Ampere's Law

Who shall teach thee, unless it be thine own eyes?

Euripides (480?-406? BC)

OBJECTIVES

To characterize a Hall probe for magnetic field measurements. To develop an operational understanding of a line integral and to examine the operation of Ampere's Law.

THEORY

At first glance Ampere's Law,

$$\oint \vec{B} \bullet d\vec{s} = \mu_0 I \tag{1}$$

just seems to be a complicated way to relate a magnetic field to a current. We are instructed to divide an imaginary closed curve into equal displacement vector of length $|d\vec{s}|$. Compute the dot products $\vec{B} \bullet d\vec{s}$ for all the segments making up the curve, add them up, and we get a number proportional to the total current I through the surface that spans the closed curve. The law becomes odder when you think about it more carefully. Most obviously, the integral does not depend on the path chosen, but only on the current encircled. Further, the current that appears on the right hand side need not be the only source of magnetic fields along the path. Somehow the field produced by other currents does not contribute to the integral. Finally, it is only the net current through the spanning surface that is important. If we choose a path so that the same current loops through the surface spanned by the path in opposite directions, then the right hand side will be zero even though there is field along the path. In this experiment we will see how these results come about.

The instrument we will use to measure magnetic fields is called a Hall probe. The Hall effect refers to the potential difference which appears at right angles to the current flow when a conductor is immersed in a magnetic field. The potential difference is a consequence of the Lorentz force, acting on the charge carriers, so it is linearly proportional to the component of the applied magnetic field normal to both the current and the line between the sensing electrodes. By finding the orientation that gives the maximum Hall voltage we can determine the magnitude and direction of $\vec{B}$.

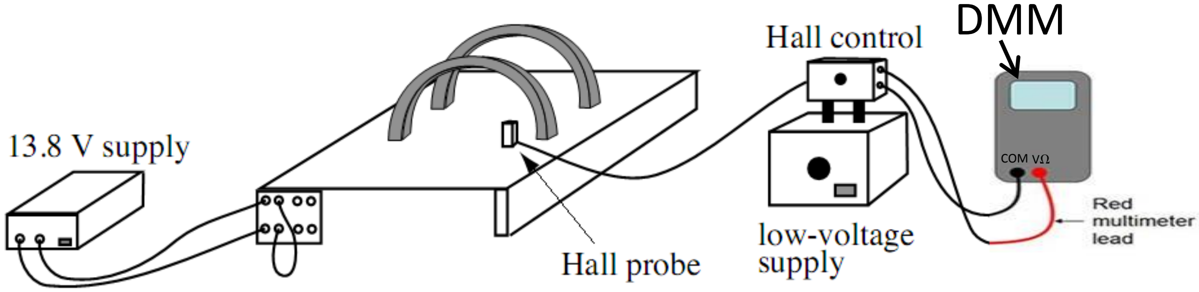

Fig. 1 Overall layout of apparatus for Ampere's Law measurements.

EXPERIMENTAL PROCEDURE

We will study the fields produced by one or two coils, mounted on a wooden plotting surface, as sketched in Fig. 1. The desired path can be marked off on a piece of paper cut to fit around the coils, and a Hall probe used to determine the dot product. By being a bit clever we can calibrate the Hall probe output relative to the current in the coil, and thereby carry out a quantitative test of Eq. 1.

The coils we will study are mounted on a wooden plotting table. Current is provided by a **fixed** 13.8 V supply (**That's the gray rectangular box, with an on/off button but NO knob**). The Hall probe is inside a clear plastic box that is connected via white wires to **a black box with a small knob, which is the Hall control box**. In order for the Hall probe to function properly, the Hall control box needs to be connected to a power supply that is outputting 8 V. Even if your Hall control box is already connected to the **adjustable low-voltage supply** (**That's the blue box with a knob on the middle that you have used in previous experiments**), you should check that the output of the blue box is 8 V by removing the Hall control box from the adjustable low-voltage supply and measure the voltage output of the power supply with your DMM (**The orange box with the digital display**). Once you have confirmed that the output of the adjustable low-voltage supply is 8 V, connect the Hall control box to the output of the power supply. **Once the output of the adjustable low-voltage supply is set to 8 V, DO NOT adjust the knob on the adjustable low-voltage supply for the rest of the lab session.** You can now use the DMM to measure the Hall voltage output from the control box by connecting leads from the Hall control box to the terminals on the DMM marked VΩ and COM and setting the large knob on the DMM to point at the V with the solid and dashed lines over it.

Hall probe set-up

The first job is to zero the Hall probe output when no magnetic field is present. **Note that the actual Hall probe where the magnetic field is measured is the white rectangular ceramic chip inside the clear plastic box. Also, the Hall probe has been designed to measure the component of the magnetic field normal to the wider surface of the white ceramic sensor chip.** Put the clear plastic box inside the cylindrical magnetic shield (that's the black cylinder) such that the white rectangular ceramic chip is facing the wall of the shield and adjust the zero control **on the Hall control box** until the DMM reads as close to zero as you can manage. When you remove the shield you should be able to detect the effect of earth's field on the readings. The voltage value you read on the DMM is proportional to the magnetic field measured by the Hall probe. **For this lab, we will record the magnetic field simply as the voltage output of the Hall probe on the DMM and not worry about converting the voltage measurement to tesla.**

Now connect one of the two coils, the 0.5 Ω resistor built into the side of the wooden plotting table, and the 13.8 V power supply in series, and turn on the supply. **[Failure to wire the 0.5 Ohm resistor in series in the circuit could lead to the total destruction of the fuse in the power supply.]** **Position the clear box containing the Hall probe, so that the white rectangular ceramic chip is at the center of the energized coil and knowing that the magnetic field of the coil must be normal to the plane of the coil, observe how the output on the DMM changes as you rotate the probe relative to the field. Note that the coils are positioned such that the top of the wooden plotting surface is BELOW the center of the coils.** The reading on the DMM should vary from nearly zero to a maximum of about 0.5 to 1.0 V. If this doesn't occur, check that there is current through the coils by measuring the voltage across the resistor and that the Hall probe is getting 8 V input from the adjustable low-voltage supply. Consult the instructor if you cannot get a satisfactory signal.

Single coil energized

To evaluate the left hand side of Eq. 1, cut a piece of paper to fit around the coils and draw a closed path on the paper. **Your closed path can be any shape as long as it is closed.** When cutting your paper remove only the portions of the paper that overlaps with the locations where the coils go into the wooden plotting surface. **DO NOT remove the portions of the paper that sit within the coils.** Mark off equal distances of about 1 cm along the chosen path. These intervals will approximate the small intervals $d\vec{s}$ in the line integral.

Recall that the output of the Hall sensor is proportional to $B\cos\theta$, where θ is the angle between $\vec{B}$ and the normal to the probe surface. This means that when the normal to the probe surface is parallel to $d\vec{s}$, the Hall output as indicated on the DMM is linearly proportional to the desired dot product we wish to sum to use Ampere's Law. **All we need to do is line up the probe at each mark so that the Hall probe is measuring the component of the magnetic field that is parallel to $d\vec{s}$, and add up the readings to get a number proportional to the value of the line integral.** (Don't forget to zero the probe inside the magnetic shield before starting measurements.)

To make a quantitative comparison in situations where the left hand side of Eq. 1 is not zero, we must calibrate the Hall probe output. An easy way to do this is to note that **the magnitude of the magnetic field measured with the Hall probe at the center of the energized coil is B_0,** which is given by

$$B_0 = \mu_0 N I_c / 2R \qquad (2)$$

when the coil has N turns each carrying current I_c with average radius R. For a path which encircles one side of the energized coil the total current appearing in Ampere's Law is NI_c, which can be found from Eq. 2, substituted into Eq. 1, and rearranged to yield

$$\frac{1}{B_0}\oint \vec{B} \bullet d\vec{s} = 2R \qquad (3)$$

This means that as long as the line integral is not zero, we can add up the contributions around the path, divide by the Hall reading at the center of the coil (**B_0**), and compare the result with $2R$ to verify Eq. 1. **For our coils, R = 12.1 cm.**

Carry out this procedure for **at least three different paths** in the vicinity of the coil you are energizing. You should include paths which encircle one side, both sides and neither side of the coil. Provide diagrams of the paths in your report, and describe how various parts of the path contribute to the final sum. Are the overall integrals quantitatively consistent with Eq. 3?

Normalization and data for two-coil geometry

You can do a similar exercise for the more complicated situation where both coils are energized, but you will need to know how much the current is reduced because the two coils together have more resistance than one alone. Before dismantling the single coil connection, measure the voltage across the 0.5 Ω resistor and call it V_1. Now connect the 0.5 Ω resistor and the two coils in series with the 13.8 V power supply (the rectangular gray box) so that a current will flow sequentially through all four components. **[Failure to wire the 0.5 Ohm resistor in series in the circuit could lead to the total destruction of the fuse in the power supply.]**

Measure the new voltage across the 0.5 Ω resistor and call it V_2. Determine if the currents in the coils are flowing in the same or opposite directions by using the Hall probe to find out if the fields tend to add or to cancel on the coil axis at the midpoint **between** the coils.

For a path which encircles one side of one coil we would get a relation like Eq. 3, except that the factor B_0 must be smaller because the current is less and B_0 is proportional to current. The voltage across the 0.5 Ω resistor is also proportional to current, so the new factor must be

$$B_0' = \frac{V_2}{V_1} B_0 \tag{4}$$

Using this value you can deduce relations like Eq. 3 for paths which encircle parts of one, both or neither coil. You could also repeat the integration along one of the paths you used before to see if the results are consistent.