

浙江大学 2019-2020 学年 秋冬 学期

《离散数学》课程期末考试试卷

课程号: 21120401 开课学院: 计算机学院

考试试卷: ☒ A卷 ☐ B卷

考试形式: ☒ 闭卷 ☐ 开卷, 允许带 _____ 入场

考试日期: 2020 年 1 月 14 日, 考试时间: 120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名 _____ 学号 _____ 所属院系 _____

题序	1	2	3	4	5	6	7	8	总分
得分									
评卷人									

ZHEJIANG UNIVERSITY
DISCRETE MATHEMATICS, FALL-WINTER 2019
FINAL EXAM

1. (20 pts) Determine whether the following statements are true or false. If it is true fill a \checkmark otherwise a \times in the bracket before the statement.

- (a) () Let A, B, C and D be arbitrary sets, then $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$.
- (b) () Let A, B be two sets. If $2^A \in 2^B$, then $A \in B$, where 2^X is the power set of X .
- (c) () Let A, B and C be arbitrary sets. If $2^A \oplus 2^C = 2^B \oplus 2^C$, then $A = B$, where \oplus denotes symmetric difference.
- (d) () Let $P(x), Q(x)$ be two predicates, then $\exists x(P(x) \vee Q(x)) \Leftrightarrow \exists xP(x) \vee \exists yQ(y)$.
- (e) () If exactly one of the assignments 000, 011, 100, and 111 make the propositional formula φ false, then φ can be converted in full disjunctive normal form $\Sigma(0, 3, 4, 7)$.
- (f) () The set of all functions from \mathbb{N} to \mathbb{N} is uncountable infinite.
- (g) () Let R be a binary relation. If R is symmetric and transitive, then R is reflexive.
- (h) () Let (S, \preceq) be a partially ordered set, if there is unique minimal element e of S , then e is the least element of S .
- (i) () If a graph contains an Hamilton circuit, then it does not have a cut-edge.
- (j) () Let G be a simple planar graph with e edges, v vertices and r regions, then $r = e - v + 2$.

2. (12 pts) Construct arguments to prove that the following reasoning is valid.

Hypothesis: $\forall x(\forall y(B(x, y) \rightarrow \neg A(y)) \rightarrow \neg C(x))$

Conclusion: $\forall x(C(x) \rightarrow \exists y(A(y) \wedge B(x, y)))$

3. (12%) Prove that $\mathbb{N} \times \mathbb{N}$ is countable infinite.

4. **(10 pts)** There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.

5. **(12 pts)** Let F be the set of all propositional formulas involving variables p_1, p_2, \dots, p_n . Define a binary relation \mathcal{R} on F :

$\forall A, B \in F, A\mathcal{R}B$ if and only if $A \leftrightarrow B$ is a tautology.

- (a) Prove that the binary relation \mathcal{R} on F is an equivalence relation.
- (b) Give a description of the equivalence classes.
- (c) How many different equivalence classes of \mathcal{R} are there?

6. **(14 pts)** Let $G = (V, E)$ be a simple graph. Define its complement \overline{G} as a graph on the vertex set V with an edge set \overline{E} (the complement of E).
- (a) What is the degree sequence of \overline{G} in terms of the degree sequence of G ?
 - (b) An automorphism of a graph G is a permutation of its vertices which preserves adjacency (i.e. $(u, v) \in E \Leftrightarrow (\varphi(u), \varphi(v)) \in E$). Let $\text{Aut}(G)$ be a set of automorphisms of G . Show that $\text{Aut}(G) = \text{Aut}(\overline{G})$.
 - (c) Prove that at least one of G and \overline{G} is connected.
7. **(10 pts)** Draw all non-isomorphism trees with exactly 7 vertices with maximum degree 3.

8. **(10 pts)** Let S be a set having n elements. Let H be the Hasse diagram for the partial ordering $\{(A, B) \mid A \subseteq B\}$ on the power set of S . Let f_n denote the number of edges in a Hasse diagram representing a set with n element.
- (a) Compute f_0 , f_1 , f_2 and f_3 .
 - (b) Find a recurrence relation for f_n , and justify your answer.