A Physical Pendulum

"Physics is experience, arranged in economical order."

E. Mach (1838-1916)

OBJECTIVES

To observe and analyze an example of simple harmonic motion.

THEORY

Simple harmonic motion can be defined as one for which the position of an object changes sinusoidally with time. This means that $x(t)$, $\theta(t)$ or some other coordinate is a sine or cosine function, repeating endlessly, or perhaps slowly decreasing in amplitude due to friction. Many systems exhibit harmonic motion, or a good approximation to it. Physics courses use a mass on a spring or a mass swinging from a string as typical examples. Vibrating bars or strings, or the vibrating air column in a flute are more complicated examples. Studying any one of these, therefore, can provide insight into a wide variety of physical systems.

A pendulum is convenient for our purposes, but the textbook example of a small mass hanging from a weightless string is too simple. Instead, we consider a physical pendulum consisting of a metal bar hanging from a pivot point, as shown in Fig. 1. If the bar is pulled aside and released, it will swing back and forth due to the gravitational force. We will test the claim that the angle from the vertical, θ, displays harmonic motion for small deflections.

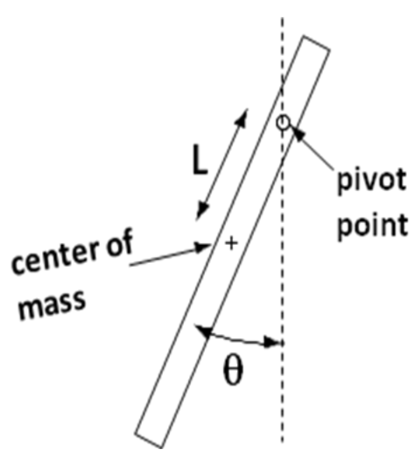


Fig. 1 Coordinates used for the physical pendulum

The equation of motion for a rigid object rotating about a fixed point is

$$\tau = I\frac{d^2\theta}{dt^2} \tag{1}$$

where τ is the torque and I is the moment of inertia about the point of rotation. The torque is given by

$$\tau = -mgL\sin\theta \tag{2}$$

where L is the distance from the point of suspension to the center of mass and m is the total mass of the pendulum. Unfortunately, the resulting equation can only be solved numerically, and predicts non-harmonic motion.

If θ, in radians, is small, then it is possible to approximate sin θ by θ itself, leading to the much simpler equation of motion

$$\frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\theta \tag{3}$$

This is the equation for simple harmonic motion. It is not an exact description of the pendulum, but can be made as accurate as desired by limiting θ to sufficiently small values. For example, θ is only 1% larger than sin θ when $\theta = 0.4$ radian, which is about 23°.

Eq. 3 predicts that the period, T, of a physical pendulum within a limited range of θ will be

$$T = 2\pi\left(\frac{I}{mgL}\right)^{1/2} \tag{4}$$

For data analysis, it is convenient to use the parallel axis theorem to write I in terms of the moment of inertia about the center of mass, I_{cm}, and the distance L between the pivot and the center of mass

$$I = I_{cm} + mL^2 \tag{5}$$

which gives

$$T = 2\pi\left(\frac{mL^2 + I_{cm}}{mgL}\right)^{1/2} \qquad (6)$$

We will attempt to verify this prediction quantitatively, as well as showing that the motion is harmonic for small angles.

EXPERIMENTAL PROCEDURE

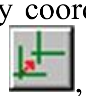
On your table you will find a long flat metal bar with a series of holes drilled through it and a metal stand with a short metal rod attached near its top. The short metal rod at the top of the stand is cylindrical at the end that is attached to the stand and triangular at the other end. **For the short metal rod, the sharpest edge of the triangle should be pointed upward.** We will refer to the sharpest edge of the short metal rod as the knife edge. Our physical pendulum is the long flat piece of metal bar with the series of holes drilled through it. The bar can be suspended from the knife edge through any of the holes on the metal bar. When pushed gently, the long flat metal bar will start to oscillate around the knife edge. **When positioning the bar, make sure the bar is resting on the knife edge only.** Using a video camera, you can then verify that the swinging motion is harmonic. A photogate provides more precise measurements of the period to check for amplitude dependence and to test the prediction of Eq. 6 for various L. **Here, *L* is the distance from the point on the metal bar where the knife edge contacts the metal bar and the center of mass of the metal bar.**

1. Position measurements

Hang the bar from a hole near one end, so that most of the length is below the pivot point. Position the photogate, or remove it entirely, so that the bar can swing through a reasonably large angle, **but less than 23°.** Arrange the camera and pendulum so that the pivot point and the colored dot at the end of the bar are both visible on the camera viewfinder, with the plane of the pendulum motion perpendicular to the camera axis. **The orientation of the camera is purposely set so that the length of the bar is along the shorter dimension of the camera when the bar is at rest. DO NOT change the orientation of the camera.** Adjust the tilt of the stand so that the pendulum swings smoothly, without excessive wobble. Complete elimination of the wobbling in the bar is probably not possible due to the bluntness of the knife edge, but you should do the best you can. **DO NOT push the metal bar all the way to the**

cylindrical portion of the short metal rod. While positioning the metal bar this way will decrease the wobbling, this will also change the contact geometry of the pendulum and increase the friction between the metal bar and its pivot.

Now open LoggerPro with the **Graph.cmbl** startup file found on the desktop of the computer (**if asked by the computer, use the file as is**), so it ignores the photogate, and obtain a movie showing at least two or three cycles of the oscillation. Mark the positions of the colored dot for **at least two full periods of the motion**. It is helpful to turn off the markers with the Toggle Trails icon, , since the motion repeats.

Next, convert the x-y coordinates of the bar end to the angle θ relative to the vertical. Click on the Set Origin icon, , located just below the Add Point icon in the movie window. Now click and drag the intersection of the yellow axis lines from the bottom left edge of the frame **to the pivot point of the pendulum. DO NOT rotate the yellow axis lines as you drag the axis lines.** This sets the origin of the coordinates at the pivot point. **The origin is the intersection of the two yellow lines. The big yellow dot denotes the direction of the positive x-axis.** Angle θ is then the arctangent of y/x. LoggerPro will calculate the angle if you go to Data > New Calculated Column... Give the column a name if you wish, and then type in or use the pull down menus to construct **atan("Y"/"X")** in the Equation box. Close the box and the new calculated column will appear in the data table.

Plot the calculated θ vs time, and use the **Sine function** in Analyze > Curve Fit... to display the sinusoidal function that best describes the data. Does the curve seem to be a good description of your data?

2. Small angle limit

Timing is done with a photogate. The photogate works by emit an invisible light beam (Infrared frequency) from a small circular aperture to a detector on the other side of the photogate and measuring the interruption time of the light beam. Hang the bar from a hole of your own choosing, and adjust the photogate so that the bar moves completely through the photogate on each swing. **The light beam must miss the holes when the bar move pass its path.** Start LoggerPro from the file **Pendulum.cmbl** that is on the computer desktop to configure the photogate to measure and display the period.

The first task is to check the amplitude dependence of the period. You need to find the small-angle limit of your physical pendulum. You can use a protractor held up to the pivot point to estimate the angular deflection. Position the photogate so that the bar can swing through at least one data points from $(30^o < \theta < 50^o)$, and determine the period. Repeat for at least three data points from $(10^o < \theta < 30^o)$, and at least two data points from $(5^o \leq \theta < 10^o)$ in order to determine, roughly, how the period changes with angular amplitude. **You should include at least two angles that are less than 10^o.** Using your data, estimate the largest amplitude that will give a period within 0.5% of the ideal, zero-amplitude period (this means assuming that the trend in your data at small amplitude continues, estimate the period of the pendulum as the amplitude approaches 0^o). You **DO NOT** need to curve fit this data set.

3. Dependence on *L*

The distance *L* is the length from the point of support, at the top of one of the holes, to the center of mass of the bar. The location of the center of mass is marked with a pencil on most bars. If the center of mass on your bar is not mark, the center of mass of the bar is the position that is directly above its point of support when the bar is perfectly balanced. **Measure and record *L* for at least eight holes on one side of the center of mass. Be sure to cover the available range of *L*.** Now obtain the average period over several cycles of the motion for **all the values of *L* you have measured**. Keep the amplitude small, consistent with your previous determination, so that you are measuring within 0.5% of the ideal, zero-amplitude period.

A direct comparison of the period measurements with Eq. 6 is awkward because I_{cm} is not known, and the form does not lend itself to direct fitting. However, squaring both sides of Eq. 6 and multiplying by *L* yields the linear relationship

$$T^2 L = \frac{4\pi^2}{g} L^2 + \frac{4\pi^2 I_{cm}}{mg} \tag{7}$$

Plot your data such that you can extract out a value for g from the slope of your plot and decide whether or not the linear relationship is a good description. Historically, pendulum devices were the most precise way to measure *g*. Obtain a value of *g* from your plot, and compare it to the accepted value 9.79285 m/s^2 for Houston. What do you think is the main source of error in this part of the experiment?