Due: Thursday, September 6

1. Sets

- If set A has a elements and set B has b elements, how many elements are in the set $A \times B$? Explain your answer.

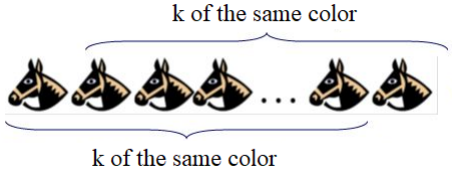
2. Proofs

(a) Find the error in the following proof.

Claim: All horses are the same color.

Proof: We prove that any collection of horses is monochromatic by induction on the number of horses in the collection.

Base Case: Obviously, a set of one horse is a set of horses all with the same color.
Induction Hypothesis: Assume that any set of k horses are all the same color.
Inductive Step: Consider a set of $k+1$ horses, and stand them all in a line.



The first k horses in the line form a set of k horses, and so by the Inductive Hypothesis, are all the same color. The same is true for the last k horses in the line. Therefore the entire set consists of $k+1$ horses of the same color.

(b) Let $S(n) = 1 + 2 + \cdots + n$ be the sum of the first n natural numbers and let $C(n) = 1^3 + 2^3 + \cdots + n^3$ be the sum of the first n cubes. Prove the following through induction on n.

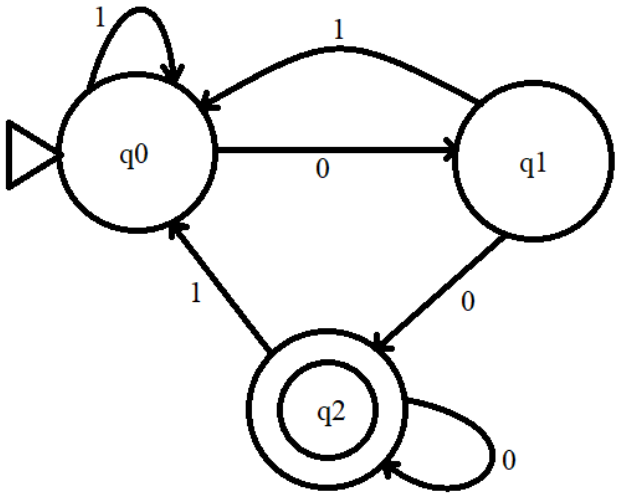
- $S(n) = \frac{1}{2}n(n+1)$.
- $C(n) = \frac{1}{4}(n^4 + 2n^3 + n^2) = \frac{1}{4}n^2(n+1)^2 = S^2(n)$.

3. **Describing DFAs**

For the following deterministic finite automaton M_1:

(a) Write out the full mathematical description of M_1.

(b) Determine what language M_1 recognizes.



4. **Creating DFAs**

Draw DFAs for the following languages (you may assume the alphabet is always $\{0, 1\}$):

(a) $\{w \mid w \neq \varepsilon\}$

(b) $\{w \mid w \neq 11 \text{ and } w \neq 111\}$

(c) $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$

(d) $\{w \mid w \text{ contains the substring 0101 } (w = x0101y \text{ for some } x \text{ and } y)\}$

(e) $\{w \mid \text{every odd position of } w \text{ is a 1}\}$