

Linear Programming

For problems 1-4, you only need to model the questions, without the need for solving them or providing specific solving algorithms, and there is no need to discuss time and space complexity.

1. Maximize profit

Imagine you're a logistics manager at a company that needs to optimize its production process. You oversee the manufacturing of two types of products, Product 1 and Product 2, which require the use of two types of resources: Material A and Material B.

Each unit of Product requires:

	A	B
Product 1	2	1
Product 2	1	3

You have a limited amount of each material available:

- You have a maximum of 6 units of Material A.
- You have a maximum of 9 units of Material B.

Your goal is to **maximize profit** from producing these two products. The profit for each unit of Product 1 is 3 units of money, and the profit for each unit of Product 2 is 5 units of money.

2. Shipping goods

A company needs to ship goods from three warehouses to four retail stores. Each warehouse has a limited inventory of goods, and each retail store has a limited demand. The cost of shipping each unit of goods is different between each warehouse and each retail store. The goal is to determine the quantity of goods shipped from each warehouse to each retail store to minimize the total transportation cost while satisfying the inventory constraints of each warehouse and the demand constraints of the retail stores.

Warehouse	Supply (Units of Goods)
Warehouse 1	50
Warehouse 2	60
Warehouse 3	40

Table 1: Warehouse Supply Limits

Retail Store	Demand (Units of Goods)
Store A	40
Store B	50
Store C	40
Store D	20

Table 2: Retail Store Demand Limits

	Store A	Store B	Store C	Store D
Warehouse 1	8	6	10	9
Warehouse 2	7	5	8	7
Warehouse 3	6	7	7	8

Table 3: Transportation Costs

3. Production

The company manufactures three products, A_1 , A_2 , and A_3 , utilizing resources such as metal sheets, labor, and machinery. The quantities of various resources required to manufacture one unit of each product are provided in the table below. Without considering fixed costs, the unit profits for each product are 40,000 yuan, 50,000 yuan, and 60,000 yuan, respectively. Available resources include: 500 tons of metal sheets, 100 workers per month, 300 machines per month. In addition to production costs, fixed costs must be paid: 1 million yuan for A_1 , 1.5 million yuan for A_2 , 2 million yuan for A_3 . The goal is to develop a production plan to maximize the company's total profit.

Resources	A_1	A_2	A_3
Metal sheets (tons)	2	4	8
Labor force (person/month)	2	3	4
Machinery (units/month)	3	6	9

Table 4: Resource Requirements per Unit of Product

Maximize the total profit while adhering to the resource constraints and including the fixed costs associated with each product.

4. Ex. Wakeup as billionaire

A company produces three products: A , B , and C . Production of B and C must follow the production of A , and a certain proportion of A is required to make B and C . The production process uses two resources, R_1 and R_2 , and takes place in two factories, F_1 and F_2 . After production, the company needs to transport the products to three markets, M_1 , M_2 , and M_3 , to meet market demand. Your goal is to maximize the total profit of this company. The following is a detailed description:

1. Product information

Product	Selling Price (USD/unit)	Required R_1 (kg/unit)	Required R_2 (kg/unit)	Required A (unit/unit)
A	50	3	2	—
B	60	2	4	0.5
C	70	4	3	0.8

2. Factory resources

Factory	R_1 (kg)	R_2 (kg)
F_1	400	300
F_2	500	400

3. Market demands

Market	A (unit)	B (unit)	C (unit)
M_1	40	50	60
M_2	50	60	70
M_3	60	70	80

4. Transportation costs

Factory \rightarrow Market	M_1 (USD/unit)	M_2 (USD/unit)	M_3 (USD/unit)
F_1	5	7	8
F_2	6	6	7