Projectile Motion

"Common sense is nothing more than a deposit of prejudices laid down by the mind before you reach eighteen."

A. Einstein

OBJECTIVES

To observe and describe some one- and two dimensional motions.

THEORY

An important insight, due to Galileo, is that a force in one direction does not affect motion at right angles to the force. The significant force in this exercise will be gravity, which causes any object near the Earth's surface to fall downward with a constant acceleration, the famous *g*. You will study in some detail how things fall vertically, with and without additional horizontal motion.

If we choose a coordinate system with the y-axis vertical and the x-axis horizontal, the gravitational force on any object will cause a constant acceleration in the y direction, and no acceleration in the x direction, assuming no other forces are present. The resulting motion is described by the following equations

$$x(t) = x_0 + v_{0x}t \qquad (1)$$

$$y(t) = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad (2)$$

We will examine situations for which v_{0x} is either zero, a vertical fall, or non-zero, a projectile trajectory.

EXPERIMENTAL PROCEDURE

The video camera and computer will again be the main measuring tool. Pick an area near your lab bench where you can drop or toss things over a range of a meter or two without impacting the neighboring group. Set up the camera at least a couple of meters away, so it has a clear view of the area. Adjust the holder so the picture is level, and the camera points horizontally. The line of sight to the camera must be perpendicular to the x-y plane of the motion, as best you can arrange. What would happen if the line of sight were not perpendicular to the plane of the motion? If the line of sight were not perpendicular to the x-y plane of the motion would the measured displacement of an object moving in the x-y plane be smaller or larger than the actual displacement of the object?

Start LoggerPro from the file **Graph.cmbl** on the computer desktop to configure it to capture the movie by going to Insert > Video Capture, and verify that you have a good image of the desired area. If you cannot open the video capture window, check that the camera is turned on and all cables are connected. You are now ready to examine several different projectile motions.

Dropping a ball. For a first exercise, drop the rubber ball so it will fall vertically from the top to the bottom of the camera image. Using LoggerPro, mark the same position on the ball in each frame, and only mark position **when the ball is in the air, NOT touching your hands, the ground or anything else.** You should collect at least 8 data points. To get more data points, you can increase the distance between the camera and where you drop the ball. **Also, you can drop the ball from where it is out of the frame of the camera. You are only interested in the motion of the ball when the only force on it is gravity, so you can't use any portion of the video where the ball is in physical contact with anything.** Use the diameter of the ball, which you will have to measure, to calibrate the length scale. **DO NOT** calibrate the length scale using anything other than the ball itself. With the marking and calibration done, click on the graph window to see the $x(t)$ and $y(t)$ plots that describe the motion. Go to Analyze > Curve Fit..., select the Y data in the popup, and choose a quadratic function in the next popup to fit it. Does the quadratic describe $y(t)$ reasonably well, as expected from Eq. 2? Deduce a value of g from the fitting parameter for the t^2 term. Is it reasonably close to what you expect? Video data is subject to a lot of systematic errors, so the numeric value might be off by 10-20%. What specific factors do you think contribute to the difference between your deduced value of g and 9.8 m/s^2 ? **DO NOT** use general description such as "human error" when answering this question. Please be as specific as you can in your description. Another way to answer this question is to describe one way that you can "cheat" to get your deduced value of g to as close to 9.8 m/s^2 as you wish by manipulating the length scale calibration on the video analysis software.

Tossing a ball. Next you can try a two-dimensional projectile motion. Capture a movie as you toss the ball to your partner. A **underhanded slow toss** will give the best movie. **Make sure the ball does not go out of the frame of the camera at the highest point of its trajectory.** Remember to keep the plane of the motion perpendicular to the camera line of sight. Again, you can position the camera so that the person tossing the ball and the person receiving the ball are not in the camera frame. When you get a good movie, mark the same position on the ball in each frame and calibrate the scale as you did before. **Remember to use only those frames in which the ball is flying freely, without touching anything.** If Eq. 1 is correct, $x(t)$ should be a straight line. Apply Analyze > Linear Fit to the X-data to see if that is the case. The *y(t)* data can be analyzed the same way you did the vertical fall. Is *y(t)* quadratic, with the expected acceleration?

Tossing an irregular object. The motion of a less symmetric object, like a flat rectangular mouse pad or a bottle, is obviously more complicated since it can tumble or spin. Nevertheless, you will learn later that one special point in the object, called the center of mass, moves according to Eqs. 1 and 2. You can demonstrate this experimentally by tossing one of the bottles or mouse mats to your partner. **You must give the object a spin in the x-y plane in your throw.** When you get a good movie, analyze it as you did the ball movie. Don't forget to recalibrate the length scale using the object you tossed. **DO NOT** calibrate the length scale using anything other than the object you tossed. The center of mass of our objects is usually near the geometric center, which you can locate fairly well by eye. Does the center of mass point move as claimed? What about some other point on the object? Go back to the movie, and mark the position of a corner or end. Demonstrate that the motion of a non-central point is not described by Eqs. 1 and 2.

Leaping humans. The center of mass of even more flexible objects will follow the same trajectory, as you can show with a jumping human. Set up the camera so you can see the center of mass of the jumper, keeping the line of sight perpendicular to the intended plane of motion, and then record some sort of jump or leap. It is not necessary to include the entire body of the jumper in the camera frame, but the center of mass of the jumper must be in the camera frame throughout the entire recording. The center of mass of a human is roughly in the middle, at the waist line. Mark the center of the jumper's waist in each frame **where his or her feet are off the ground** and see if it follows the expected trajectory. The data will be a bit sloppy, since the center of mass can actually shift significantly as arms and legs flex, but you should get approximate agreement if the jumper is not too floppy. Use a part of the jumper's body to calibrate the length scale. **DO NOT** calibrate the length scale using anything other than a part of the jumper's body.

Unfree fall. For all the objects studied so far, gravity dominated the motion and led to the constant vertical acceleration assumed in deriving the equations of motion. If other forces, such as air resistance, are also significant, the motion can be very different. To see one example of this, analyze the motion of a falling styrofoam bowl. **The styroform bowl will be easier to track with the camera if the wider face of the bowl faces upward during the fall.** Don't forget to recalibrate the length scale using the bowl. **DO NOT** calibrate the length scale using anything other than the bowl itself. Using your graphs [*y(t)* and $v_y(t)$], describe how the acceleration changes as the bowl falls toward the ground. It is actually easier to see how the acceleration of the bowl changes with time if you look at the plot of the vertical velocity as a function of time. Later in the course you will be able to derive the equations for this motion by including the force of air resistance.

Clean up. When you have finished using your data, please drag any files you saved to the trash so the next student has a clean work space. Don’t delete any files that are not yours, as they will be needed by others.