : 0.01$. What is the average number of bits required to encode a character?