Energy Conversions

"The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them."

W. L. Bragg

OBJECTIVE

To see the work-energy theorem in action.

THEORY

The ideas of work and kinetic energy are connected by the work-energy theorem, a consequence of Newton's second law. For an object which behaves like a single particle the theorem claims that the work done by external forces is equal to the change in kinetic energy

$$\int_1^2 \vec{F}_{\text{net}} \cdot d\vec{s} = \frac{1}{2} m\left(v_2^2 - v_1^2\right) \tag{1}$$

The integration runs over some definite path from point 1 to point 2, $\vec{F}_{\text{net}}$ is the net external force acting on the object, and v_1, v_2 are the initial and final speeds.

To see how this works, we will deal with a slightly simplified situation in which the motion is restricted to a straight line. Forces will be applied by hanging mass, m, as in Fig. 1 or by springs as in Fig. 2 and, inevitably, by friction. Calling the force pushing the object of interest F and the friction force f, Eq. 1 reduces to

$$\int_{x_1}^{x_2} (F - f)dx = \frac{1}{2} m\left(v_2^2 - v_1^2\right) \tag{2}$$

or, identifying the various terms as work or change in kinetic energy,

$$W_F - W_{fric} = \Delta K \tag{3}$$

To check this, we need to measure each term ($\boldsymbol{W_F, W_{fric}, and\ \Delta K}$) independently as explained below and then see if they add up properly.

EXPERIMENTAL PROCEDURE

The object in motion is one of the heavy carts. It can be pulled across the table where a pair of photogates will determine the velocity at two positions, and hence the change in kinetic energy, ΔK, between those positions. The forces due to friction and the weight or springs can be measured over the same range of positions, and the work done by these forces can be computed using the definition of work.

1. Force from hanging weights

Set up the cart and hanging mass, as suggested by Fig. 1. Attach the metal flag vertically to the magnet at the side of the cart (make sure the long sides of the metal flag sits as perpendicularly as possible to the cart) and adjust the height of the gates so that the metal flag goes through the photogates without hitting. **Plug the power adopter for the "Vernier Lab pro" box into a power outlet if you have not already done so.** Start LoggerPro from the file **Work.cmbl** from the computer desktop to get the proper settings for the photogates.

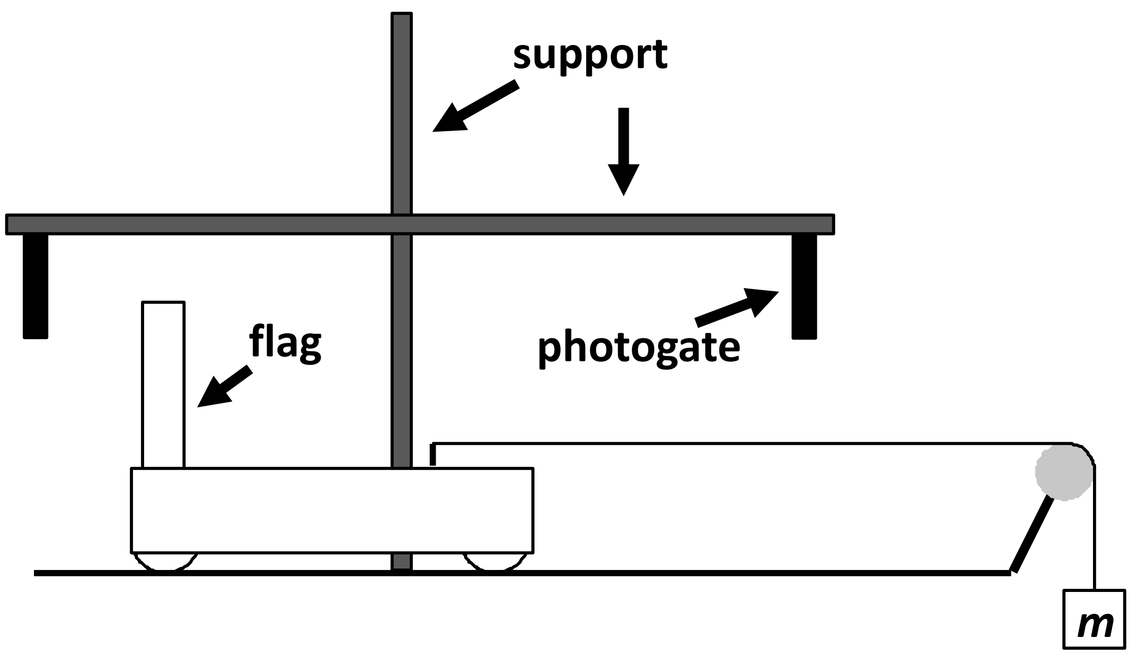

Fig. 1 Arrangement for pulling the cart with a weight (side view).

The work done by friction, W_{fric}, can be found by putting some mass on the string, start data collection, and release the cart before the first photogate (It is okay to give the cart a push to help it to start moving). **If properly set up, the rope should be level between where it is connected to the cart and the top of the pulley.** The data table on the computer will show the velocity of the cart as it goes through photogate number 1 and photogate number 2. Change the amount of hanging mass on the string until there is **little or no change in velocity measured at the two photogates.** At this point there is no acceleration, so the gravitational force on the hanging mass is equal in magnitude to the frictional force f on the cart. If we assume f to be constant, then

$$W_{fric} = \int_{x_1}^{x_2} f dx = m_0 g L \tag{4}$$

, where m_0 is the mass needed to get constant velocity. (Note: experimentally, $m_0 g L$ accounts for both the work done by the frictional force and the work done by gravity due to the tilt of the table surface.) **To get an error estimate on m_0, see how much you can change m_0 while still keeping essentially constant velocity between the two photogates.**

Once you found W_{fric}, you can add more mass on the string, so the hanging mass, m, is greater than m_0. The net force on the cart should now be constant and **non-zero**. Find the velocity of the cart at photogate number 1 and photogate number 2 as the mass m falls and using these measurements to compute ΔK. **Don't forget to add m to the cart's mass when calculating ΔK, since the system that is moving is both the cart and the hanging mass. You will need to repeat the measurement three times with the same m to get an average for ΔK.**

If we call the distance between the photogates L, then m falls a distance L while the cart moves between the gates, so the gravitational work done on the cart-hanging mass system is

$$W_F = mgL \tag{5}$$

You can now add up all the pieces and see if your results follow Eq. 3 within 20% of what is expected. If you see a discrepancy, what do you think could be the cause? Repeat the process for a **different** m, to be sure there is nothing special about your initial choice.

2. Force from springs

Now set up the cart and spring apparatus as in Fig. 2, fastening the springs to the anchor at one end of the bench and connecting the spring assembly to the cart with a string. Pull the cart back to stretch the springs and release the cart so that it travels straight toward the anchor, being sure that the flag goes through the photogates. The net force on the cart in this set up is non-constant and non-zero. **Do not extend the spring apparatus over 85 cm as indicated in Figure 2** to avoid permanent deformation. You can find the change in kinetic energy with the photogates as you did previously.

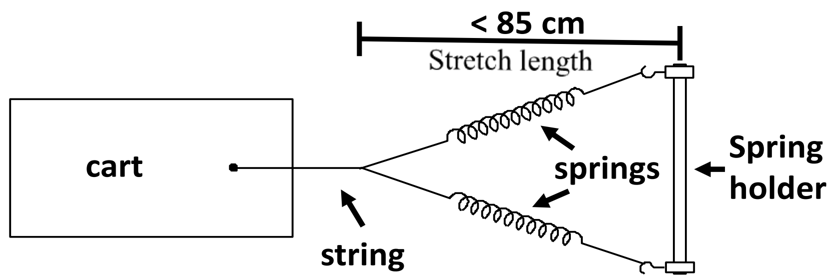

Fig. 2 Arrangement for pulling the cart with the spring apparatus.

When setting up your cart and spring apparatus for this part of the experiment, **the stretch length of the spring apparatus needs to be at least 16 cm when the flag of the cart is at the photogate closest to the end of the table where the spring holder is attached to the table. The stretch length is the distance between the spring holder and the knot that ties the springs to the string as indicated in figure 2.**

You will need to repeat the measurement three times with the same initial stretch length to get an average for ΔK. To find the work done by the springs you will need to measure their force as a function of their stretch length, and then integrate over the distance between the photogates. **The stretch length is the distance between the spring holder and the knot that ties the springs to the string as indicated in figure 2.** You need to first find the stretch lengths of the spring apparatus when the flag is in each photogate. This can be done by positioning the cart in each gate and measuring the distances from the spring holder to the knot in the string when the metal flag is in each photogate. **The integration will be done over this range of stretch length.**

Next, you can determine the spring force as a function of stretch length. There are vertical supports at the two tables at the back of the room near the printer which you can use to suspend weights from your spring assembly. The stretched length is defined the same way as shown in figure 2 **(From the spring holder to the knot that ties the springs to the string)**. The weight you attached to the spring apparatus is the applied force to the spring apparatus. **Plot the applied force to the spring apparatus *(F)* vs stretched length**. **The applied force F must be the values plotted on the vertical axis.** Following Eq. 2, use LoggerPro **to integrate** the force over the range of stretch length between which the cart passed through the two photogates and thereby find W_{Spring}.

To integrate in LoggerPro, start LoggerPro from the file **Graph.cmbl on the computer desktop** (If you start *LoggerPro* from the taskbar it will be slow and somewhat confusing. If asked by LoggerPro, please choose "Use file as is"). First, enter the stretch length data in the first column and the *F* data in the second column. **Your data in LoggerPro needs to be entered in ascending order in term of the stretched length in order for the "Integral" function to work properly.** Next, plot ***F* vs stretched length (This means plotting F on the vertical axis)**. Finally, select the region of interest in the plot by holding down the mouse key and dragging the cursor across the area. Then choose "Integral" from the "Analyze" menu. Describe a way to estimate the uncertainty for W_{Spring} as calculated from integrating the F vs Stretch length data (You do not have to actually carry out the calculation.)

The frictional work W_{fric} should be the same as before, so you can now find W_{Spring} -W_{fric} and compare the total with the change in kinetic energy, as required by Eq. 3. Considering the likely errors of measurement, do your results agree with Eq. 3? Discuss any discrepancies you notice, including sources of error.