# RC Circuits

"Nature, to be commanded, must be obeyed."

F. Bacon (1561-1626)

**OBJECTIVE**

To study a simple circuit that has time-dependent voltages and current.

**THEORY**

Circuits with steady currents and voltages are of some importance, but time-varying currents are more interesting. Consider, for example, the series connection of a resistor, capacitor and optional battery, shown in Fig. 1. If we flip the switch to connect the resistor to the battery current starts to flow, building up charge on the capacitor until the capacitor voltage equals the battery voltage. In the opposite switch position, charge slowly flows from one plate to the other until the capacitor voltage is zero. Your text demonstrates that the voltage across the capacitor changes exponentially in both cases. The expressions are

$$V_C(t) = V_B\left(1 - e^{-t/\tau}\right) \qquad \text{charging} \quad (1)$$

$$V_C(t) = V_C(t = 0)e^{-t/\tau} \quad \text{discharging} \quad (2)$$

where $\tau = RC$ is the characteristic time of the circuit.

In order to actually measure $V_C$ we need to use a non-ideal voltmeter, which will not have infinite resistance. **For example, the computer interface we use can be modeled as an ideal voltmeter in parallel with a 1 MΩ resistance, which is not negligible when R is on the order of 100 kΩ.** This means that we need to understand a circuit like Fig. 2 in order to interpret measurements.
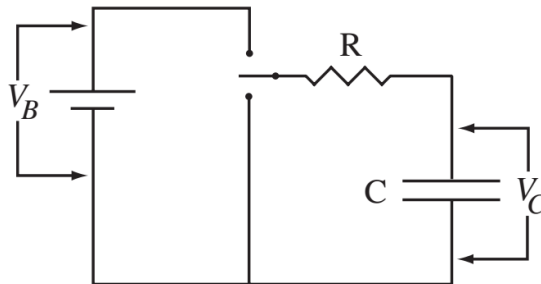


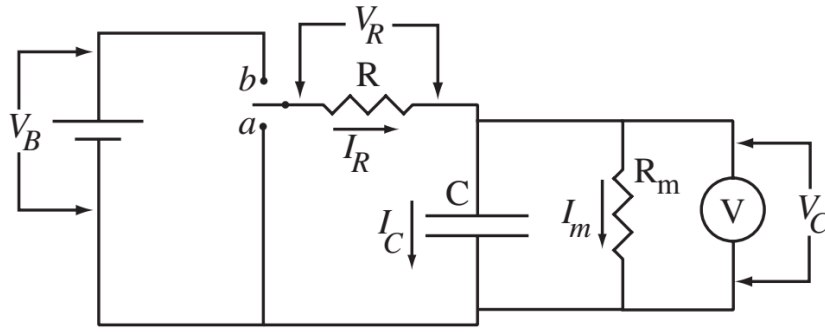*Fig. 1 Ideal circuit for displaying capacitor charging and discharging.*

*Fig. 2 Actual circuit for displaying capacitor charging and discharging.*

In the circuit described in figure 2, when the switch is set to position $a$ to observe a discharge, $R_m$ is in parallel with R, so the discharge still follows Eq. 2, it just occurs faster. The new time constant is $\tau' = R'C$ where $R'$ is the parallel combination of $R_m$ and $R$, given by

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R_m} \qquad or \qquad R' = \frac{R_m R}{R_m + R} \qquad\qquad (3)$$

Note that as $R_m$ becomes very large, $R'$ approaches $R$ and we recover the ideal result.

The charging circuit is more complicated. We expect the final voltage to be less than $V_B$ because some current will always flow through $R_m$, causing a voltage drop across R. The time constant will turn out to be $\tau'$, but this is not obvious, so a complete analysis is needed. Applying Kirchoff's laws, we get the three relations

$$I_R = I_C + I_m \qquad\qquad (4)$$

$$V_B = I_R R + V_C \qquad\qquad (5)$$

$$V_C = I_m R_m \qquad\qquad (6)$$

These can be combined to get an equation with only $V_C$ and $I_C$ as unknowns

$$V_B = I_C R + V_C \frac{R}{R_m} + V_C \qquad\qquad (7)$$

We can then use the usual relation for a capacitor

$$I_C = C \frac{dV_C}{dt} \qquad\qquad (8)$$

to get a differential equation for $V_C$

$$\frac{dV_C}{dt} = \frac{1}{RC}\left[V_B - V_C\left(1 + \frac{R}{R_m}\right)\right]$$ (9)

This looks daunting, but the solution is fairly simple,

$$V_C(t) = V_B\left(\frac{R_m}{R+R_m}\right)\left(1 - e^{-t/\tau'}\right)$$ (10)

which you can verify by substituting into Eq. 9 and using the earlier definitions. As promised, the final asymptotic voltage is less than $V_B$, and the time constant is the same as for the discharge, $\tau' = R'C$.

## EXPERIMENTAL PROCEDURE

You should first verify that the charge and discharge of a capacitor are exponential, as suggested by Eqs. 10 and 2. If this is true, you can then try to demonstrate that $\tau' = R'C$. Finally, you can use measured time constants and resistances to determine the capacitance of two capacitors connected in series or parallel. The measurements will be automated with the use of a voltage interface on the lab computers.

*Charging*

The circuit of Fig. 3, with $R = 470$ k$\Omega$, $C = 10$ μF, and the power supply output set to 10 V, is convenient for measuring charging. **Neither the 10 μF nor the 22 μF capacitors that you will be using later in the lab are one of the capacitors that are permanently attached to the component board; they are the small <span style="color:orange">orange capacitors</span> that are attached to the silver posts at the right-hand side of the board.** When the switch is set to $a$ the capacitor will discharge quickly, setting the initial condition. Flipping the switch to $b$ allows the power supply to charge the capacitor through R. *Be sure to connect the capacitor with positive and negative as shown.* **The capacitor will not behave properly if connected in reverse.**
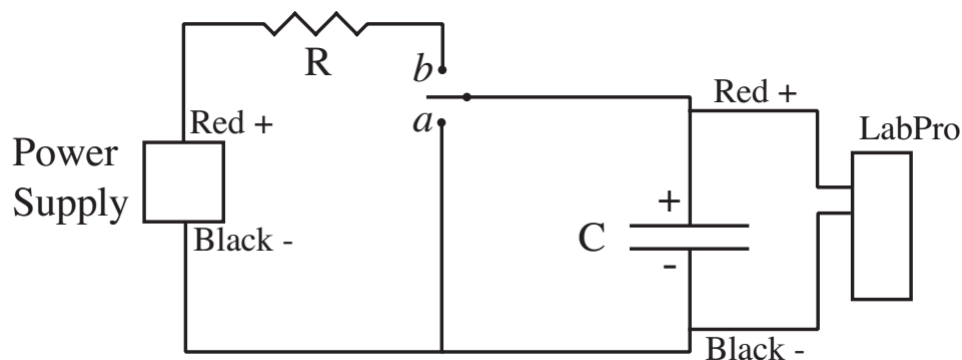


*Fig. 3 Practical circuit for measuring capacitor charging.*

To compare with Eq. 10, you need to measure the voltage at regular time intervals over the full range of the charging. As a start, you might choose to measure for about 10 time constants $(10\tau')$, and obtain 50-100 points **per time constant**. To do this, start LoggerPro from the file **RC.cmbl on the computer desktop**, and connect the voltage leads across the capacitor as shown in figure 3. Select Experiment > Data Collection.... on the menu bar. Click on the Collection tab in the window, and set Mode: Time Based, the desired total measurement time, and the speed in samples per second. You should start collecting data only when the capacitor begins to charge, so click on the Triggering tab. **Check Trigger on sensor value and select Increasing, Across, 0.1V**. This will start data acquisition when the input voltage increases from less than 0.1 V to more than 0.1 V. Click on Done to save the settings.

To collect data, put the switch in position *a* so that the initial voltage is zero, click on Collect, wait five seconds and flip the switch to begin charging. You should see a rising curve on the screen, starting near zero time.

When you have a good curve, use Analyze > Curve fit.... to fit the data with the pre-defined function called (RC Charge)

$$A*(1-exp(-G*t))+B \tag{11}$$

Except for the constant B, this is Eq. 10 in a form that the computer program recognizes. The additive constant has two purposes. It compensates for the fact that the triggering is not perfect, so the measurements may not start at zero voltage at zero time. A more interesting reason is that some charge may be retained in the insulation of the capacitor, even though the plates have been shorted together, causing a small voltage even before the charging starts. You may have noticed a similar effect when you studied electrostatics at the start of the semester.

Does it appear that your data are well-described by Eq. 10? Do the points follow the expected curve? Is the voltage at long times, as read from the data table, close to the expected value, $V_B \left( \frac{R_m}{R+R_m} \right)$ within 5%? Is the time constant, $\tau'$, close to the expected value within 5%? It will improve your accuracy for the expected values if you measure the 470 k$\Omega$ resistor with the ohmmeter function of the DMM, rather than relying on the marked values.

*Discharging*

Use the circuit of Fig. 4, with the same component values, to study the discharge. The switch is wired to permit rapid charging in position *b*, and discharging through R in position *a*. Set up LoggerPro as before, but **change the triggering to Decreasing, Across a voltage slightly less than the power supply setting**. This will cause the system to start data acquisition as soon as the voltage starts to decrease.
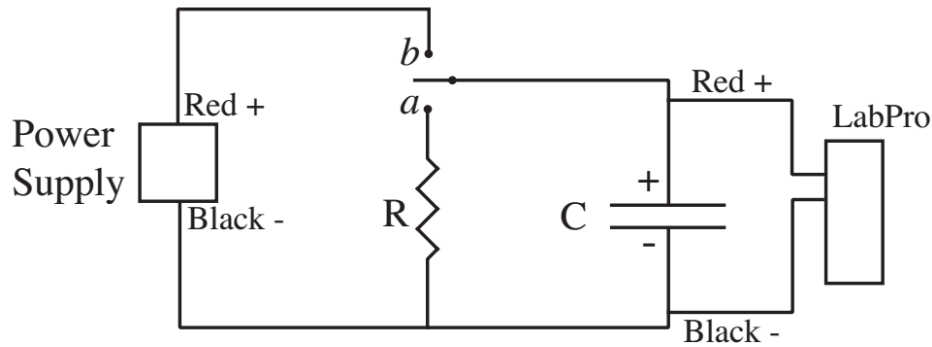
*Fig. 4 Practical circuit for measuring capacitor discharge.*

Measure $V_C(t)$ as you did before, and fit your data to Eq. 2 using the pre-defined function called (RC discharge)

$$A*exp(-G*t)+B \qquad (12)$$

Here the constant **B** accounts only for residual charge in the capacitor dielectric. Does the equation fit accurately, with the same value for the time constant, $\tau'$, as you found for charging within 5%?

You can also check that $\tau' = R'C$ for the discharge circuit, by using the relation

$$\frac{1}{\tau'} = \frac{1}{CR'} = \frac{1}{CR} + \frac{1}{CR_m} \qquad (13)$$

This says that a plot of the fitting coefficient **G** (which is just $1/\tau'$) vs $1/R$ should be a straight line with slope $1/C$ and intercept $1/CR_m$. If correct, this verifies the relation and provides values for $C$ and $R_m$.

Measure the decay time for the 10 µF capacitor with at least 5 different resistors, and make the plot. Your choice of resistors should cover a fairly wide range of time constants, so you will have to adjust the acquisition time and rate to get a sufficient number of points over the whole decay time for each resistor. Since the computer system becomes somewhat inaccurate at high data rates, you should only use time constants **longer than 1 second, which would limit you to using only resistors with resistance greater than 100 kΩ**. It will also improve your accuracy if you measure the resistances with the ohmmeter function of the DMM, rather than relying on the marked values. Is your plot straight? Are the values of $C$ and $R_m$ that you find agree with the manufacturer's claims within 10%?

*Capacitance measurements*

Having characterized your RC circuit, you can use it as a tool to measure capacitance. Pick a resistor that should give a time constant of 2-3 seconds with the 22 µF capacitor, measure the decay rate, and deduce its actual capacitance. Next, connect the 10 µF and 22 µF capacitors in parallel, being careful to get the polarities correct, and measure the capacitance of the combination. Do you get the expected result within 10% for the combination if you use your measured values of the individual capacitors? Repeat for the series combination. Do you get the textbook result within 10%?