诚信考试，沉着应考，杜绝违纪。

考生姓名：_____学号：________ 任课教师：_____所属院系：_______

题序	一	二	三	四	五	六	七	八	总 分
得分									
评卷人									

1、(20 marks) Determine whether the following statements are true or false. If it is true write a √, otherwise a × in the blank before the statement.

1) (×) $n! = \Theta(2^n)$,where n is a nonzero natural number.
2) (√) Assume that $\forall x \exists y P(x,y)$ is true and that the domain of discourse is nonempty. Then the statement $\exists x \exists y P(x,y)$ must also be true.
3) (√) If n is a positive integer, then $\left\lceil \frac{n}{2} \right\rceil \left\lfloor \frac{n}{2} \right\rfloor = \left\lfloor \frac{n^2}{4} \right\rfloor$.
4) (√)Let A, B and C are sets, ⊕ is symmetric difference, if $A \oplus C = B \oplus C$, then A= B.
5) (×) Let f be a function from set A to set B, S and T be subsets of A, then $f(S \cap T) = f(S) \cap f(T)$
6) (√) Suppose S={1,2,3}. The graphical representation of relation R on S is given in Fig.1. Then R is reflexive, symmetric, anti-symmetric and transitive.

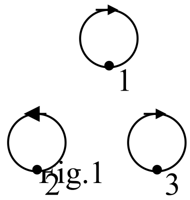


Fig.1

7) (×)Let R and S be relations on nonempty set A，if R and S are transitive, then so is $R \cup S$.
8) (√) if G = (V,E) is a simple connected non-planar undirected graph, then |V|+|E| ≥ 15.
9) (√)The chromatic number of a simple connected non-bipartite undirected graph is no less than 3.
10) (×) If both planar graphs G_1 and planar graphs G_2 each have v vertices, e edges, and r regions, these two graphs are isomorphic.

2. (30 marks) Filling in the blanks.

1) Selecting 3 numbers from 1,2,3, …,100, the sum of these 3 numbers must be divided by 4, then the total ways of different selecting is $\underline{C_{25}^3 + 3C_{25}^1 C_{25}^2 + 25^3}$

2) Suppose $|A| = 3$ ， there are_$\underline{2^9}$___binary relations on the set A. Among all binary relations on A, there are ___5__.equivalence relations on A.

3) Assume that you have 20 balls and three boxes(labeled A, B, and C).
(a) Assuming that the balls are distinguishable, In how many ways can you put the balls in the boxes, __3^{20}_
(b) In how many ways can you put the balls in the boxes, assuming that the balls are identical and each box must have at least two balls put into it? _C(3-1+14,14)__

4)If $G(x)$ is the generating function for $a_0, a_1, a_2, a_3 \ldots$, the generating function for $0, a_1, 2a_2, 3a_3 \ldots$,in terms of G(x) is $\underline{xG'(x)}$

5) Let A be the adjacency matrix for the graph G. The number of triangles (cycles with three edges) that contain v_j is $\underline{0.5\,[A^3]_{j,j}}$.

6) The full disjunctive normal form of $\neg r \vee (p \leftrightarrow q)$ is

$$(\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r)$$

.

7) The particular solution of $a_n = 4a_{n-1} - 3a_{n-2} + 4n$ is __$-n^2-4n$_______

8) Let G be a planar graph with k connected components, v vertices and e edges, then Euler's Formula for this graph is $\underline{v - e + r = 1+k}$.

4. (10 marks) Compute the ways that the digits 0,1,2,3,4,5,6,7,8,9 are arranged so that the first 4 digits are not in their original positions?

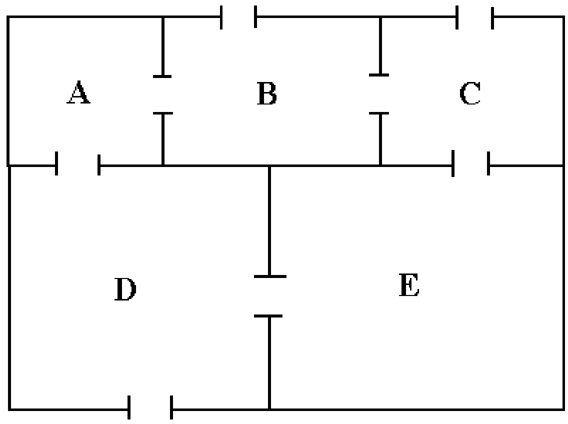
Solution:

Let A_0 be the set 0 is arranged in its original position. Let A_1 be the set 1 is arranged in its original position. Let A_2 be the set 2 is arranged in its original position. Let A_3 be the set 3 is arranged in its original position. Then what we want to obtain is:

$$|\overline{A_0} \cap \overline{A_1} \cap \overline{A_2} \cap \overline{A3}| = |U| - |A_0 \cup A_1 \cup A_2 \cup A_3|$$

=10! -4*9!+6*8!-4*7!+ 6!

5. (10 marks) The diagram below represents a floor plan with the doors between the rooms and the outside indicated. The real estate agent would like to be able to tour the house, starting and ending outside, by going through each door exactly once. What is the fewest number of doors that should be added, and where should they be placed in order to make this tour possible? Give reasons for your answer.



Solution: Letting the rooms be vertices (with the outside as a vertex also) and the doors be edges between these vertices, the tour corresponds to an Eulerian circuit in this graph. This is possible if and only if every vertex has even degree. Since rooms B, C, D and the outside have odd degree, the tour is not possible. We would have to add at least two edges between these vertices (i.e., at least 2 new doors) to make it possible. Since we can not join C to D (there is no common wall to put a door in), we could join C to B and D to the outside. Thus, it is possible to take the tour with the addition of two doors, a first one between rooms C and B and a second door from D to the outside.

6. (10 marks) Suppose A is the set of months in the year. That is,

A = {January, February, March, April, May, June, July, August, September, October, November, December }.

We will say that month x is related to month y (written as xRy) if month x and month y begin with the same letter (so, for example, January$R$June).

1) Show that R is an equivalence relation on A.
2) Find its equivalence class.
3) Into how many non-empty sets is the set A partitioned by the equivalence relation R?

Solution:

1）reflexive: Since month x has the same first letter in its name as month x, then xRx.

symmetric: If xRy, then month x and month y begin with the same letter. That is month y and month x begin with the same letter. It follows that yRx.

Transitive: If xRy and yRz, then xRz.

(3 points)

2) Equivalence class:

[January]={ January, June, July }

[February]={ February}

[March]={ March, May}

[April]={ April, August }

[September]={ September }

[October]={ October }

[November]={ November }

[December]={ December } (2 points)

3) 8 (1 points)

7(10 marks) Suppose that Q(a,b,c,x) is a quadratic equation in the form of "$ax^2 + bx + c = 0$", where $a(a \neq 0)$, b, c are integers, and x is a real number.
1) (1 marks) Use quantifiers and logical connectives to express the fact that any Q(a,b,c,x) has at most two real roots.
2) (5 marks) Determine whether the set of real roots of all Q(a,b,c,x)s is countable and explain why.
Solution：1)

$$\forall a \forall b \forall c(a \neq 0 \rightarrow \forall x_1 \forall x_2 \forall x_3((ax_1^2 + bx_1 + c = 0 \land ax_2^2 + bx_2 + c = 0 \land ax_3^2 + bx_3 + c = 0) \rightarrow (x_1 = x_2 \lor x_1 = x_3 \lor x_2 = x_3))$$
)

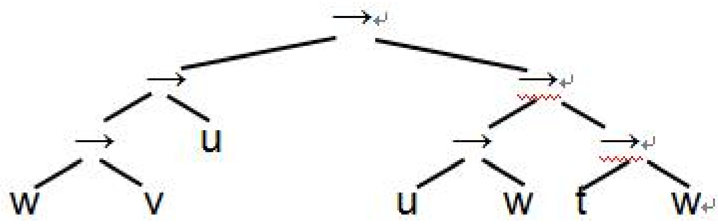
2) There are at most two real roots of each quadratic equation, so the number of solutions is countable as long as the number of triples (a, b, c), with a, b, and c integers, is countable. There are a countable number of pairs (b, c), since for each b (and there are countably many b's) there are only a countable number of pairs with that b as its first coordinate. Now for each a (and there are countably many a's) there are only a countable number of triples with that a as its first coordinate (since we just showed that there are only a countable number of pairs (b, c)). Thus there are only countably many triples.

8.(10 marks): Given the following prefix form T:

$$\rightarrow\rightarrow\rightarrow wvu \rightarrow\rightarrow uw \rightarrow tw$$

(a) Build the corresponding binary expression tree for T.
(b) Give the postfix form for T.
(c) Prove that T is a tautology.

A:
(a)



(3 marks)

(b)
wv→u→uw→tw→→→
(3 marks)

(c)
$$((w \to v) \to u) \to ((u \to w) \to (t \to w))$$
$$\equiv \neg((w \to v) \to u) \vee ((u \to w) \to (t \to w))$$
$$\equiv \neg(\neg(w \to v) \vee u) \vee (\neg(u \to w) \vee (t \to w))$$
$$\equiv \neg(\neg(\neg w \vee v) \vee u) \vee (\neg(\neg u \vee w) \vee (\neg t \vee w))$$
$$\equiv \neg((w \wedge \neg v) \vee u) \vee ((u \wedge \neg w) \vee (\neg t \vee w))$$
$$\equiv \neg((w \vee \neg v) \wedge (\neg v \vee u)) \vee ((u \vee \neg t \vee w) \wedge (\neg w \vee \neg t \vee w))$$
$$\equiv \neg(w \vee u) \vee \neg(\neg v \vee u) \vee (u \vee \neg t \vee w)$$
$$\equiv (\neg w \wedge \neg u) \vee (v \wedge \neg u) \vee (u \vee \neg t \vee w)$$
$$\equiv (\neg w \vee (v \wedge \neg u) \vee u \vee \neg t \vee w) \wedge (\neg u \vee (v \wedge \neg u) \vee u \vee \neg t \vee w)$$
$$\equiv T \wedge T$$
$$\equiv T$$
(4 marks)