Angular Dynamics

"The researches of many commentators have already thrown much darkness on this subject, and it is probable that, if they continue, we shall soon know nothing at all about it."

Mark Twain

OBJECTIVES

To quantitatively examine the relation between torque and angular acceleration.

THEORY

In this experiment we will study the angular acceleration due to the combination of an applied torque and a frictional torque. In the apparatus sketched in Fig. 1 a rotating bar is free to rotate about a vertical axis. A string wrapped around a rotor pulley and attached to mass M, which hangs from a deflection pulley, produces a torque which will cause the rotating bar to speed up. Additionally, there will be a frictional torque τ_f due to imperfections in the bearings which will tend to stop the rotating bar. Putting all the forces together, we can write Newton's equations for the system in the form

$$\tau = I\alpha = r_s T - \tau_f \quad (1)$$

$$Ma = Mg - T \quad (2)$$

where I is the moment of inertia of the apparatus, T is the tension in the string, and r_s is the radius of the rotor pulley. **The rotor pulley has three different radii (r_s), and they are 1.25, 2.50 and 3.75 cm. The cylindrical masses, C, are 900 grams each.**

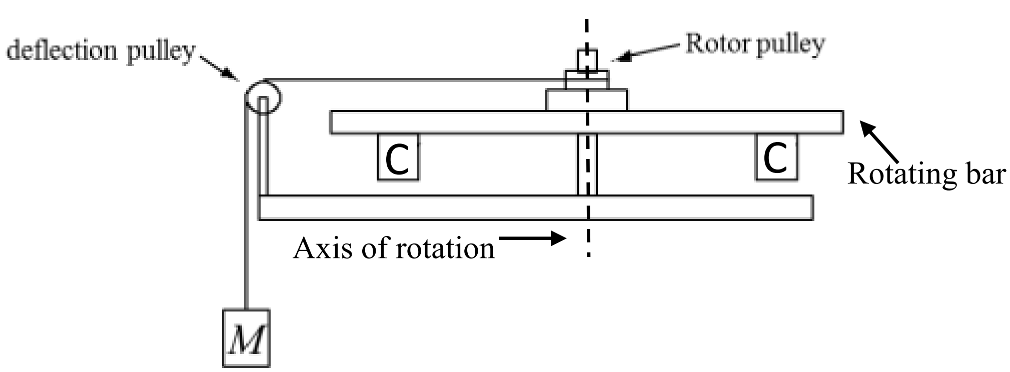

Fig. 1 The angular acceleration apparatus (side view). **The cylindrical masses, *C*, are 900 grams each.** The rotor pulley radii are r_s = 1.25, 2.50 and 3.75 cm.

Because the string does not stretch, the linear acceleration, a, of the hanging mass, M, is related to the angular acceleration, α, of the rotor pulley by

$$a = r_s \alpha \tag{3}$$

These equations can be solved for the angular acceleration, yielding

$$\alpha = \frac{M r_s g - \tau_f}{I + M r_s^2} \tag{4}$$

This expression is somewhat messy, but fortunately **<u>for our apparatus, we can make the approximation that $\boldsymbol{M r_s^{\,2}}$ is much smaller than $\boldsymbol{I}$.</u>** With that simplification we find

$$\alpha \approx \frac{M r_s g}{I} - \frac{\tau_f}{I} \tag{5}$$

Assuming that τ_f is constant, we see that for a particular value of I, the angular acceleration of the rotating bar, α, would be a linear function of $M r_s$. If we could determine the angular acceleration, we could check this relation by plotting α vs. $M r_s$. The plot should be a straight line.

One way to measure the angular acceleration is to measure the angular velocity, ω, of the rotating bar as a function of time. The angular acceleration can then be found from the angular analog of the familiar velocity-time relation for constant acceleration

$$\omega = \omega_0 + \alpha t \tag{6}$$

The angular velocity ω can be measured with a photogate, and a plot of ω vs t will then give a value for α from the slope.

EXPERIMENTAL PROCEDURE

As indicated in Fig. 1, the rotor pulley has three different radii. This allows you to vary M and r_s independently. Also, the two large cylindrical masses, C, attached to the rotating bar can be moved to various radii, so the moment of inertia can be changed. You will want to investigate the effects of all these possibilities.

<u>To take data, plug the power adopter for the “Vernier Lab pro” box into a power outlet if you have not already done so.</u> A photogate is positioned so that the end of the rotating bar will block the beam at each half-turn. If you start *LoggerPro* from the file ***Angular.cmbl*** **on the desktop of the computer**, then *LoggerPro* will measure the duration Δt of each interruption and calculate ω from

$$\omega = \frac{w}{R_{bar}\Delta t} \tag{7}$$

where w is the width of the bar and R_{bar} is the radius of the bar's rotation as measured from the axis of rotation to where the bar goes through the photogate. **You DO NOT need to measure either w, the width of the bar or R_{bar}, the radius of the bar's rotation, these values are already written into Angular.cmbl.** Since the time of each angular velocity measurement is known, this allows you to make the needed plot of ω vs t to find the angular acceleration.

To make an angular acceleration measurement, hook the string from M over one of the pegs on the rotor pulley, and wind a convenient number of turns of string onto the rotor pulley. **Be sure to adjust the height and angle of the deflection pulley so that the string is tangent to the rotor pulley. If your string is not tangent to the rotor pulley, you will have a hard time keeping the string from slipping off the rotor pulley as it unravels.** Also, **DO NOT** wrap the string below the peg on the rotor pulley. Doing so will add friction to the motion of the string. Start the data acquisition and let the hanging mass fall until the string slips off the peg. Stop the acquisition and then fit a straight line to the constant-acceleration part of the ω vs t graph to get α. **You DO NOT need to print out any of the ω vs t plots. You only need to write down the value of the slope on the data table on the lab report template.**

Make measurements using **three different masses**, M, at each r_S. Be sure you are covering the available range of M. You should obtain two series of measurements, one with the cylindrical masses positioned in such a way as to minimize I, and the other with the cylindrical masses set for a larger I. **When positioning the cylindrical masses to get the larger I, make sure that the cylindrical masses don't go through the photogate (This means DO NOT use the last hole on the rotating bar), as they are much wider than the bar and will invalidate the angular velocity calculation.**

Analyze your data by making a plot of α vs. Mr_S, as suggested by Eq. 5. You will need to submit a copy of the plot of α vs. Mr_S with the appropriate linear fits. Are your results consistent with Eq. 5? Can you explain any discrepancies that you observe? Is the change in moment of inertia that you deduce from the slopes consistent with the change in moment of inertia that you estimated from the mass and change in position of the cylindrical masses? You will also need to estimate τ_f from this graph, and explain the sign of your value of τ_f.