Due: Thursday, September 13

1. **Building NFAs**

   Draw NFAs for the following languages, with the specified number of states. You may assume the alphabet is always $\{0, 1\}$.

   (a) The language $\{w \mid w \text{ ends in } 00\}$, 3 states.

   (b) The language $\{w \mid w \text{ contains an even number of 0s or exactly two 1s}\}$, 6 states.

   (c) The language $0^*1^*0^+$, 3 states.

   (d) The language $1^*(001^+)^*$, 3 states.
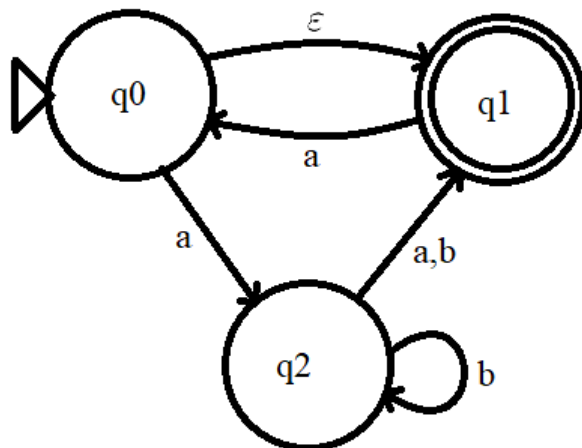
   (e) The language 0^*, 1 state.

2. **Subset Construction**

   Turn the following NFAs into DFAs using the subset construction. It should be clear from your drawing of a DFA how the subset construction was applied.

   (a)
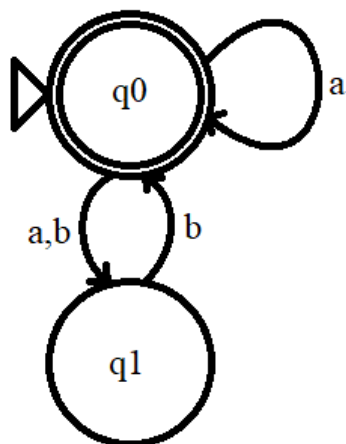
   (b)

3. **Reversing a Language and Building a Binary Adder**

If A is a language, we define its reverse, $reverse(A)$, as the language of all strings in A written in reverse, i.e. $reverse(A) = \{w^{\mathcal{R}} \mid w \in A\}$. For example, if the string '00110' is in language A, then the string '01100' is in $reverse(A)$.

(a) Prove that the set of regular languages are closed under the reverse operation, i.e., prove that if language A is regular, then $reverse(A)$ is also regular. (Hint: if A is regular, then there is an NFA M_1 that recognizes it. How would you change that NFA into a new machine M_2 that recognizes the reverse?)

(b) Define the alphabet

$$\Sigma_3 := \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

The alphabet Σ_3 contains eight size-3 columns of 0s and 1s. A string of symbols in Σ_3 thus builds threes rows of 0s and 1s. Consider each row to be a binary number and define the language

$$B = \{w \in \Sigma_3 \mid \text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example,

$$011 + 001 = 100 \quad \text{but} \quad 01 + 00 \neq 11,$$

so

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: show that $reverse(B)$ is regular, and by the previous problem conclude that $reverse(reverse(B)) = B$ is also regular.)