Kinematics in 1-D

"The ill and unfit choice of words wonderfully obstructs the understanding"
F. Bacon (1561-1626)

OBJECTIVES

To learn to describe one-dimensional motion. To learn how to use video motion analysis.

THEORY

The motion of an object along a straight line, as suggested in Fig. 1, can be described by the position, velocity and acceleration of the object, any or all of which may change with time. Although these words are familiar, they have more limited and specific meanings in science than they do in common usage.

Position is simply the distance from a chosen point, called the origin. For motion on a line, the position at any time can be specified by a single value, x, with units of distance. In Fig. 1, positions to the right of the origin are given a positive sign, positions to the left are negative. We might specify the position of an object by giving a mathematical function, $x(t)$, or by giving a table of values of x at various times t.

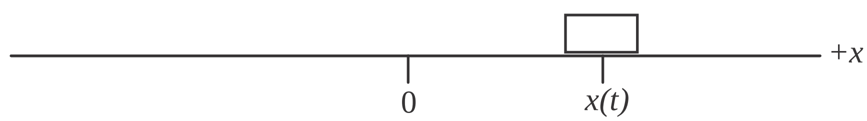

Fig. 1 Coordinate axis for one-dimensional motion, showing an object located at position x at time t.

Velocity is a calculus concept, the derivative of position with respect to time. If $x(t)$ is a known function, the velocity can be found by differentiation

$$v(t) = \frac{dx(t)}{dt} \tag{1}$$

More commonly, the position is measured at several different times and the velocity is approximated by

$$v(t) = \frac{\Delta x}{\Delta t} \tag{2}$$

where Δx is the distance moved in the short time Δt. As Δt becomes very small this becomes exactly equal to the derivative. Finally, if a plot of x vs t is made, $v(t)$ is the slope of the line tangent to the curve at time t. Again, this is a familiar result from calculus, and will be particularly useful in this exercise.

For motion along a line, the sign of v indicates the direction of motion but not where the particle is relative to the origin. With the axes in Fig. 1, v is positive for motion to the right, negative for motion to the left. The magnitude of v is a positive number often called the speed.

Acceleration is also defined as a derivative,

$$a = \frac{dv}{dt} \tag{3}$$

the rate of change of velocity. The acceleration can be found from measurements or plots of $v(t)$ in much the same way as velocity is found from position data.

The sign of a indicates the direction of the acceleration, but doesn't directly show whether the speed is increasing or decreasing. For motion along a line, if v and a have the same sign, the object is speeding up. If they have the opposite sign, it is slowing down. (It's even trickier for circular motion, where the magnitude of acceleration for an object can be constant and non-zero, but its speed never changes.)

EXPERIMENTAL PROCEDURE

The main instrument is a video camera and a computer that captures a movie of the motion. Like any movie, this one is a series of individual images taken at regular time intervals. (For US standard video, 29.97 images per second.) The computer software allows us to look at each successive image and mark the position of a point in the picture. The x-y coordinates of the points are presented in a data table and as a graph vs time.

The camera should already be set up so that it looks directly down on the lab table. Check that the camera power adapter is plugged in and all cables are connected. Start the LoggerPro video capture program, following the instructions in *Using LoggerPro*, and locate the area of the lab table that the camera sees.

Position the cart so that it is near one side of the camera screen, and practice rolling it smoothly along the bench. Note that it is heavy, about 15 kg (33 lbs), and will require a noticeable force to start or stop. *Do not let it fall off the table, and do not let it run into the computer.* The bumpers will help confine the cart, but don't count on them to stop a fast-moving cart. Stop or at least slow the cart by hand before it hits the bumper.

Analysis of a graph. Follow the instructions in *Using Logger Pro* to capture a movie of the cart as it moves in a straight line across the picture. Push on it to vary the speed or even change direction, but be fairly quick. Analyzing more than two or three seconds of video is tedious. By default, the side of the screen where the computer sits is in *the positive x* direction for the movies captured by *Logger Pro*.

Once you capture an **interesting** motion, use the analysis window to mark the position of a convenient point on the cart in each frame. **A simple linear *x(t)* is not consider interesting motion.** Use the length of the cart, which you will have to measure, to calibrate the length scale. **DO NOT** use anything else other than the cart itself to calibrate the length scale. With the marking and calibration done, click on the graph window to see the *x(t)* and *y(t)* plots that describe the motion. Only the *x* data is of interest, so click on the axis label and select just the X variable for display.

Print a copy of the *x(t)* graph for each student with the File > Print Graph... command. (**DO NOT** use Print. It prints the whole screen, and is very slow.) In the space at the bottom of the page, each student should sketch the *v(t)* graph corresponding to the printed *x(t)*. For convenience, align the time scales, and **provide a rough vertical scale**, using the points of zero and maximum speed for reference. You will probably find it easiest to get the speeds by drawing tangent lines at the desired locations and finding their slopes.

When you finish your sketch, choose X Velocity on the LoggerPro graph to see how your result compares with what the computer calculated using the measured $\Delta x/\Delta t$. If you find substantial discrepancies, review your work so you understand what went wrong.

Constant velocity. Next, record a movie as you push the cart along the bench at constant velocity, and then produce the *x(t)* plot. To see how well you did, click and drag to select the points where you think the speed was nearly constant. Use Analyze > Linear Fit to display the straight line that best fits your chosen data points. If the data points are all close to the line, the motion occurred at constant velocity. Explain why a linear *x(t)* implies constant velocity motion.

It is fairly hard to move at constant velocity using your hand, so you should also try an alternative method. Start the cart at one edge of the camera screen with a hard push, and then let it coast until it almost hits the barrier. Record the movie and analyze the data as before, but for this part of the lab **uses only the points where you were not touching the cart.** Is this undisturbed motion more nearly constant-velocity than when you actively guided the cart?

Reproduce a motion. Fig. 2 graphs the *x* position of a cart as it moves along a line. Describe the motion in each of the time intervals marked I, II and III. The graphs in the conceptual questions are designed to help you with this part of the lab. Now test your explanation by recording and analyzing a movie of the cart as you move it to reproduce Fig. 2. If the position vs. time plot of your cart looks similar to an inverted image of Fig. 2, then that means your camera's x-axis is defined in the opposite sense than what you assumed. To change the orientation of the x-axis, see the third page of the document "Using LoggerPro". You can find the document in the same section of OwlSpace that you found the lab manual. Can you create the sharp corners between intervals? Why not? (Hint: Think about accelerations.)

Reproduce a velocity. The velocity of a cart moving along a line is shown in Fig. 3. Describe the motion in each of the marked time intervals and test your explanation by plotting *v(t)* for a trial run. You may find it helpful to have a look at the graphs in the conceptual question and plot their *v(t)* before starting the video capture. This is surprisingly tricky, and you may have to do it a few times to get a fair approximation of Fig. 3. If the velocity vs. time plot of your cart looks similar to an inverted image of Fig. 3, then that means your camera's x-axis is defined in the opposite sense than what you assumed. Do you again have troubles getting the sharp corners? Why?

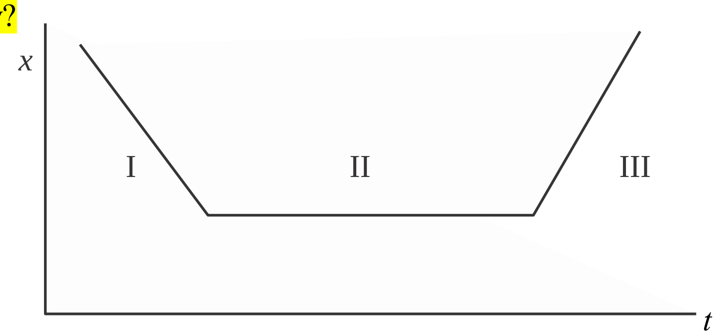

Fig. 2 Sketch of position vs time to be reproduced.

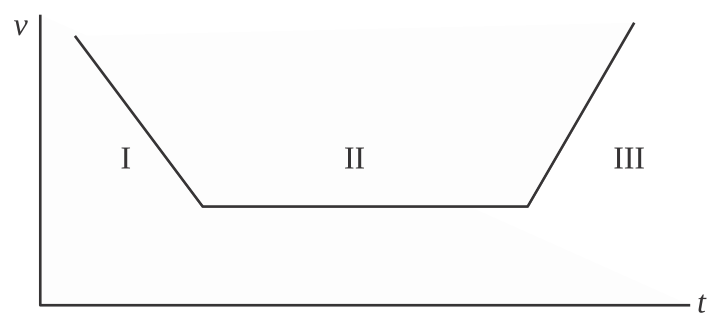

Fig. 3 Sketch of velocity vs time to be reproduced.

Clean up. When you have finished using your data, please drag any files you saved to the trash so the next student has a clean work space. Don't delete any files that are not yours, as they will be needed by others.