Generating Permutations and Combinations

Section 6.6

Section Summary

- Generating Permutations
- Generating Combinations

Generating Permutations

Problem:

List the permutations of any set of n elements.

Solution:

- Any set with n elements can be placed in one-to-one correspondence with the set {1, 2, …, n}
- Generate the permutation of the n smallest positive integers, and then replace these integers with the corresponding elements.

Generating Permutations

The lexicographic ordering of the set of permutations of {1, 2, …, n}

The permutation $a_1a_2\ldots a_n$ precedes the permutation of $b_1b_2\ldots b_n$, if for some k, with $1\le k\le n$, $a_1=b_1$, $a_2=b_2$, …, $a_{k-1}=b_{k-1}$, and $a_k<b_k$,

For example:

123465 124635

Generating Permutations

Algorithm of producing the n! permutations of the integers 1, 2, …, n

- Begin with the smallest permutation in lexicographic order, namely $1234\ldots n$.
- Produce the next larger permutation.
- Continue until all $n!$ permutations have been found.

Generating Permutations

Given permutation $a_1 a_2 \ldots a_n$, find the next larger permutation in increasing order:

(1) Find the integers

a_j, a_{j+1} *with* $a_j < a_{j+1}$ *and* $a_{j+1} > a_{j+2} > \cdots > a_n$

(2) Put in the jth position the least integer among

$a_{j+1}, a_{j+2}, \cdots, a_n$ that is greater than a_j

(3) List in increasing order the rest of the integers

$a_j, a_{j+1}, \cdots, a_n$

Generating Permutations

Example 1:

What is the next larger permutation in lexicographic order after 124653?

Solution:

- The next largest permutation of 124653 in lexicographic order is 125346

Generating Combinations

Problem 1:

Generate all combinations of the elements of a finite set .

Solution:

- A combination is just a subset. $\Rightarrow$ We need to list all subsets of the finite set.
- Use bit strings of length n to represent a subset of a set with n elements. $\Rightarrow$ We need to list all bit strings of length n.
- The 2^n bit strings can be listed in order of their increasing size as integers in their binary expansions.

Generating Combinations

Algorithm of producing all bit strings

- Start with the bit string 000…00,with n zeros.
- Then, successively find the next larger expansion until the bit string 111…11 is obtained.

The method to find the next larger binary expansion:

Locate the first position from the right that is not a 1, then changing all the 1s to the right of this position to 0s and making this first 0 a 1.

For example:

1000110011 → 1000110100

Generating Combinations

Problem 1:

Generate all r-combinations of the set {1, 2, ..., n}

Solution:

The algorithm for generating the r-combination of the set $\{1, 2, \ldots, n\}$

(1) $S_1 = \{1, 2, \ldots, r\}$

(2) If $S_i = \{a_1, a_2, \cdots, a_r\}, 1 \le i \le C_n^r - 1$ has found, then the next combination can be obtained using the following rules.

First, locate the last element a_i in the sequence such that $a_i \ne n - r + i$. Then replace a_i with $a_i + 1$ and a_j with $a_i + j - i + 1$, for $j = i + 1, i + 2, \ldots, r$.

Generating Permutations

Example 2:

$S_i = \{2,3,5,6,9,10\}$ is given from the set $\{1,2,3,4,5,6,7,8,9,10\}$. Find S_{i+1} .

Solution:

$$S_{i+1} = \{2,3,5,7,8,9\}$$

Homework

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